Electrostatics

I. Charge and Force
- Coulomb's Law
- conservation of charge
- quantization of charge

II. Electric Fields
- field strength
- motion of charged particles
- field lines
- **distributions of charge**

	The student will be able to:	HW:
1	Relate electrical phenomena to the motion and position of the fundamental charge found on electrons and protons and explain process of charging in terms of conduction or induction and relate to properties of conductors and insulators.	1 – 6 ✓
2	State and apply Coulomb's Law to solve problems relating force and separation of discrete charges and recognize the Coulomb as the SI unit of charge and *e* as the elementary quantum of charge.	7 – 15 ✓
3	Define and apply the concept of electric field in terms of force acting on a charge within the field and solve related problems	16 – 19 ✓
4	Solve problems involving the motion of a charged particle in a uniform electric field.	20 – 24 ✓
5	Determine the electric field produced by a discrete charge or a set of such charges and solve related problems.	25 – 29 ✓
6	State and apply the rules for sketching electric fields.	30 ✓
7	Determine the electric field produced by a continuous charge distribution and solve related problems.	31 – 42

Electric Fields of Charge Distributions

Electric field of a distribution of point charges:

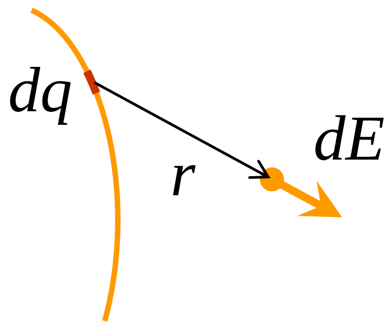
$$\vec{E} = \sum_i \frac{1}{4\pi\varepsilon_0} \frac{q_i}{r_i^2} \hat{r}$$

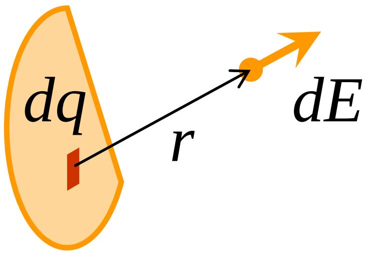
Electric field of a continuous distribution of charge:

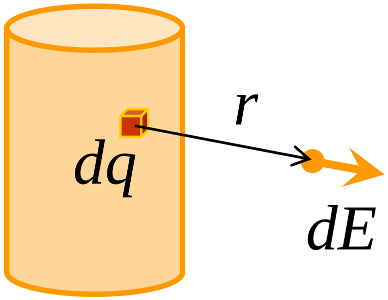
$$\vec{E} = \int \frac{dq}{4\pi\varepsilon_0 r^2} \hat{r}$$

This is an infinite sum of field vectors contributed by infinitesimal charges!

Electric Fields of Charge Distributions







$dq = \lambda d\ell$

λ = charge per length

$dq = \sigma dA$

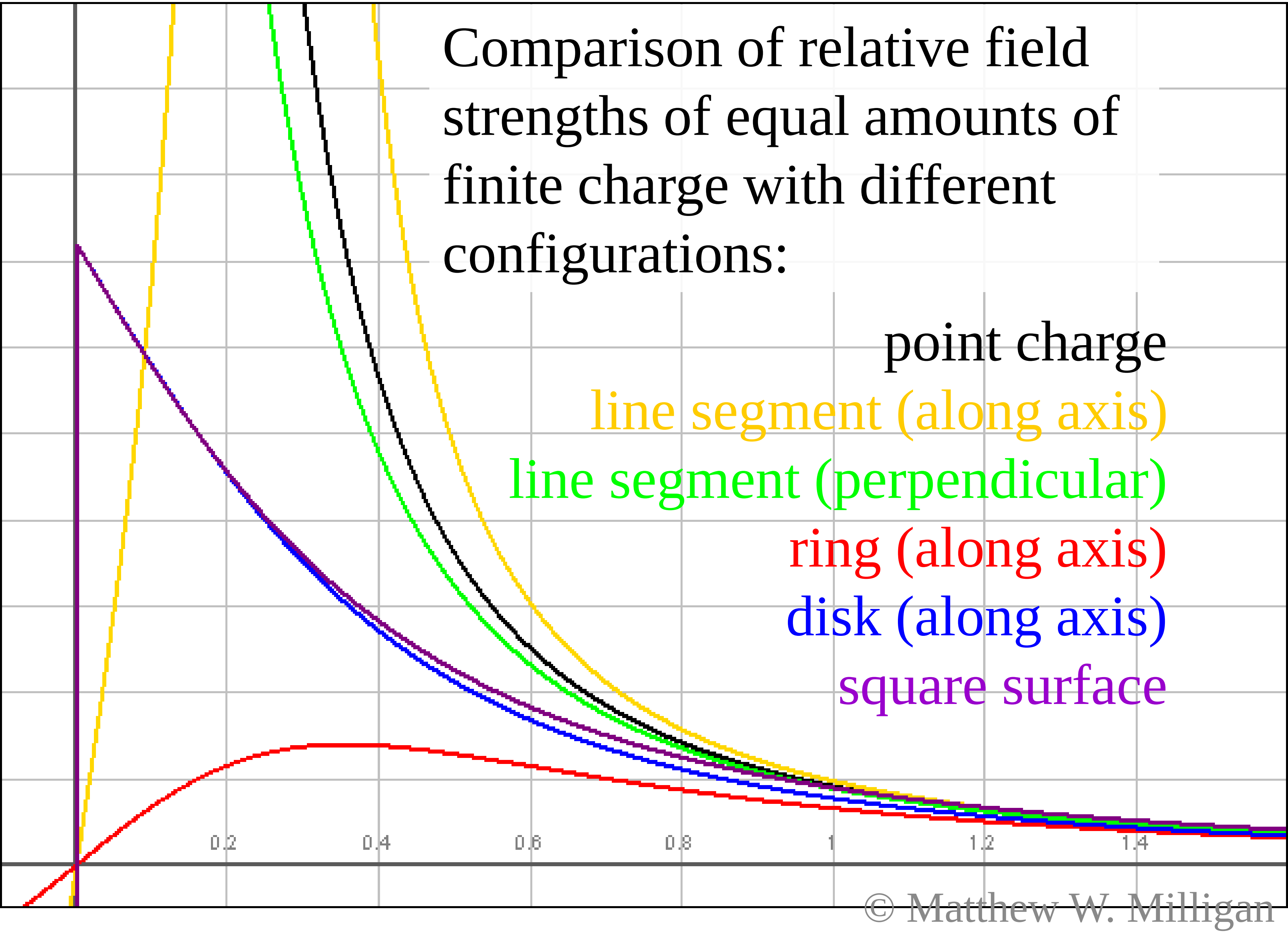
σ = charge per area

$dq = \rho dV$

ρ = charge per volume

$$\vec{E} = \int \frac{dq}{4\pi\varepsilon_0 r^2}\hat{r}$$

Usually the infinitesimal *dq* is expressed in terms of charge density, depending on choice of the variable of integration.



Comparison of relative field strengths of equal amounts of finite charge with different configurations:
point charge
line segment (along axis)
line segment (perpendicular)
ring (along axis)
disk (along axis)
square surface
0.2
0.4
0.6
0.8
1
1.2
1.4
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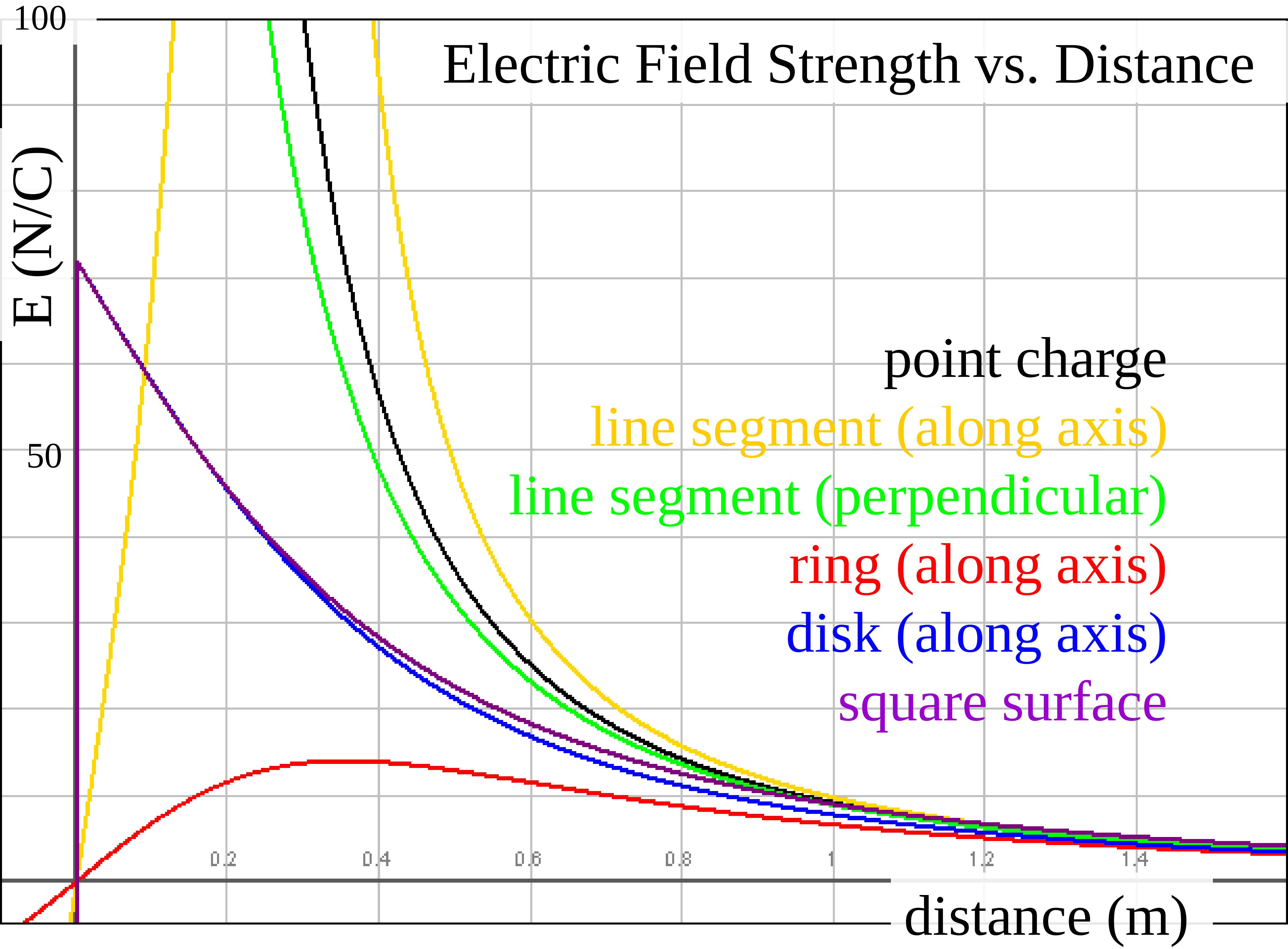
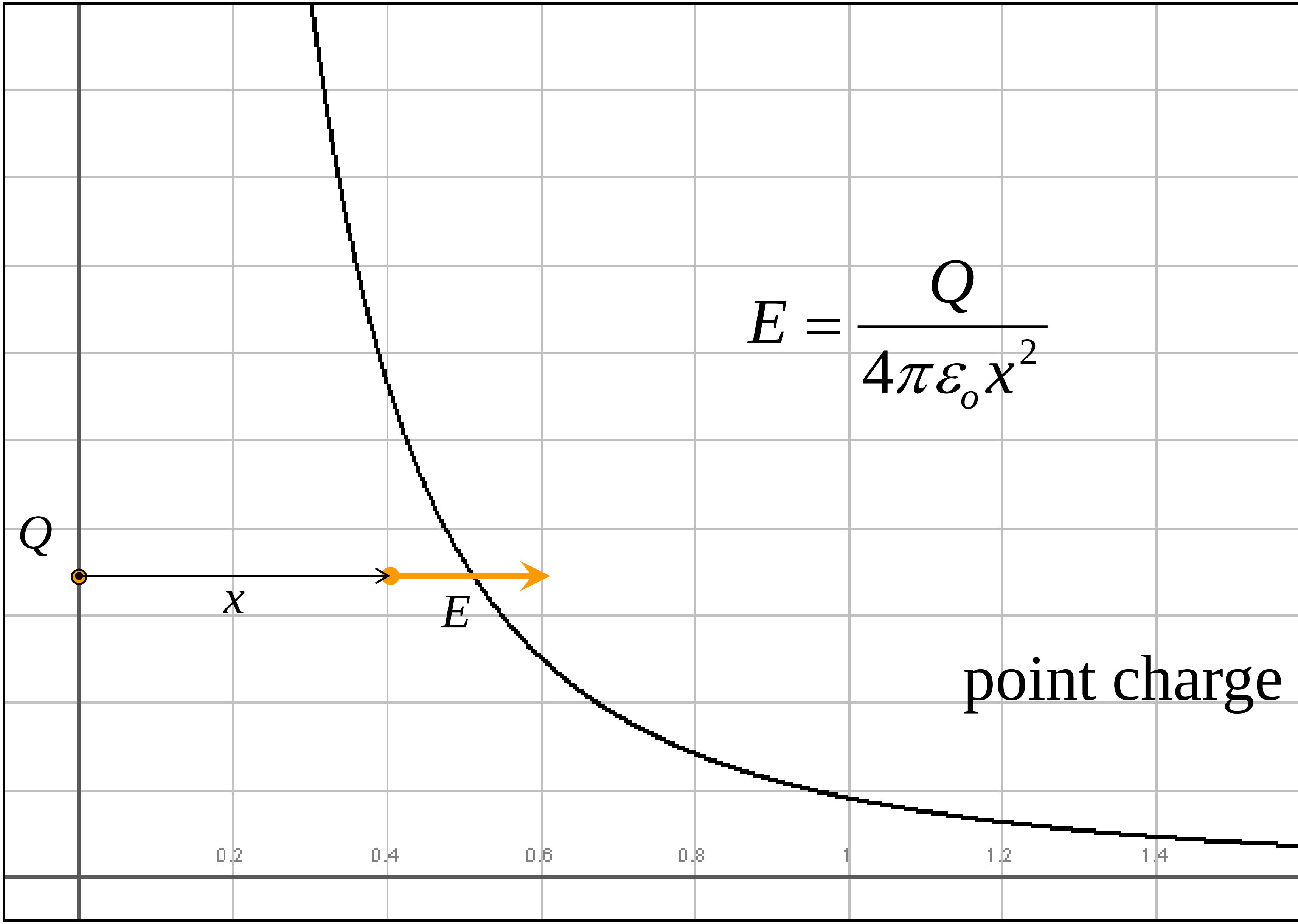

Electric Field Strength vs. Distance
100
50
E (N/C)
point charge
line segment (along axis)
line segment (perpendicular)
ring (along axis)
disk (along axis)
square surface
0.2
0.4
0.6
0.8
1
1.2
1.4
distance (m)

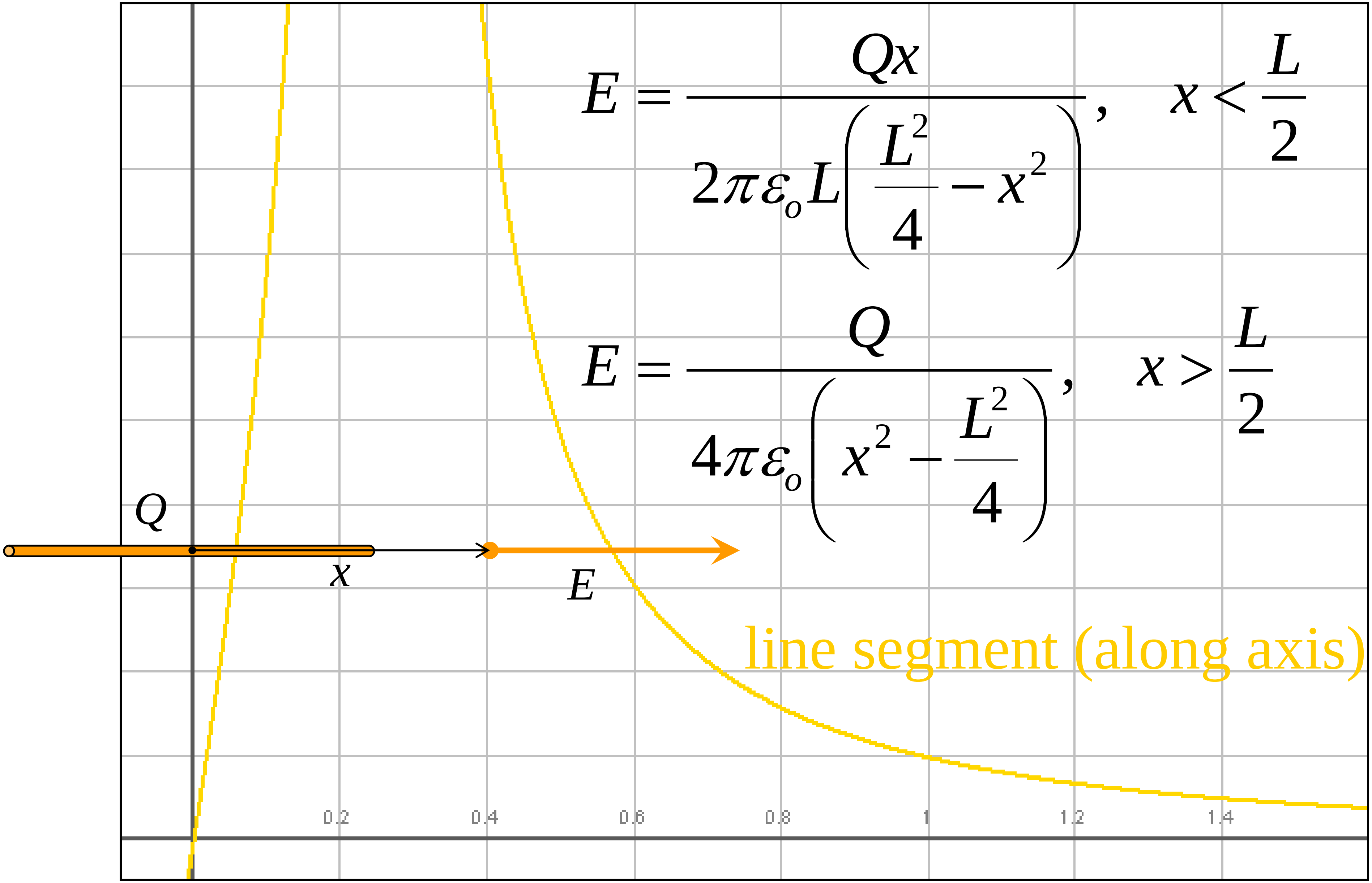
Finite charges, each $q = 1$ nC:

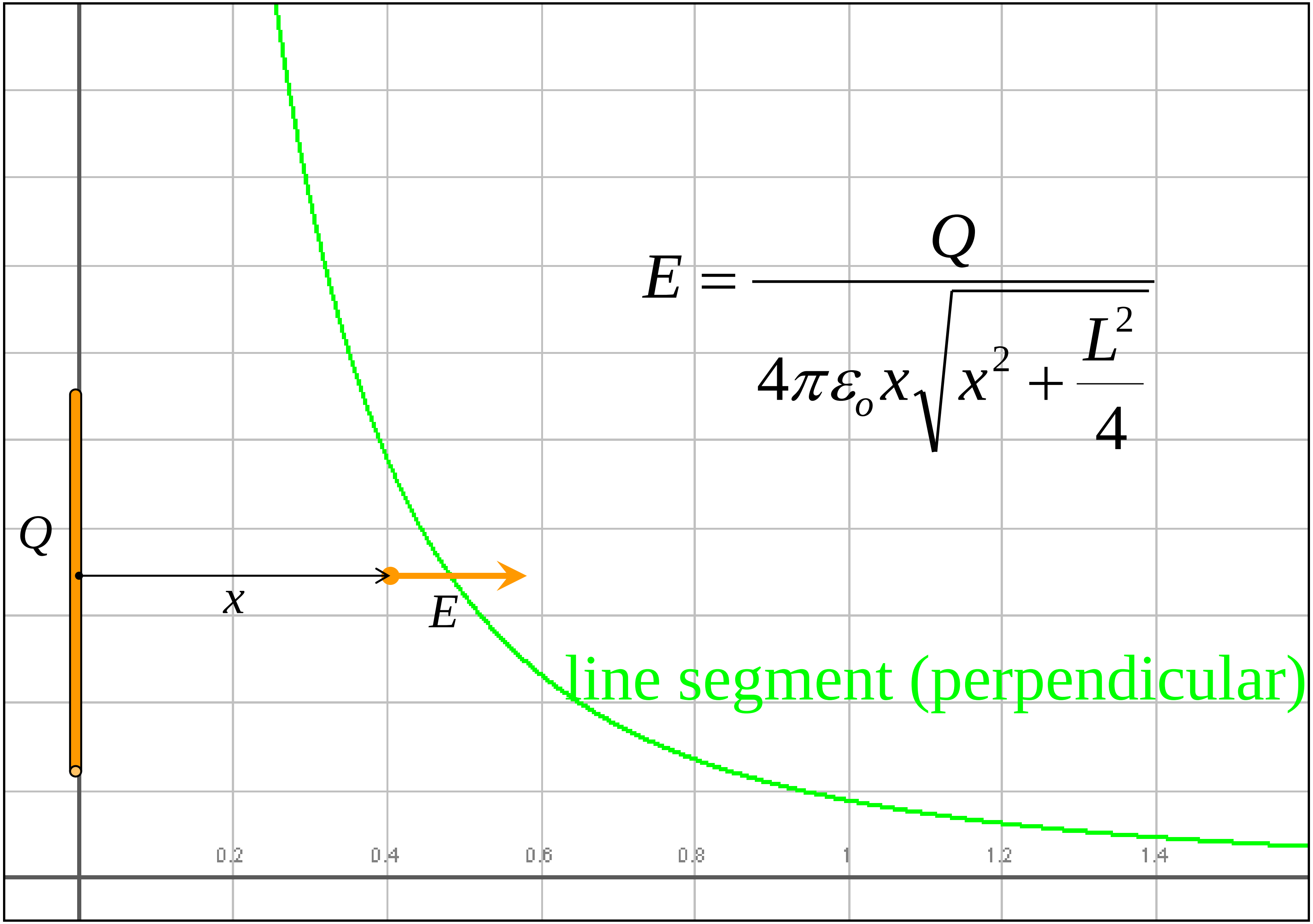
1. point charge
2. finite line of charge $L = 0.5$ m, along axis, distance x from center
3. finite line of charge $L = 0.5$ m, along perpendicular bisector, distance x from center
4. finite ring of charge $R = 0.5$ m, along axis, distance x from center
5. finite disk of charge $R = 0.5$ m, along axis, distance x from center
6. finite square area of charge, sides $L = 0.886$ m so that charge per area is same as disk, normal distance x from center


$E = \frac{Q}{4\pi\varepsilon_o x^2}$
Q
x
E
point charge
0.2
0.4
0.6
0.8
1
1.2
1.4

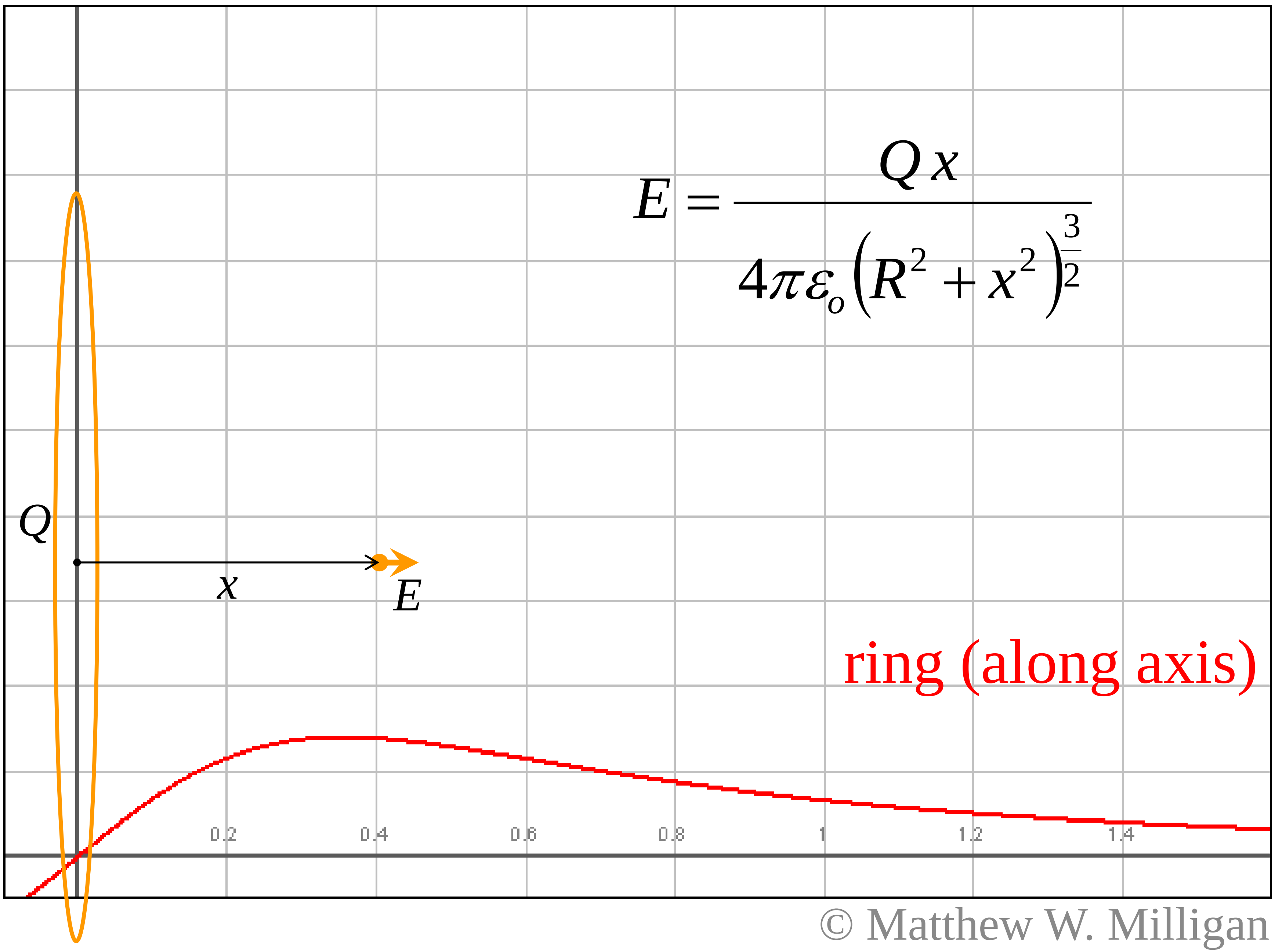
$$E = \frac{Qx}{2\pi\varepsilon_o L\left(\frac{L^2}{4} - x^2\right)}, \quad x < \frac{L}{2}$$

$$E = \frac{Q}{4\pi\varepsilon_o \left(x^2 - \frac{L^2}{4}\right)}, \quad x > \frac{L}{2}$$

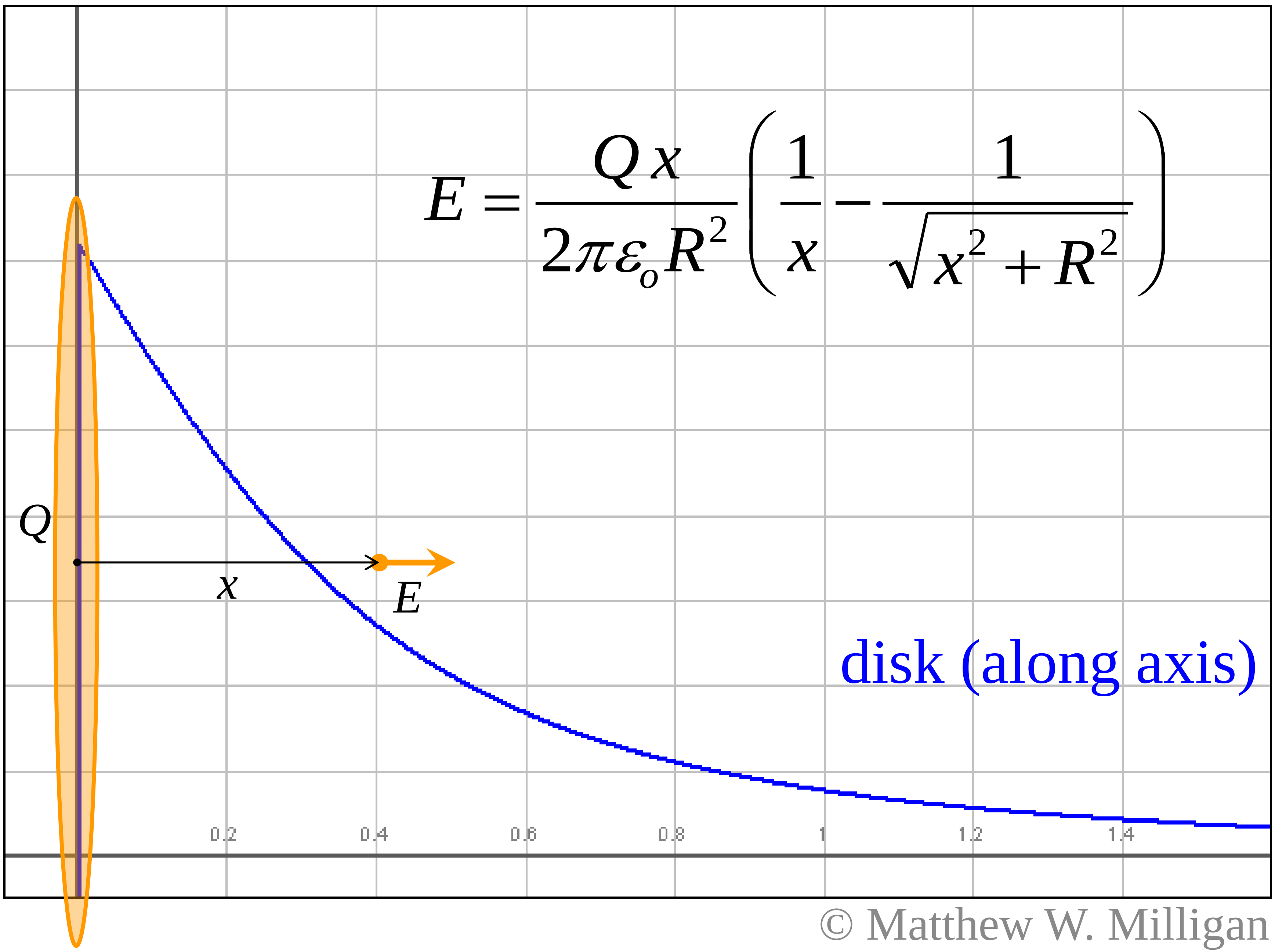




$$E = \frac{Q}{4\pi\varepsilon_o x\sqrt{x^2 + \frac{L^2}{4}}}$$
Q
x
E
line segment (perpendicular)
0.2
0.4
0.6
0.8
1
1.2
1.4



$E = \frac{Q\,x}{4\pi\varepsilon_o\left(R^2 + x^2\right)^{\frac{3}{2}}}$
Q
x
E
ring (along axis)
0.2
0.4
0.6
0.8
1
1.2
1.4



$E = \frac{Qx}{2\pi\varepsilon_o R^2}\left(\frac{1}{x} - \frac{1}{\sqrt{x^2 + R^2}}\right)$
Q
x
E
disk (along axis)
0.2
0.4
0.6
0.8
1
1.2
1.4

$$E = \frac{Q}{2\pi\varepsilon_o L^2}\left(\tan^{-1}\left(\frac{2L - \sqrt{2\left(L^2 + 2x^2\right)}}{2x}\right) - \tan^{-1}\left(\frac{2L + \sqrt{2\left(L^2 + 2x^2\right)}}{2x}\right) + 2\tan^{-1}\left(\frac{\sqrt{2\left(L^2 + 2x^2\right)}}{2x}\right)\right)$$