Capacitance

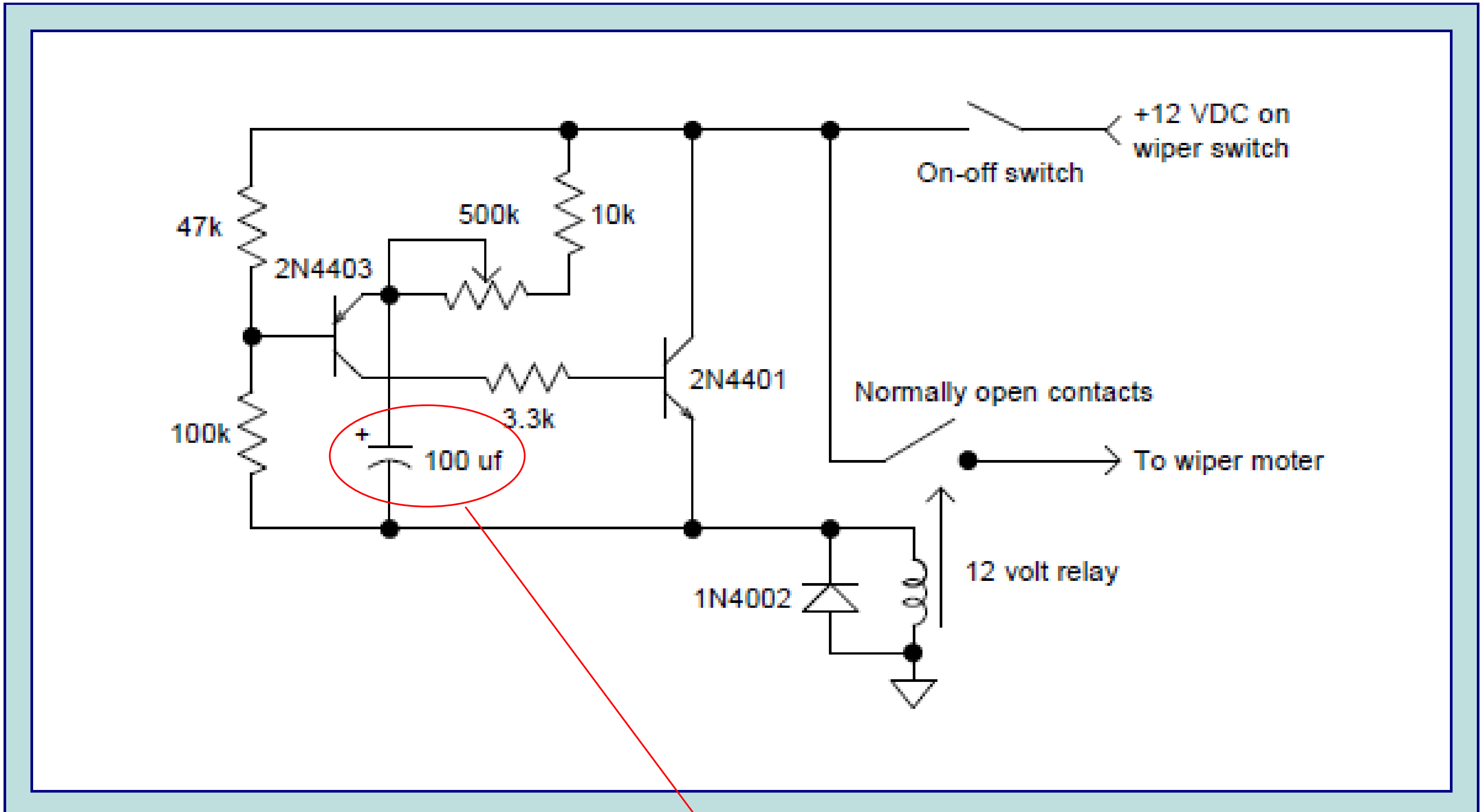
I. Basic Concepts
- definition
- steady state behavior

II. Capacitor Design
- capacitance vs. geometry
- materials & dielectrics

III. RC Circuits
- dynamic behavior
- charging/discharging

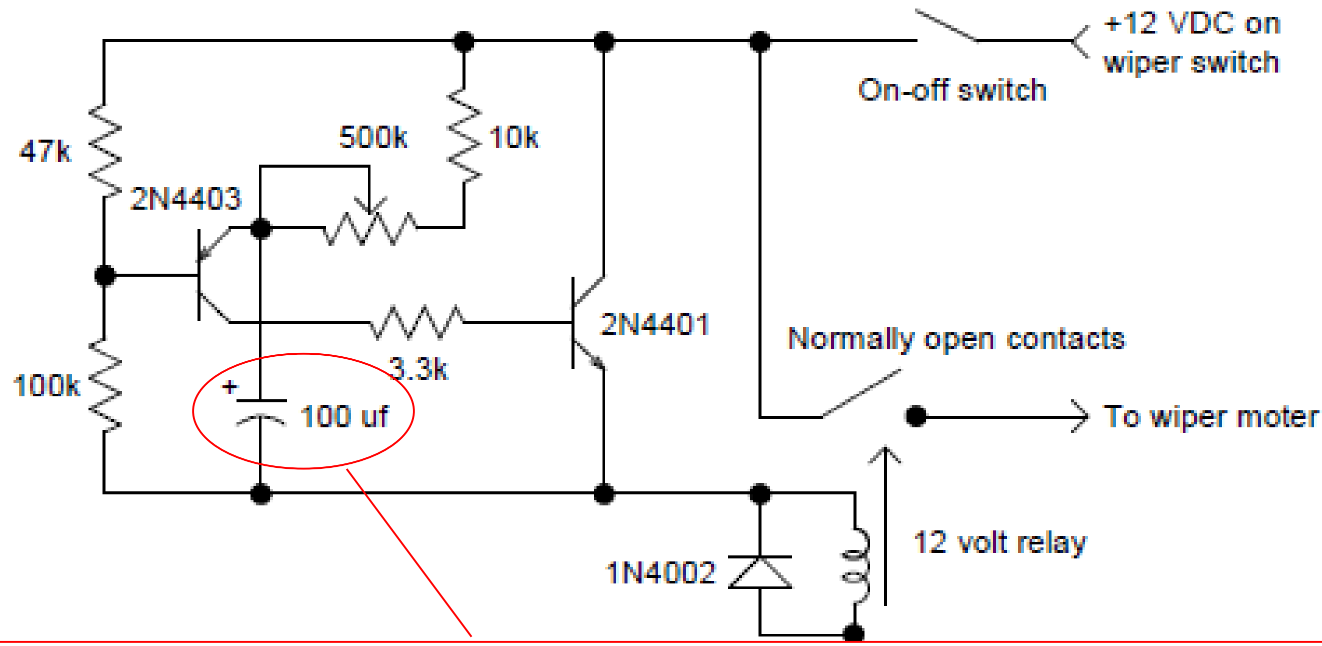
	The student will be able to:	HW:
1	Define and calculate capacitance in terms of voltage and charge and solve related problems.	1 – 3
2	Solve steady-state problems involving series and parallel connections of capacitors and batteries.	4 – 6
3	Solve problems relating capacitance to geometry and dielectrics for parallel plate, cylindrical, and spherical capacitors.	7 – 17
4	Analyze RC circuits in terms of the appropriate differential equation and resulting exponential functions for charge, current, voltage, etc.	18 – 22

Intermittent wiper controller.



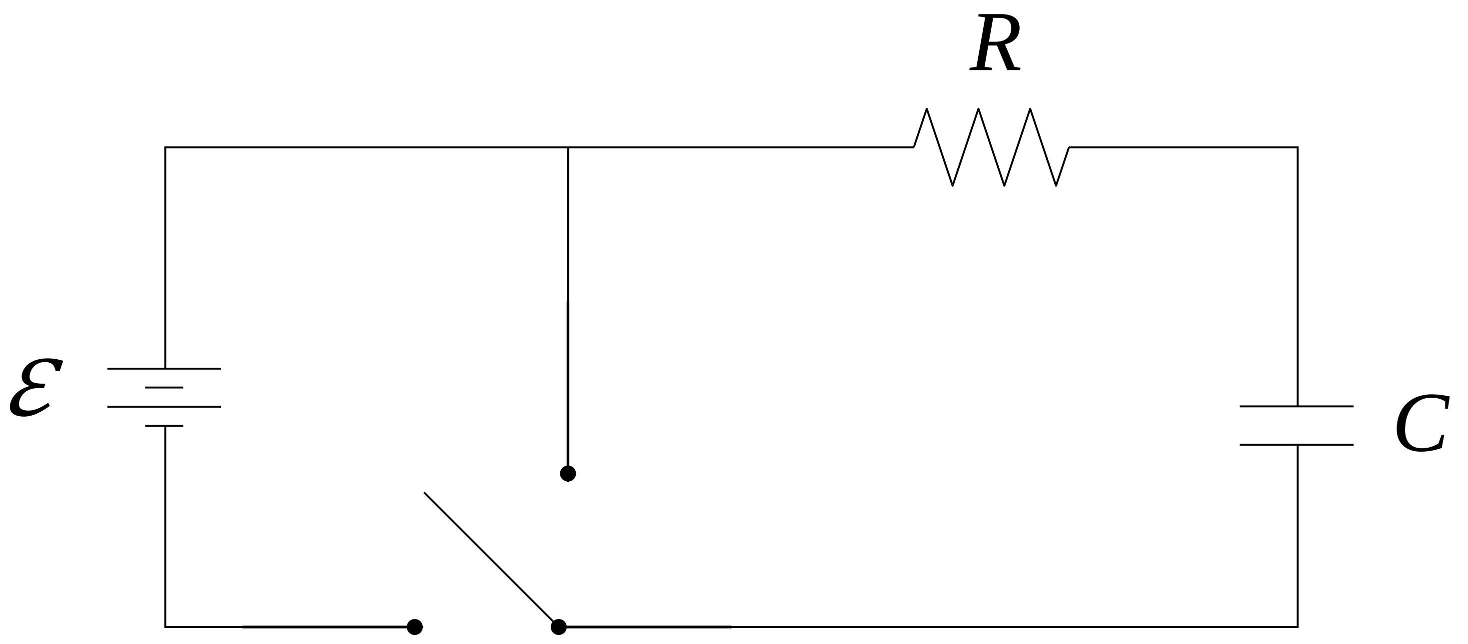
What is the role of the capacitor in this circuit?

Intermittent wiper controller.



The wipers are turned on and off at a rate controlled and synchronized by the time it takes for the capacitor to charge and discharge. The time for this to happen can be adjusted with the variable 500 kΩ resistor. This is a good example of an application of an *RC* circuit.

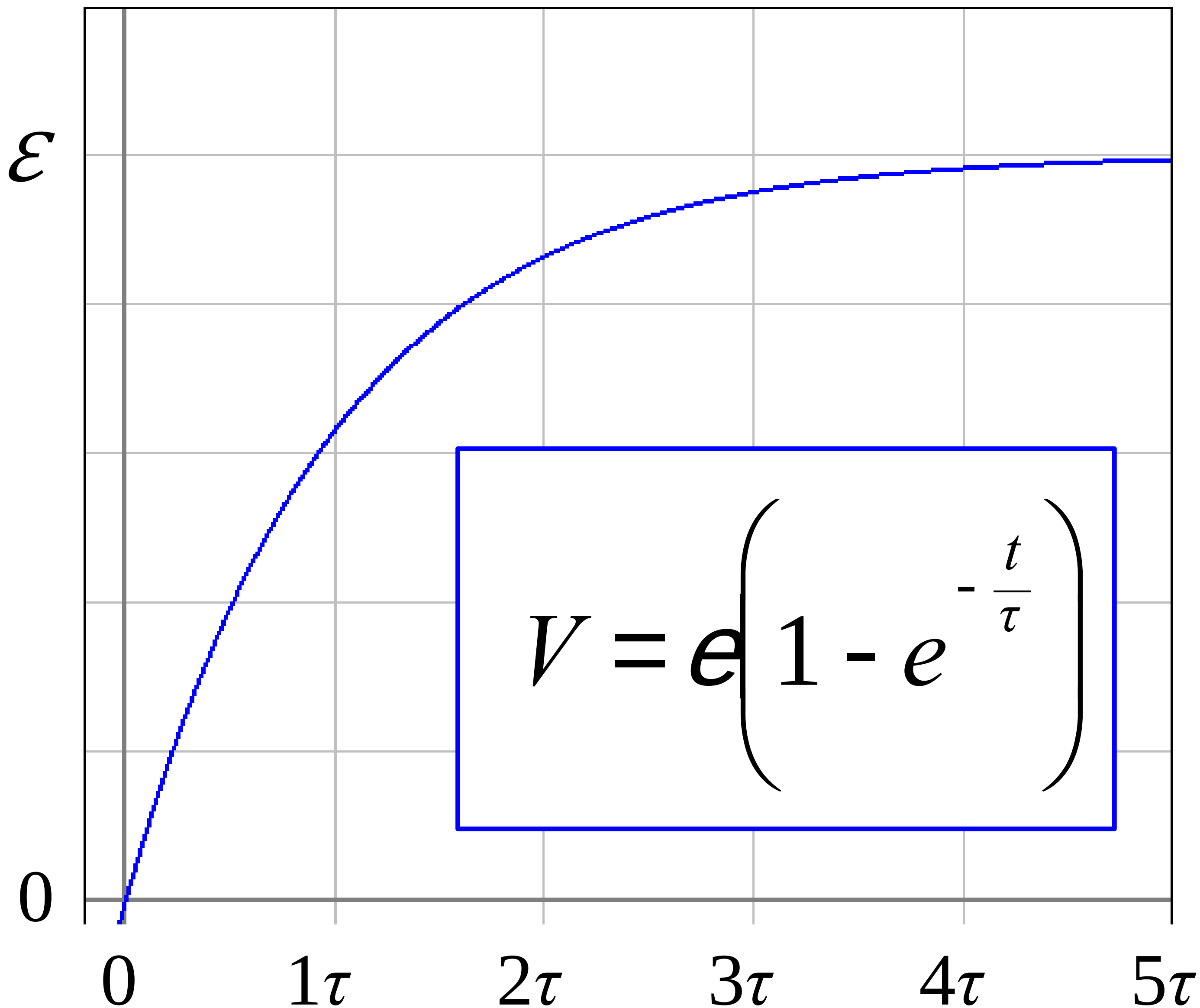
The capacitor in the circuit below is charged and discharged using the SPDT switch. Find expressions for voltage and current as functions of time.



Behavior of basic *RC* Circuit: Charging

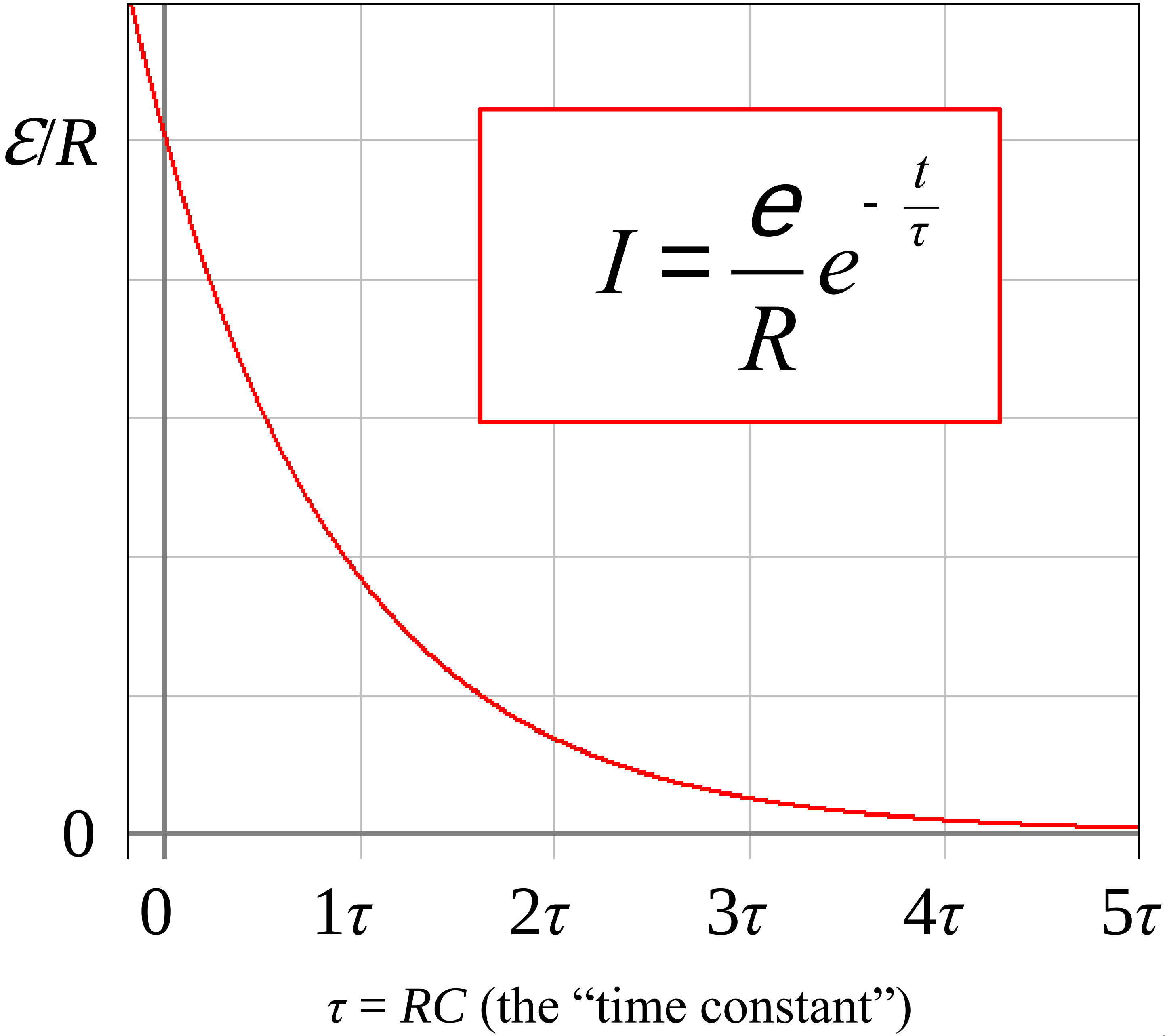
- The loop equation $\mathcal{E} - IR - q/C = 0$ becomes the differential equation $\mathcal{E} - (dq/dt)R - q/C = 0$.
- The solution for $q(t)$ is an inverse exponential function with the time constant $\tau = RC$ that "plateaus" with the capacitor reaching a full charge $\mathcal{E}/C$.
- The product of resistance and capacitance is an amount of time that governs the rate at which the capacitor will charge. A nearly full charge is achieved in about 5τ. A greater resistance and or a greater capacitance will cause a proportional increase in the time for charging to occur.
- As the charge and voltage of the capacitor are rising, the current is undergoing an exponential decrease with the same time constant $\tau = RC$.

Voltage vs. Time – Charging Capacitor



$\tau = RC$ (the "time constant")

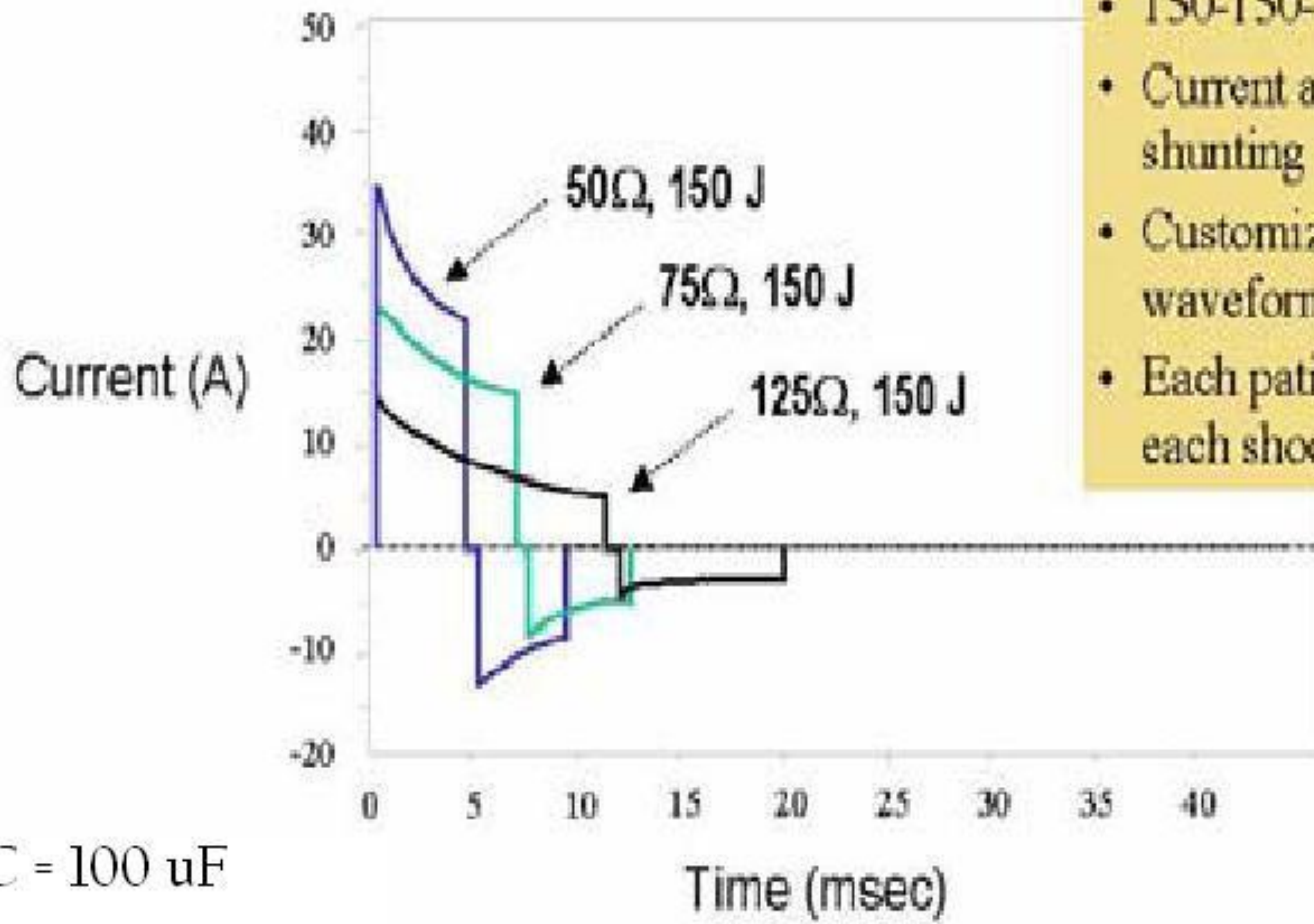
Current vs. Time – Charging Capacitor



Behavior basic *RC* Circuit: Discharging

- The loop equation $-IR - q/C = 0$ becomes the differential equation $-(dq/dt)R - q/C = 0$.
- The solution for $q(t)$ is an exponential decay function with the time constant $\tau = RC$ that drops to zero as the capacitor loses its charge.
- The product of resistance and capacitance is an amount of time that governs the rate at which the capacitor will charge. A near complete discharge occurs in about 5τ. A greater resistance and or a greater capacitance will cause a proportional increase in the time for discharging to occur.
- As the charge and voltage of the capacitor are falling, the current is also undergoing an exponential decrease with the same time constant $\tau = RC$.

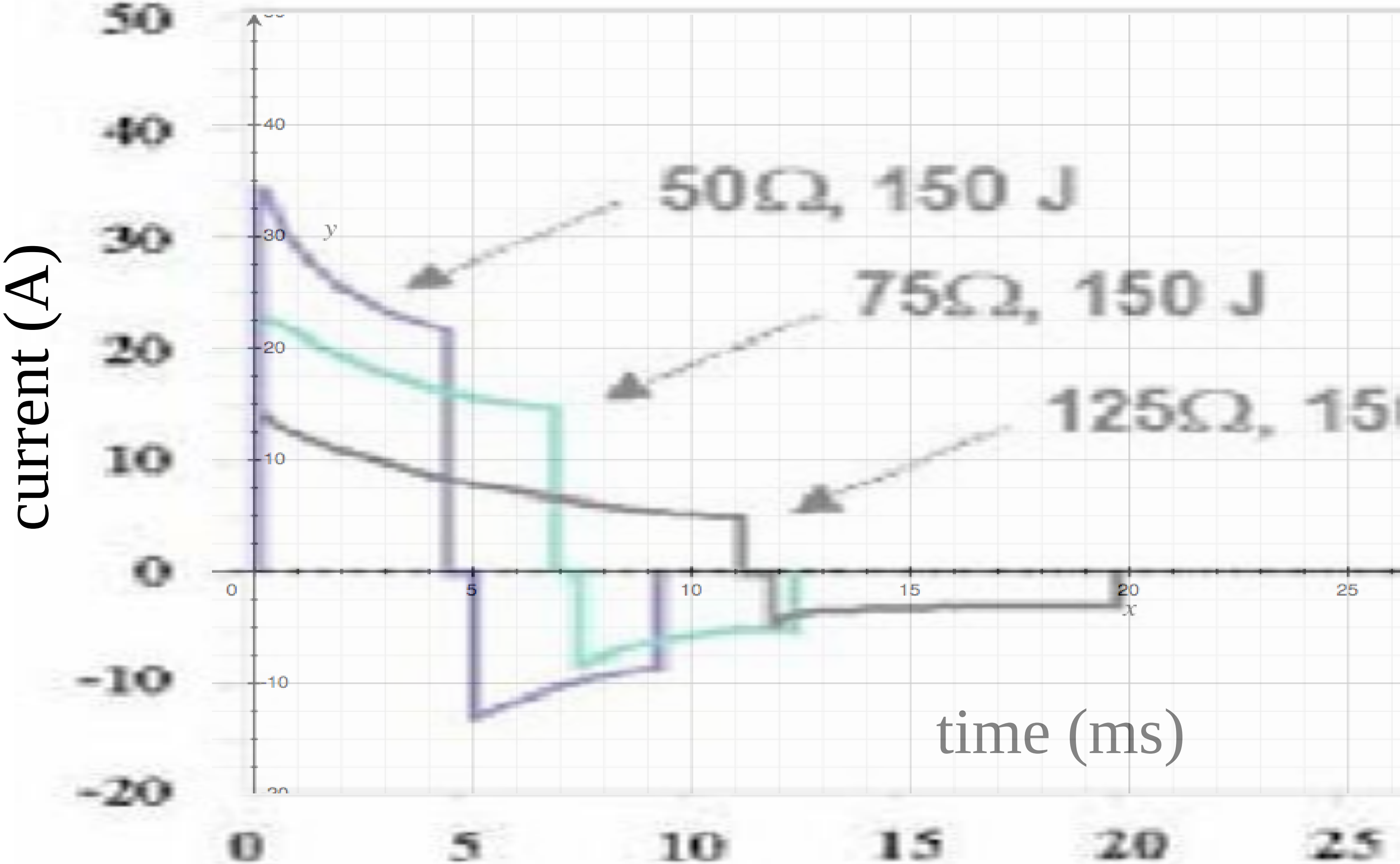
Charge or Voltage of a Capacitor		
time	charging	discharging
0	0 %	100 %
1τ	63 %	37 %
2τ	86 %	14 %
3τ	95 %	5 %
4τ	98 %	2 %
5τ	99 %	1 %



- 150-150-150 J
- Current adjusted for shunting
- Customized waveform shape
- Each patient and each shock

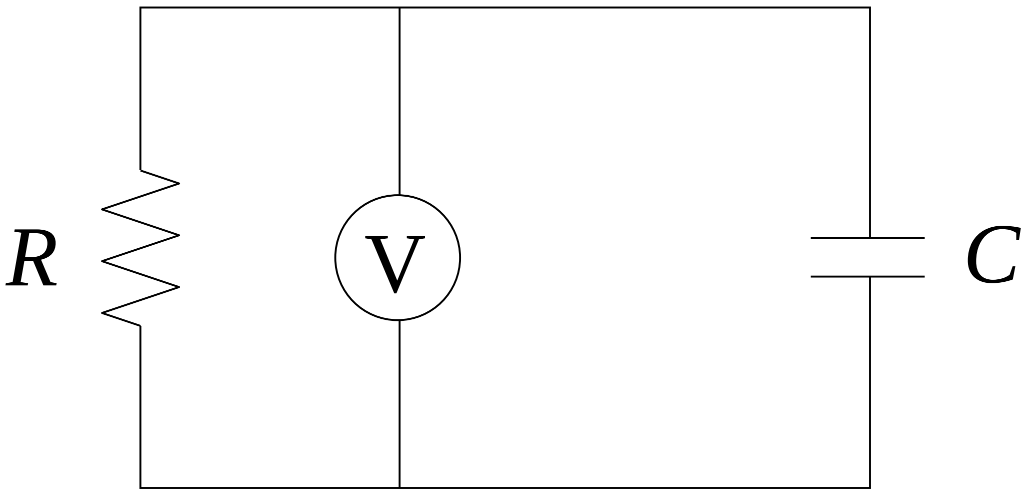
C = 100 uF

This is information for a defibrillator – essentially an *RC* circuit! The resistance is that of the patient’s chest between the two paddles that are used to deliver the shock as the capacitor discharges.



Use the information on the graph to determine the capacitance and compare to published value.

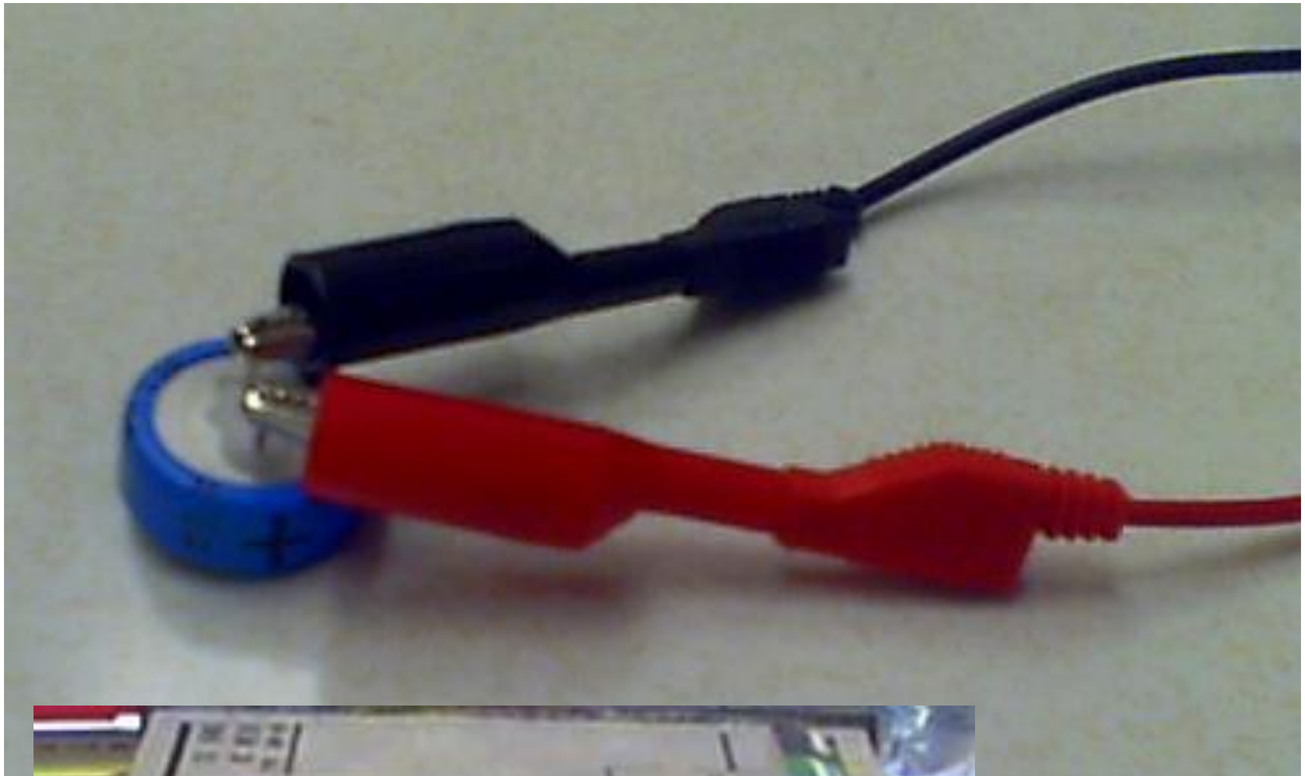
Mini-Lab: Time a Capacitor Discharge



Voltage (V)	Time (s)
4.00	0.00
3.00	
2.00	
1.00	
0.50	

Collect data using a stopwatch. Use the "Lap" button to pause and unpause the total elapsed time displayed. The capacitor has been previously charged by a 6 V battery and will begin to discharge as soon as the circuit is completed.

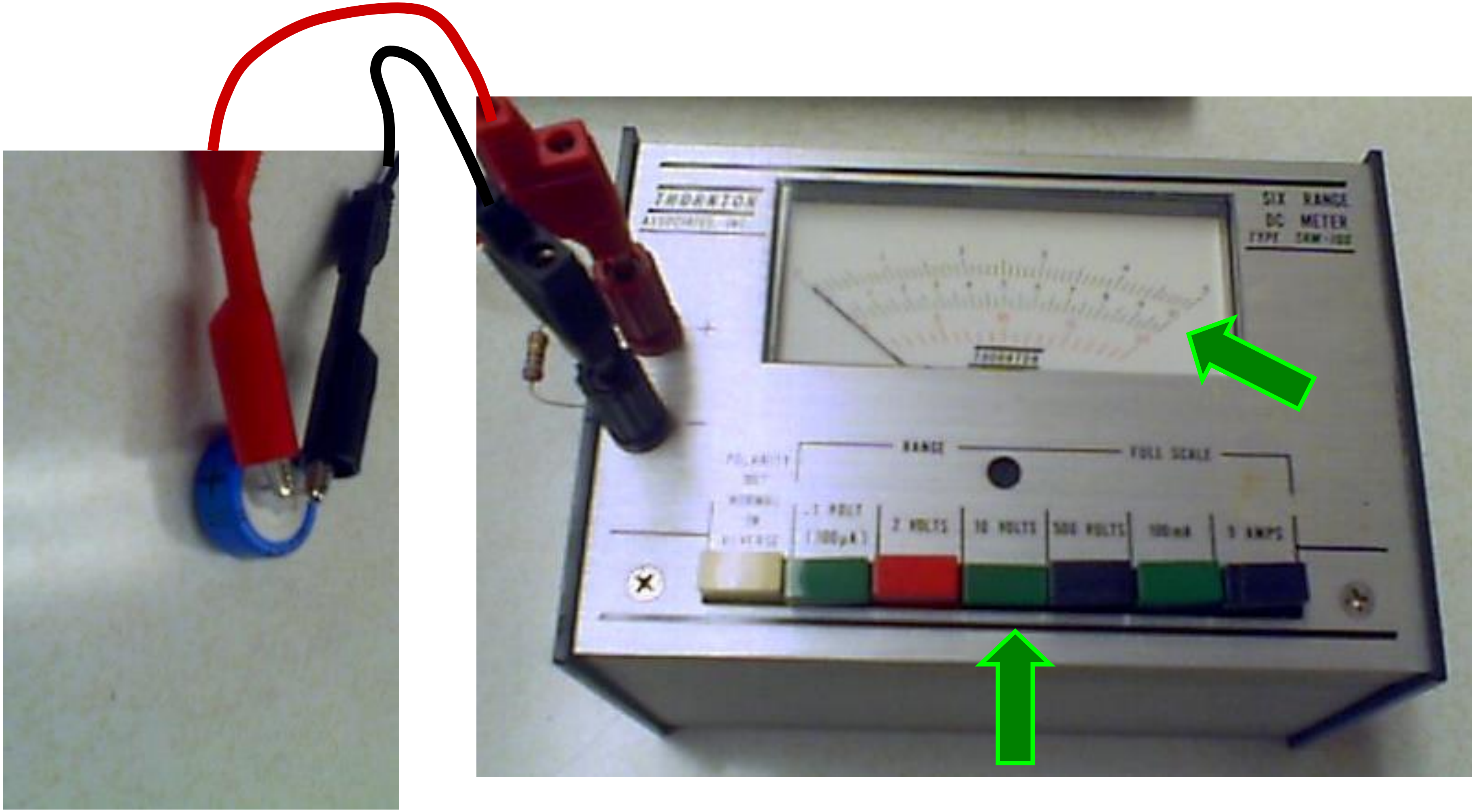
For convenience let $t = 0$ be the point in time when the capacitor reaches 4 volts as it discharges.



Notice the polarity of the capacitor!

Do **NOT** yet connect!

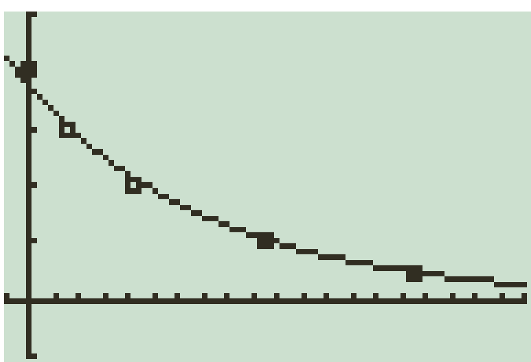
Gently connect the resistor to the terminals of the meter – tighten *very little*!



Do **<u>NOT</u>** connect until you are *ready to time* with stopwatch!

Use the 10 volts scale.

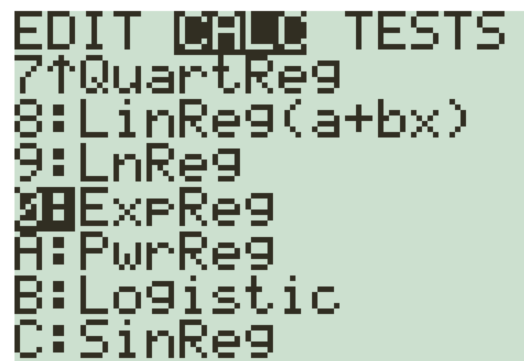
Voltage (V)	Time (s)
4.00	0.00
3.00	14.55
2.00	42.02
1.00	94.70
0.50	154.98



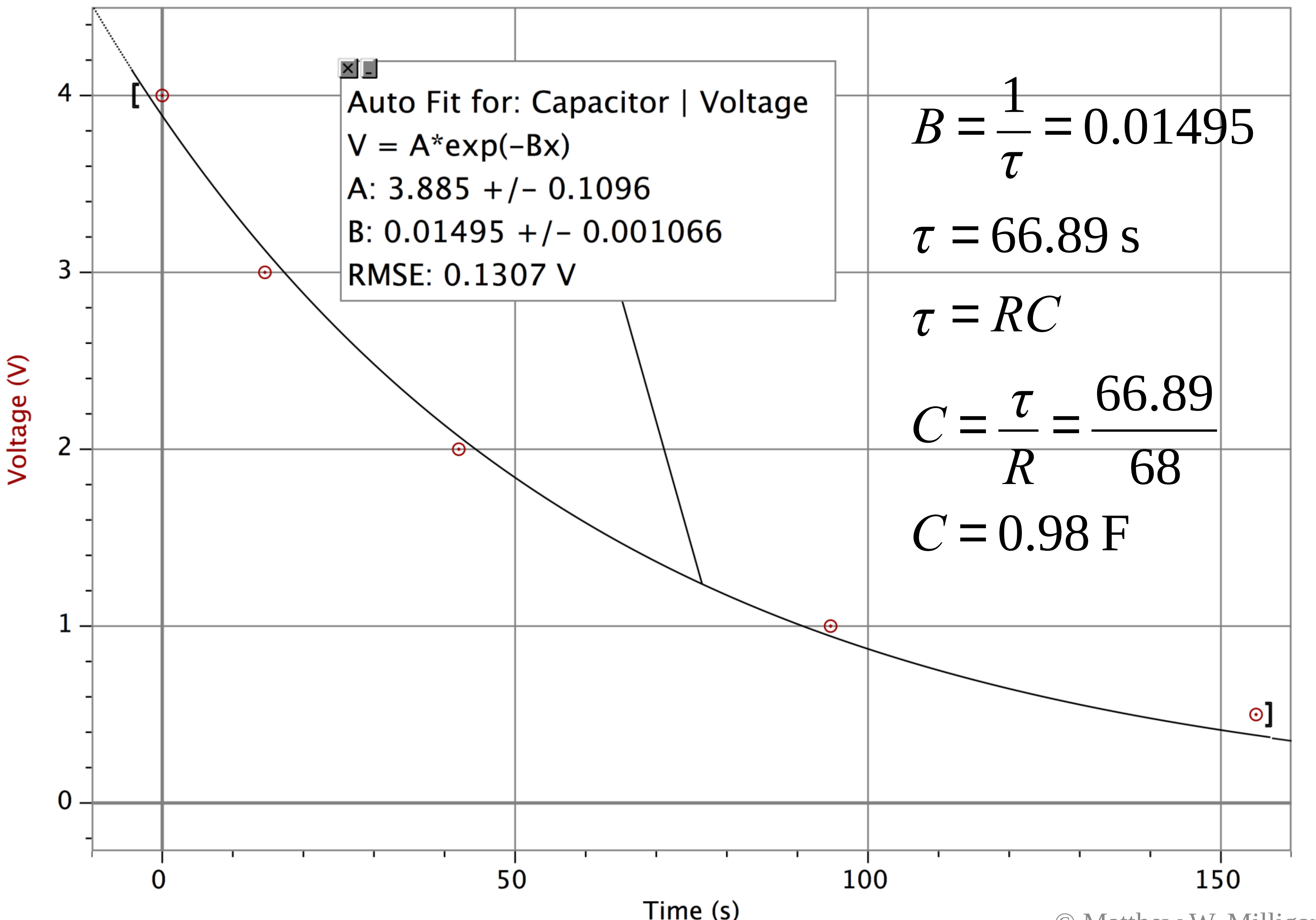
1. How much time does it take for the voltage to halve? Use this time and the value of *R* to solve for *C*.

2. Plot the data and perform an exponential regression.

3. Use the regression equation to determine the time constant and use to find *C*.

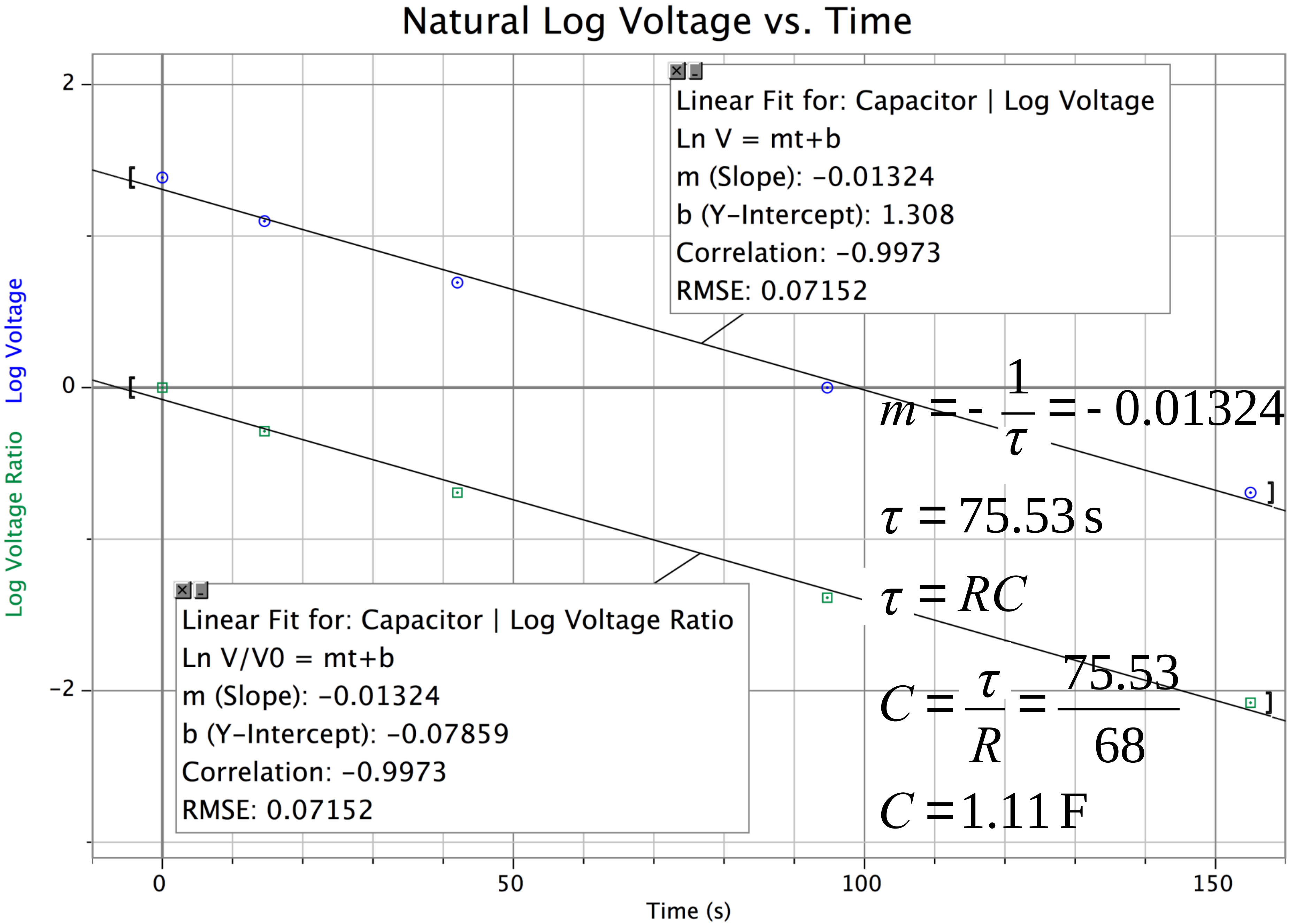


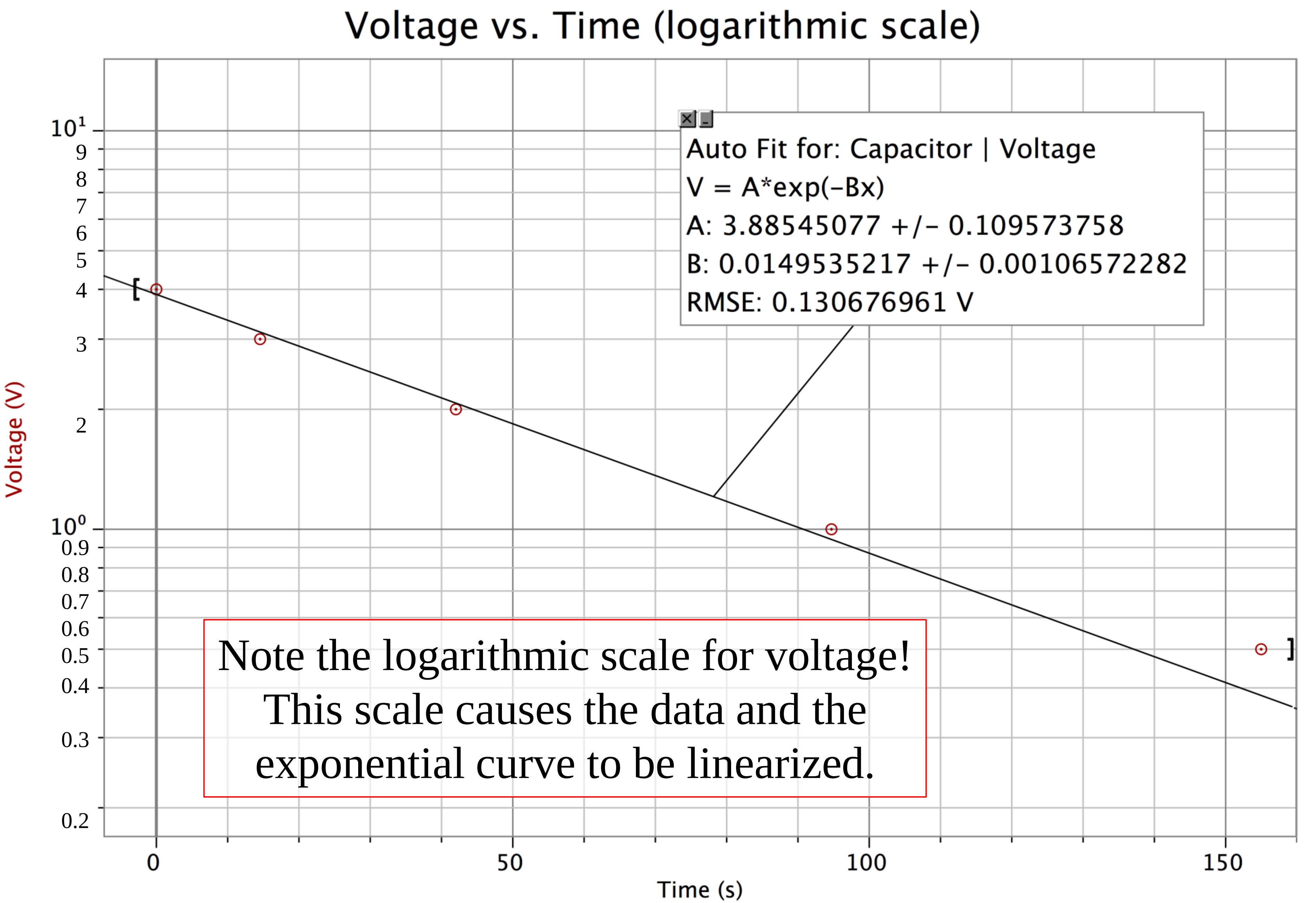
Voltage vs. Time for 1 F Capacitor



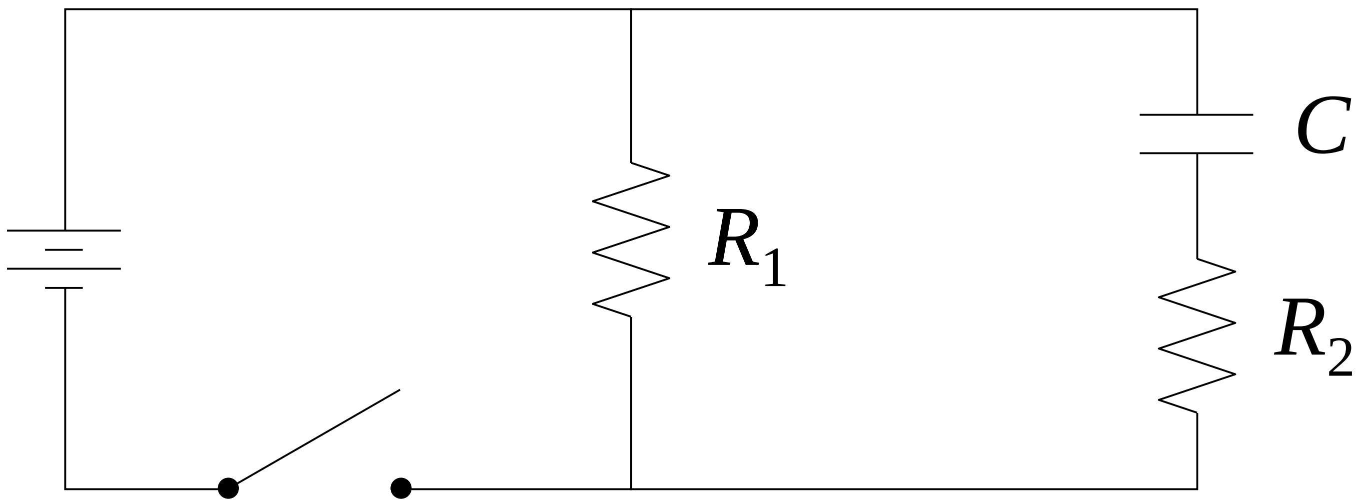
Voltage (V)	Time (s)	$\ln(V)$	$\ln(V/V_0)$
4.00	0.00	1.386	0.000
3.00	14.55	1.099	–0.288
2.00	42.02	0.693	–0.693
1.00	94.70	0.000	–1.386
0.50	154.98	–0.693	–2.079

An alternate way to analyze exponential data is to use logarithms…


Natural Log Voltage vs. Time
Linear Fit for: Capacitor | Log Voltage
Ln V = mt+b
m (Slope): −0.01324
b (Y−Intercept): 1.308
Correlation: −0.9973
RMSE: 0.07152
Linear Fit for: Capacitor | Log Voltage Ratio
Ln V/V0 = mt+b
m (Slope): −0.01324
b (Y−Intercept): −0.07859
Correlation: −0.9973
RMSE: 0.07152
$m = -\frac{1}{\tau} = -0.01324$
$\tau = 75.53\ \mathrm{s}$
$\tau = RC$
$C = \frac{\tau}{R} = \frac{75.53}{68}$
$C = 1.11\ \mathrm{F}$
Log Voltage
Log Voltage Ratio
Time (s)
2
0
−2
0
50
100
150

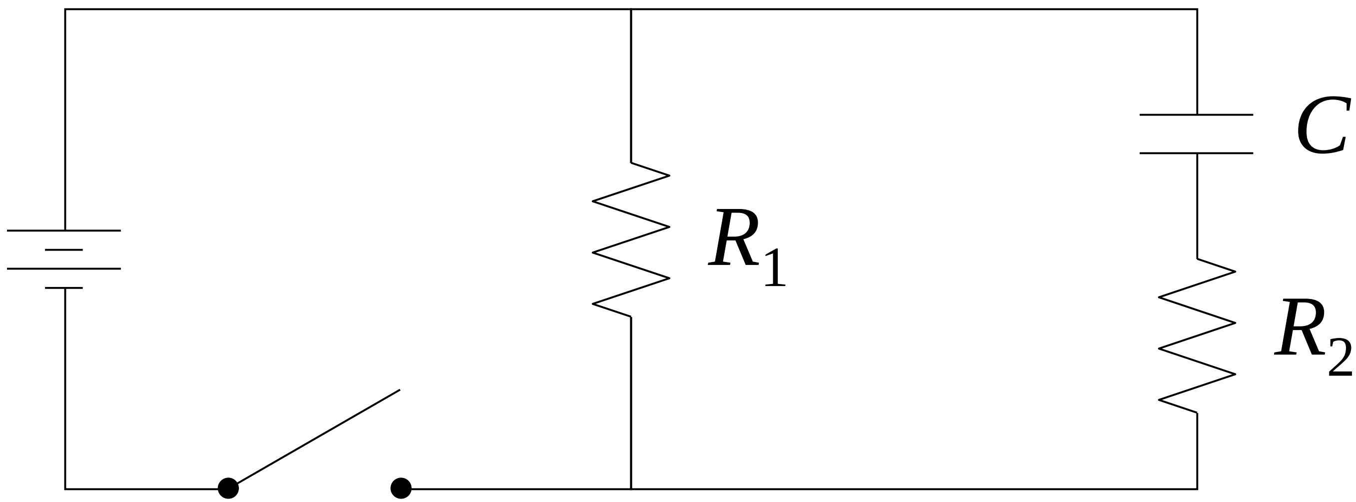

Voltage vs. Time (logarithmic scale)
Auto Fit for: Capacitor | Voltage
V = A*exp(–Bx)
A: 3.88545077 +/– 0.109573758
B: 0.0149535217 +/– 0.00106572282
RMSE: 0.130676961 V
Voltage (V)
Time (s)
10^1
10^0
0
50
100
150
Note the logarithmic scale for voltage! This scale causes the data and the exponential curve to be linearized.

Resistors R_1 = 330 Ω, R_2 = 470 Ω, capacitor C = 2.2 mF, and a 1.50 V battery are connected as shown. The switch is closed at t = 0 and then opened at t = 100.0 s. (a) Find the times at which the voltage of the capacitor is 0.75 V. (b) Make a careful sketch of current vs. time for the battery and for R_2.

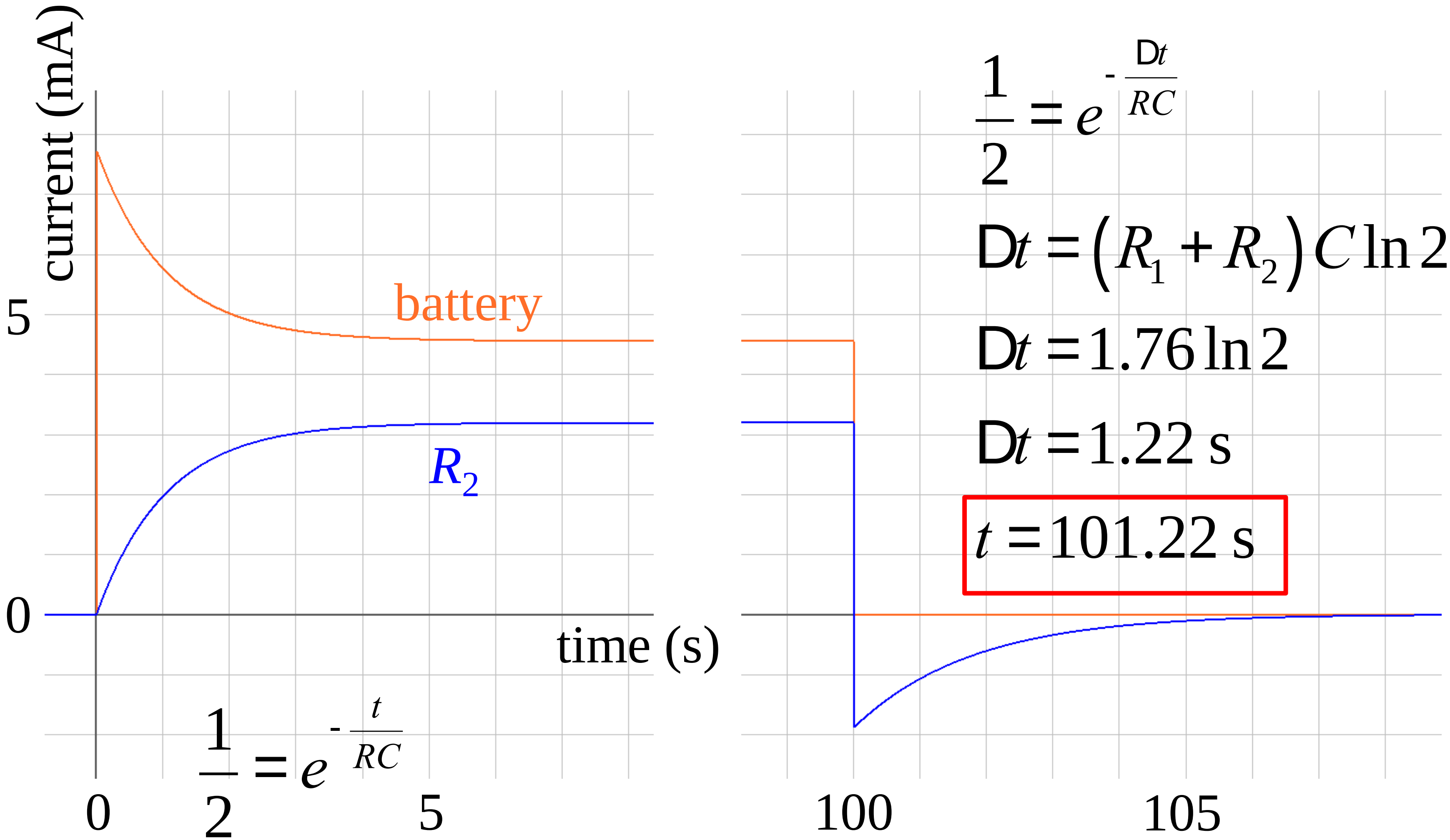


In principle any and every capacitor circuit problem can be solved by writing Kirchoff node and loop equations, substituting $I = dq/dt$, and solving the system of differential equations. However, there is often an easier way using only the notion of the time constant $\tau = RC$ and appropriate exponential functions… (go to the next page to see)

Resistors R_1 = 330 Ω, R_2 = 470 Ω, capacitor C = 2.2 mF, and a 1.50 V battery are connected as shown. The switch is closed at $t = 0$ and then opened at t = 100.0 s. (a) Find the times at which the voltage of the capacitor is 0.75 V. (b) Make a careful sketch of current vs. time for the battery and for R_2.



Many circuits are like this – solve by comparison to the "basic *RC* circuit". When the switch closes the time constant will be R_2C (R_1 does not affect the charging of the capacitor). And when the switch opens the time constant will be $(R_1 + R_2)C$. All that remains is to apply logical exponential functions using these time constants…



$$\frac{1}{2} = e^{-\frac{t}{RC}}$$

$$t = R_2 C \ln 2$$

$$t = 1.034 \ln 2$$

$$t = 0.717\ \text{s}$$

$$\frac{1}{2} = e^{-\frac{\Delta t}{RC}}$$

$$\Delta t = \left(R_1 + R_2\right) C \ln 2$$

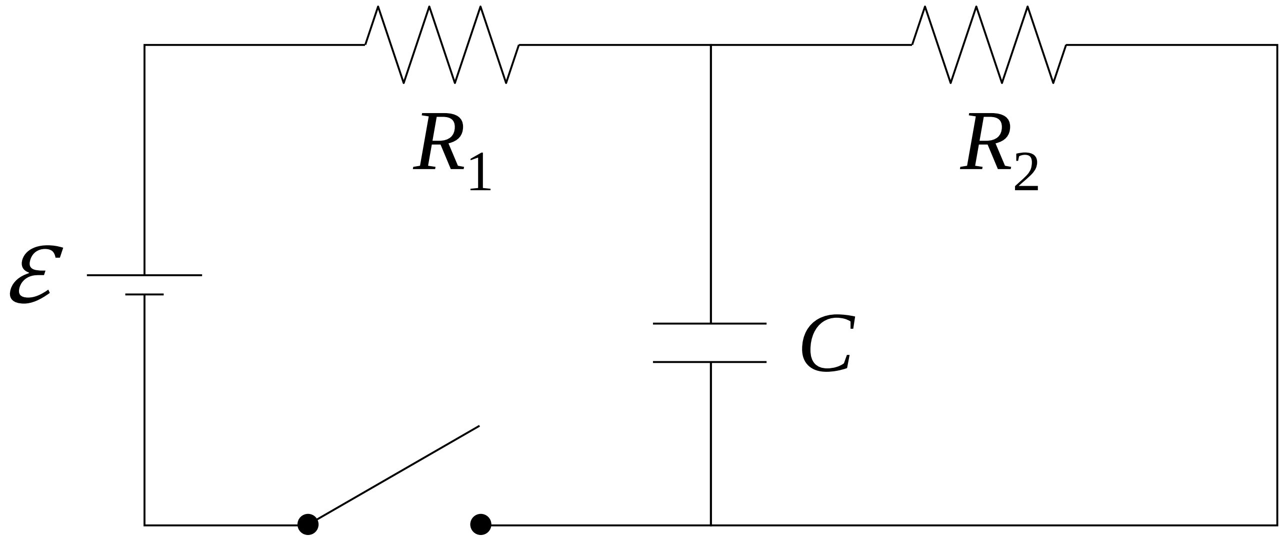
$$\Delta t = 1.76 \ln 2$$

$$\Delta t = 1.22\ \text{s}$$

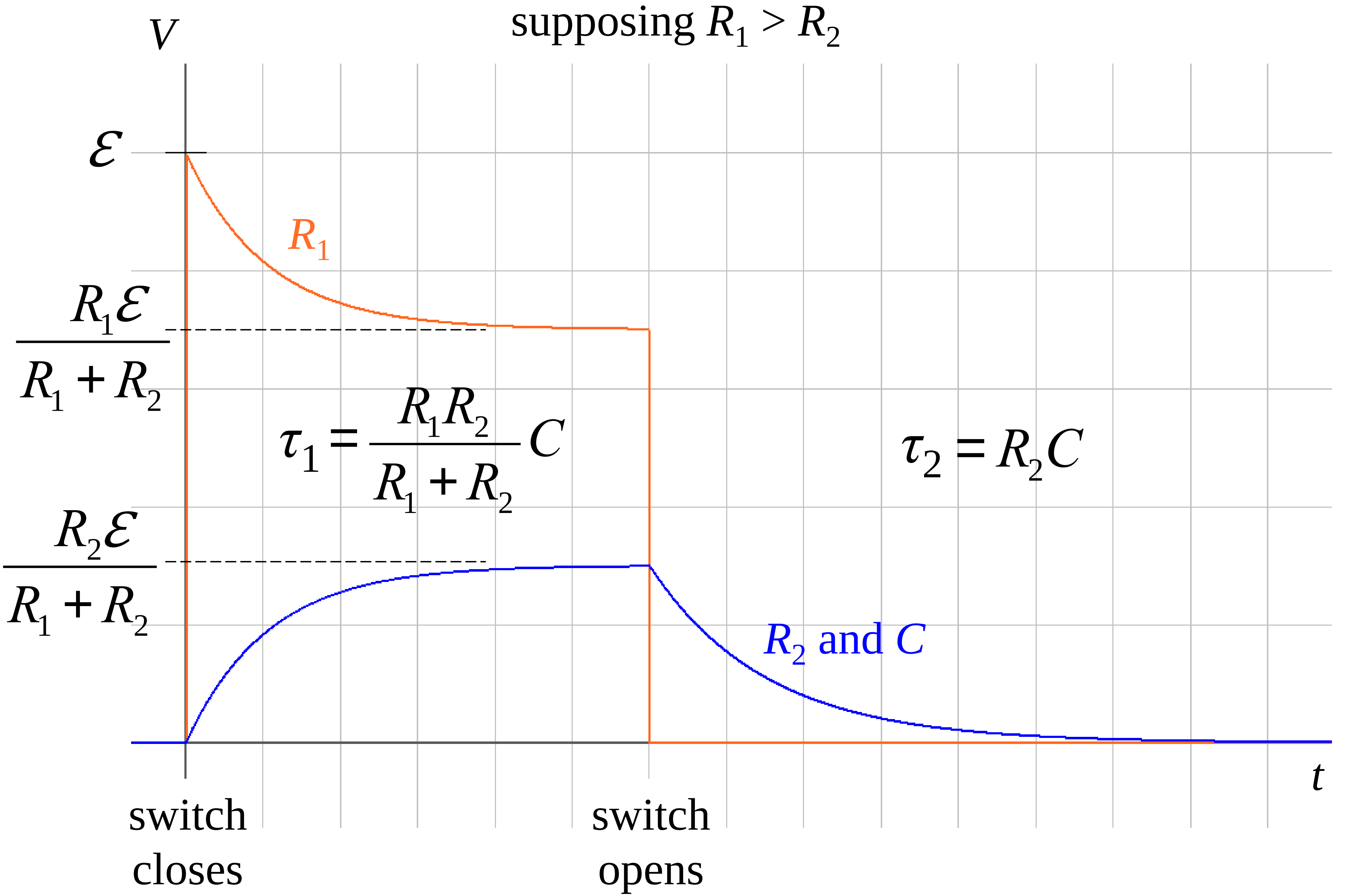
$$t = 101.22\ \text{s}$$

The two times when the voltage of the capacitor is 0.75 volts.

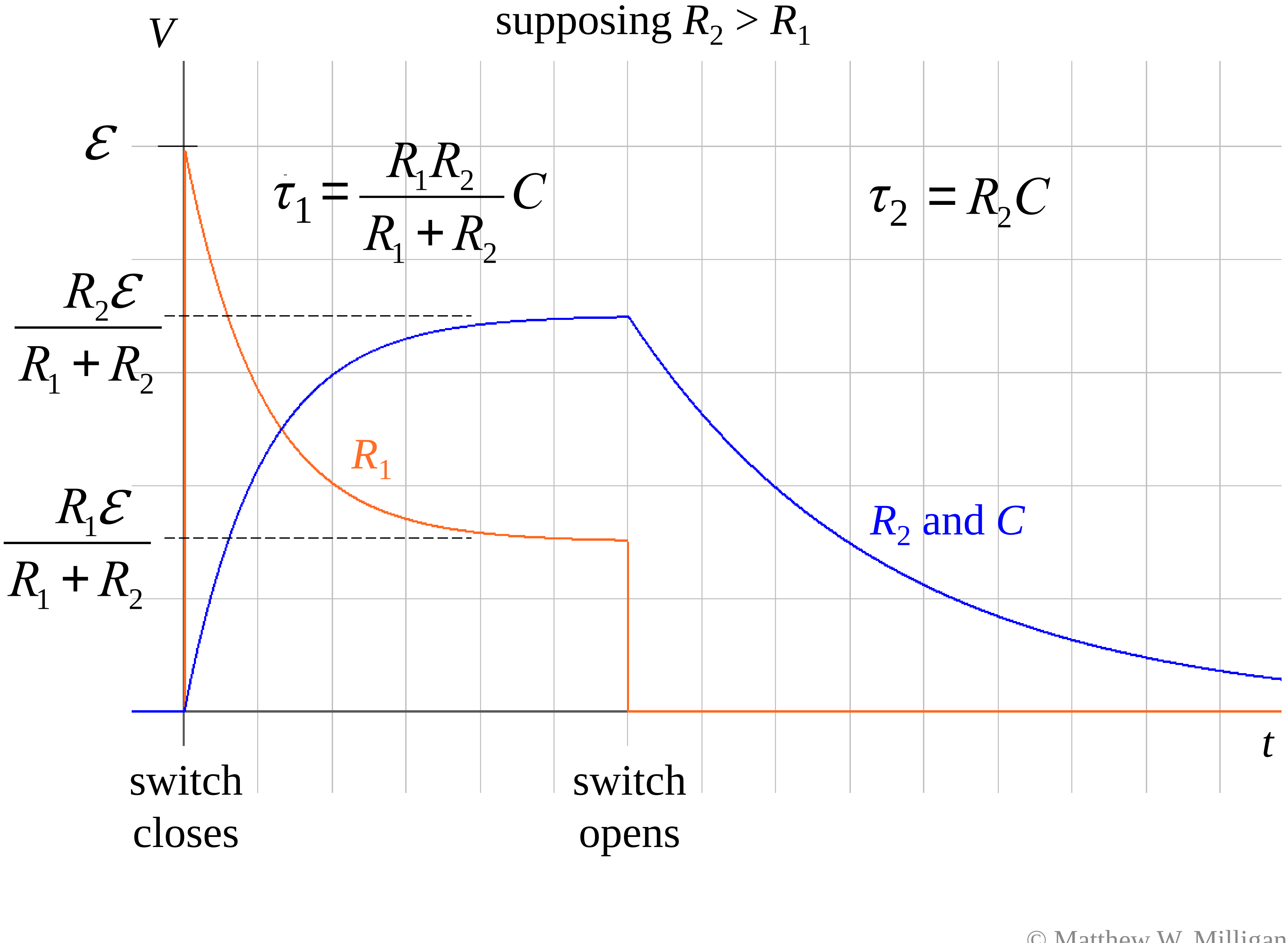
The capacitor in the circuit below is charged and discharged by closing and opening the switch. (a) Sketch voltage vs. time graphs for each resistor and the capacitor. Find the time constant for (b) discharging and (c) charging.



When the switch opens the time constant is R_2C as discharging occurs. However when the switch closes it is not obvious what happens! Sometimes it may be necessary to apply node and loop differential equations! It can be shown for this particular circuit that the time constant will be given by $\tau = R_1R_2C/(R_1 + R_2)$ as charging occurs.



supposing $R_1 > R_2$
V
$\mathcal{E}$
R_1
$\frac{R_1\mathcal{E}}{R_1+R_2}$
$\tau_1 = \frac{R_1R_2}{R_1+R_2}C$
$\tau_2 = R_2C$
$\frac{R_2\mathcal{E}}{R_1+R_2}$
R_2 and C
t
switch closes
switch opens



supposing $R_2 > R_1$
V
$\mathcal{E}$
$\tau_1 = \frac{R_1 R_2}{R_1 + R_2} C$
$\tau_2 = R_2 C$
$\frac{R_2 \mathcal{E}}{R_1 + R_2}$
$\frac{R_1 \mathcal{E}}{R_1 + R_2}$
R_1
R_2 and C
t
switch closes
switch opens