Electrical Circuits

I. Current and Resistance
 - Ohm's Law

II. Resistivity
 - resistance factors
 - conductivity
 - drift velocity & current density

III. Electrical Power

IV. Circuits
 - series and parallel
 - Kirchoff's Laws

V. Batteries and Meters
 - internal resistance

	The student will be able to:	HW:
1	Define current and the ampere, conventional positive current flow, and solve related problems, including those with both positive and negative charge carriers. ✓	1 – 3
2	Define resistance and the ohm, state Ohm's Law, and solve related problems involving ohmic and/or nonohmic devices. ✓	4 – 7
3	Describe and explain factors influencing resistance, state mathematical relation between resistance, length, area, and resistivity or conductivity, and solve related problems. ✓	8 – 13
4	Solve problems involving current density, electric field, resistivity, drift velocity and/or use these concepts to explain the nature of resistance. ✓	14 – 16
5	Solve problems involving electric power. ✓	17 – 21
6	Determine effective resistance of a network of series and/or parallel resistors. ✓	22 – 24
7	Solve for voltage, current, resistance, and power in DC circuits using Kirchoff's Laws and/or effective resistance.	25 – 38
8	Model a cell or battery as an ideal voltage source or as an EMF with internal resistance and a certain terminal voltage, and solve related problems.	39 – 43
9	Understand operation and properties of voltmeters and ammeters and illustrate proper connections thereof, and solve related problems.	44 – 46

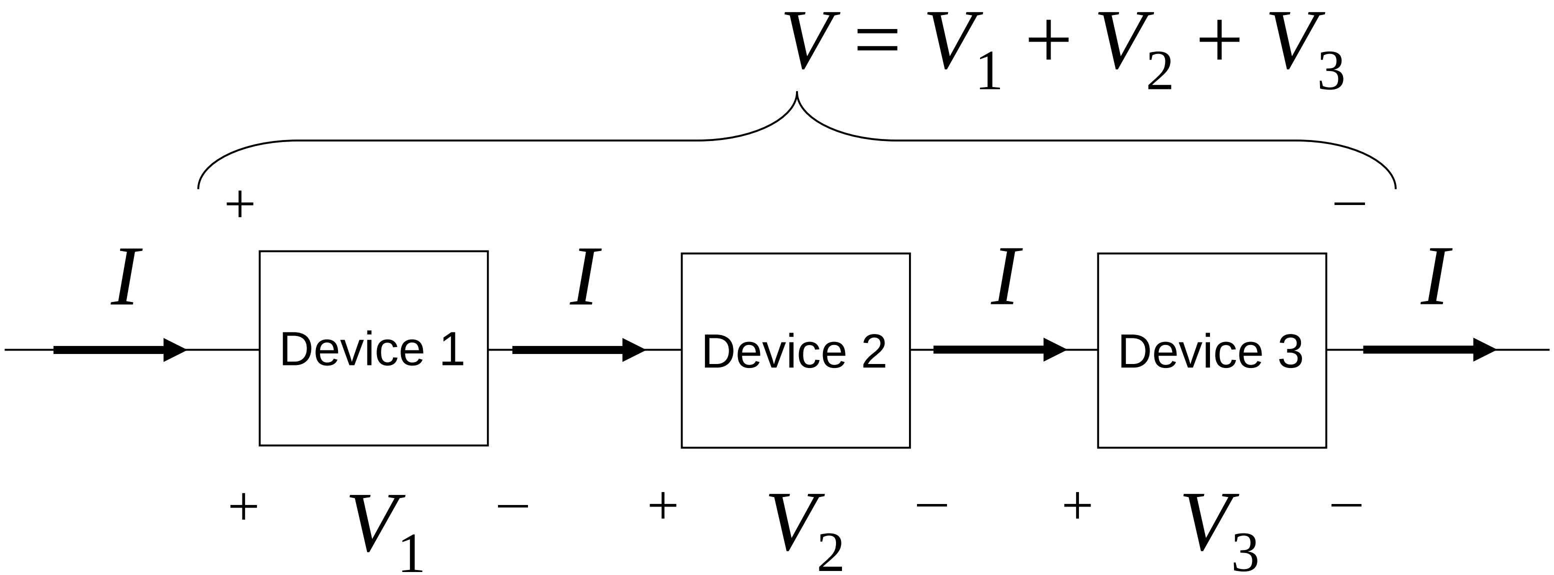
There are primarily two ways to connect electrical devices: **series** and **parallel**.

The behavior of a circuit is dependent upon the type of connections within it. There are important applications of both series and parallel circuits.

In many ways the two types of circuit connections are "opposites" or "inverses" of one another.

Series Connections

Current passes through each device in a particular order. There is only one path for current to follow.

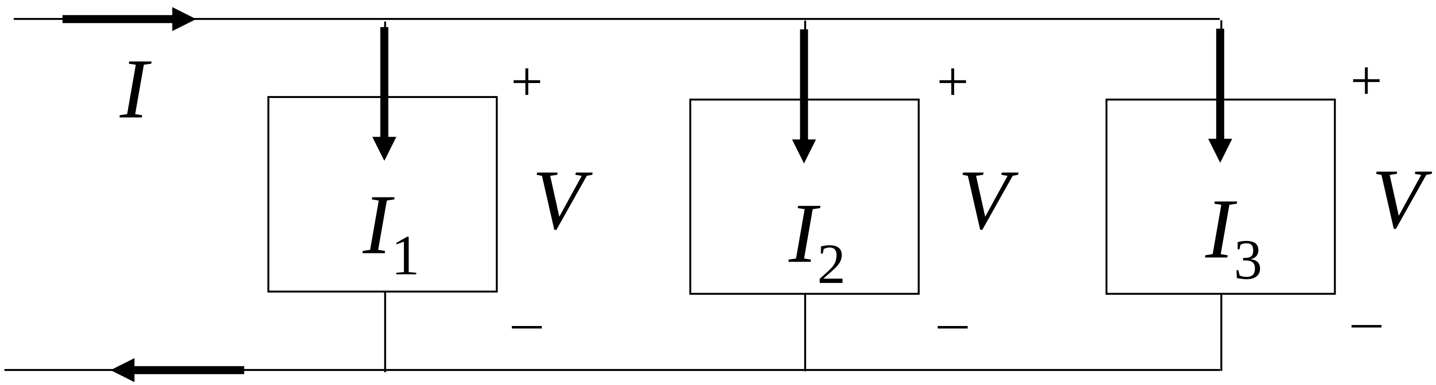


The **current** is the same through each device.

The **voltage** across any set of devices is equal to the sum of the individual voltages.

Parallel Connections

Current passes through the devices simultaneously. There are multiple paths for current to follow.

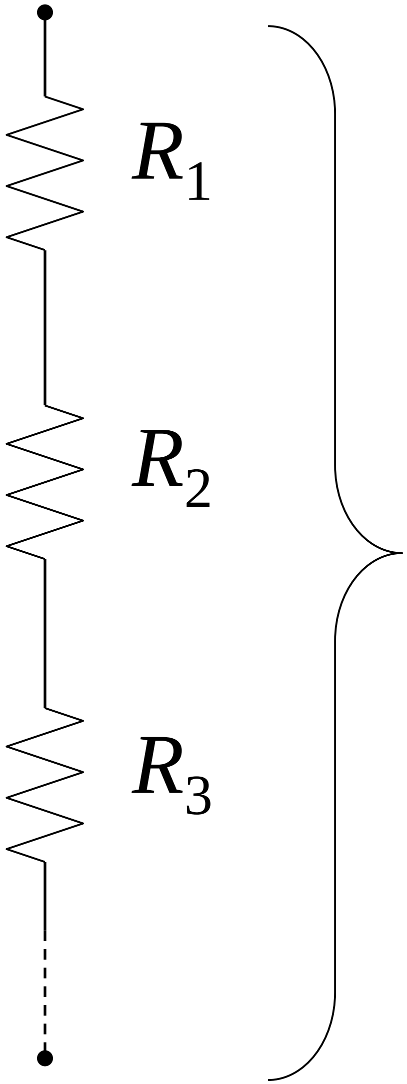


$$I = I_1 + I_2 + I_3$$

The **voltage** is the same across each device.

The **current** through any set of devices is equal to the sum of the individual currents.

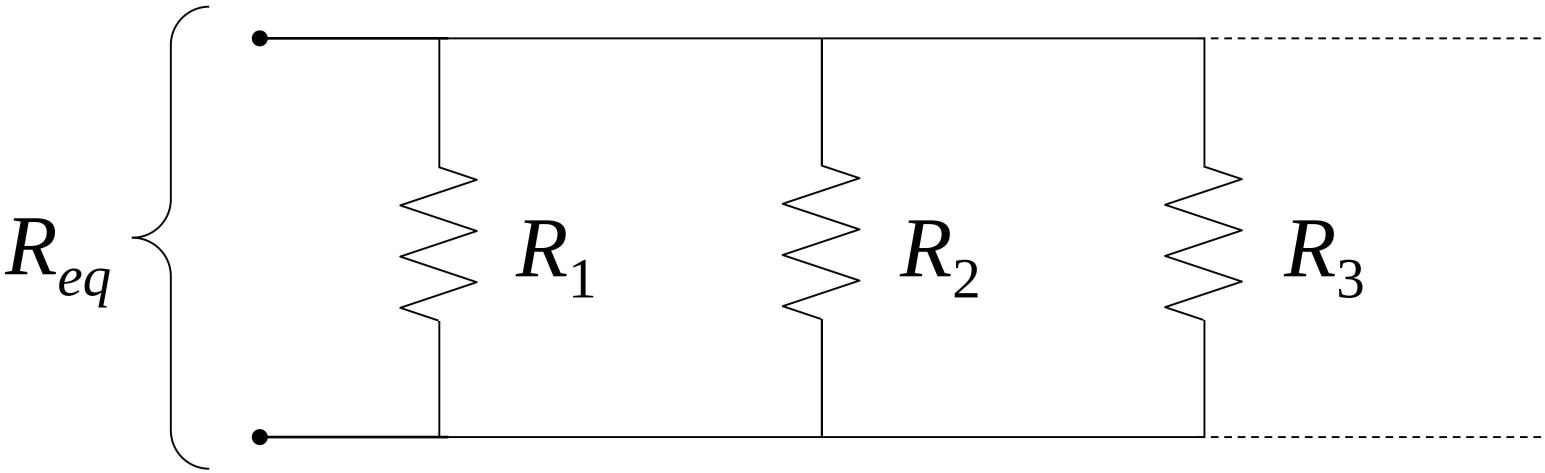
Equivalent Resistance of Series Resistors



$$R_{eq} = R_1 + R_2 + R_3 + \ldots$$

The equivalent resistance is *greater* than that of any single resistor in the set.

Equivalent Resistance of Parallel Resistors



$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$$

The equivalent resistance is *less* than that of any single resistor in the set.

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- **Kirchoff's Laws**

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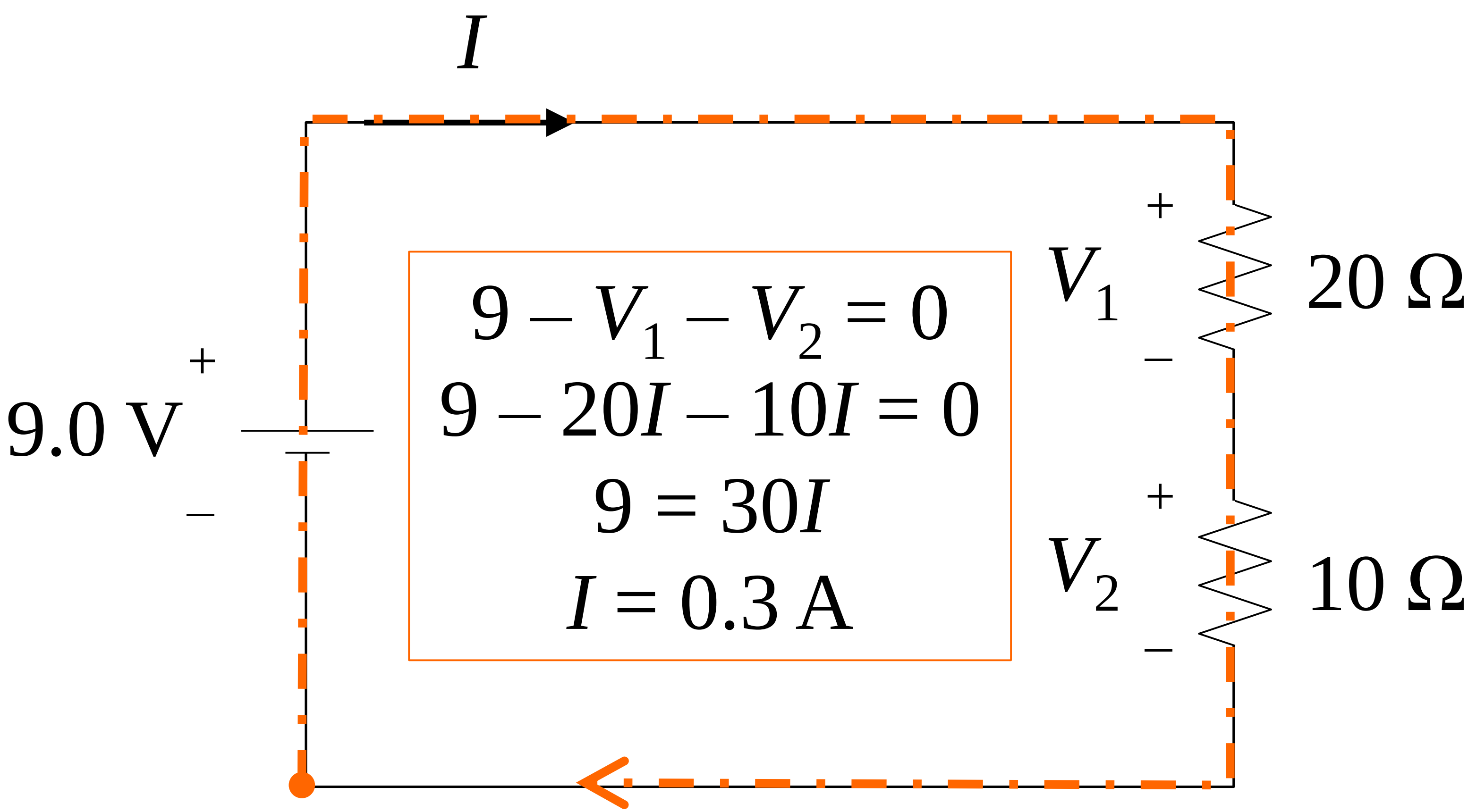
Kirchhoff’s Laws

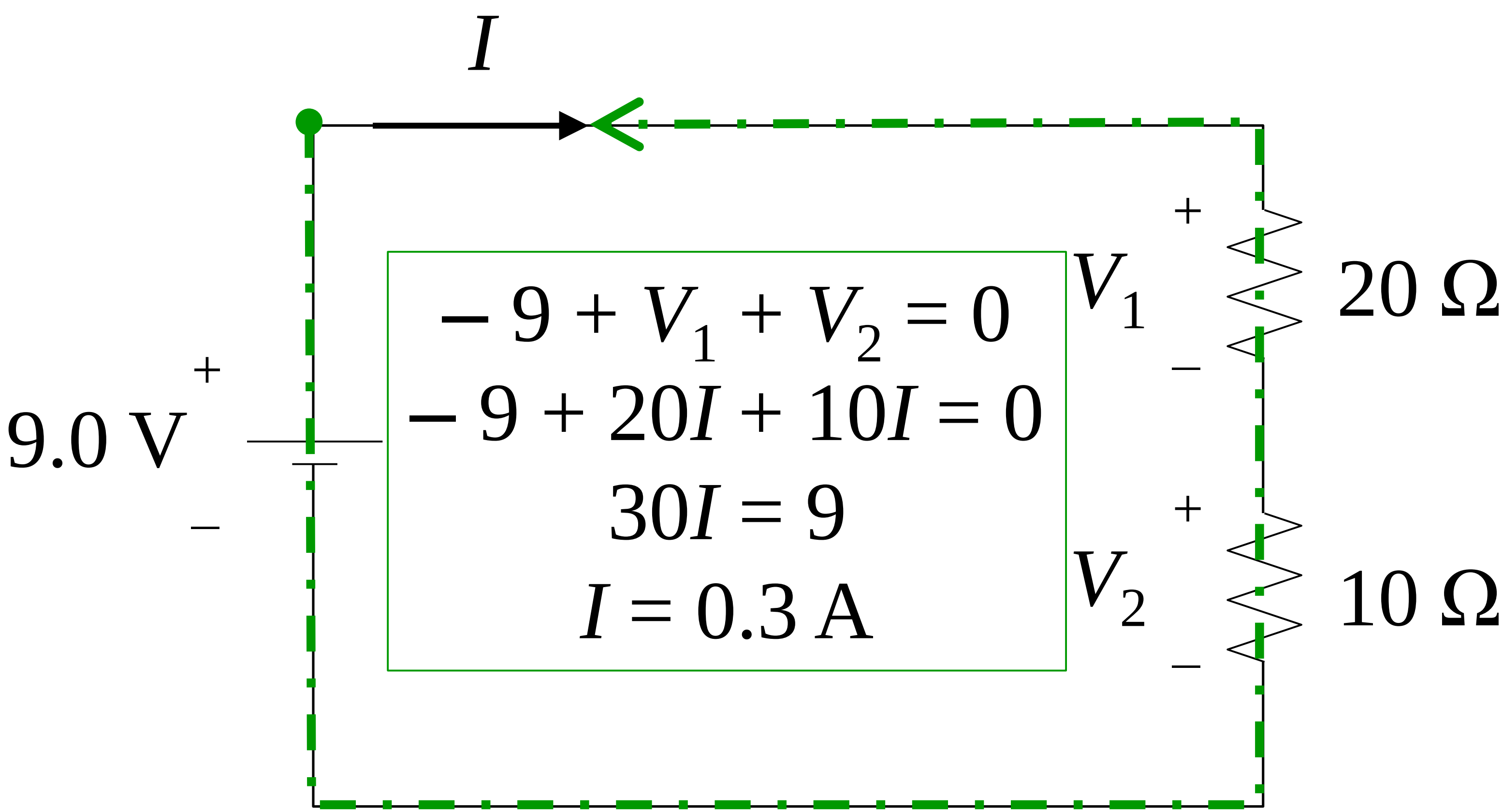
- Two simple rules or laws may be used to analyze any circuit, including circuits that are not “reducible” to series and parallel “pieces”.
- Node Rule: The sum of currents entering a node must equal the sum of currents exiting a node.
- Loop Rule: The sum of the potential differences across all elements around any loop within a circuit must equal zero.
- To solve a circuit set up a system of equations based on these rules and solve for unknowns.

Following the orange path clockwise around the circuit, the potential increases by 9 volts and then drops by amount *IR* for each resistor. This leads to a "loop equation" from which current can be determined.

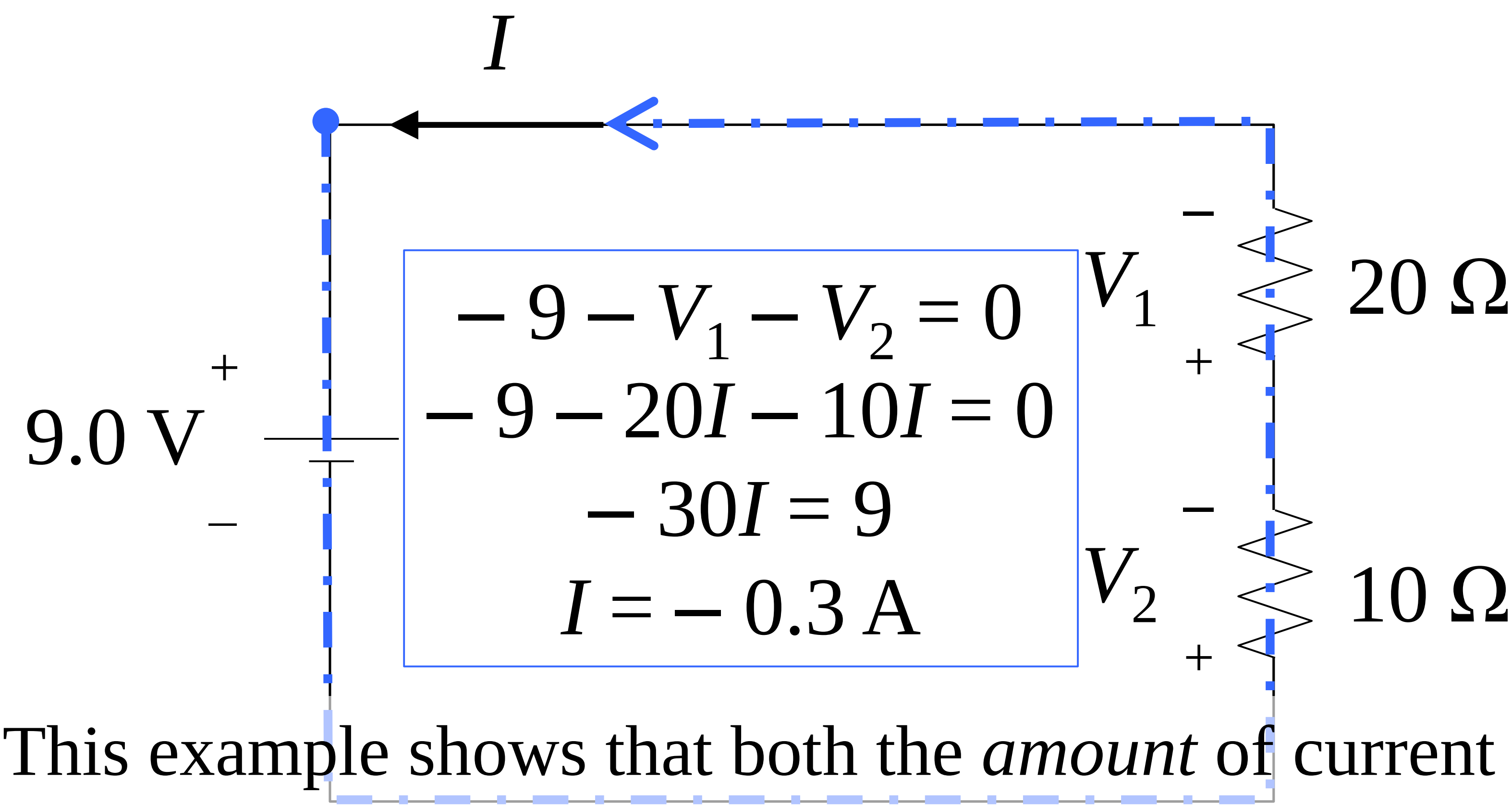


$$9 - V_1 - V_2 = 0$$
$$9 - 20I - 10I = 0$$
$$9 = 30I$$
$$I = 0.3\ \text{A}$$

Following the green path counterclockwise around the circuit, the potential decreases by 9 volts and then goes up by amount *IR* for each resistor. This leads to a “loop equation” from which current can be determined with the same result.

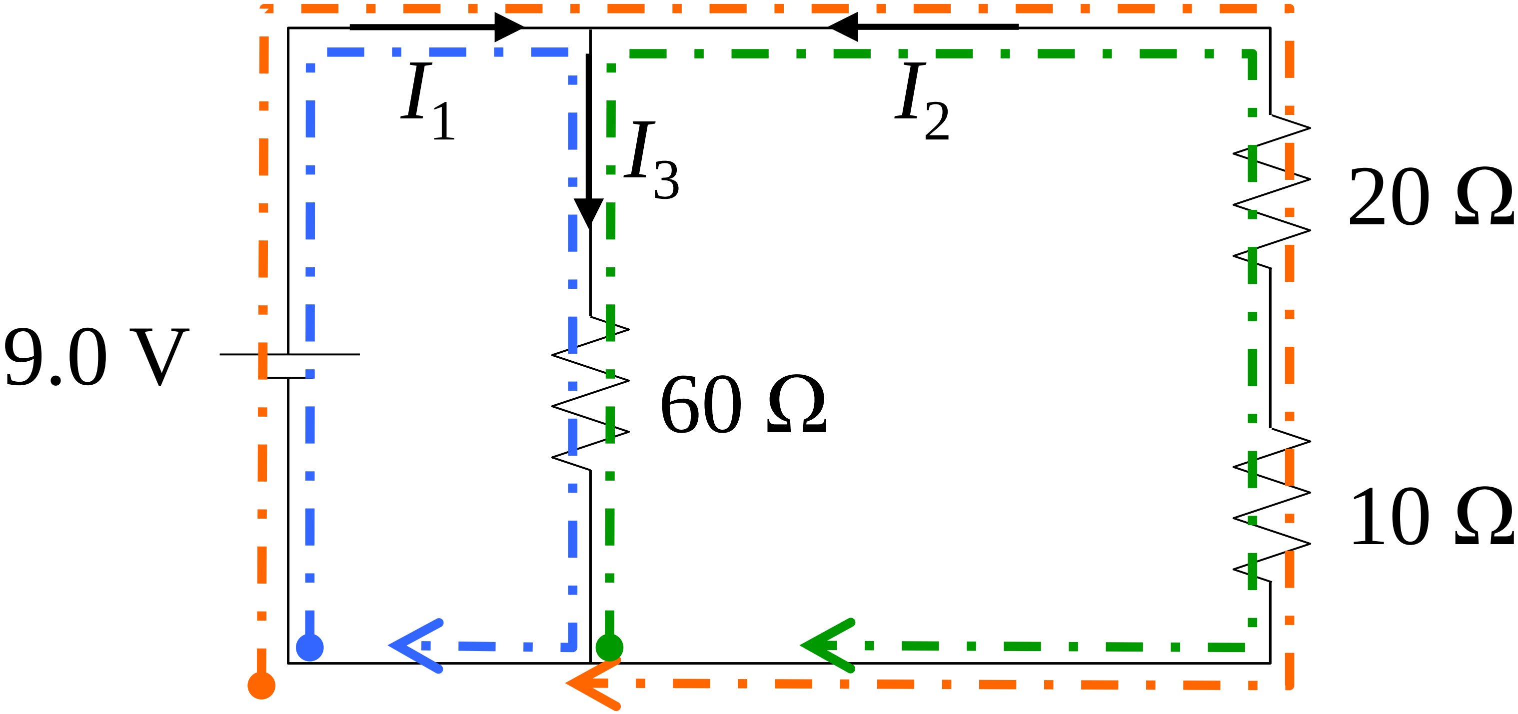


Here the direction of the current has been incorrectly drawn. *If* current *were* flowing thusly *all* of the voltages would be negative along of the blue path. Correctly analyzing the loop reveals that current must be in the opposite direction.



$$-9 - V_1 - V_2 = 0$$
$$-9 - 20I - 10I = 0$$
$$-30I = 9$$
$$I = -0.3\text{ A}$$

This example shows that both the *amount* of current and the *direction* can be determined with Kirchoff's Laws.



node

$$I_1 + I_2 = I_3$$

loop

$$9 - 60I_3 = 0$$

$$9 + 20I_2 + 10I_2 = 0$$

$$60I_3 + 20I_2 + 10I_2 = 0$$

$I_1 = 0.45$ A

$I_3 = 0.15$ A

$I_2 = -\,0.3$ A

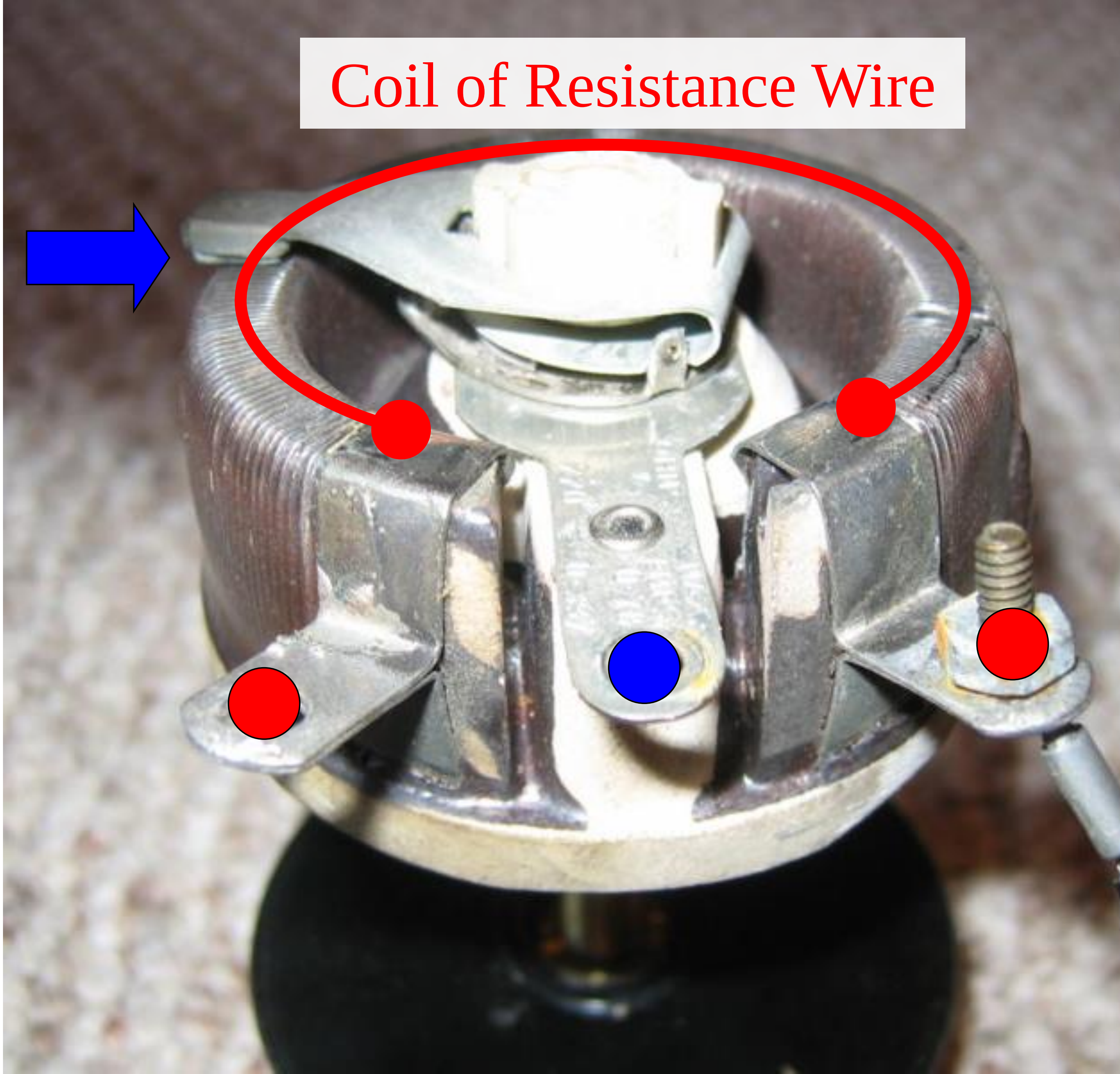
Kirchhoff's Laws – Tips

- Start by drawing and labeling any and all currents, each in an assumed direction.
- Write out as many unique node equations as possible.
- Write out *enough* unique loop equations to solve the system of unknowns.
- When there are three or more unknown currents it is often best to use matrices!

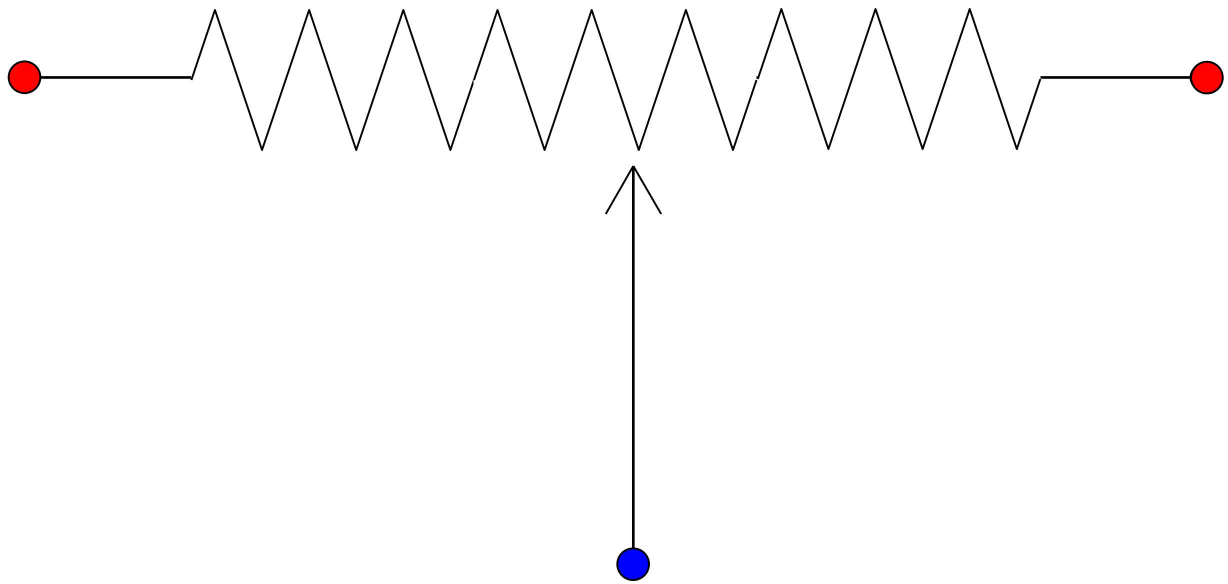
Potentiometer

AKA:
Pot,
Trim Pot,
Rheostat,
Variable Resistor

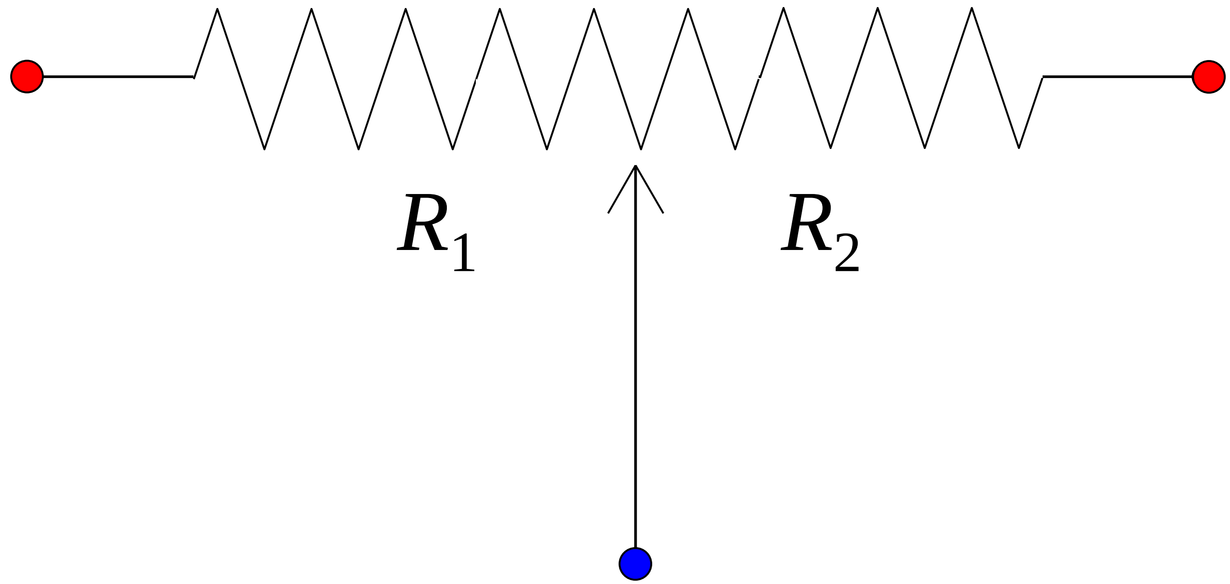
Sliding Contact



Potentiometer



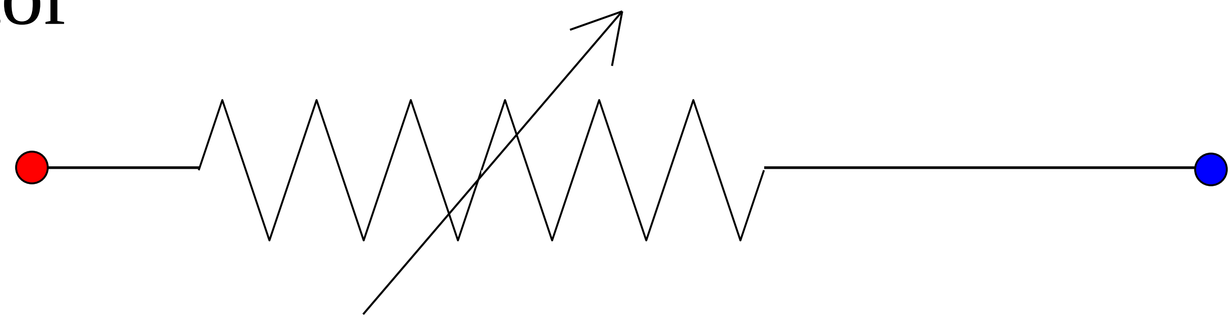
Potentiometer



$$R = \rho \frac{L}{A}$$

The total resistance ($R_1 + R_2$) between the two ends (red terminals) is constant. A certain part (R_1 or R_2) of the total resistance exists between either end and the point of contact (blue terminal) depending on the length of wire between.

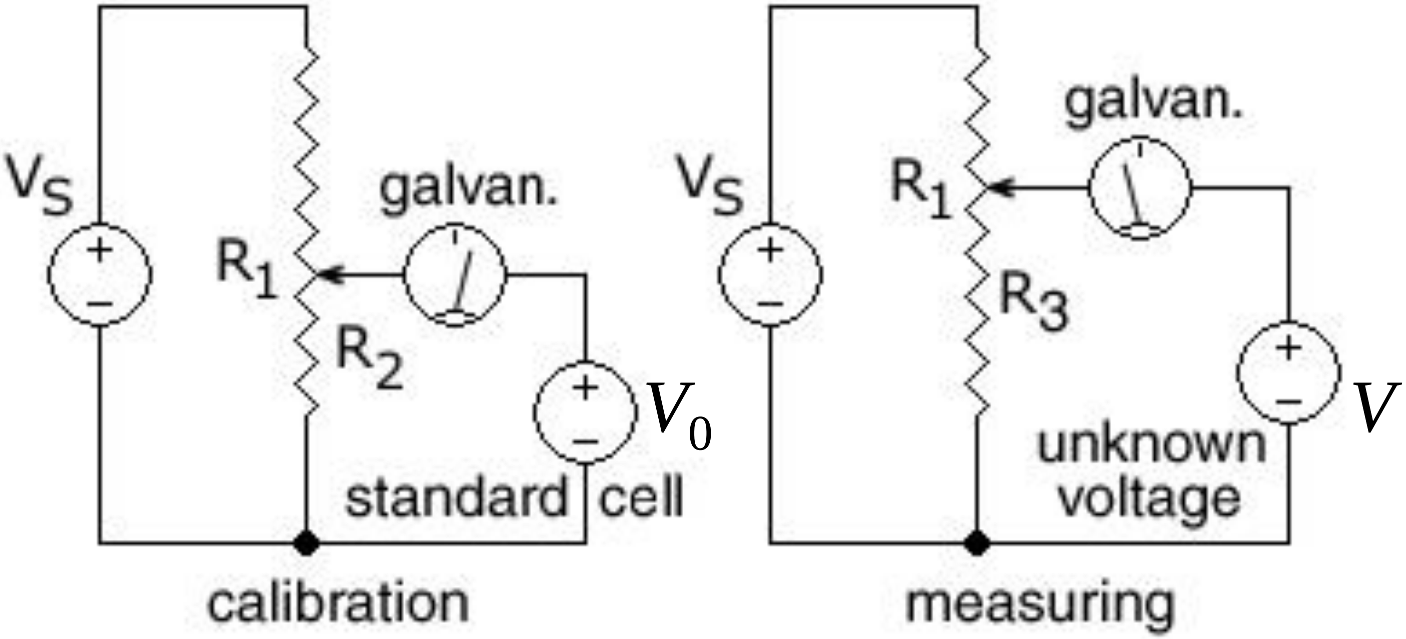
Variable Resistor



If only the blue and one of the red terminals are connected the device becomes simply a variable resistor that can be controlled by turning a knob or otherwise sliding the point of contact along the resistance wire.

Potentiometers and variable resistors are useful for controlling current and/or voltage and are found in many different types of circuits.

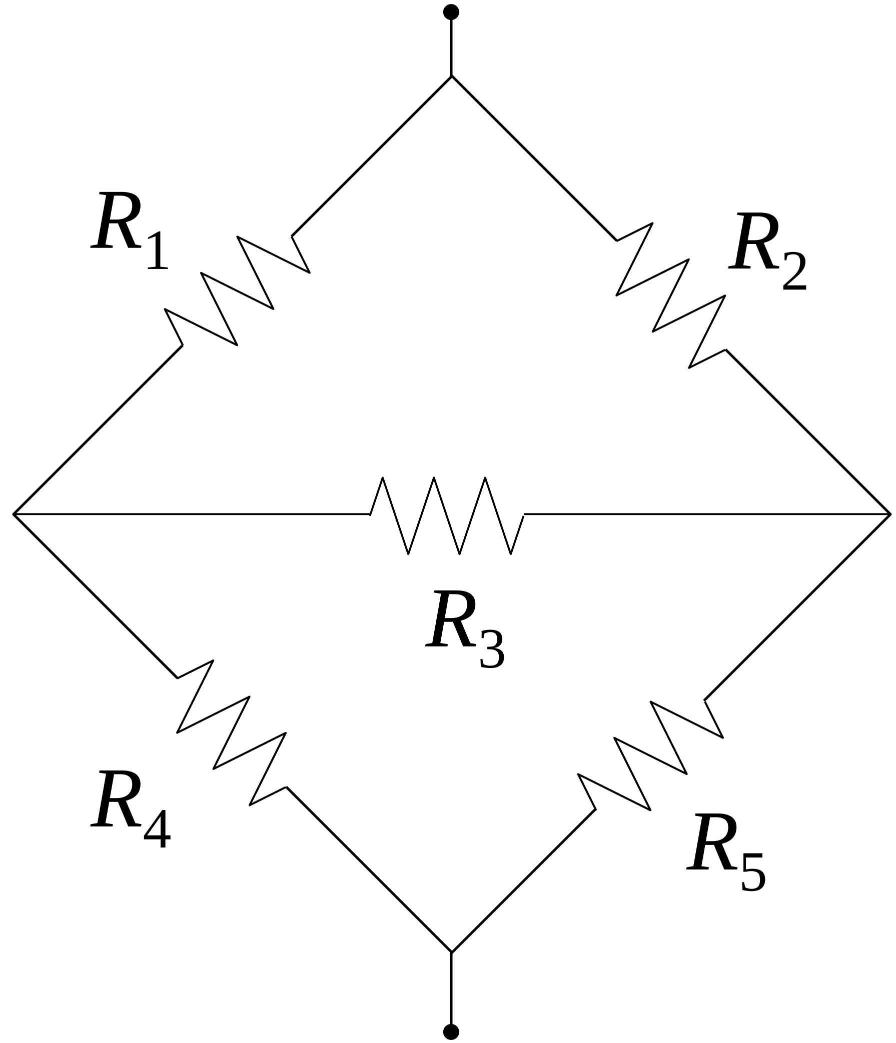
the "original" "Potentiometer" Circuit (aka "Infinite Resistance Voltmeter")



The sliding point of contact is adjusted until the galvanometer shows zero current. Using Kirchoff's Laws it can be shown that the unknown voltage V can is related to the standard cell by $V = (R_3/R_2)V_0$. The voltage V_S can be any amount. This circuit allows an unknown potential to be measured without any current through its source.

Wheatstone Bridge

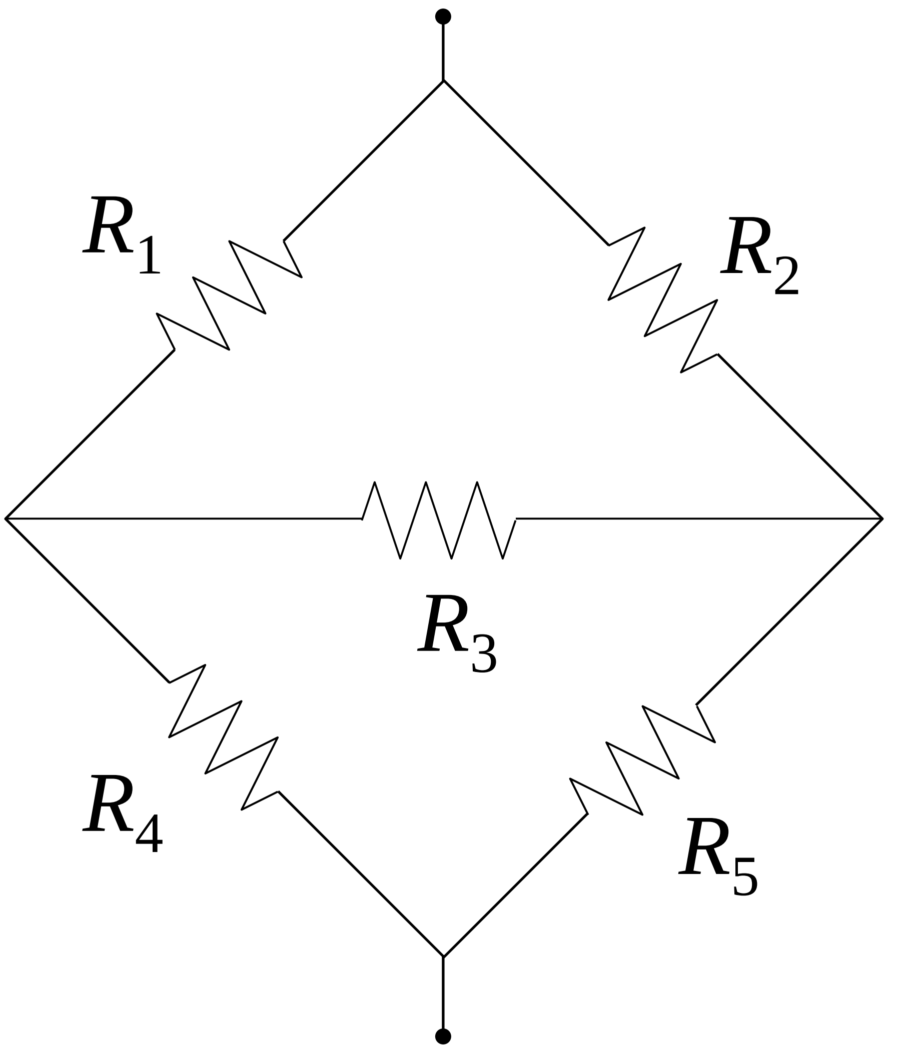
This is a type of circuit that is neither series nor parallel. One application is to adjust the resistances until the current through the "bridge" resistor R_3 is zero. Using Kirchoff's Laws it can be shown that in that special case $R_1/R_4 = R_2/R_5$ (if this ratio is true then any voltage applied results in zero current through the bridge).



$R_{eq} = ?$

Wheatstone Bridge

It is not possible to reduce this circuit to any combination of series and/or parallel resistor "pieces". Instead this is a situation where it is necessary to use Kirchoff's Laws and there is no other choice. The equation below (true even if there is a current through R_3) was determined using node and loop equations!



$$R_{eq} = \frac{R_1R_2R_3 + R_1R_2R_4 + R_1R_2R_5 + R_1R_3R_5 + R_1R_4R_5 + R_2R_3R_4 + R_2R_4R_5 + R_3R_4R_5}{R_1R_3 + R_1R_4 + R_1R_5 + R_2R_3 + R_2R_4 + R_2R_5 + R_3R_4 + R_3R_5}$$