Circular Motion & Gravitation

I. Circular Motion
- kinematics & dynamics
- centripetal acceleration
- centripetal force
- nonuniform circular motion
- parametric equations

II. Universal Gravitation
- **Newton's "4th" Law**
- **force fields & orbits**
- Kepler's Laws

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	The student will be able to:	HW:
1	Solve problems of uniform circular motion involving period, frequency, speed, velocity, acceleration, force.	1 – 10
2	Distinguish, explain, and apply the concepts of centripetal and centrifugal force.	11 – 13
3	Solve problems of uniform circular motion or cycloid motion by use of parametric equations.	14 – 15
4	Solve problems of nonuniform circular motion involving constant rate of change in speed in which there are radial and tangential components of acceleration.	16 – 18
5	State and apply Newton's Law of Universal Gravitation.	19 – 23
6	Define and apply gravitational field strength.	24 – 28
7	Solve problems involving circular orbits.	29 – 34
8	State, apply, and derive Kepler's 3rd Law	35 – 36

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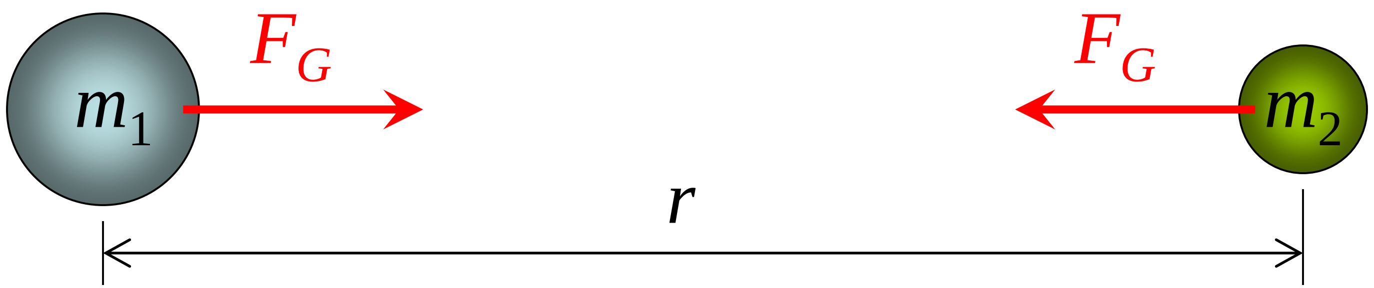
Newton's Law of Universal Gravitation (1687):



$$F_G = G\frac{m_1 m_2}{r^2}$$

where: m = mass ("point-like" or spherical)
r = distance between centers
G = universal gravitational constant:
$6.674 \times 10^{-11}\ \mathrm{m^3/kg\ s^2}$

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Henry Cavendish determined *G* in 1798

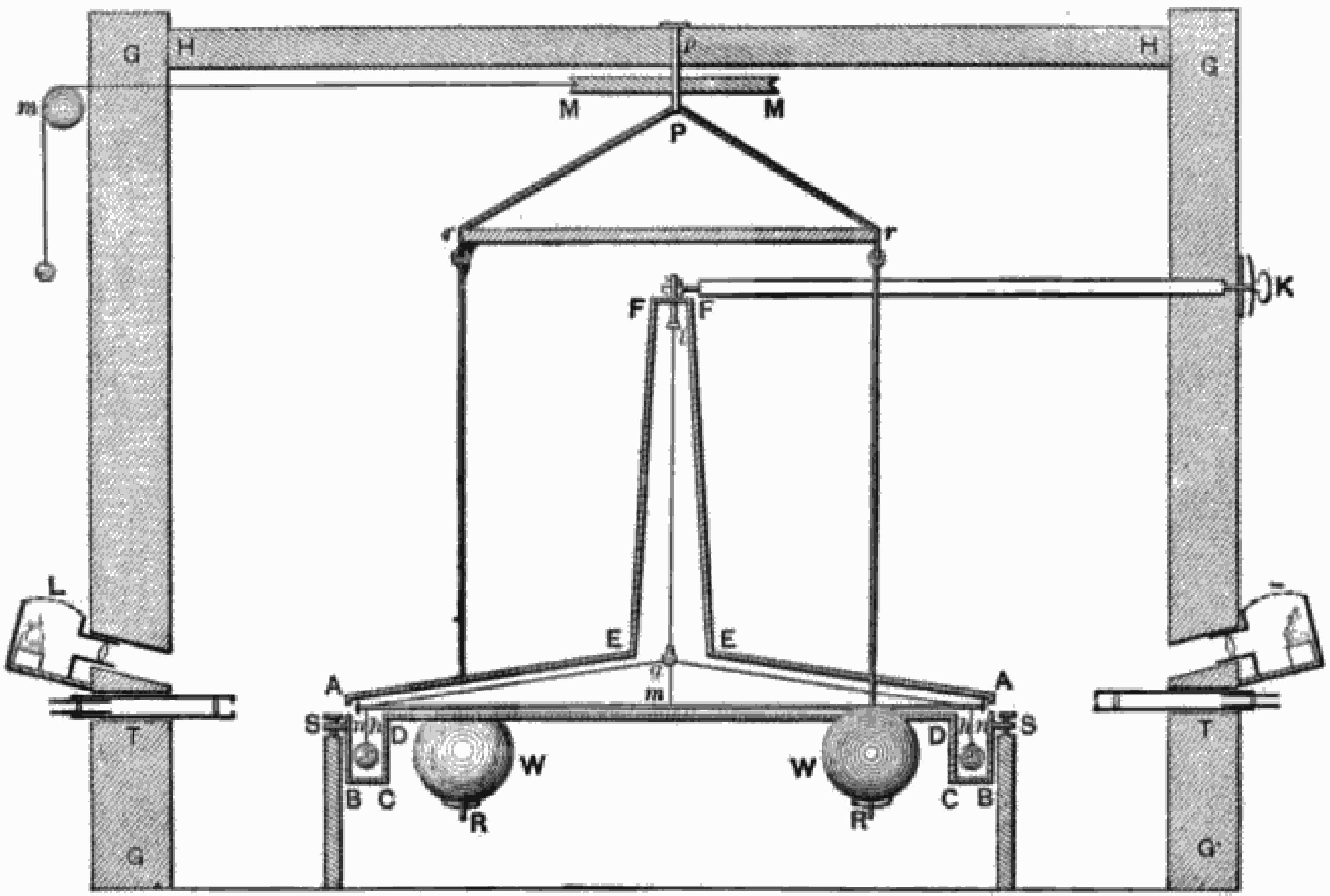
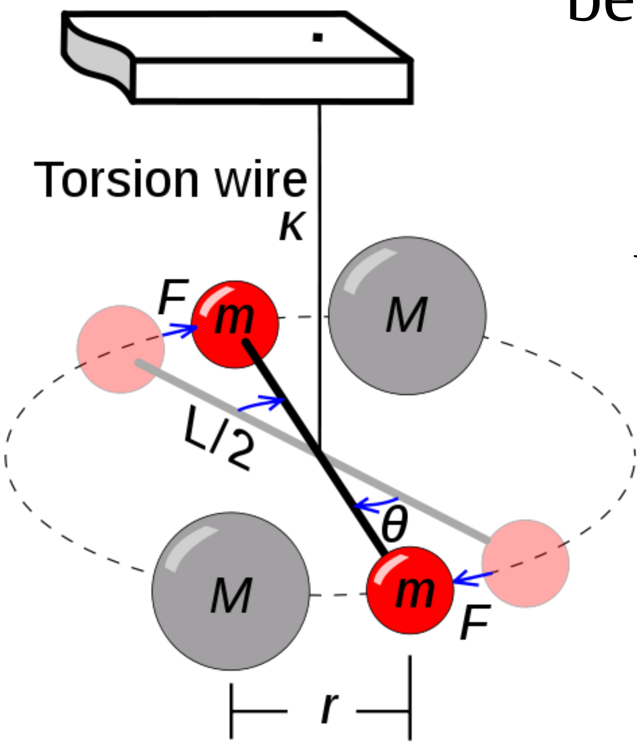
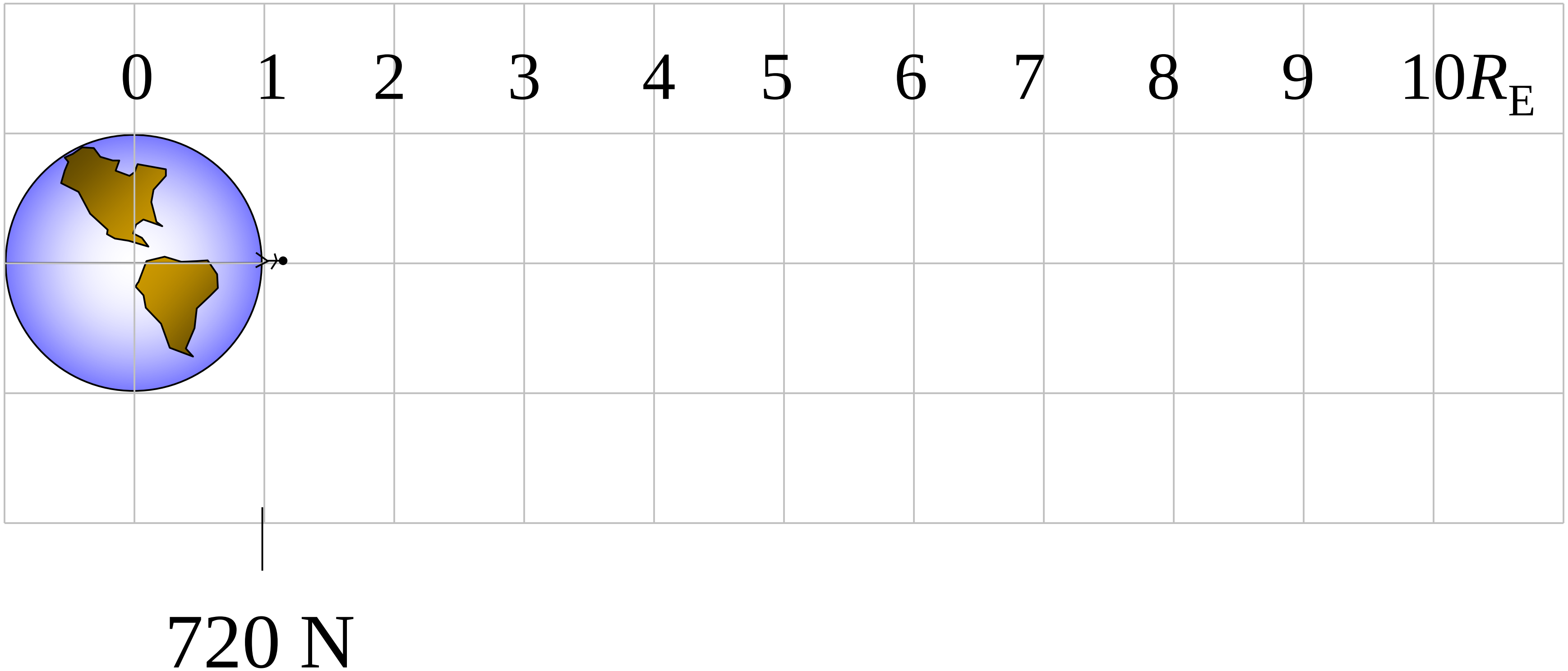


Fig. 1



The effect of universal gravitation between masses *M* and *m* causes the wire to twist (greatly exaggerated in this diagram) and can be observed experimentally. Based on the amount of twist in the wire the force of attraction *F* can be determined. Then using the separation *r* of the masses it is possible to determine an experimental value of the constant *G* found in Newton's law of universal gravitation.

“Inverse Square Law” – Weight of 73.5 kg person



At the surface of Earth the person has weight 720 N.

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“Inverse Square Law” – Weight of 73.5 kg person

At the surface of Earth the person has weight 720 N.

At *twice* the distance the person’s mass is unchanged, but his weight is one *fourth* its original value: 180 N.

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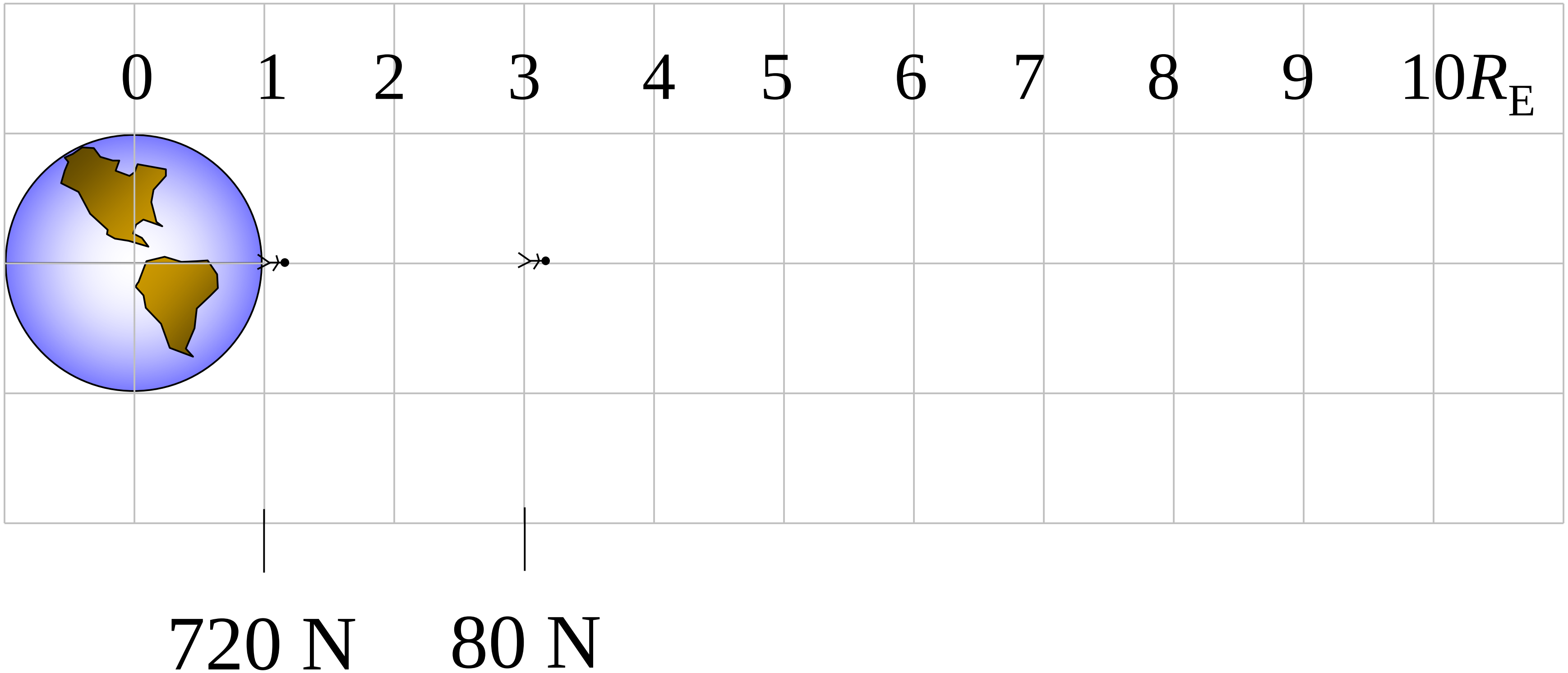
"Inverse Square Law" – Weight of 73.5 kg person



At the surface of Earth the person has weight 720 N.

At *three times* distance the weight is *one ninth*: 80 N.

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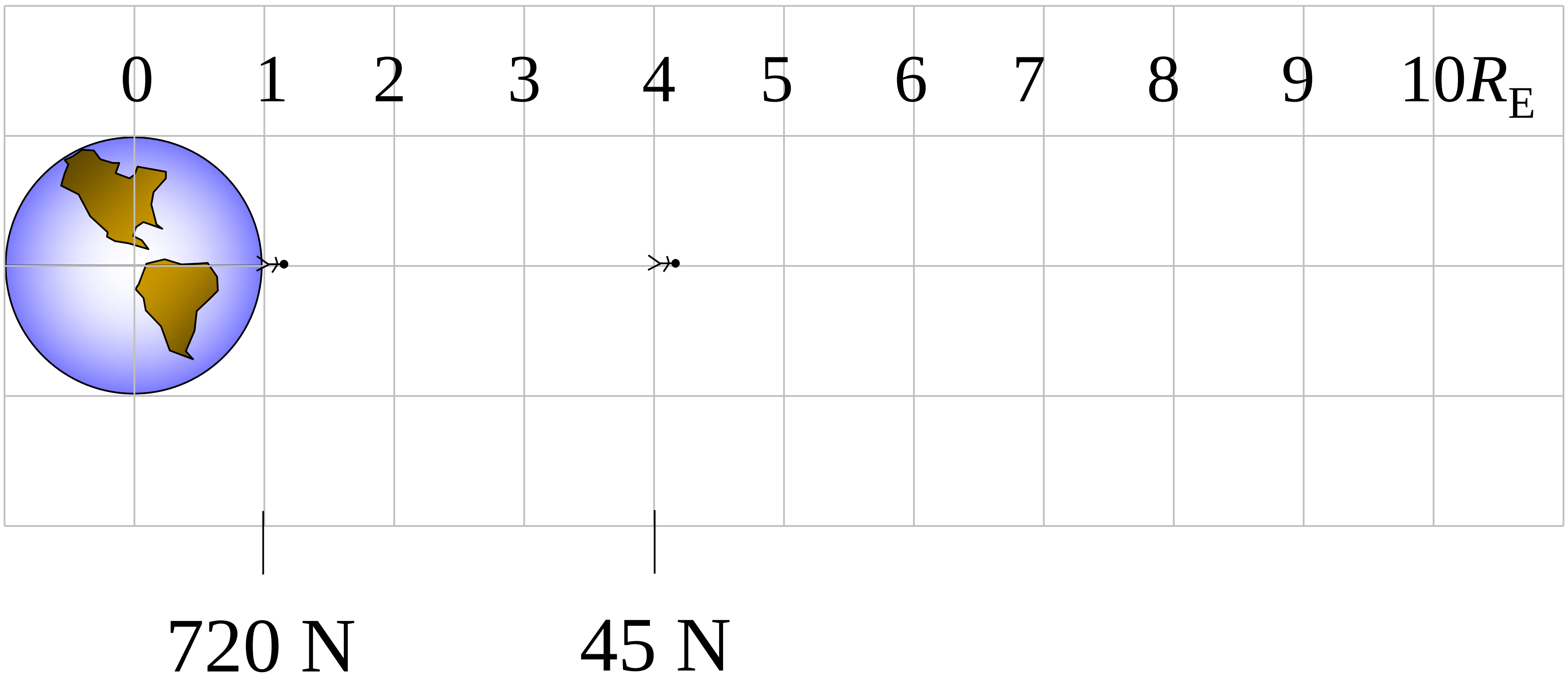
"Inverse Square Law" – Weight of 73.5 kg person



At *four times* distance the weight is a *sixteenth*: 45 N.

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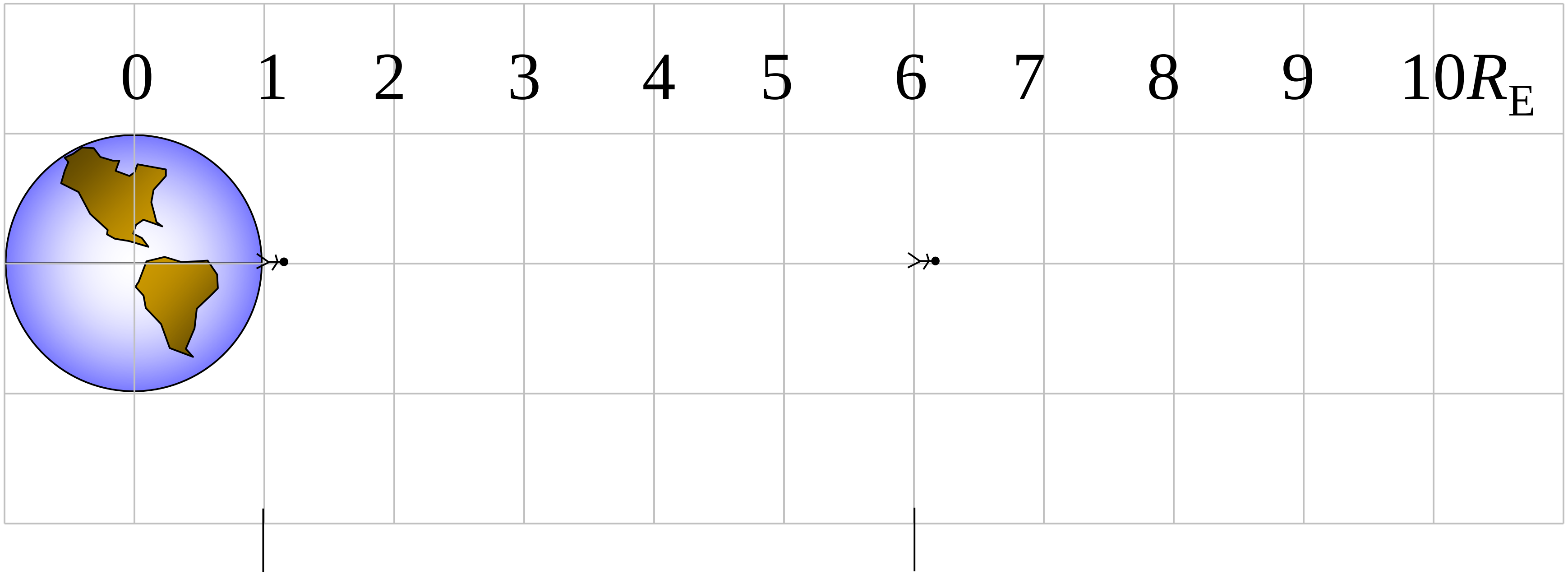
"Inverse Square Law" – Weight of 73.5 kg person



720 N

20 N

$$\frac{720}{6^2} = 20$$

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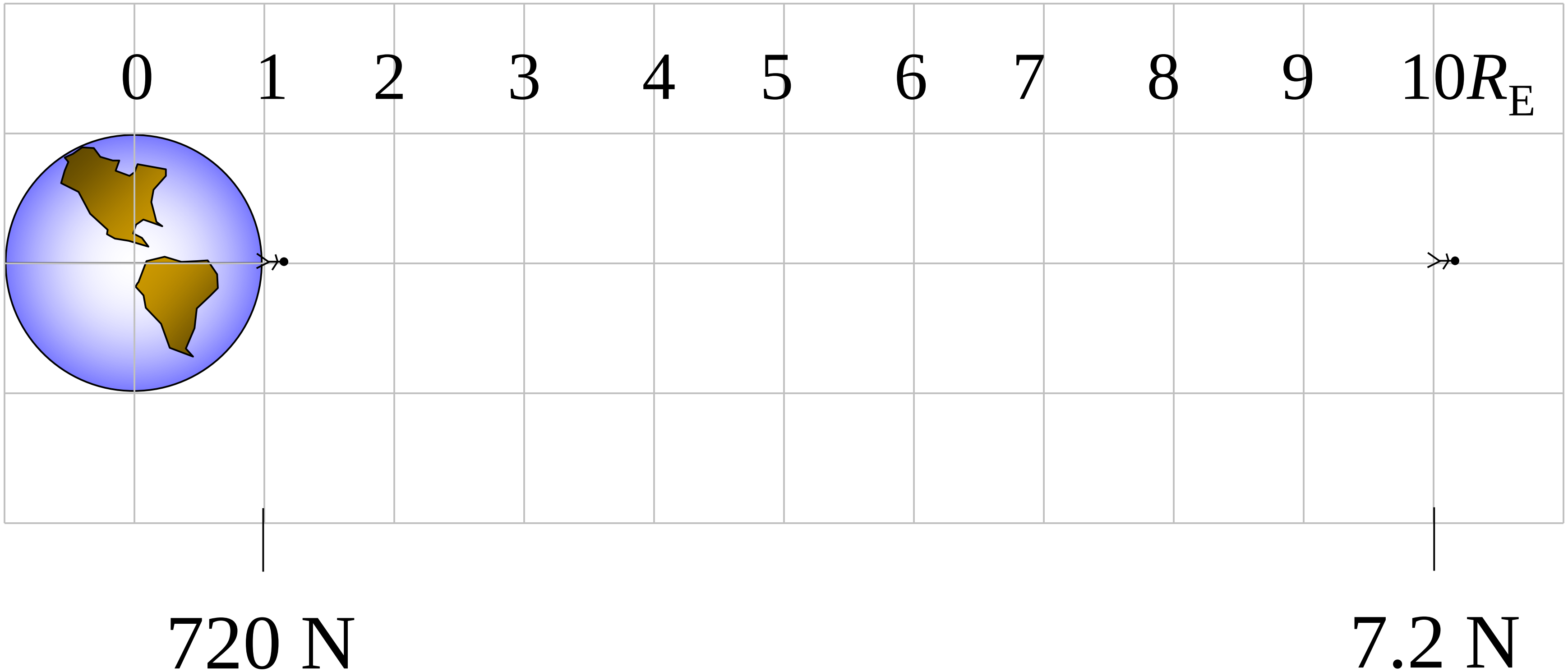
“Inverse Square Law” – Weight of 73.5 kg person



$$\frac{720}{10^2} = 7.2$$

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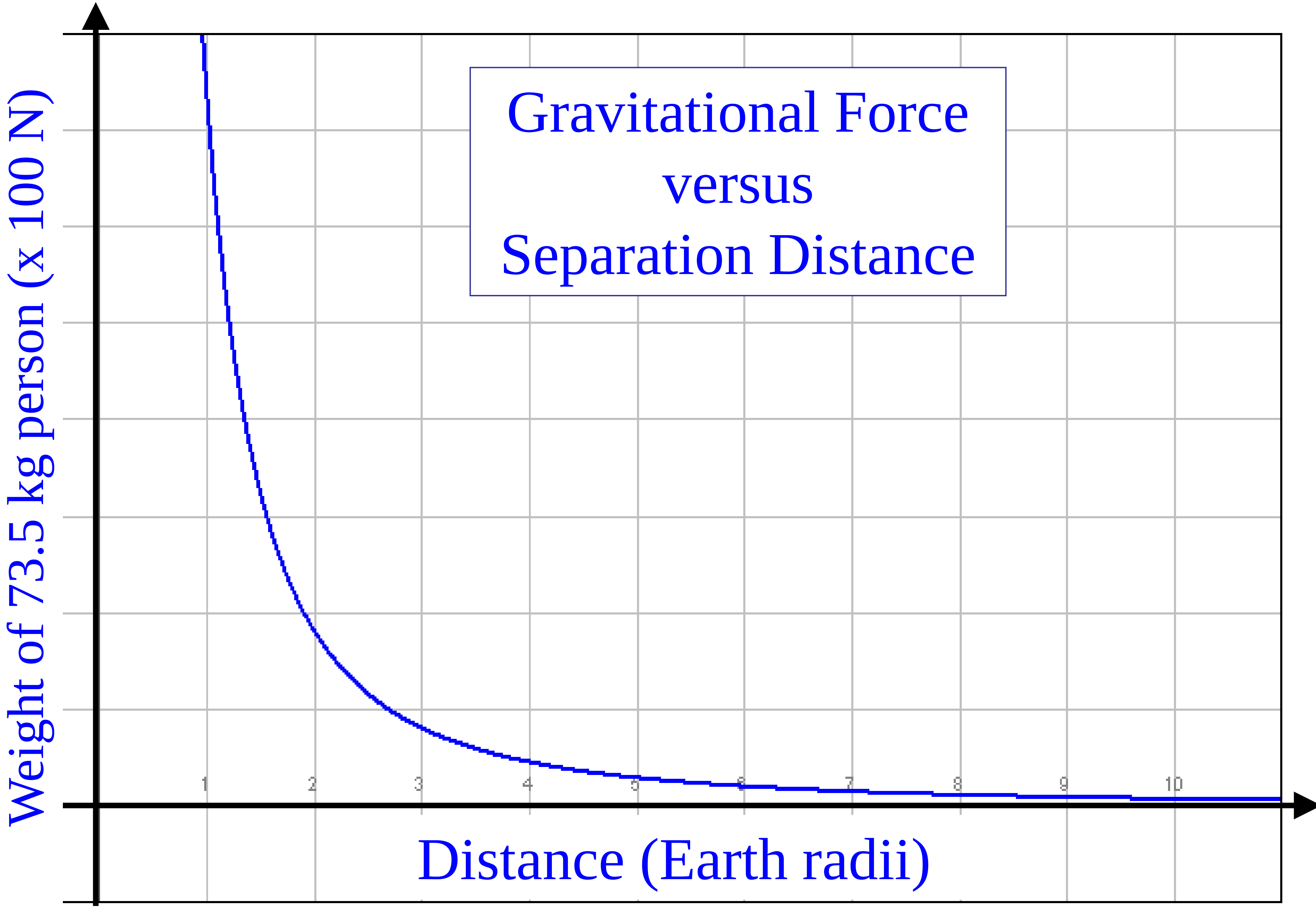

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What would happen to the person's freefall acceleration at greater and greater altitudes?

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Gravitational Field Strength, g

$$g = G\frac{M}{r^2}$$

where: M = mass of the field's *source*

r = distance between centers

G = universal gravitational constant:

$6.674 \times 10^{-11}\ \mathrm{m^3/kg\ s^2}$

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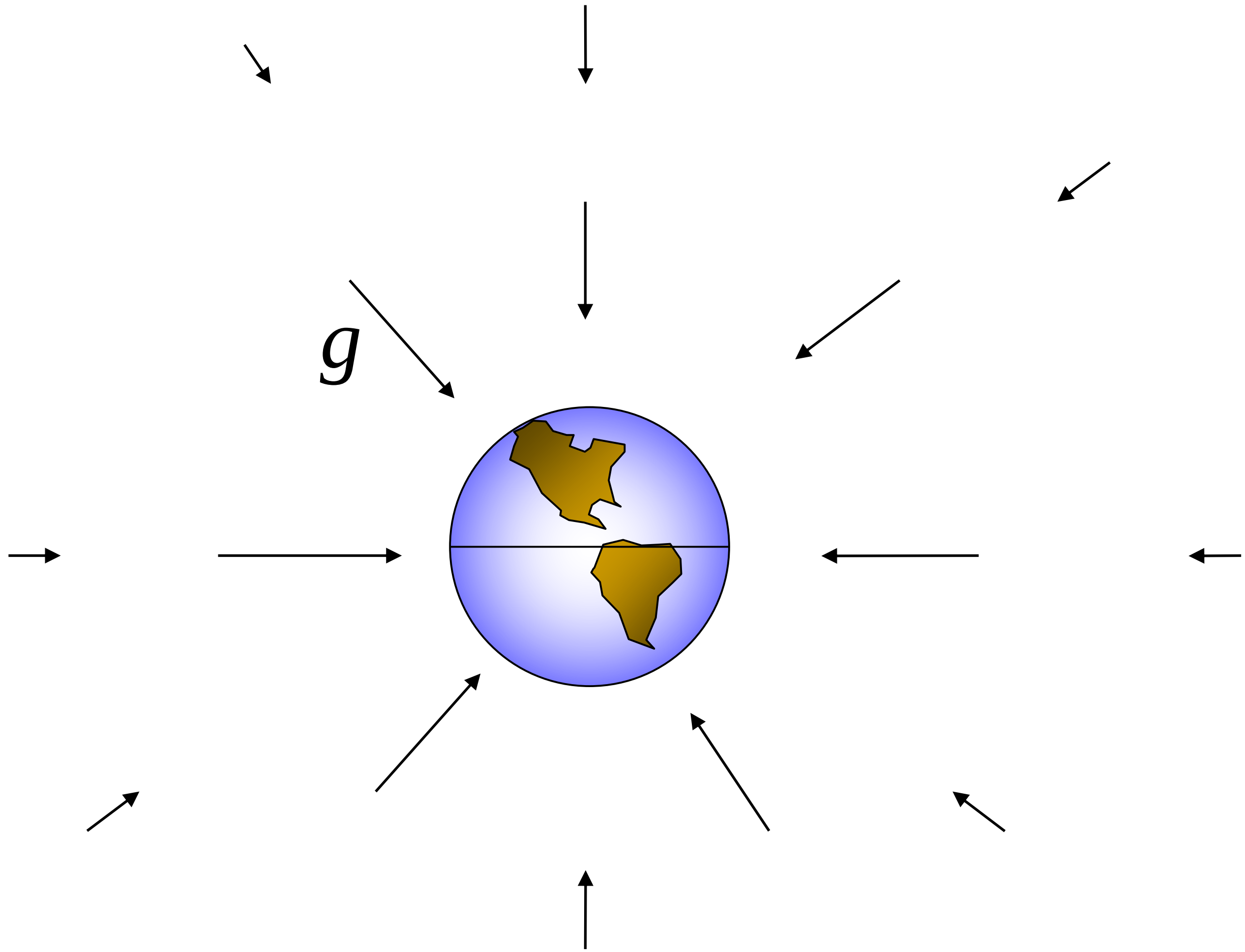
Earth's Gravitational Field



g = 9.80 N/kg at Earth's surface

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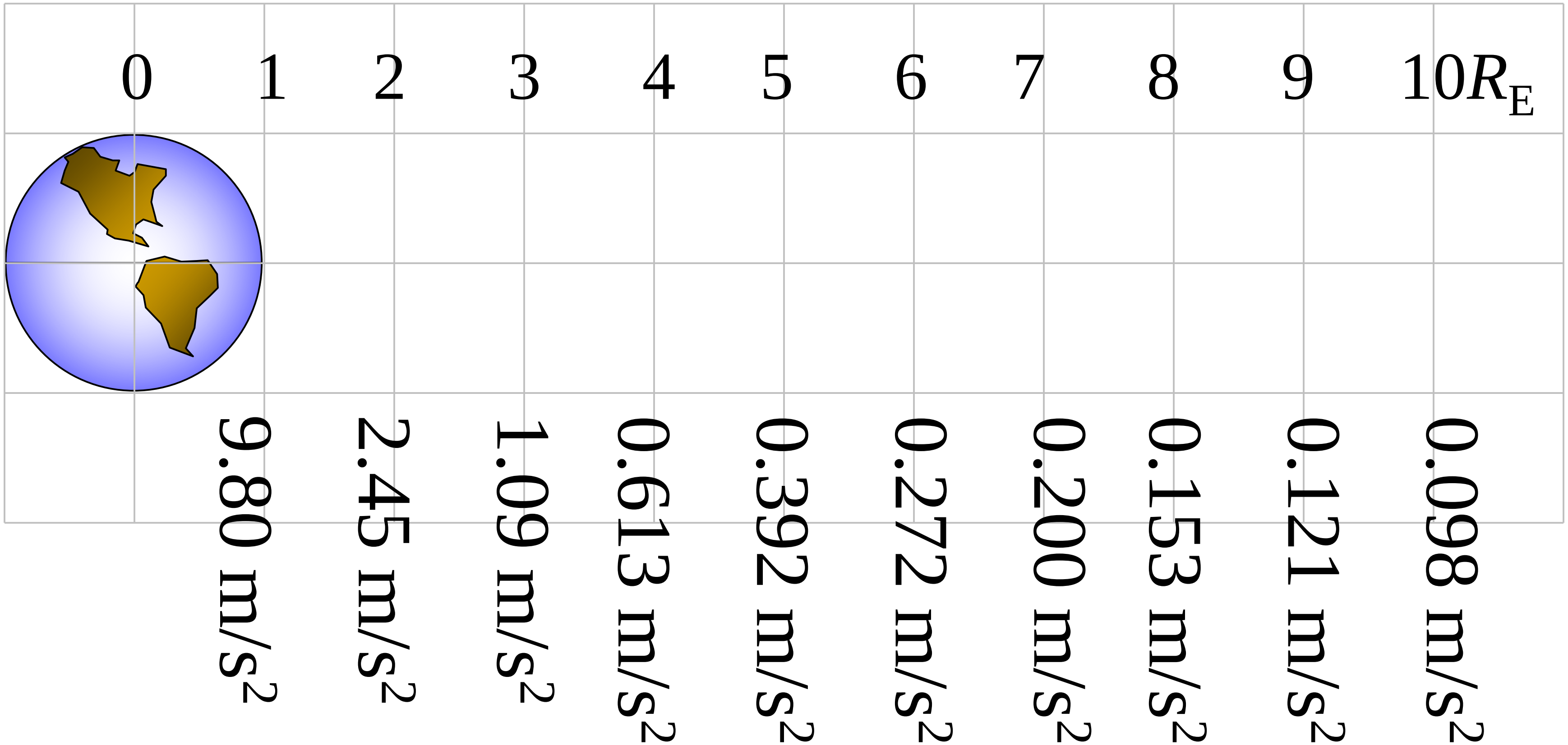
“Inverse Square Law” – Value of g



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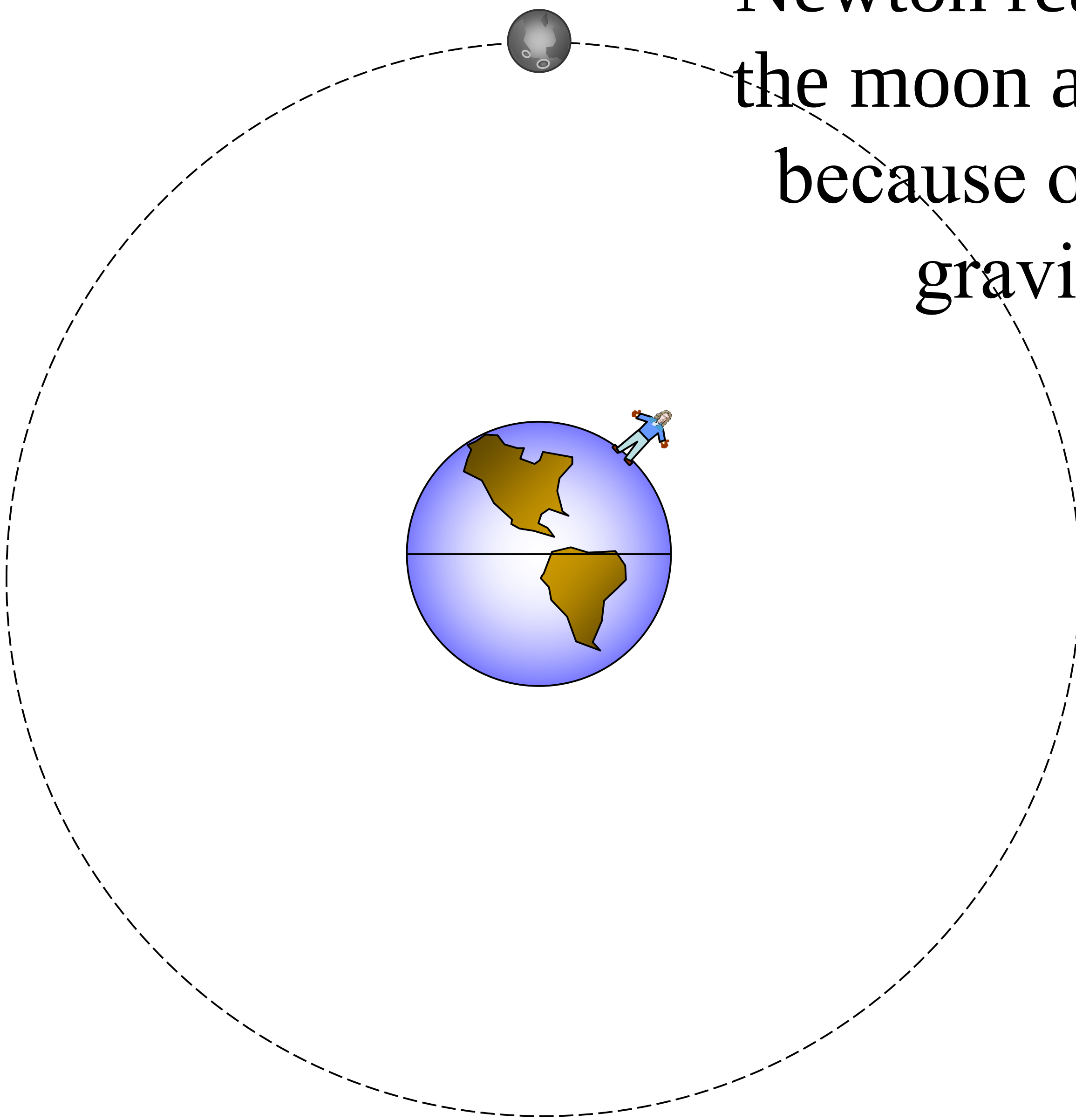
Newton realized that the moon accelerates because of Earth's gravity…



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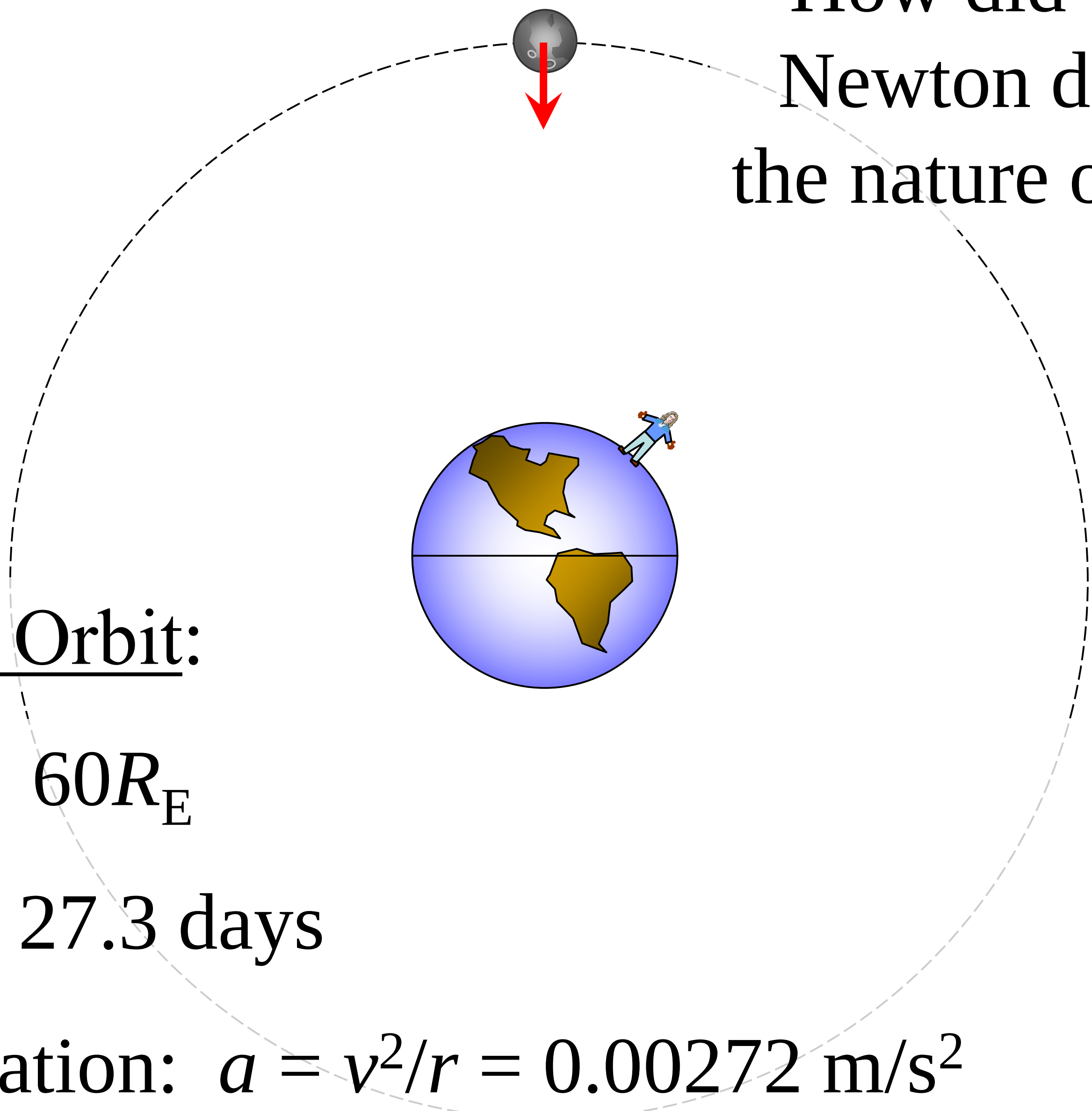
How did this help Newton determine the nature of gravity?



Moon's Orbit:

Radius: $60R_E$

Period: 27.3 days

Acceleration: $a = v^2/r = 0.00272\ \text{m/s}^2$

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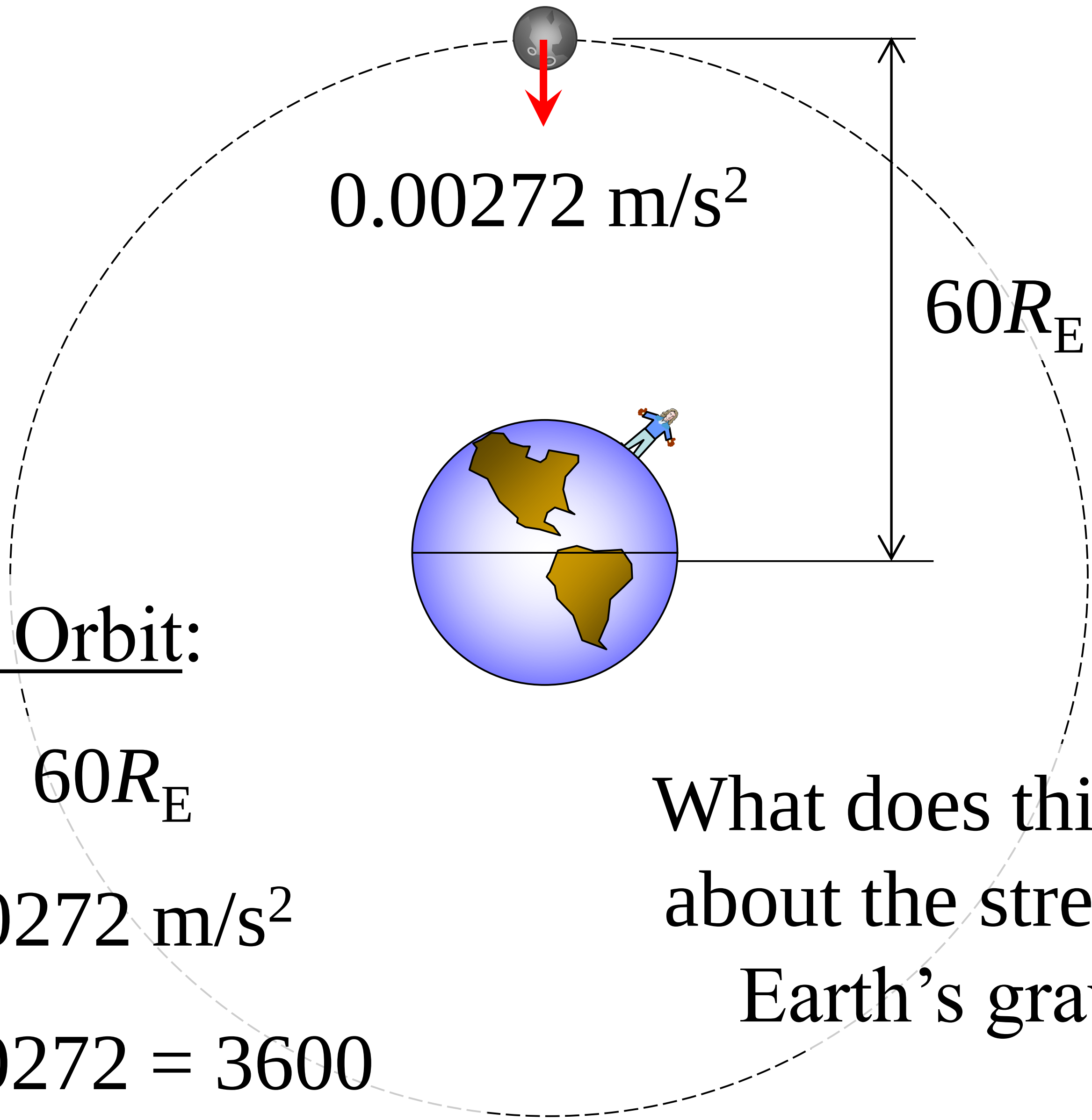
Even more accurate:



<u>Moon's Orbit</u>:

Radius: $60R_E$

$g = 0.00272\ \text{m/s}^2$

$9.8/0.00272 = 3600$

What does this imply about the strength of Earth's gravity?

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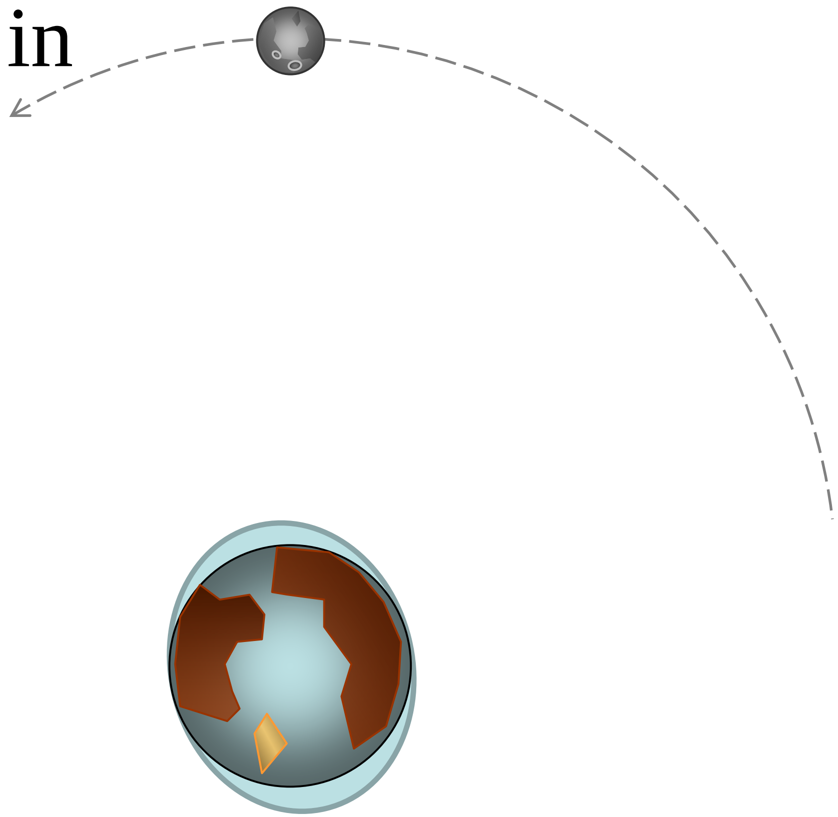
The Moon's gravity causes tidal bulges in the Earth's oceans:



One bulge is toward the Moon but there is also a bulge away from the Moon! Why?!

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Imagine a stack of boxes, initially at rest, released from a great altitude above Earth…

$$g = G\frac{M}{r^2}$$

The value of *g* is greatest for the green box and least for the yellow box. Because of this variance in the gravitational field the green accelerates away from the blue and the blue accelerates away from the yellow. The boxes will become separated more and more from one another as the distance to Earth decreases.

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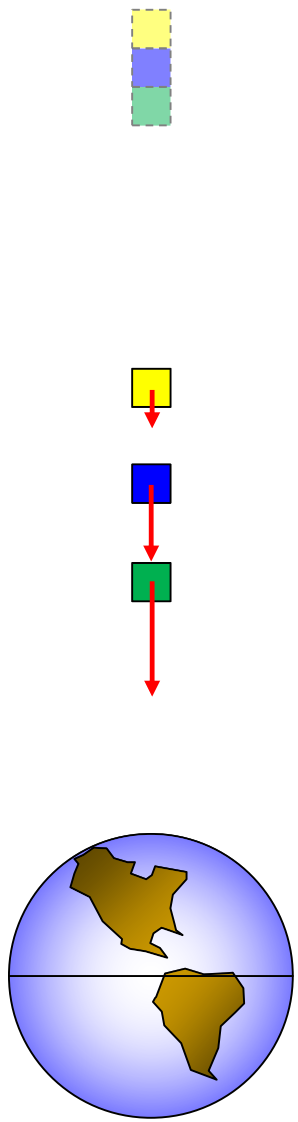
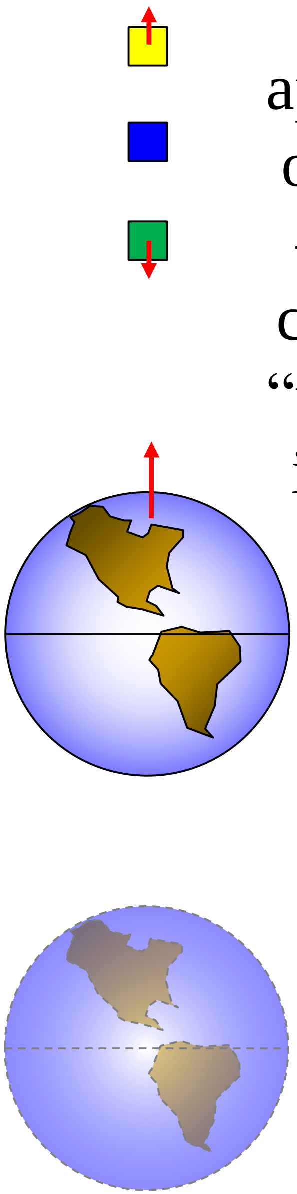
Relative to Earth:



Relative to the blue box:



In this frame of reference it appears there is a force pulling or stretching the objects apart – this is the "tidal effect". It can be viewed as a (fictitious) "tidal force". This same effect is responsible for the tides in Earth's ocean.

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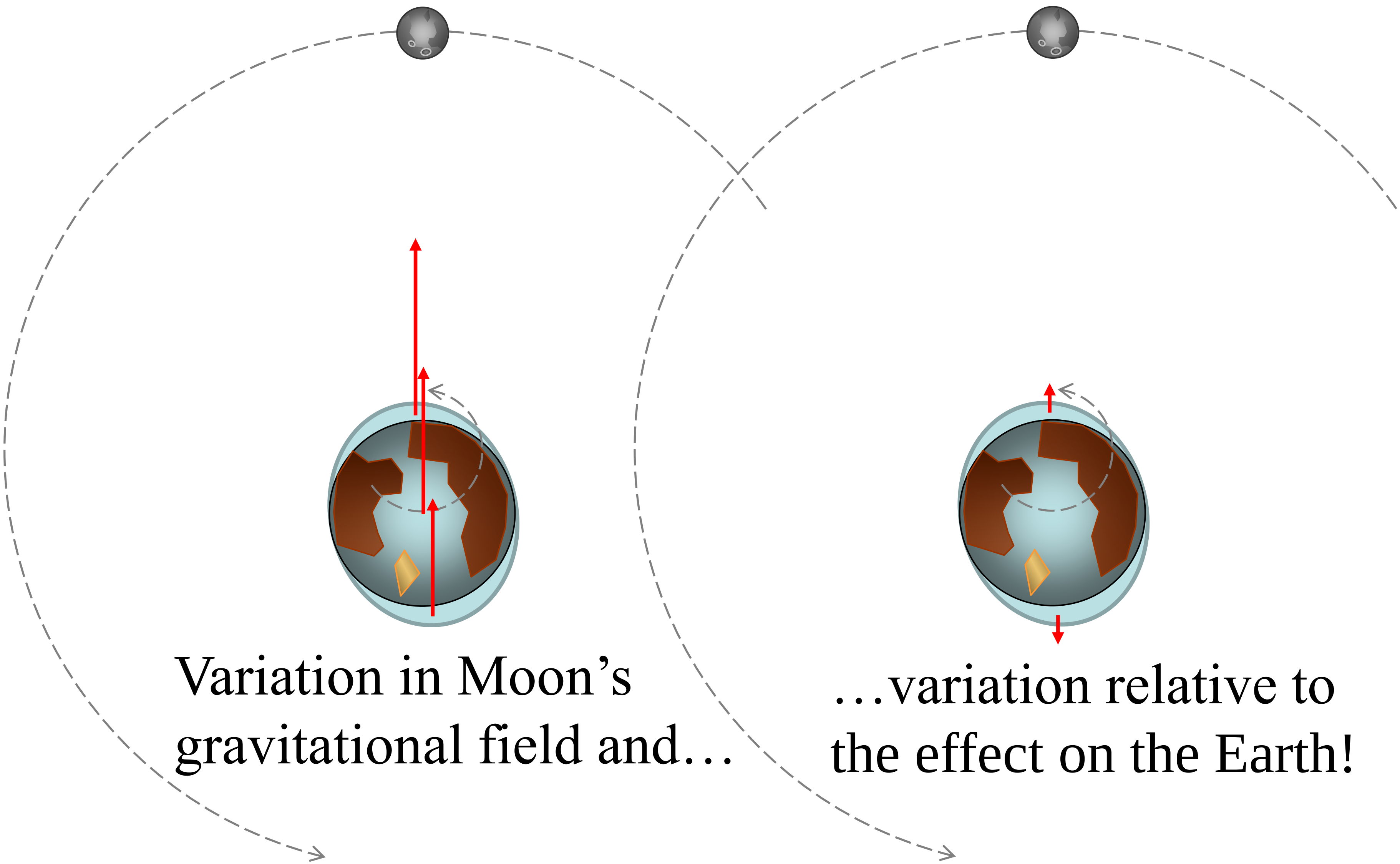

Moon's gravity causes Earth to wobble and the variation in its field raises tides in oceans on the near and far side of Earth!

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