Electrical Circuits

I. Current and Resistance
- Ohm's Law

II. Resistivity
- resistance factors
- conductivity
- **drift velocity & current density**

III. Electrical Power

IV. Circuits
- series and parallel
- Kirchoff's Laws

V. Batteries and Meters
- internal resistance

	The student will be able to:	HW:
1	Define current and the ampere, conventional positive current flow, and solve related problems, including those with both positive and negative charge carriers. ✓	1 – 3
2	Define resistance and the ohm, state Ohm's Law, and solve related problems involving ohmic and/or nonohmic devices. ✓	4 – 7
3	Describe and explain factors influencing resistance, state mathematical relation between resistance, length, area, and resistivity or conductivity, and solve related problems. ✓	8 – 13
4	Solve problems involving current density, electric field, resistivity, drift velocity and/or use these concepts to explain the nature of resistance.	14 – 16
5	Solve problems involving electric power.	17 – 21
6	Determine effective resistance of a network of series and/or parallel resistors.	22 – 24
7	Solve for voltage, current, resistance, and power in DC circuits using Kirchoff's Laws and/or effective resistance.	25 – 38
8	Model a cell or battery as an ideal voltage source or as an EMF with internal resistance and a certain terminal voltage, and solve related problems.	39 – 43
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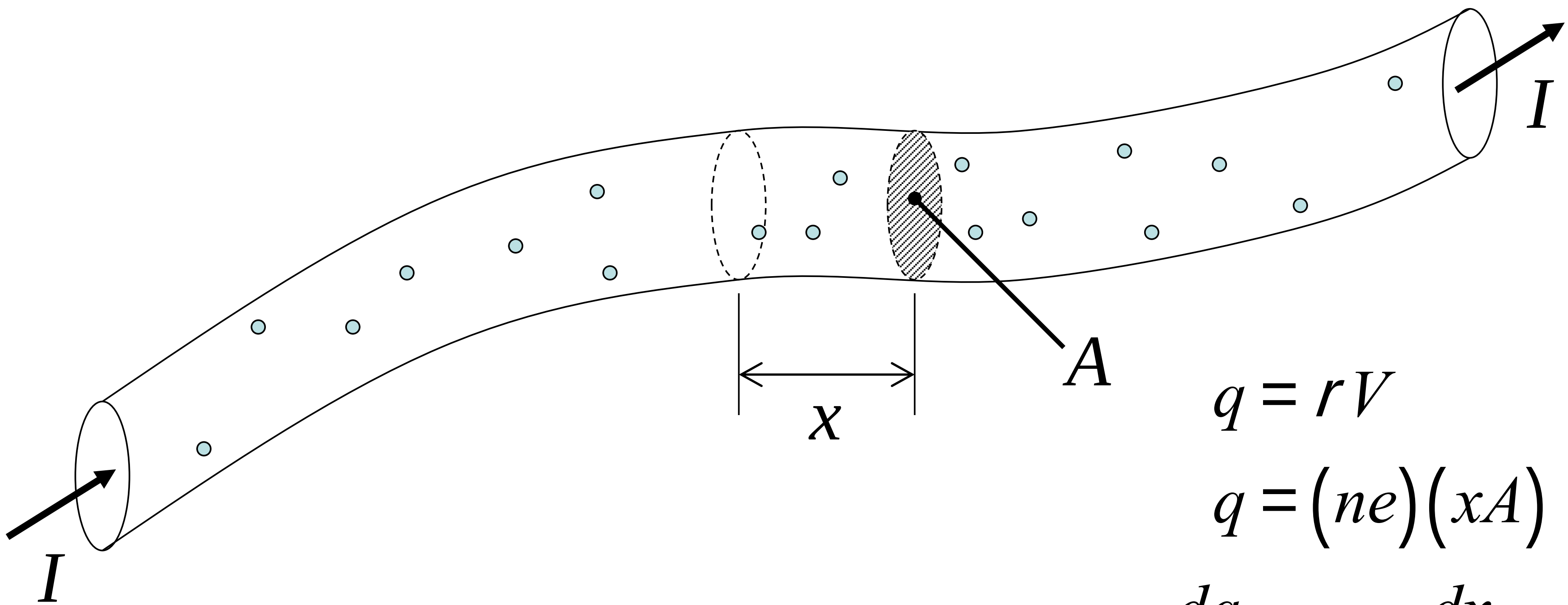
Current – a Microscopic Model

What actually happens *inside* a current?

Is there a theoretical basis for Ohm's Law?
How about resistance and resistivity?

To answer these questions we will attempt to apply classical Newtonian mechanics to the charged particles that constitute the current…

Drift Velocity



$$q = rV$$

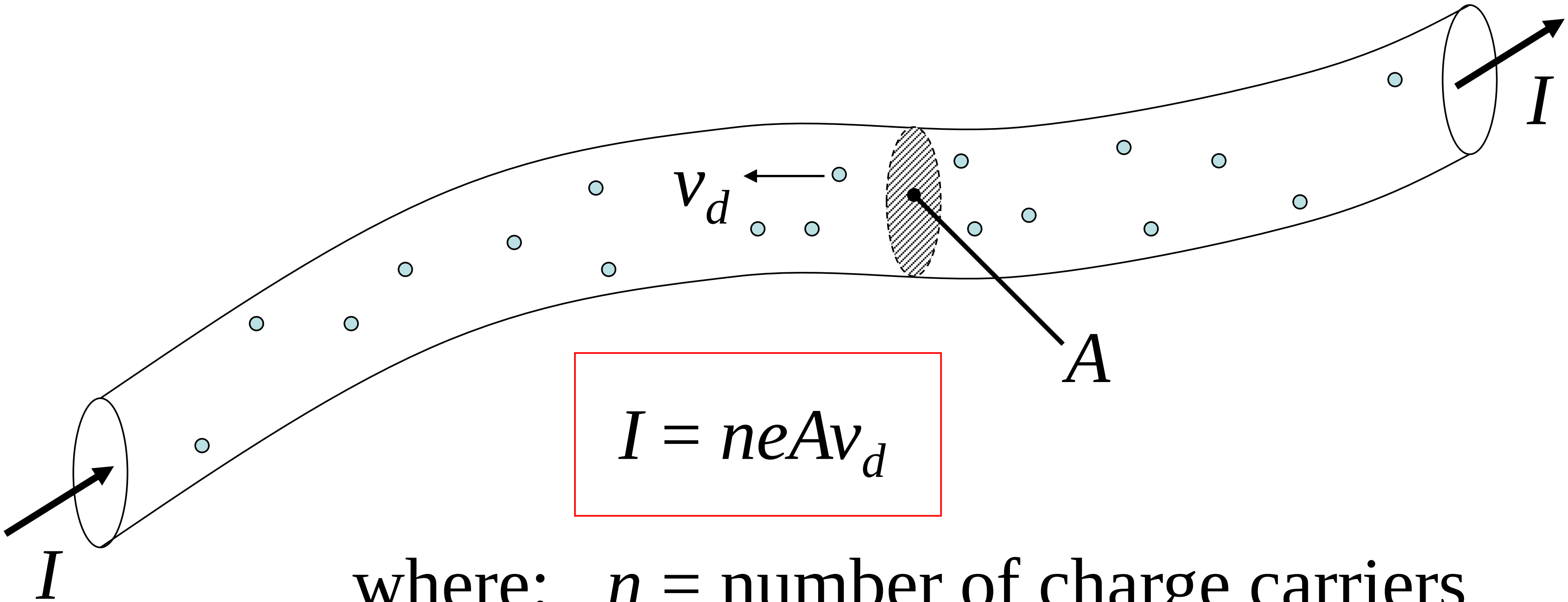
$$q = (ne)(xA)$$

$$\frac{dq}{dt} = neA\frac{dx}{dt}$$

$$I = neAv_d$$

Relate the current to the charge density, the cross-sectional area, and the speed of charged particles…

Drift Velocity



$$I = neAv_d$$

where: n = number of charge carriers per volume

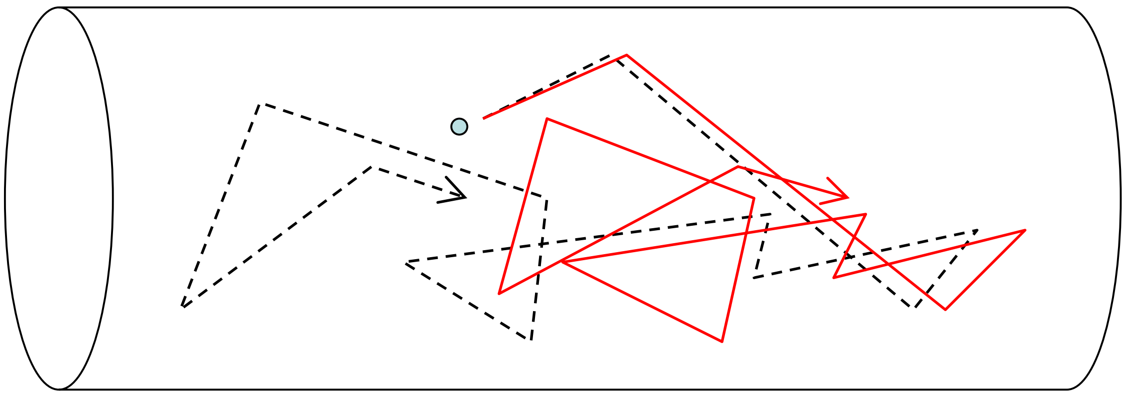
e = amt of charge each carrier

A = cross-sectional area

v_d = "drift velocity"

	density (g/cm^3)	mass (g/mol)	conduction electrons per atom	n ($1/m^3$)
copper	8.96	63.54	1	8.49×10^{28}
silver	10.5	107.87	1	5.86×10^{28}
gold	19.3	196.97	1	5.90×10^{28}
iron	7.86	55.85	2	1.70×10^{29}
aluminum	2.70	26.98	?	2.10×10^{29}

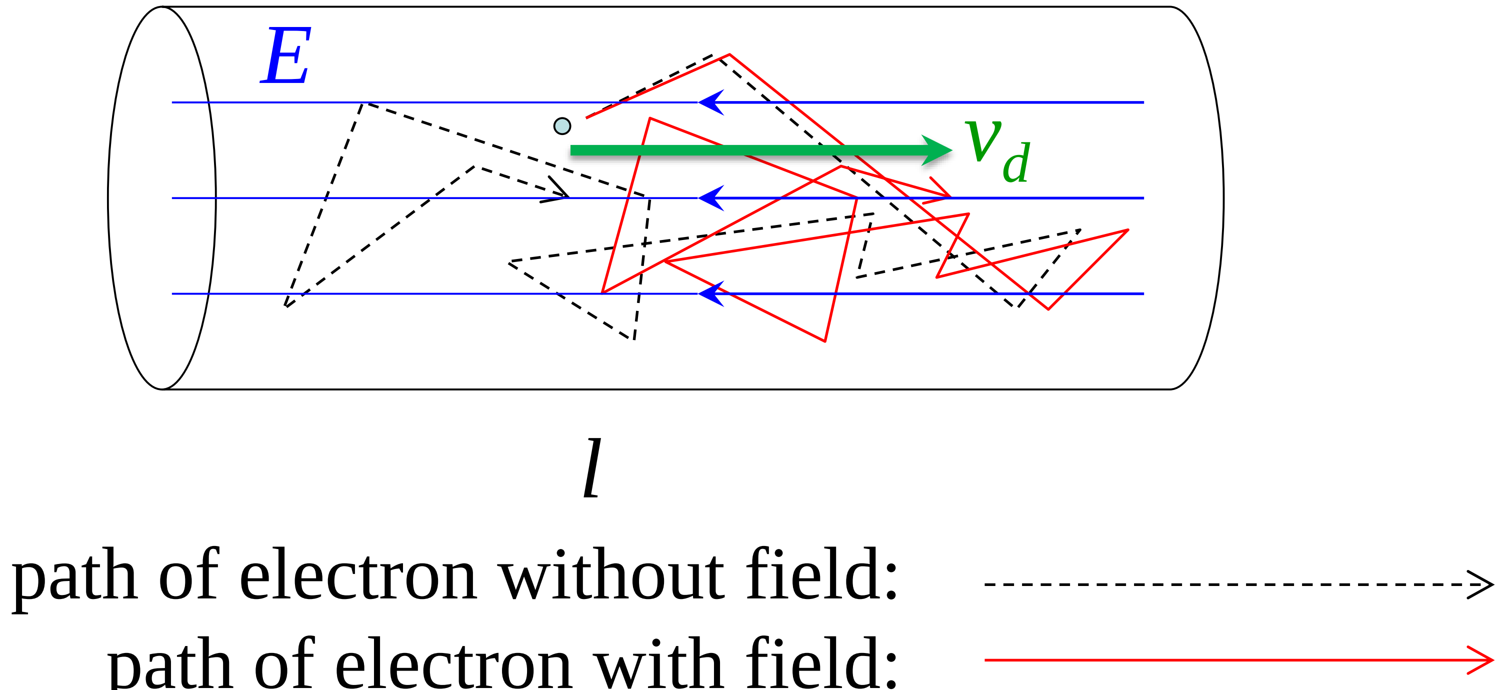
Deriving Ohm's Law



Start with 2nd Law of Motion and determine voltage in terms of current…

Take drift velocity to be *mean* velocity (drift velocity changes with constant acceleration between collisions …)

Deriving Ohm's Law



The drift velocity v_d is due to acceleration of the electron that occurs between collisions. This acceleration is caused by an electric field *E*. Normally the velocity of a conduction electron is random, but the presence of the field causes a small net change that occurs repeatedly between collisions.

2nd Law: $F = ma$

applied to an electron: $eE\ell = ma\ell$

$$E\ell = \frac{m\ell}{e} a$$

$$E\ell = \frac{m\ell}{e} \frac{v_d}{t}$$

$$E\ell = \frac{m\ell}{et} \frac{I}{neA}$$

$$V = \left(\frac{m}{ne^2 t}\right)\left(\frac{\ell}{A}\right) I$$

recognize resistance: $V = \frac{r\ell}{A} I$

Ohm's Law: $V = RI$

incorporating drift velocity:

$$Dv = at$$

$$v_d = at$$

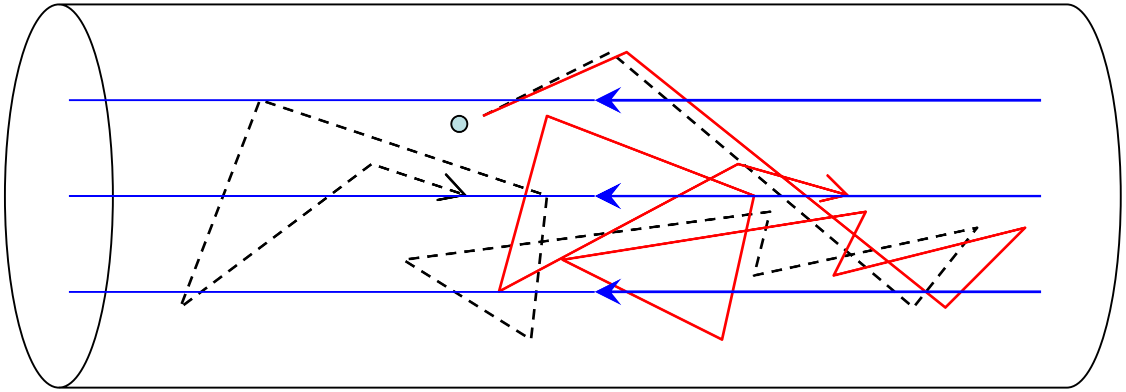
$$a = \frac{v_d}{t}$$

$$I = neAv_d$$

$$v_d = \frac{I}{neA}$$

let $r = \frac{m}{ne^2 t}$

Deriving Ohm's Law



$$V = I\left(\frac{m}{ne^2\tau}\cdot\frac{l}{A}\right)$$

where: m = mass per charge carrier

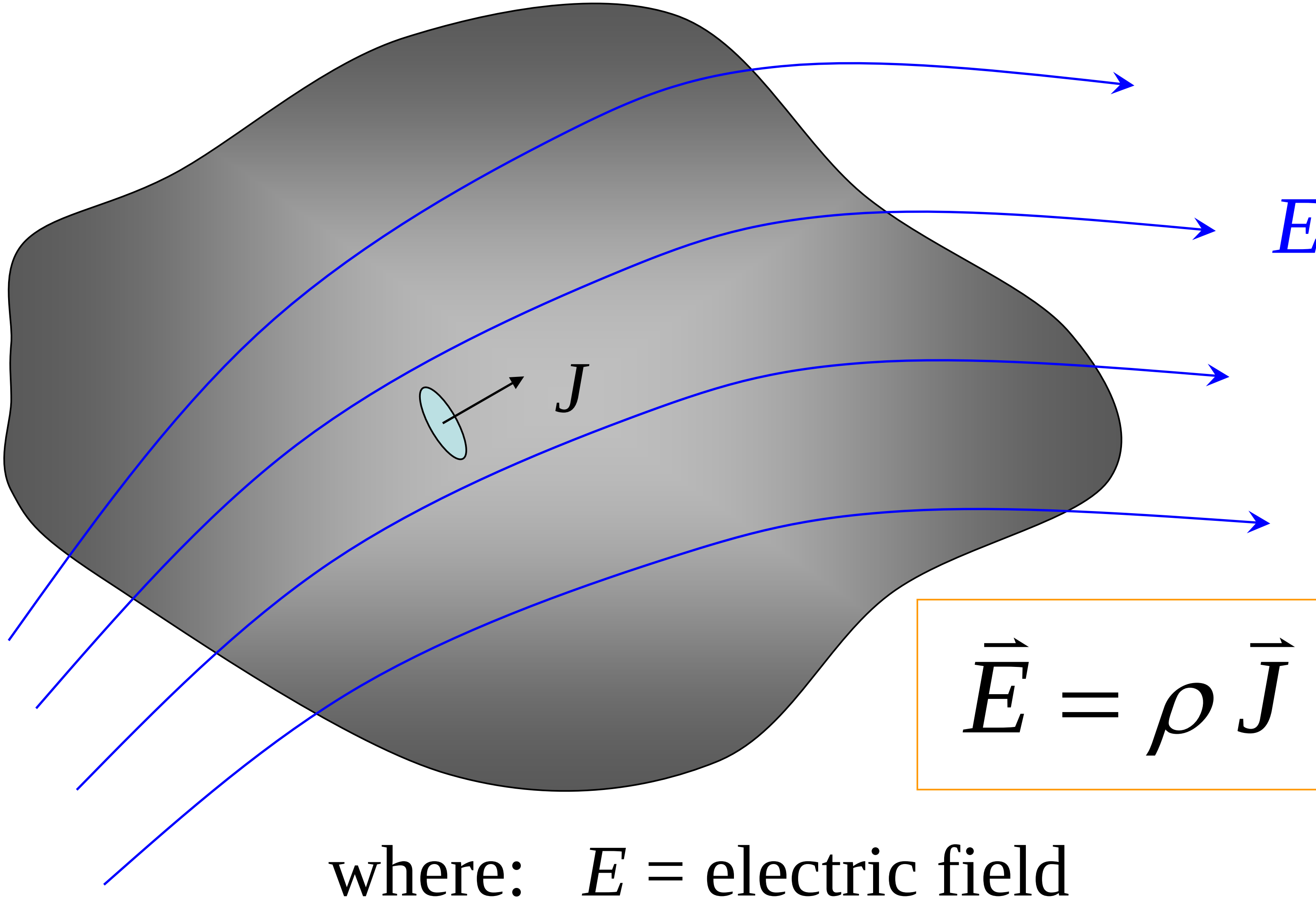
l = length of conductor

τ = time constant (average time between collisions)

Resistivity "Explained"

$$\rho = \frac{m}{ne^2\tau}$$

Based on classical physics, this combination of quantities should equate with resistivity. Note that these quantities are indeed properties of a material: the number of charge carriers per volume *n* and the time between collisions τ. As temperature increases this time would decrease and cause an increase in resistivity as expected. However, not all materials exhibit this property and it should be stressed that this is "only a model" and that atoms and subatomic particles are best understood using quantum properties and the Standard Model (not classical physics).



$$\vec{E} = \rho \vec{J}$$

where: E = electric field

ρ = resistivity of substance

J = current density (current per cross-sectional area)

Current density J is a vector defined such that:

$$I = \int \vec{J} \cdot d\vec{A}$$

$$\left|\vec{J}\right| = \frac{dI}{dA}$$

$$V = IR$$

$$V = \frac{r\,\ell}{A} I$$

$$\frac{V}{\ell} = r\,\frac{I}{A}$$

This relation is essentially a version of Ohm’s Law. It has the advantage of being a point-by-point relation – true at any given location where current exists.

$$E = r\,J$$

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Power – Conceptual

- Recall that power is the rate at which work is done.
- For electrical components it is perhaps easier to think of power as the rate at which energy is transformed.
- Electrical power is most directly related to voltage (energy per charge) and current (charge per time)…

Electric Power

$$P = VI$$

where: P = power
V = potential difference “across”
I = electric current “through”

Note: depending on the usage this could represent power input or output of an electrical device.

Electric Power and Resistance

$$P = \frac{V^2}{R}$$

$$P = I^2 R$$

This type of calculation yields rate at which energy is "dissipated" by a circuit element due to its resistance. This is the rate at which electric energy is transformed into thermal energy (heat and/or light).