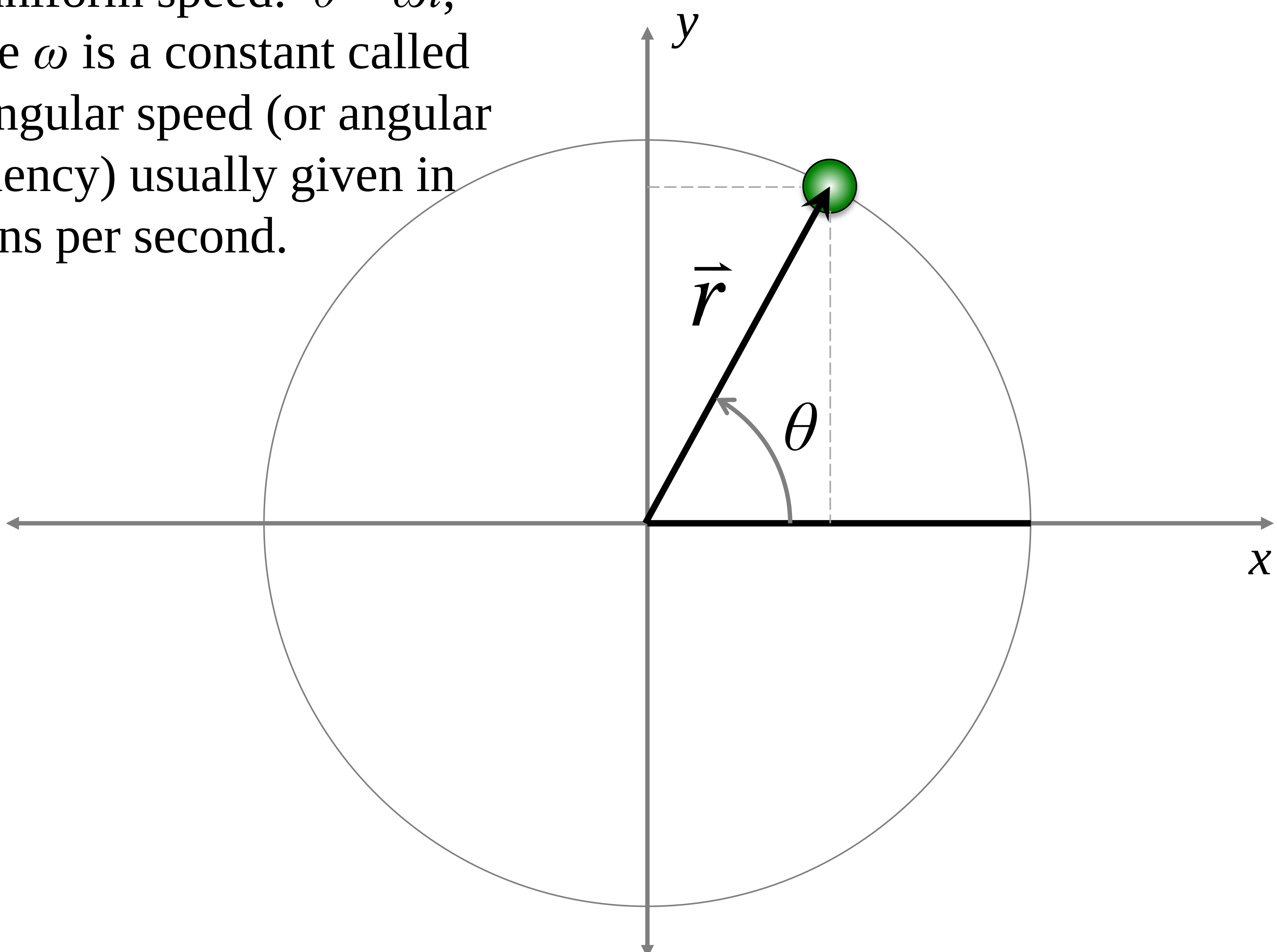
Circular Motion & Gravitation

I. Circular Motion
- kinematics & dynamics
- centripetal acceleration
- centripetal force
- **nonuniform circular motion**
- **parametric equations**

II. Universal Gravitation
- Newton's "4th" Law
- force fields & orbits
- Kepler's Laws

	The student will be able to:	HW:
1	Solve problems of uniform circular motion involving period, frequency, speed, velocity, acceleration, force.	1 – 10
2	Distinguish, explain, and apply the concepts of centripetal and centrifugal force.	11 – 13
3	Solve problems of uniform circular motion or cycloid motion by use of parametric equations.	14 – 15
4	Solve problems of nonuniform circular motion involving constant rate of change in speed in which there are radial and tangential components of acceleration.	16 – 18
5	State and apply Newton's Law of Universal Gravitation.	19 – 23
6	Define and apply gravitational field strength.	24 – 28
7	Solve problems involving circular orbits.	29 – 34
8	State, apply, and derive Kepler's 3rd Law	35 – 36

For uniform speed: $\theta = \omega t$, where ω is a constant called the angular speed (or angular frequency) usually given in radians per second.



Derive position, velocity, acceleration as functions of angular speed and time using *x* and *y* coordinates…

Parametric Form of Uniform Circular Motion

$$x = r\cos(\omega t + \delta) \qquad y = r\sin(\omega t + \delta)$$

$$v_x = -r\omega\sin(\omega t + \delta) \qquad v_y = r\omega\cos(\omega t + \delta)$$

$$a_x = -r\omega^2\cos(\omega t + \delta) \qquad a_y = -r\omega^2\sin(\omega t + \delta)$$

$$\omega = \frac{2\pi}{T} = 2\pi f$$

$$v = r\omega = \frac{2\pi r}{T}$$

$$a = r\omega^2 = \frac{v^2}{r}$$

where:

r = radius

ω = angular speed

δ = phase angle

Parametric Form of Uniform Circular Motion

$$\frac{v_x}{v_y} = \frac{-r\omega\sin(\omega t + \delta)}{r\omega\cos(\omega t + \delta)} = -\tan(\omega t + \delta)$$

$$\frac{a_y}{a_x} = \frac{-r\omega^2\sin(\omega t + \delta)}{-r\omega^2\cos(\omega t + \delta)} = \tan(\omega t + \delta)$$

$$\frac{a_y}{a_x} = -\frac{v_x}{v_y}$$

What is the significance of this?

This shows that velocity and acceleration vectors are always perpendicular to one another.

Parametric Form of Uniform Circular Motion

$$\vec{r} = \left(r\cos\left(\omega t + \delta\right)\right)\hat{i} + \left(r\sin\left(\omega t + \delta\right)\right)\hat{j}$$

$$\vec{r} = r\left(\cos\left(\omega t + \delta\right)\hat{i} + \sin\left(\omega t + \delta\right)\hat{j}\right)$$

$$\vec{a} = \left(-r\omega^2\cos\left(\omega t + \delta\right)\right)\hat{i} + \left(-r\omega^2\sin\left(\omega t + \delta\right)\right)\hat{j}$$

$$\vec{a} = -\omega^2 r\left(\cos\left(\omega t + \delta\right)\hat{i} + \sin\left(\omega t + \delta\right)\hat{j}\right)$$

$$\vec{a} = -\omega^2\vec{r}$$

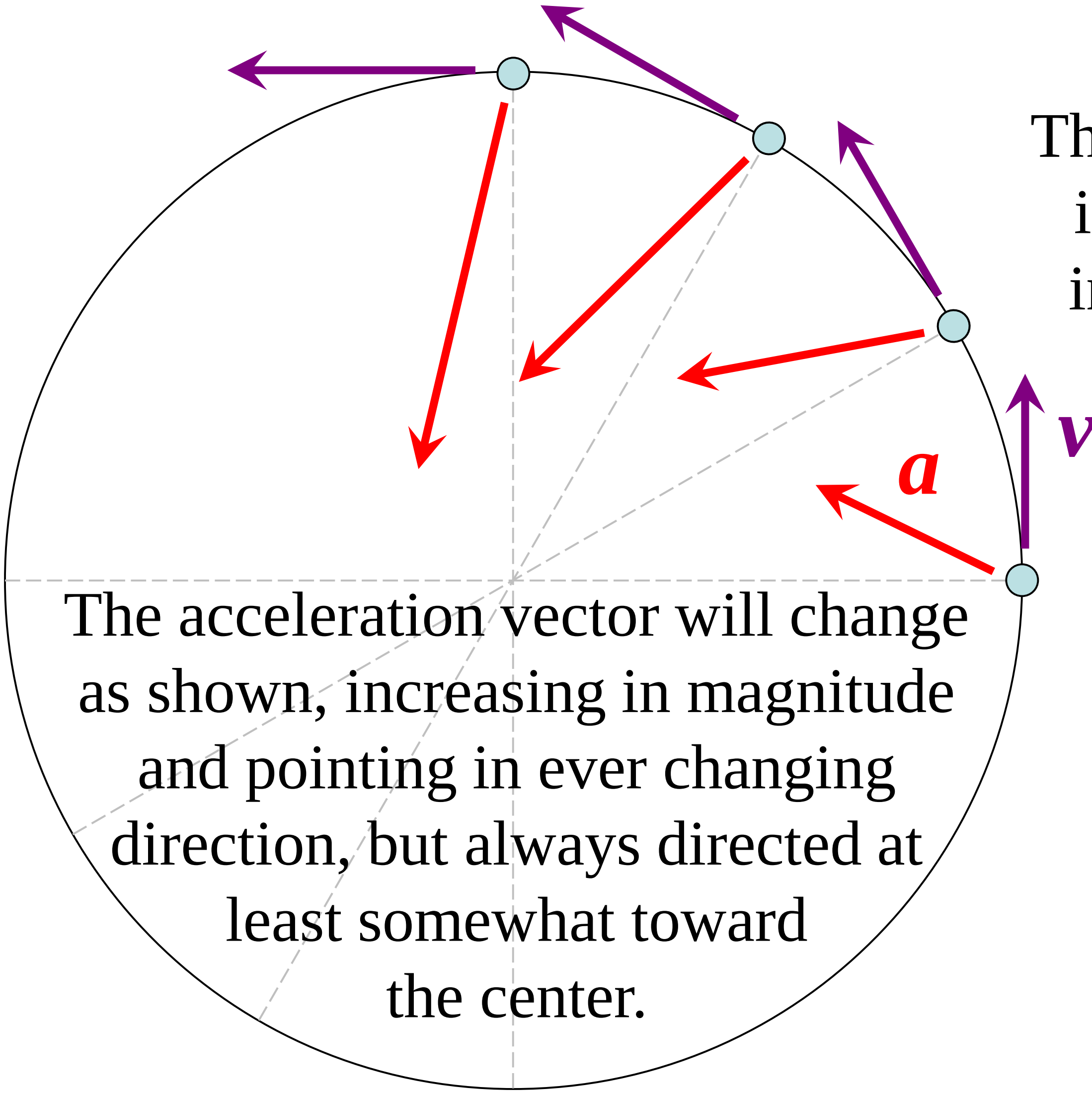
What is the significance of this?

This shows that acceleration is always pointing opposite the position – *i.e.* acceleration is always pointing directly toward the center of the circle as long as the speed is uniform.

Nonuniform Circular Motion

- This type of motion involves two kinds of acceleration: changing speed <u>and</u> changing direction – occurring at the same time.
- These changes in the velocity vector constitute *components* of acceleration that are radial and tangential.
- The object’s acceleration vector will point “generally inward” but not exactly toward the center of the circle.

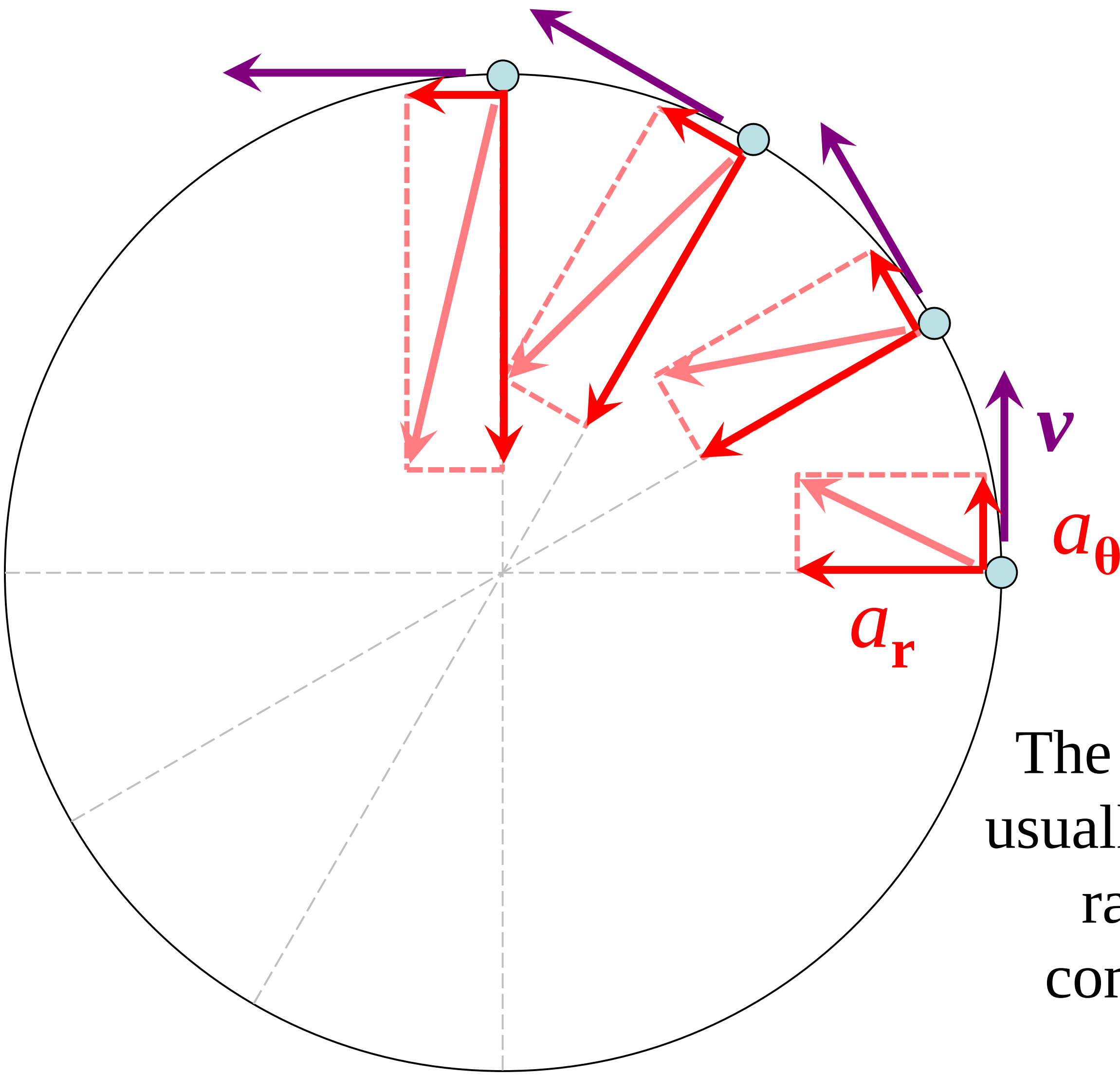
Consider an object that is moving along a circular path and gaining speed at a steady rate. The vectors are shown correct to scale and direction.



The velocity vector increases in magnitude as the speed increases. At all times the velocity vector remains tangent to the circle.

The acceleration vector will change as shown, increasing in magnitude and pointing in ever changing direction, but always directed at least somewhat toward the center.

Consider an object that is moving along a circular path and gaining speed at a steady rate. The vectors are shown correct to scale and direction.



The acceleration vector is usually analyzed in terms of radial and tangential components: a_r and a_θ.

Radial Acceleration

Regardless of changes in speed, an object moving in a circle must have a component of acceleration directed toward the center of the circle:

$$a_r = \frac{v^2}{r}$$

where: v = speed
r = radius

Note: speed does <u>not</u> have to be constant for this to be true!

Tangential Acceleration

Increase or decrease in speed results in a component of acceleration tangent to the circle, aligned with the velocity vector

$$a_\theta = \frac{dv}{dt}$$

where: v = speed
t = time

Note: this is a derivative of *speed*, not velocity!

An object that is moving along a circular path and *decreasing* speed at a constant rate would have vectors as shown here; dv/dt would be negative.