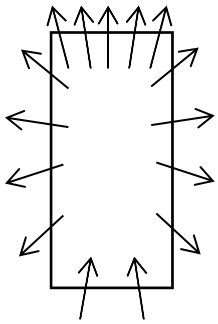
Answers – Physics C Potential Assignment

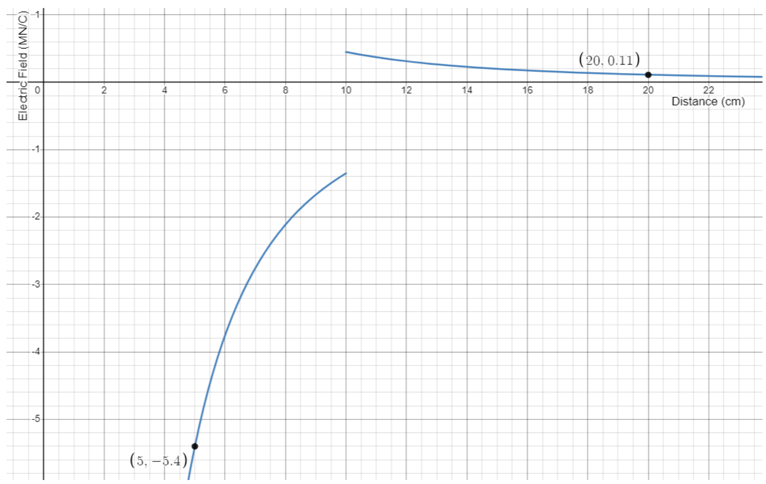
1. a. 63 N m^2/C
   b. 48 N m^2/C
   c. 0
2. a. If rain is falling straight down the gutters collect equal amounts. Imagine the roofs are removed from each house and the rain simply falls inside the identical houses – in this situation it is obvious that the amount of water is the same. But any drop of rain that would have fallen into the house without its roof, would instead land on the roof (of any amount of slope) and then end up in the gutter. Therefore the amounts are the same with any amount of incline to the roofs. In physics terms the flux is the same because the vertical component of the roof's "area vector" is the same for each house and has magnitude equal to the area of the "footprint" of the house.
   b. If the rain falls at an angle then the amounts of rain collected in the gutters could be different. Imagine an extreme example with the rain moving nearly horizontally. If one roof has a very low amount of slope then it would collect very little of this rain, whereas if the other is highly inclined it might collect most or all of the rain. In physics terms the flux would depend on how well the roof's "area vector" is aligned with the direction of the rain. The flux would be greatest with the roof's surface tilted such that the rain's direction is normal.
3. 110 N m^2/C
4. a. Certain regions of the *xy*-plane would have field lines passing upward, while other regions would have field lines passing downward. Any particular "patch" would have a nonzero flux proportional to the number of lines passing through that particular region.
   b. Flux is proportional to the number of field lines crossing through the plane. Every line that passes downward through the plane will curve around and at some point return upward through the plane. Every field line connected to one of the charges will also connect to the other if it is equal and opposite.
5. 160 N m^2/C
6. a. 9$\underline{0}$0 N m^2/C
   b. 9$\underline{0}$0 N m^2/C
   c. The half of the sphere closest to the negative charge would have the greatest flux because there would be a greater number of lines passing through this hemisphere than the one opposite the negative charge. Alternatively, it is clear that the field strength is greater in between the two charges than beyond the two charges. The hemispheres have equal area, but the greater field strength results in greater flux through the side closest to the negative charge.
7. a. −340 N m^2/C
   b. −56 N m^2/C
8. $\Phi = -\dfrac{q}{3\varepsilon_0}$

9. a. 4.2 nC
   b. There is no way to know exactly what is in the box, but there must be 4.2 nC *more* positive charge than negative charge.
   c.

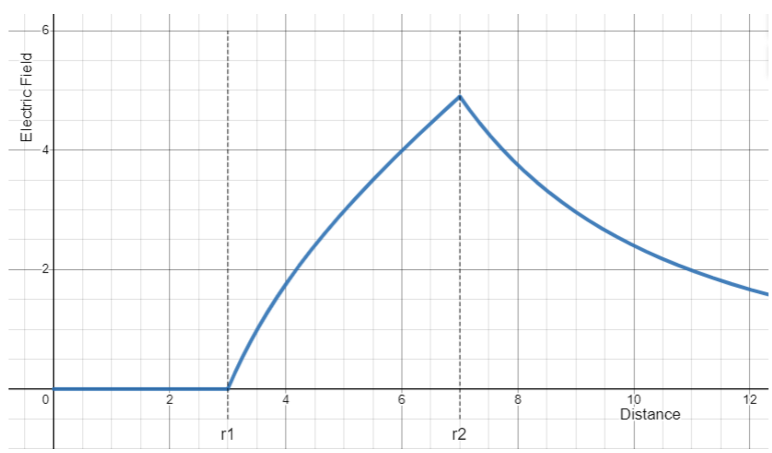


   d. It would appear there is a strong positive charge near the top end of the box and a weaker negative charge near the bottom end of the box.

10. a. -5.4×10^6 N/C $\hat{r}$
    b. 1.1×10^5 N/C $\hat{r}$
    c.



11. a.
$$\vec{E} = 0, \quad r \le r_1$$
$$\vec{E} = \frac{Q\left(r^3 - r_1^3\right)}{4\pi\varepsilon_0 r^2\left(r_2^3 - r_1^3\right)}\hat{r}, \quad r_1 \le r \le r_2$$
$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}, \quad r \ge r_2$$
    b.



12. a. $\sigma = 1.99 \times 10^{-6}$ C/m^2
    b. 2.50×10^{-7} C
    c. 1.3×10^{-7} C

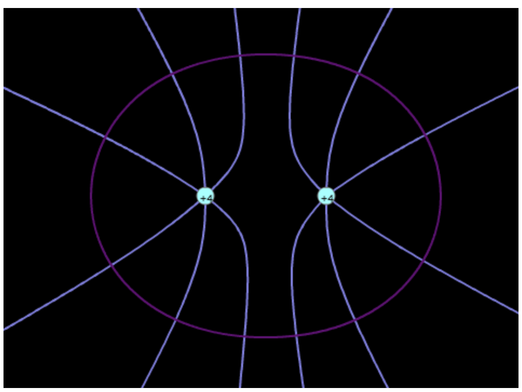
13. a. 2.3 MN/C
b. 1.3 mC/m^3

14.
$$\vec{E} = \frac{br^2}{3\varepsilon_0}\hat{r}, \quad r \le R$$
$$\vec{E} = \frac{bR^3}{3\varepsilon_0 r}\hat{r}, \quad r \ge R$$

15.
$$\vec{E} = -\frac{\rho h}{2\varepsilon_0}\hat{k}, \quad z \le -\frac{h}{2}$$
$$\vec{E} = \frac{\rho z}{\varepsilon_0}\hat{k}, \quad |z| \le \frac{h}{2}$$
$$\vec{E} = \frac{\rho h}{2\varepsilon_0}\hat{k}, \quad z \ge \frac{h}{2}$$

16. a. sketch
b. 4700 N/C, toward line
c. 4.7 cm from line, 1.3 cm from plane

17. a.



b. The surface must be normal to the field at all locations and enclose the two charges. Also, to simplify the integral for flux, the electric field strength must be uniform at all points on the surface, and the area of the surface must be calculatable.
c. It seems unlikely that all of the requirements can be met. Perhaps there is a surface that is normal, but there is no reason to conclude the field is uniform on such a surface, nor any formula for determining its area.

18. a. 0.54 μJ
b. decrease separation to 6.4 cm
c. −0.54 μJ
increase separation to 22.5 cm

19. a. −11 μJ
b. 9.0 μJ
c. 2.2 μJ

20. a. Decreasing the separation of two like charges increases the potential energy of the system. The force of electrostatic repulsion increases with less separation and therefore can do more work should the charges be released and move off in opposite directions. The decreased separation also gives the force more distance over which to do work if the charges move apart.
b. Increasing the separation of two opposite charges increases the potential energy of the system. The increased separation increases the distance over which the force of electrostatic attraction can act should the charges be released and allowed to come together. With more distance over which to act the force can do more work.

21. a. $W = \dfrac{q^2}{4\pi\varepsilon_0 s}$

b. $W = -\dfrac{q^2}{2\pi\varepsilon_0 s}$

c . $E = \dfrac{3q^2}{4\pi\varepsilon_0 s}$

22. a. 0.91 mJ
b. 1.8 mJ

23. a. 4.8×10^{-15} J, 1.0×10^{8} m/s
b. 1.8×10^{8} m/s

24. a. $E = -\dfrac{e^2}{8\pi\varepsilon_0 r}$

b. 1.64×10^{-18} J

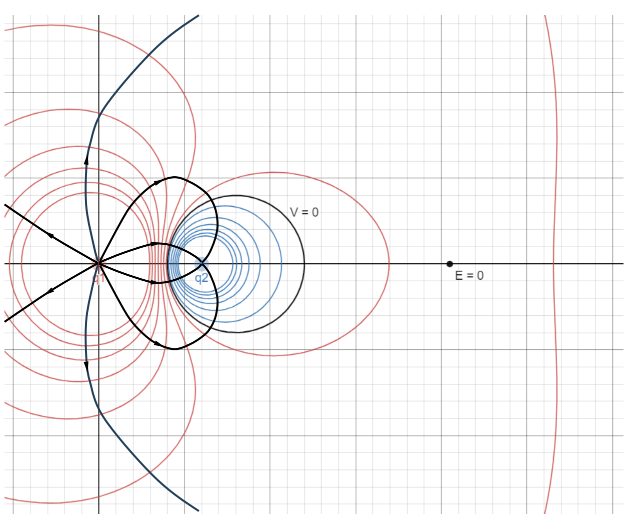
25. a. $a_1 = 0.78$ mm/s^2, $a_2 = 0.52$ mm/s^2,
opposite directions
b. $v_1 = 2.9$ cm/s, $v_2 = 1.9$ cm/s

26. −540 V, − 270 V

27. 180 V, $9\underline{0}$ V

28. a. 1500 V
b. 2300 V
c. −158 V
d. $x = 4.0$ cm, 12 cm (and all along a circle of radius 4.0 cm centered on $x = 8.0$ cm)

29.



30. $V = \frac{Q}{4\pi\varepsilon_0 L}\ln\left(\frac{x+L}{x}\right)$

31. a. −5250 V
    b. x = 18 cm
    c. −40.1 V, −40.5 V

32. 1.0 Mm/s

33. a. −78 kV
    b. −3.0 nC

34. a. 110 V
    b. 340 V

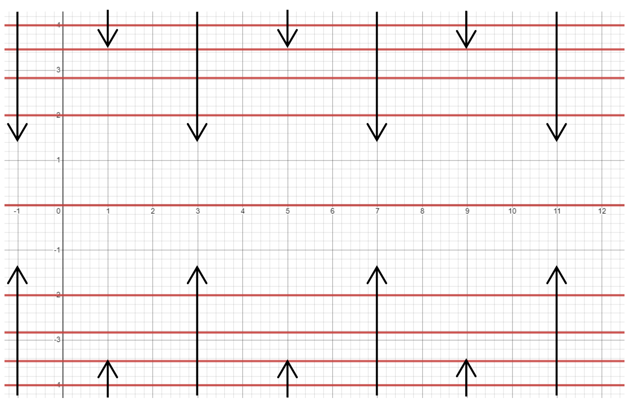
35. $V = \frac{3Q}{8\pi\varepsilon_0 R}$

36. −5100 V

37. a. 8.8×10^7 m/s
    b. 2.4×10^6 V/m
    c. 4.9×10^6 V/m

38. $\vec{E} = \frac{Q}{4\pi\varepsilon_0 x(x+L)}\,\hat{i}$

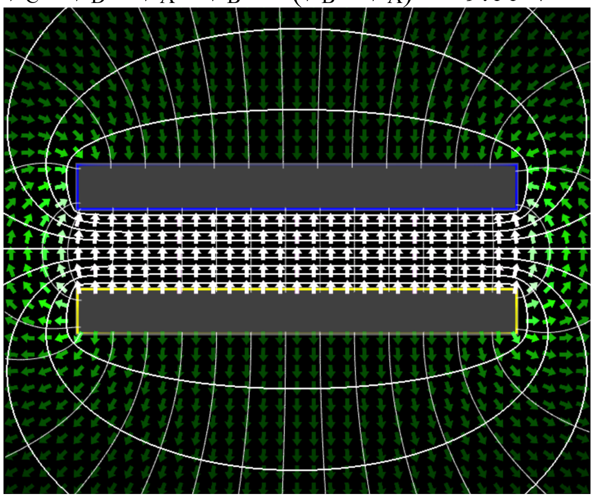
39. a. c.



b. $\vec{E} = -\frac{y}{2}\,\hat{j}$

d. For such a region to exist there must be positive charges at greater density the further away from the x-axis, above and below – however, if that were the case it is difficult to imagine how the potential on the y-axis can be zero relative to infinity.

40. a. Negative charge will be found only on the outermost surfaces (6 faces) of the cube.
   b. The charge will *not* be uniform, but instead will have greatest density at the corners of the cube. In order for the electric potential of the cube to be uniform charge density must increase where radius of curvature of the surface is decreased.
   c. In order for Gauss's Law to be satisfied there must be a charge of –1.5 μC induced on the inner surfaces of the box. This is because the field inside the metal of the box must be zero and hence the charge enclosed is zero. By conservation of charge the outer surfaces of the box will then have a charge of –3.5 μC (the box overall still has net charge 5 μC).
   d. The charge outside the box does not depend on exactly where the point charge is located inside the box. This is because the point charge and the (now charged) inner surface of the box produce a net field of zero for points anywhere outside of those two charges. Therefore the charge on the outer surfaces will be undisturbed.
41. a. The collection point is inside the sphere where the electric field is nearly zero as "required" by Gauss's Law. Once charges are located inside the sphere there is very little repulsion from the other charges already on the sphere which are on the outer surface. To add charge directly to the outer surface a charge must "make it through" an intense electric field and a relatively great repulsion.
   b. An electron on the belt is one of many that have been picked up at the base of the Van de Graaff. However, the belt is an insulator and so the electron is not able to freely move. As it approaches the negatively charged sphere there is a repulsive force, but only while it is outside of the sphere. The belt does work to force the electron into the sphere. Once it is inside, the negatively charged sphere does not affect the electron. As the belt reaches the metallic brush inside the sphere, the electron is repelled from other electrons on the belt and attracted to the induced positive charge of the brush. Once inside the metal of the sphere the electron is free to move and goes to the outer surface, spread evenly from other excess electrons by mutual repulsion.
42. a. $U = \dfrac{Q^2}{8\pi\varepsilon_0 R}$
   b. It does not matter if the sphere is hollow or solid, not to which part of the sphere the charge is conducted. The entire sphere is an equal potential because the electric field within is zero. Charges being added to the sphere "from infinity" are repelled by the sphere and so work must be done to bring the charges to it. Because the electrostatic force is conservative it doesn't matter what path is followed by the additional charges – the work done is the same.
43. $$E = 0, \quad r < R \qquad V = 0, \quad r \le R$$
   $$\vec{E} = \frac{\sigma R}{\varepsilon_0 r}\hat{r}, \quad r > R \quad V = \frac{\sigma R}{\varepsilon_0}\ln\left(\frac{R}{r}\right), \quad r \ge R$$
44. a. $q_1 = 25.0$ nC, $q_2 = 50.0$ nC
   b. $\sigma_1 = 3.11 \times 10^{-7}$ C/m^2
   $\sigma_2 = 1.55 \times 10^{-7}$ C/m^2

45. a. $V_A - V_C = V_B - V_D = 0$ because the path between the respective points is a conductor and therefore is an equipotential path along which the electric field is zero.
b. $V_C - V_D = V_A - V_B = -(V_B - V_A) = -9.00$ V
c. 
d. 6.00 kV/m, up
e. 2.12 nC

46. a. 81.0 nC
b. 9210 V
c. 4.48×10^{-8} C/m^2