1. Determine the location of the center of mass of the Pluto-Charon system.
masses: 1.303×10^{22} kg, 1.586×10^{21} kg
separation: 19570 km

2. Determine the position of the center of mass for the following set of masses:
$m_1 = 200.0$ g, $\mathbf{r}_1 = (25.0\,\mathbf{i} + 13.0\,\mathbf{j})$ cm
$m_2 = 300.0$ g, $\mathbf{r}_2 = (-15.0\,\mathbf{i} + 8.00\,\mathbf{j})$ cm
$m_3 = 900.0$ g, $\mathbf{r}_3 = (5.00\,\mathbf{i} - 10.0\,\mathbf{j})$ cm

3. Suppose a fourth mass is added to the set of masses in the previous example so that the center of mass is relocated to the origin. (a) Show that this fourth mass must lie on a line with equation $y = -0.8x$. (b) What would be the value of the fourth mass if it is located at $\mathbf{r}_4 = (-5.00\,\mathbf{i} + 4.00\,\mathbf{j})$ cm?

4. Determine the center of mass of a thin sheet of material bound by: $y \leq x^2$, $y \geq 0$, $x \leq 2$.

5. A certain nonuniform "ruler" with length 0.5 m is described by $\lambda = 0.5\sqrt{x}$, where x is position in meters and λ is mass per length in grams per meter. (a) Determine its mass. (b) Determine its center of mass.

6. Find the center of mass of an isosceles triangle cut from a uniform sheet of material.

7. Find the center of mass of a solid cone with uniform density.

8. Determine the center of mass of a semicircle of radius R made of a thin sheet of material.

9. A man, mass 90.0 kg, and a woman, who is lighter, are seated at rest in a 20.0 kg canoe that floats upon a placid frictionless lake. The seats are 2.80 m apart and are symmetrically located on each side of the canoe's center of mass. The man and woman decide to swap seats and the man notices that the canoe moves 30.0 cm relative to a submerged log during the exchange. The man uses this fact to determine the woman's mass (the guy is a physics nerd – he has his trusty calculator in his pocket and hey, he can't think of anything to say to the woman anyway). (a) What is the woman's mass? (b) Will the nerd completely ruin the date by showing the woman his calculations?

10. Two balls are attached to one another by an elastic cord. Ball A of mass 0.500 kg is picked up and thrown by a person and its motion eventually stretches the cord until ball B of mass 0.300 kg begins to move. At $t = 0$: $\mathbf{v}_A = 10.0$ m/s, 45.0°, $\mathbf{v}_B = 0$; $\mathbf{r}_A = (0, 2)$ m, $\mathbf{r}_B = (0, 0)$ m. Solve for the missing info at $t = 0.23$ s: $\mathbf{v}_A = 5.43$ m/s, 0.0°, $\mathbf{v}_B = ?$; $\mathbf{r}_A = (1.51, 2.95)$ m, $\mathbf{r}_B = (?, ?)$ m

11. Lab cart **A**, mass 2.00 kg and initial velocity 9.00 m/s, 0.0°, collides with a lab cart **B**, mass 4.00 kg, initially at rest. Find the resulting velocity of cart **B** for each scenario: (a) cart **A** bounces off with velocity 3.00 m/s, 180.0°, (b) cart **A** stops, (c) cart **A** sticks to cart **B**.

12. Find the change in momentum of each cart in all three scenarios of the previous problem.

13. A fullback of mass 100.0 kg is headed for the end zone with speed 8.00 m/s. He is grabbed and tackled by a defensive end of mass 120.0 kg moving in the opposite direction at 4.00 m/s. (a) Find the resulting speed right after the fullback is grabbed. (b) What mass defensive end at the same speed would have stopped the runner? (c) Conversely, what initial speed of the given defensive back would have stopped the runner? (d) Could the given defensive player stop the fullback using a different technique? If so, what must he do and what happens to him?

14. Two ice skaters, masses m_1 and m_2, stand initially at rest. The two skaters push off one another and move in opposite directions. Determine the ratios of their speeds and of their kinetic energy as they move apart.

15. Two disks glide across an air table and collide. Disk **A**: mass 10.0 kg, velocity 0.500 m/s, 0.0°; disk **B**: mass 4.00 kg, velocity 3.50 m/s, 90.0°. (a) The collision causes disk **A** to stop. Find the velocity of disk **B** after the collision. (b) Suppose the two disks stick together – find the velocity afterward.

16. Two disks glide across an air table and collide. Disk **A**: mass 8.00 kg, velocity 4.00 m/s, 20.0°; disk **B**: mass 10.0 kg, velocity 4.00 m/s, 0.0°. After the collision disk **A** has velocity 3.40 m/s, 0.0°. Find the velocity of disk **B** after the collision.

17. Two gliders undergo an elastic collision on an air track. Glider 1 has mass 250.0 g and initial velocity 5.00 m/s, right; glider 2 has mass 75.0 g and initial velocity 2.00 m/s, left. (a) Find the velocity of each glider after the collision. (b) Determine how much momentum and how much kinetic energy is transferred.

18. Repeat the previous problem but assume the collision is perfectly inelastic and the gliders stick together.

19. When the cue ball hits the eight ball, one possible outcome is that the cue ball stops and the eight ball "takes on" the original speed of the cue ball. (a) Show that this is consistent with an elastic collision. (b) Another possibility is that the cue ball and eight ball move in perpendicular directions – again show that this is consistent with an elastic collision.

20. Suppose two objects collide head-on elastically. If the total momentum of the system is zero, show that each object will rebound with a speed equal to its original speed.

21. Repeat problem 16 using the center of mass frame.

22. Analyze the following elastic collisions using the CM frame. (a) Show that a relatively small mass that hits a stationary large mass at speed v will rebound at speed v. (b) Show that a relatively large mass moving at speed v that hits a stationary small mass will not slow appreciably and the small mass will be "kicked away" at speed $2v$.

23. A tennis ball of mass 57.0 g is served at a speed of 50.0 m/s. (a) Find the change in momentum of the ball assuming it is hit at the peak of toss. (b) What is the net impulse on the ball? (c) If the impact with the racket lasts 2.0 ms, what is the average force on the ball? the maximum force?

24. A certain rocket engine burns fuel at a rate of 1900 kg/s and the exhaust gases exit with speed 3100 m/s. (a) Find the amount of thrust produced. (b) If the rocket carries 120000 kg of fuel what is the total amount of impulse it can deliver?

25. A model rocket engine is rated at 17 Ns and burns 25 g of fuel in 2.0 s. (a) Find the average amount of thrust during the burn. (b) Find the speed with which exhaust gases leave the nozzle.

26. A ball is dropped from a height of 70.0 cm and rebounds to a certain height. (a) Measure the rebound height and mass of the ball and determine the net impulse on the ball during the bounce. (b) Estimate the average and maximum amount of force that the ball exerts on the floor. (c) Measure with a force plate and compare.

27. As it moves along the x-axis an object's momentum varies according to $p = 3t^2 - 4t$, where p is in kg m/s and t is in s. (a) Find force as a function of time. (b) Find the force at $t = 2$ s. (c) Find the change in momentum between $t = 1$ s and $t = 3$ s. (d) Integrate the force function to get the impulse between $t = 1$ s and $t = 3$ s.

28. As it is thrown a ball is subject to a net horizontal force given by $F = 125t - 1000t^4$, where F is in newtons and $t < 0.50$ seconds. (a) Find the net positive impulse on the ball. (b) Find the change in momentum of the ball. (c) If the ball has mass 525 g, with what speed is it thrown?

29. Deep Space 1 began its mission with a total mass of 486 kg including the xenon fuel for its ion engine. The ions were accelerated by electric fields and exited the spacecraft with speed 30.0 km/s. It "burned" 71.5 kg of fuel at a rate of 1.23 mg/s. (a) Determine the thrust of the engine. (b) Find the acceleration of the spacecraft at the beginning and end of the "burn" period. (c) Estimate the change in velocity. (d) Determine explicitly the change in velocity.

30. The space shuttle's total mass at launch is 2.04×10^6 kg. Its engines develop a total thrust of 28.6 MN and burn fuel at a rate of 9230 kg/s for 2.00 minutes until the SRB's separate. Find the height and speed at the end of the burn assuming it climbs straight up.

31. Consider a planet orbiting the sun. There exist five "Lagrange points" at which a third body, such as an asteroid, can orbit in sync with the planet. In other words the third body will have the same period as the planet and maintain its position relative to the planet as it orbits the sun. This is possible due to the mutual gravity of the sun and the planet on the third body. Two of these Lagrangian points, L_4 and L_5, form perfect equilateral triangles with the Sun and the planet. Show that an object at L_4 or L_5 will indeed orbit in sync with the planet.

32. Graph the Sun's "wobble" due to the planets' gravity. Make the simplifying assumption that each planet moves in a circular orbit at a constant speed (all in the same plane) about the solar system's center of mass. The Sun moves about this center of mass as well. Use parametric equations of the form $x(t) = r\cos(\omega t + \delta)$ and $y(t) = r\sin(\omega t + \delta)$ to model the motion of each planet. Let t be measured in years elapsed since Jan. 1, 1960. The chart below shows information needed, including phase angles that specify the positions at $t = 0$. Can you show that Mercury, Mars, and Pluto have a negligible effect on this problem? Note: this exercise is based upon and inspired by a diagram in the May 2003 Sky & Telescope.

	Mass (kg)	Orbit radius (AU)	Period (yrs)	Phase Angle
Sun	1.99×10^{30}	n/a	n/a	n/a
Venus	4.87×10^{24}	0.723	0.6152	175°
Earth	5.974×10^{24}	1.000	1.000	100°
Jupiter	1.90×10^{27}	5.20	11.86	255°
Saturn	5.68×10^{26}	9.54	29.42	280°
Uranus	8.69×10^{25}	19.19	83.75	139°
Neptune	1.02×10^{26}	30.07	163.7	217°

Answers:

1. $\mathbf{r}_{cm} = (3.57\ \mathbf{i} - 2.86\ \mathbf{j})$ cm
2. a.

   b. $m_4 = 10\underline{0}0$ g
3. $x_{cm} = 1.5$, $y_{cm} = 1.2$
4. $y_{cm} = h/3$ above the base of the triangle
5. $y_{cm} = h/4$ above the base of the cone
6. $4R/3\pi$ from the center of the circle
7. a. 70.6 kg

   b. um, yes.
8. Hint: All three bodies move in circles of different radii about the system center of mass while moving "in formation" of an equilateral triangle. The Sun moves in the smallest circle and the asteroid will move in the largest circle.
9. Here is a plot of the Sun's wobble during the years 1960 through 2025. The scales give values in AU. The green curve is the path of the Sun's center and the yellow circle represents the Sun at correct scale in its final

location during this interval.