Capacitance

I. Basic Concepts
- definition
- steady state behavior

II. Capacitor Design
- capacitance vs. geometry
- materials & dielectrics

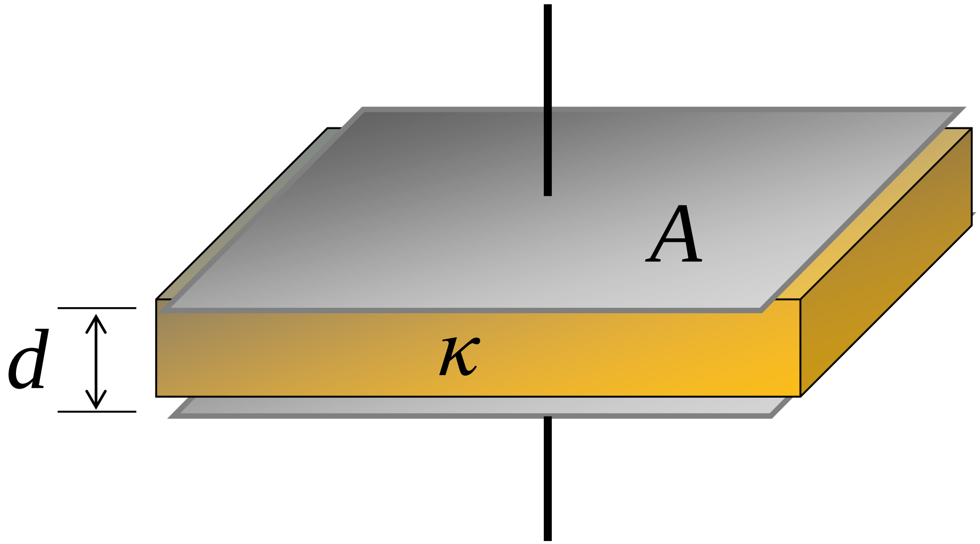
III. RC Circuits
- dynamic behavior
- charging/discharging

	The student will be able to:	HW:
1	Define and calculate capacitance in terms of voltage and charge and solve related problems.	1 – 3
2	Solve steady-state problems involving series and parallel connections of capacitors and batteries.	4 – 6
3	Solve problems relating capacitance to geometry and dielectrics for parallel plate, cylindrical, and spherical capacitors.	7 – 17
4	Analyze RC circuits in terms of the appropriate differential equation and resulting exponential functions for charge, current, voltage, etc.	18 – 22

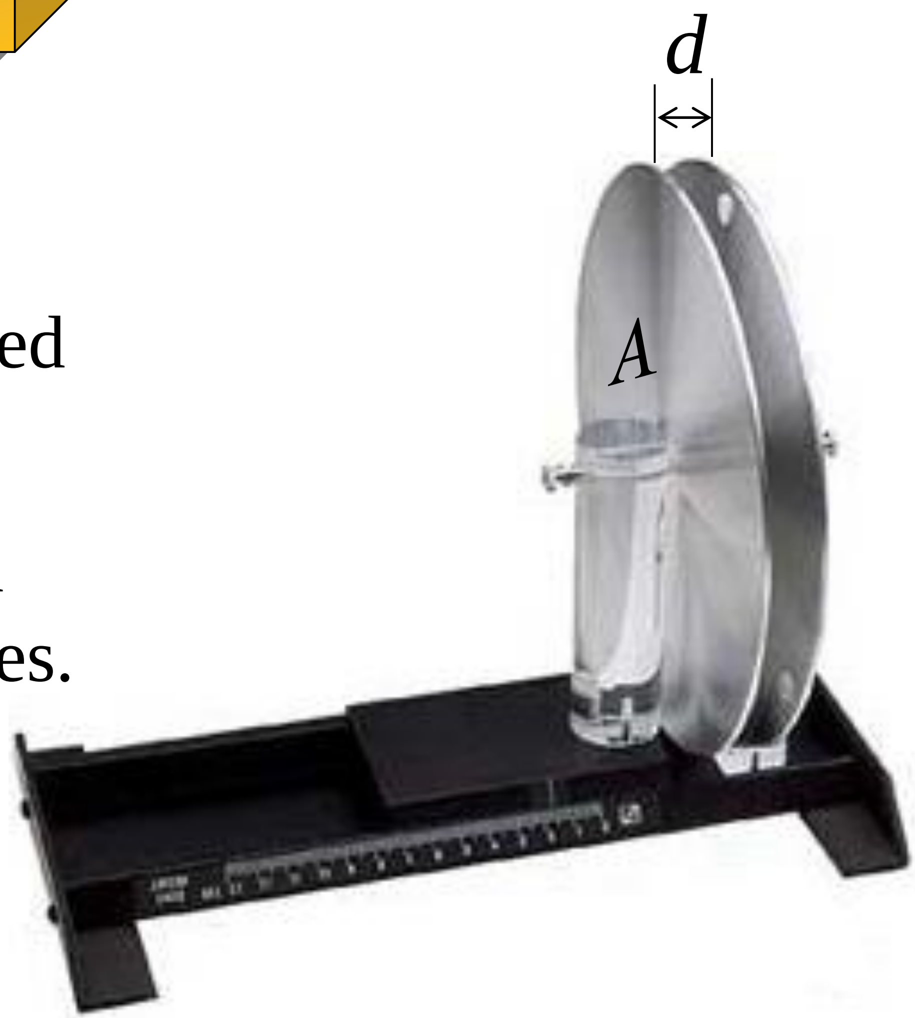
The capacitance of a parallel-plate capacitor is given by:

$$C = \frac{\kappa \varepsilon_0 A}{d}$$

where: C = capacitance
A = area of either plate
d = distance separating plates
κ = dielectric constant of material between plates



The area *A* can be any shape, but commonly is a rectangle or circle.

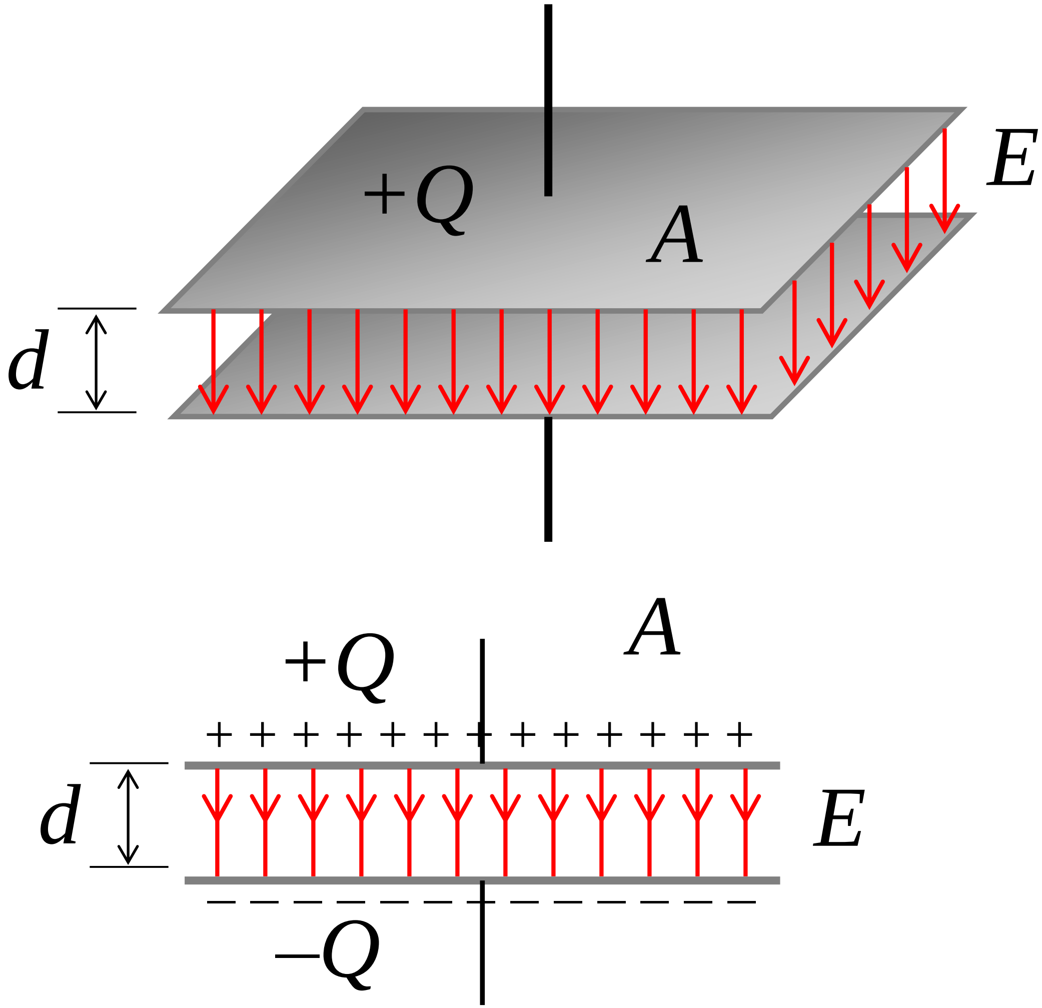


The insulating material between the plates is called the “dielectric” slab.

There can also be air or a vacuum between the plates.

Dielectric Constants and Strengths

Material	Dielectric Constant, κ	Dielectric Strength (MV/m)
Vacuum	1	N/A
Air	1.0006	3
Pyrex	5	14
Paper	3.7	16
Teflon	2.1	60
Paraffin	2.3	11
Polystyrene	2.6	24
Strontium Titanate	230	8



$$E = \frac{\sigma}{\varepsilon_0}$$

$$E = \frac{Q}{\varepsilon_0 A}$$

$$Ed = \frac{Qd}{\varepsilon_0 A}$$

$$V = \frac{Qd}{\varepsilon_0 A}$$

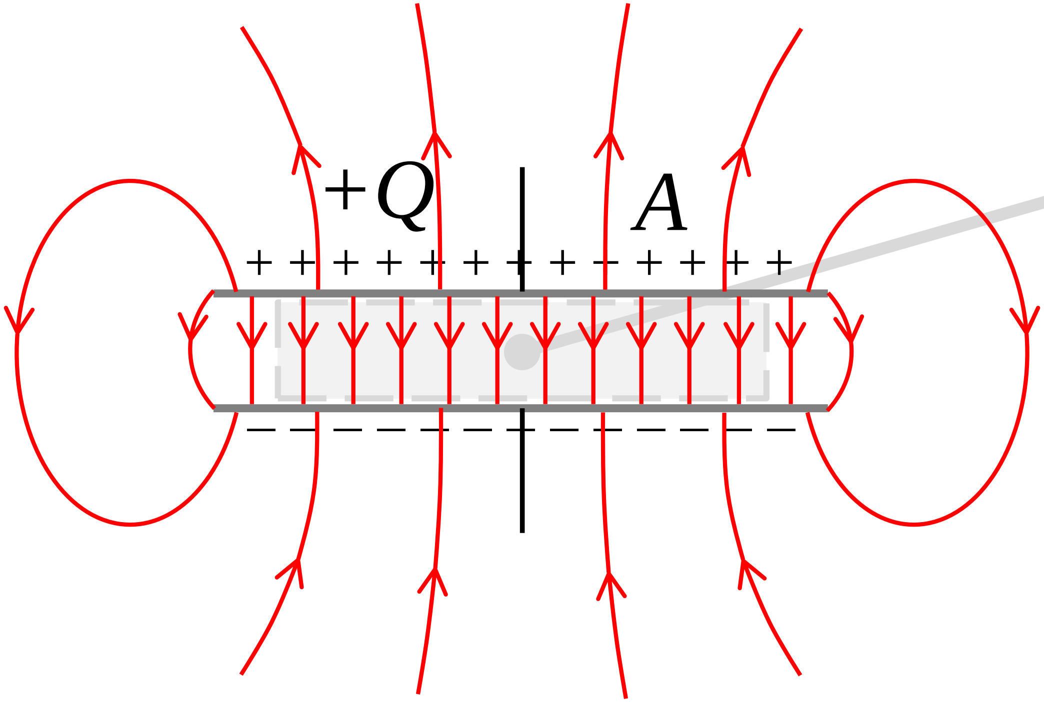
$$C = \frac{Q}{V}$$

$$C = Q\frac{1}{V}$$

$$C = Q\frac{\varepsilon_0 A}{Qd}$$

$$\boxed{C = \frac{\varepsilon_0 A}{d}}$$

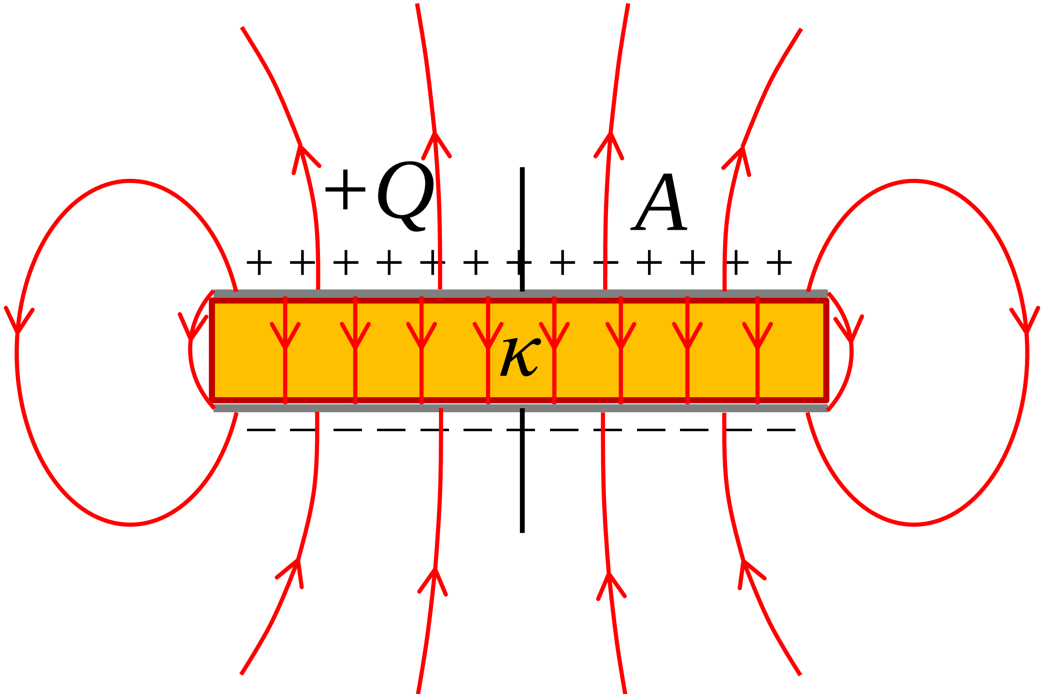
To derive capacitance in "terms of geometry" solve for the electric field and voltage for a given amount of charge. Then apply the basic definition of capacitance.



$$E = \frac{s}{e_0}$$

most accurate for
the shaded region

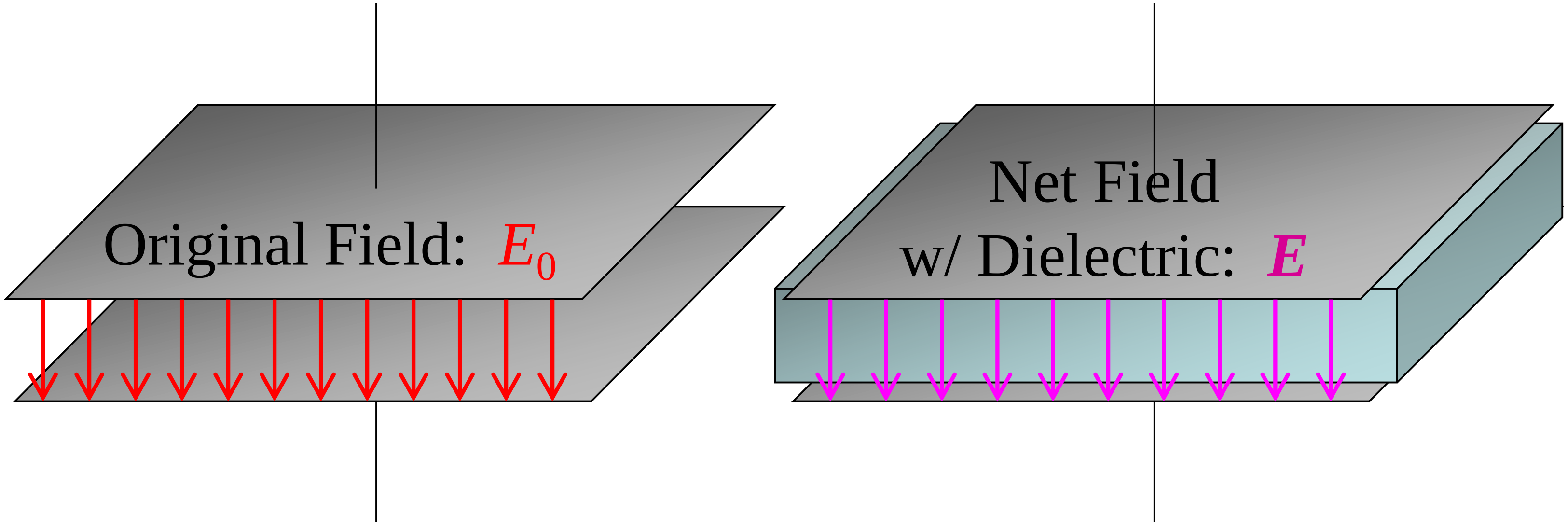
This expression applies *only* to the region *between* two oppositely charged surfaces separated by a relatively small gap. In reality the field weakens and curves near the edges of the plates and *outside* of the capacitor it is very weak and often assumed to be essentially nonexistent.



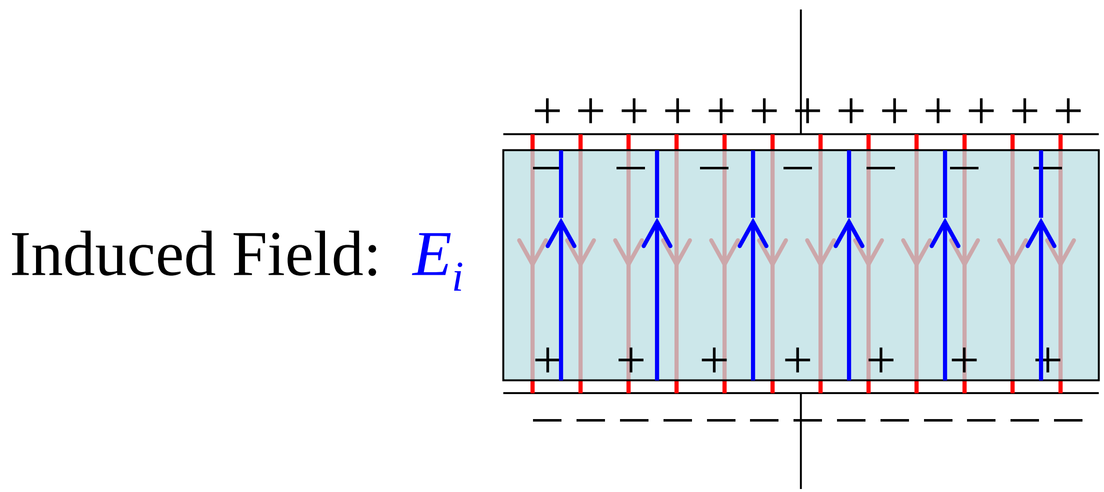
$$E = \frac{\sigma}{\kappa \varepsilon_0}$$

gives the net electric field existing in the material between plates

If the gap is filled with air or a vacuum the value of κ can be taken to be 1.00. Presence of a dielectric material causes the electric field to be weakened by the factor κ, which is a particular value depending on the characteristics of the insulating material. This is due to induced charges in the dielectric which create an opposing field. This allows greater charge to be stored on the plates for a given voltage, and hence increases capacitance by the factor κ.



A dielectric decreases the effective electric field by a factor κ.

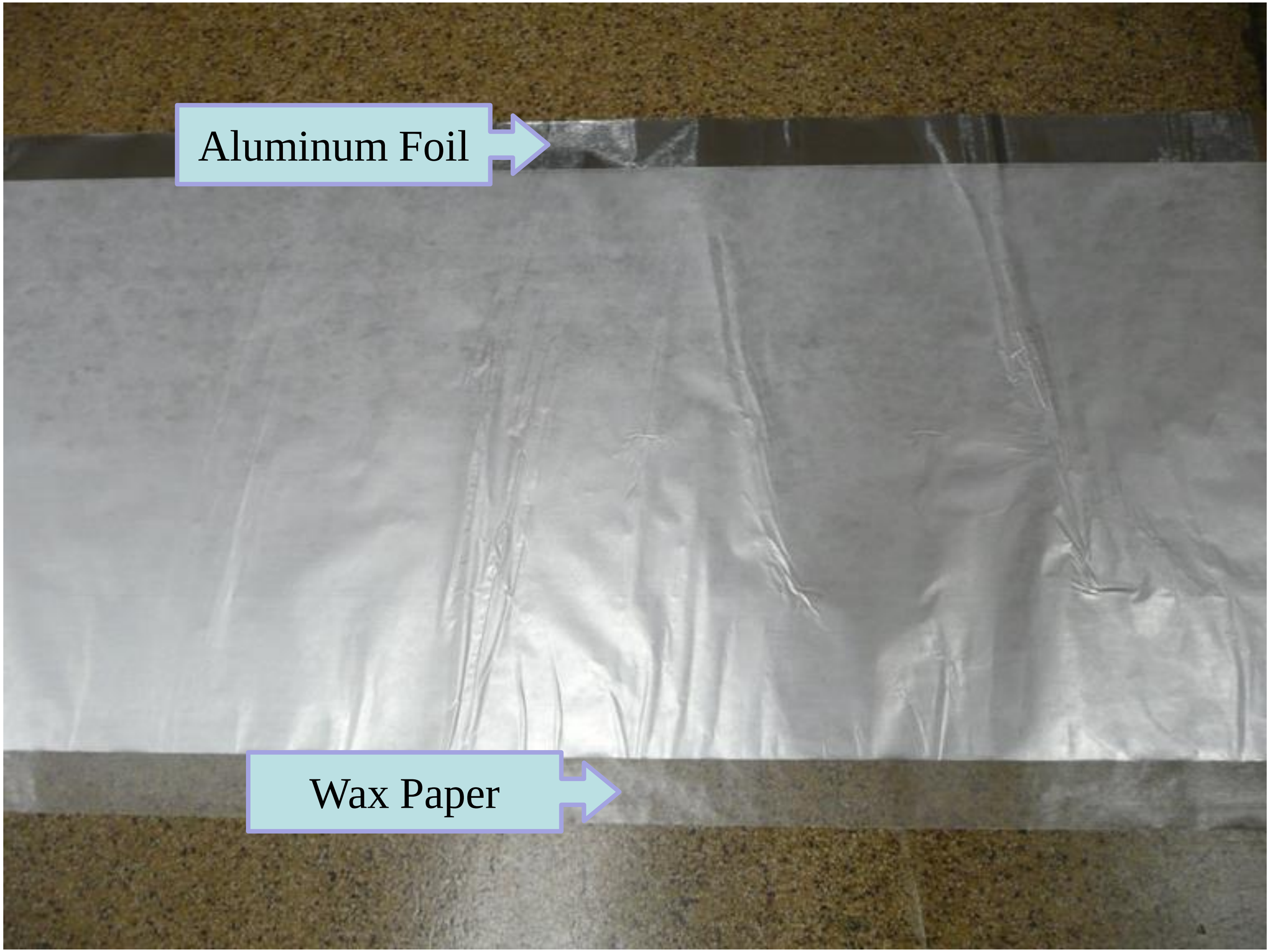


$$E = \frac{E_0}{\kappa}$$

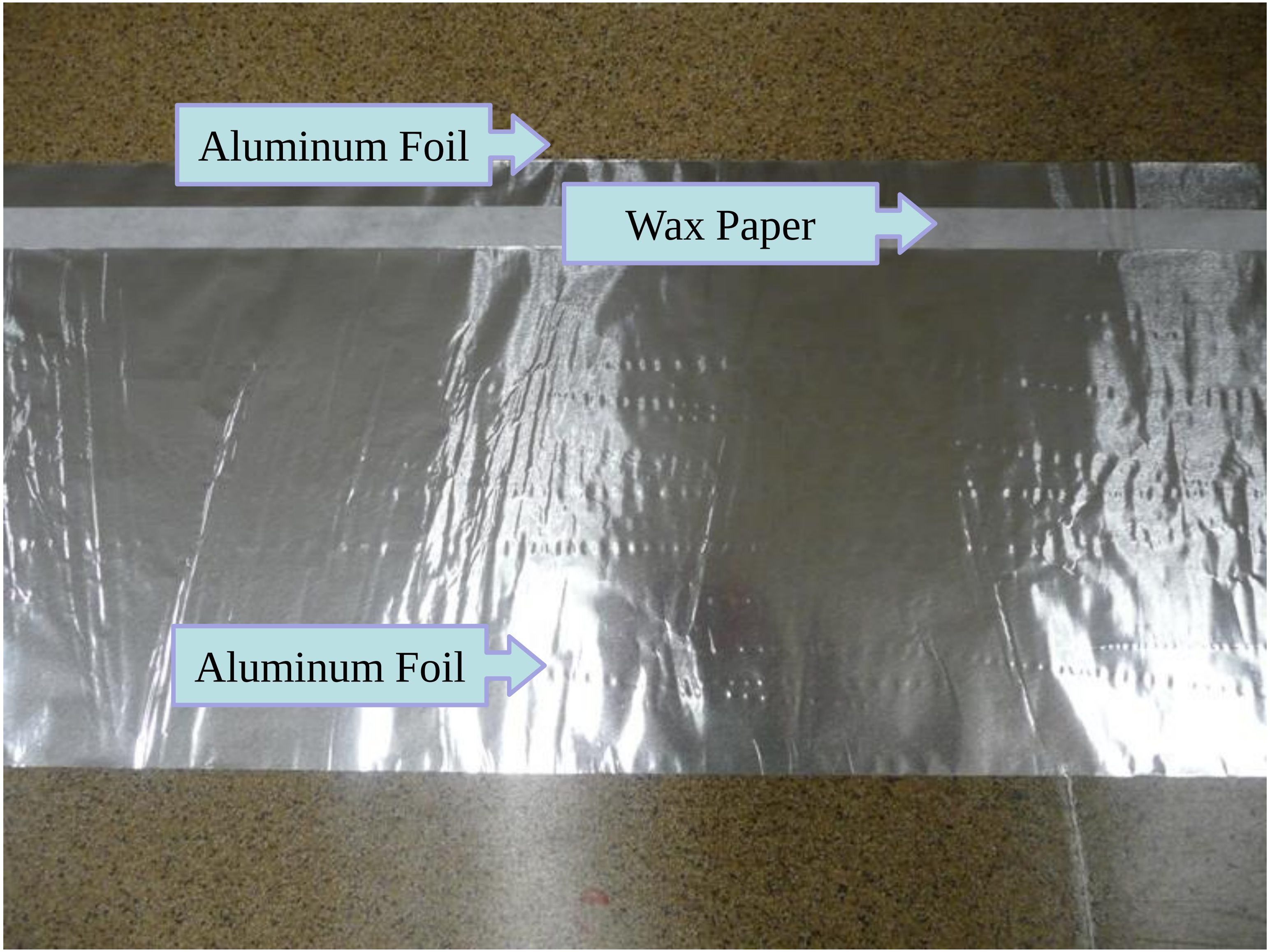
$$\boldsymbol{E} = E_0 - E_i$$

The net electric field E is the superposition of two fields:
E_0 (produced by the charges on the two plates) and
E_i (produced by the induced charges in the dielectric material).

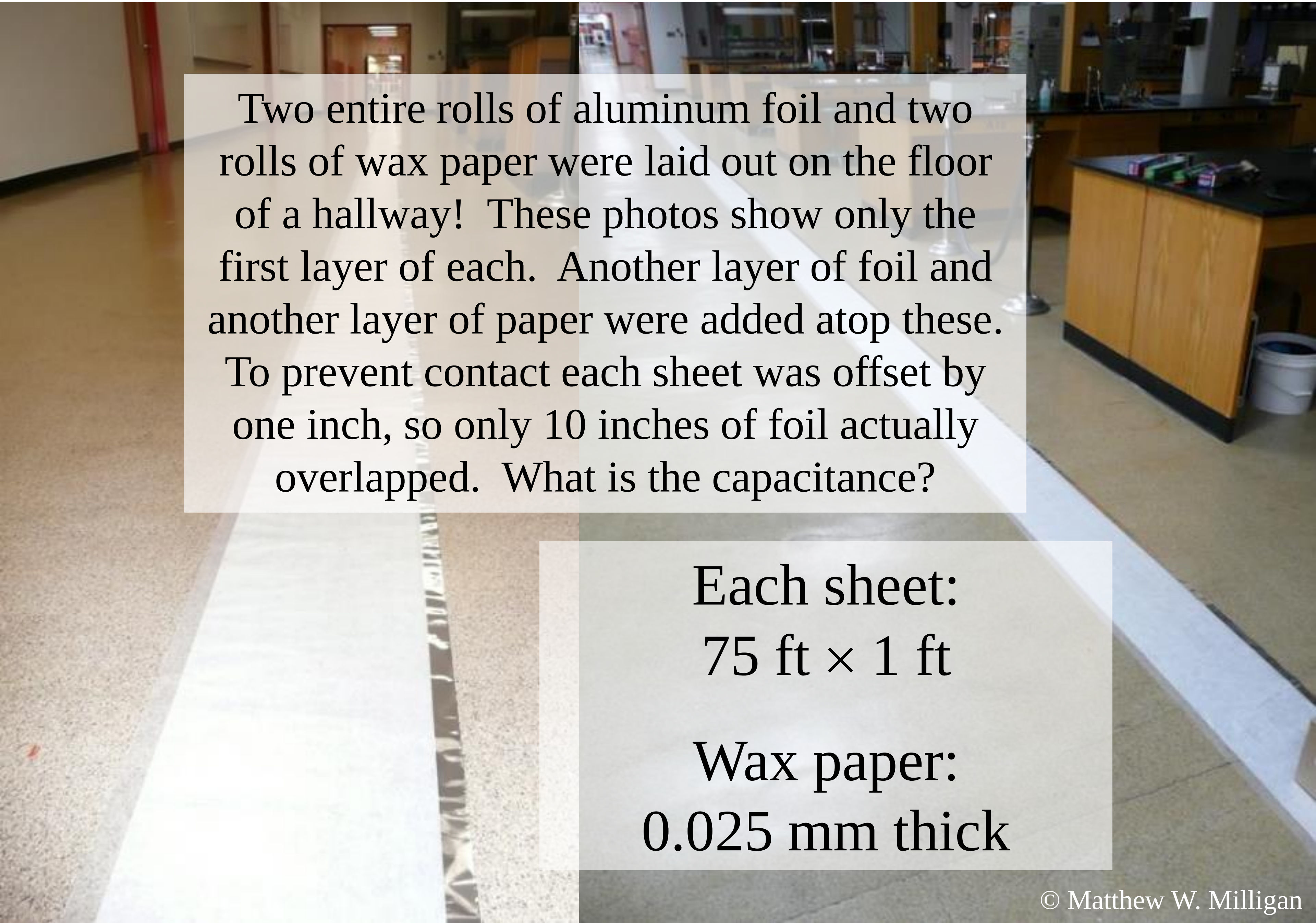
A homemade capacitor using aluminum foil and wax paper…



A homemade capacitor using aluminum foil and wax paper…



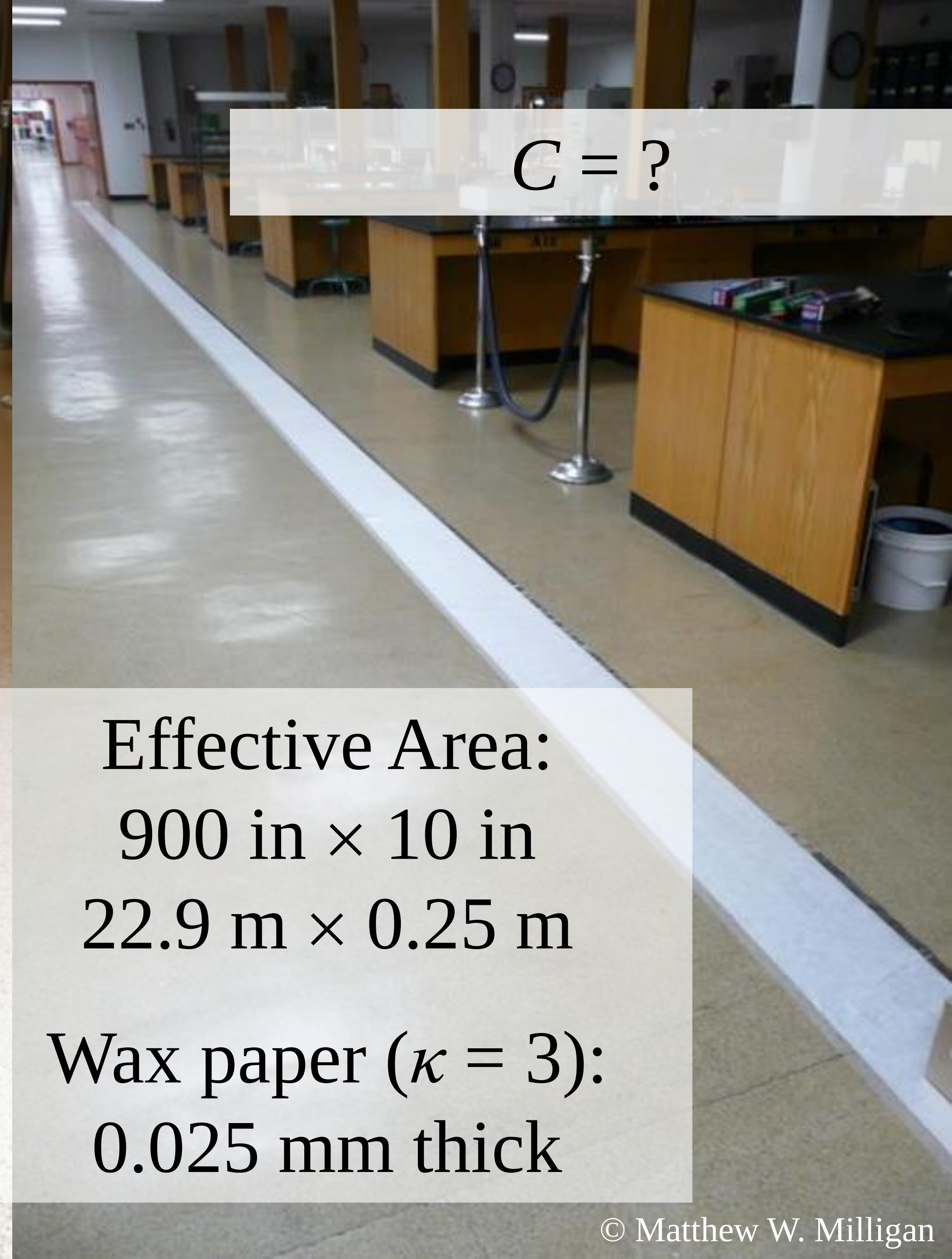
A homemade capacitor using aluminum foil and wax paper…



Two entire rolls of aluminum foil and two rolls of wax paper were laid out on the floor of a hallway! These photos show only the first layer of each. Another layer of foil and another layer of paper were added atop these. To prevent contact each sheet was offset by one inch, so only 10 inches of foil actually overlapped. What is the capacitance?

Each sheet:
75 ft × 1 ft

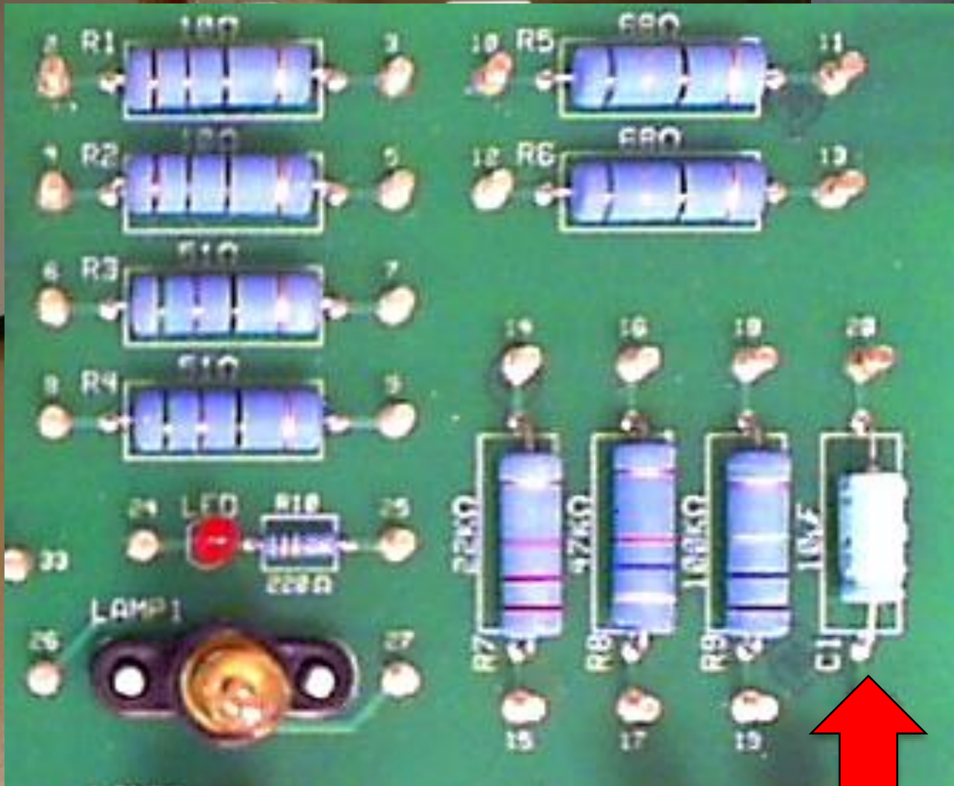
Wax paper:
0.025 mm thick



$C = ?$

Effective Area:
900 in × 10 in
22.9 m × 0.25 m

Wax paper ($\kappa = 3$):
0.025 mm thick



Almost as good as this one! (C = 10 μF)

$$C = \frac{\kappa \varepsilon_0 A}{d}$$

$$C = \frac{3\varepsilon_0 (5.81)}{0.000025}$$

C = 6.2 μF

(if left "unrolled"!)

The four sheets were then rolled up (like a "jelly roll"). This changed the capacitance! The formula shown here is technically only valid for two flat and parallel surfaces. When the sheets are rolled up it is no longer a flat surface and the ratio of charge to voltage (the capacitance) would be different because the electric field is different. In the rolled up configuration the field is more akin to that of a cylinder or a line of charge. Nonetheless the rolled up capacitor has been shown experimentally to have a capacitance of about 4 μF – approximately the same as calculated here. The rolled up capacitor looks like a huge burrito!

$$C = \frac{\kappa \varepsilon_0 A}{d}$$

$$C = \frac{3\varepsilon_0 (5.81)}{0.000025}$$

$C = 6.2\ \mu\text{F}$

(if left "unrolled"!)