## AP Physics C - Circular Motion Complete Answers

1. a. If acceleration is zero then an object's velocity must remain constant, including the direction of travel. Therefore a car that does not accelerate would move in a straight line, not a curve.
b. An object undergoing a constant acceleration moves either in a straight line or a parabolic path. In principle a car could move along a parabolic stretch of roadway with a constant acceleration. However, if a car is launched off a ramp and goes airborne then it is definitely following a parabolic path with a constant acceleration of $g$ (because it is then a projectile!) c. A circular curve involves acceleration toward the center of the circle at all times and therefore the direction of the acceleration vector must continuously change and point in different directions relative to earth. This means that acceleration (which includes direction) is not a constant value.
2. a. $120 \mathrm{~m} / \mathrm{s}^{2}$ toward center
b. 110 N toward center
c. $6.6 \mathrm{~m} / \mathrm{s}$
3. a. $7.9 \mathrm{~m} / \mathrm{s}$
b. $210 \mathrm{~m} / \mathrm{s}^{2}$ toward center
c. 1.0 N toward center
d. 0.048
4. $f=\frac{1}{2 \pi} \sqrt{\frac{\mu_{s} g}{r}}$
5. a. $1.05 \mathrm{~m} / \mathrm{s}^{2}$
b. $a=\frac{\pi}{180} \frac{v \theta}{t} \quad(\theta$ in degrees $)$
$a=\frac{v \theta}{t} \quad(\theta$ in radians $)$
6. a. The centripetal force is the horizontal component of the tension in the string and this equals mass times acceleration. The vertical component of the tension in the string balances with the weight of the mass and therefore equals $m g$. By similar triangles the ratio of these components is equal to the ratio of radius to height: $\frac{T_{x}}{T_{y}}=\frac{m a}{m g}=\frac{r}{h}$ Therefore: $a=g \frac{r}{h}$
b. $T=2 \pi \sqrt{\frac{h}{g}}$
7. a. 18 kN
b. $20 \mathrm{~m} / \mathrm{s}$
c. 3.8 s
8. a. $13.1 \mathrm{~m} / \mathrm{s}$
b. $35 \mathrm{~m} / \mathrm{s}$
9. a. 350 m
b. 640 m
10. a. 6.2 m
b. $16 \mathrm{~m} / \mathrm{s}$
11. a. The ball has inertia. When the ball is released it is already moving with a certain velocity. With no significant force to cause any acceleration it continues on a tangent path with a certain velocity due to its inertia (not due to a force). From the perspective of the boy it appears to accelerate away from the center (when in reality it is the effect of the boy accelerating inwardly away from the ball).
b. $1.9 \mathrm{~m} / \mathrm{s}^{2}$ away from center
c. 0.46 N
12. a. $0.034 \mathrm{~m} / \mathrm{s}^{2}$ toward center or down
b. 2.7 N
c. 1.4 h
13. a. $T=\pi \sqrt{\frac{2 d}{g}}$
b. 450 m
14. a. $2.0 \mathrm{~m}, 106^{\circ}$ from origin
b. $6.0 \mathrm{~m} / \mathrm{s}, 16^{\circ}$
c. $18 \mathrm{~m} / \mathrm{s}^{2}, 286^{\circ}$
d. 2.1 s

$$
x=-6 t+0.33 \cos \left(18.18 t-\frac{\pi}{2}\right)
$$

15. a.

$$
y=0.33+0.33 \sin \left(18.18 t-\frac{\pi}{2}\right)
$$

OR

$$
\begin{aligned}
& x=-6 t+0.33 \sin (18.18 t) \\
& y=0.33-0.33 \cos (18.18 t)
\end{aligned}
$$

b. $12 \mathrm{~m} / \mathrm{s}$ (to the left)

$$
\text { at } t=0.173 \mathrm{~s}, 0.519 \mathrm{~s}, 0.864 \mathrm{~s} \text {, etc }
$$

c.

16. a. $2.97 \mathrm{~m} / \mathrm{s}$
b. $6.64 \mathrm{~m} / \mathrm{s}$
c. $0 ; 7.35 \mathrm{~N}$
d.

17. a. Friction is initially tangent, then becoming more and more radial
b. Friction points in the direction of acceleration. The initial radial component of acceleration is zero, so friction is initially tangent. As the speed of the object increases, the radial component of acceleration increases causing friction to tilt away from tangent and point more toward the center as the surface rotates.
c. $\theta=\frac{1}{2} \sqrt{\frac{\mu_{s}^{2} g^{2}}{a^{2}}-1}$
18. a. 9.42 s
b. $4.46 \mathrm{~m} / \mathrm{s}^{2}, 328.8^{\circ}$
19. a. 5.6 nN
b. 30.0 cm
20. a. 1470 kg
b. No! - explain
21. a. $0.0107 \mathrm{~m} / \mathrm{s}^{2}$ toward Earth
b. 346 Mm from Earth
38.3 Mm from Moon
22. a. $2.39 \times 10^{20} \mathrm{~N}$ toward Sun
b. The Moon's "orbit about Earth" is a description of its apparent motion in the Earth frame of reference - we see it "go around us". However, in the "bigger picture" (i.e. relative to the Sun), the Moon is moving around the Sun (along with the Earth). The net force pulling Moon toward the Sun found in part (a), even when Earth's gravity pulls it in the opposite direction, indicates that the Moon is at all times accelerating at least somewhat toward the Sun and follows a path that at all times curves toward the Sun (even as its path relative to Earth is an ellipse). The path of the Moon relative to the Sun is very similar to the path of the Earth's orbit around the Sun, except that it "weaves in and out" due to Earth's gravity.
23. a. 7400 N , down
b. 37 kN tension
24. a. $3.70 \mathrm{~m} / \mathrm{s}^{2}$
b. $3.72 \mathrm{~m} / \mathrm{s}^{2}$
c. Mars is about twice the mass of Mercury, but the radius is only about 1.4 times greater.

All other things being equal, doubling the mass would cause gravitation to double in strength. However, when standing on the surface of Mars one is located farther from the center. By the inverse square law, the distance being 1.4 times greater reduces the strength of gravity by that factor squared, which is very nearly 2.0 - i.e. this causes gravity to be half as strong. The combination of the two factors results in the values of $g$ being nearly identical.
25. a. $g_{Q}=\frac{g_{E}}{2}$
b. $g_{X}=2 g_{E}$
c. $g_{S}=g_{E}$
d. $g_{M}=\frac{4}{10} g_{E}$
e. $g_{L}=0.16 g_{E}$
26. a. 2640 km
b. 6380 km
c. 6 km
27. a. $g=\frac{G M_{E} r}{R_{E}^{3}}$ or $g=\frac{r}{R_{E}} g_{\text {surface }}$
b.

28. a. $2.21 \times 10^{-6} \mathrm{~m} / \mathrm{s}^{2}$
b. 14 m
29. a. $8.94 \mathrm{~m} / \mathrm{s}^{2}$
b. $7730 \mathrm{~m} / \mathrm{s}$
c. 90.5 minutes
d. 626 N , down
e. An astronaut appears "weightless" in orbit because the person does not accelerate relative to the spacecraft. Both the astronaut and the spacecraft are moving at the same speed, and with the same acceleration toward Earth. Both objects are in freefall under the sole influence of gravity.
f. Rockets should fire in a "forward" direction, with exhaust pointing in the same direction as the spacecraft's velocity. By Newton's $3^{\text {rd }}$ Law this produces a force "backward" on the spacecraft, which decreases its speed to a value less than what is required to stay in orbit.
30. In order to put a satellite into orbit rocket engines are used to lift the object above the Earth's atmosphere. But, in order to remain in orbit at any given altitude the object must be in motion at a high rate of speed parallel to Earth's surface. The rocket engines must also be used to accelerate the satellite and increase its speed to the value required for the intended orbit. Both of these actions occur at the same time - the launch trajectory is a path that initially points directly away from the Earth, but which then curves into a direction parallel to the surface of the Earth.
31. a. $0.0026 \mathrm{~m} / \mathrm{s}^{2}, 0.0166 \mathrm{~m} / \mathrm{s}^{2}$, $0.0851 \mathrm{~m} / \mathrm{s}^{2}, 0.280 \mathrm{~m} / \mathrm{s}^{2}$
b. $110 \mathrm{~m} / \mathrm{s}, 18 \underline{0} \mathrm{~m} / \mathrm{s}, 27 \underline{0} \mathrm{~m} / \mathrm{s}$
32. a. 1400 km
b. 13
33. a. 35787 km
b. $3074.8 \mathrm{~m} / \mathrm{s}$
34. a. $7.8 \times 10^{36} \mathrm{~kg}$
b. 3.9 million suns!
35. a. $\frac{r_{A}}{r_{B}}=9$
b. $\frac{T_{A}}{T_{B}}=27$
36. a. $4.5 \times 10^{15} \mathrm{~kg}$
b. $5.0 \times 10^{14} \mathrm{~kg}, 1.7 \mathrm{~m} / \mathrm{s}$
37. a. 0.675 days, 239 days, 315 Jup. radii
b. $r^{3}$ vs. $T^{2}$ is linear with slope 65.4

