Kepler's Laws

All you *need* to know and *MORE*

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	The student will be able to:	HW:
1	Solve problems of uniform circular motion involving period, frequency, speed, velocity, acceleration, force.	1 – 10
2	Distinguish, explain, and apply the concepts of centripetal and centrifugal force.	11 – 13
3	Solve problems of uniform circular motion or cycloid motion by use of parametric equations.	14 – 15
4	Solve problems of nonuniform circular motion involving constant rate of change in speed in which there are radial and tangential components of acceleration.	16 – 18
5	State and apply Newton's Law of Universal Gravitation.	19 – 23
6	Define and apply gravitational field strength.	24 – 28
7	Solve problems involving circular orbits.	29 – 34
8	State, apply, and derive Kepler's 3rd Law	35 – 36

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Kepler
1571 – 1630 AD

Johannes Kepler was a German astronomer and mathematician. He worked with Tycho Brahe for a brief period, and eventually took over Tycho's position.

Using Tycho's detailed planetary data, Kepler was able to determine 3 Laws of Planetary Motion that defined a new cosmological model.

This model is *<u>still in use</u>*!

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Kepler's Laws of Planetary Motion

1. The orbits of the planets are ellipses (not circles). The Sun is at a focus of each elliptical orbit.
2. An imaginary line connecting the Sun and any planet sweeps out equal areas of the ellipse in equal intervals of time.
3. The square of a planet's orbital period is proportional to the cube of the semi-major axis of its orbit.

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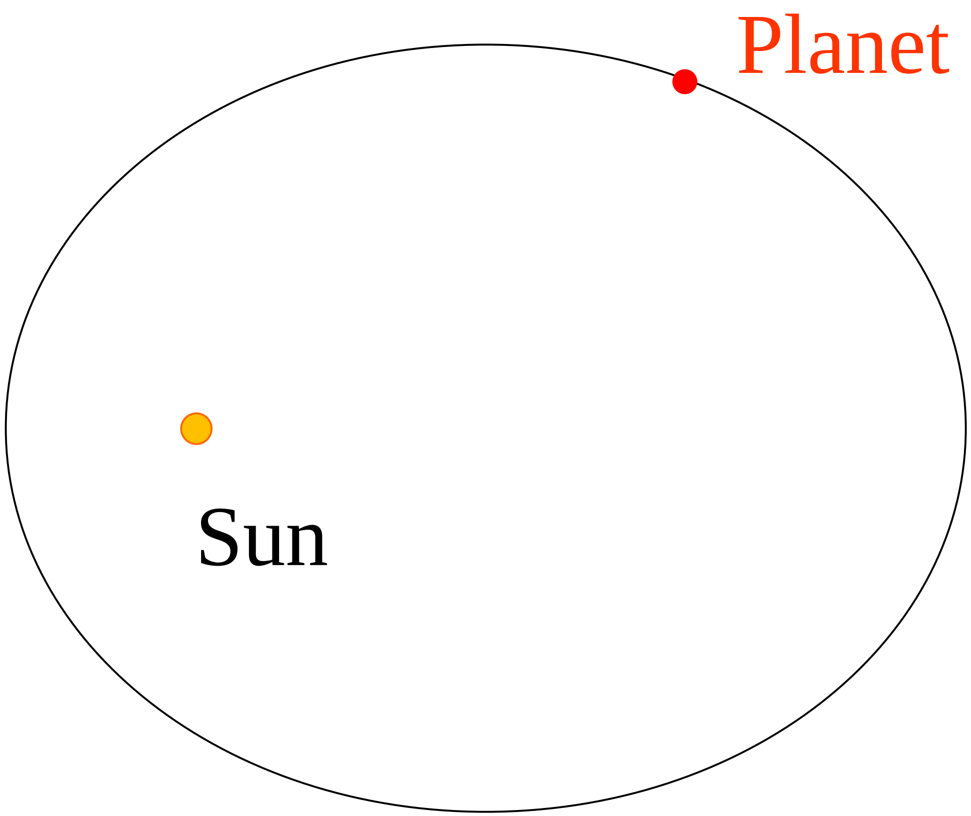
1st Law – Orbits are Elliptical with Sun at Focus



Gone is the Aristotlean idea of “perfect circles”!

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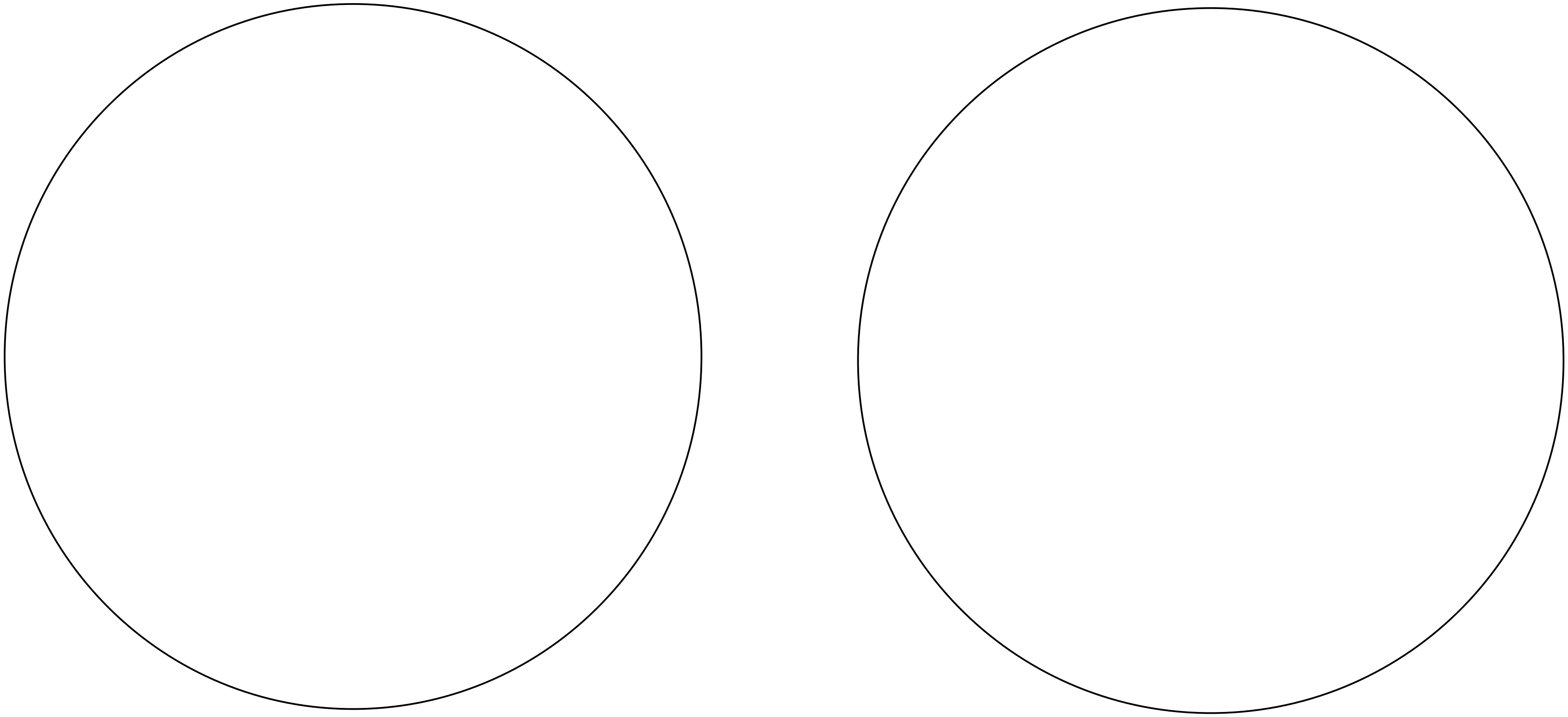

Which one is elliptical?

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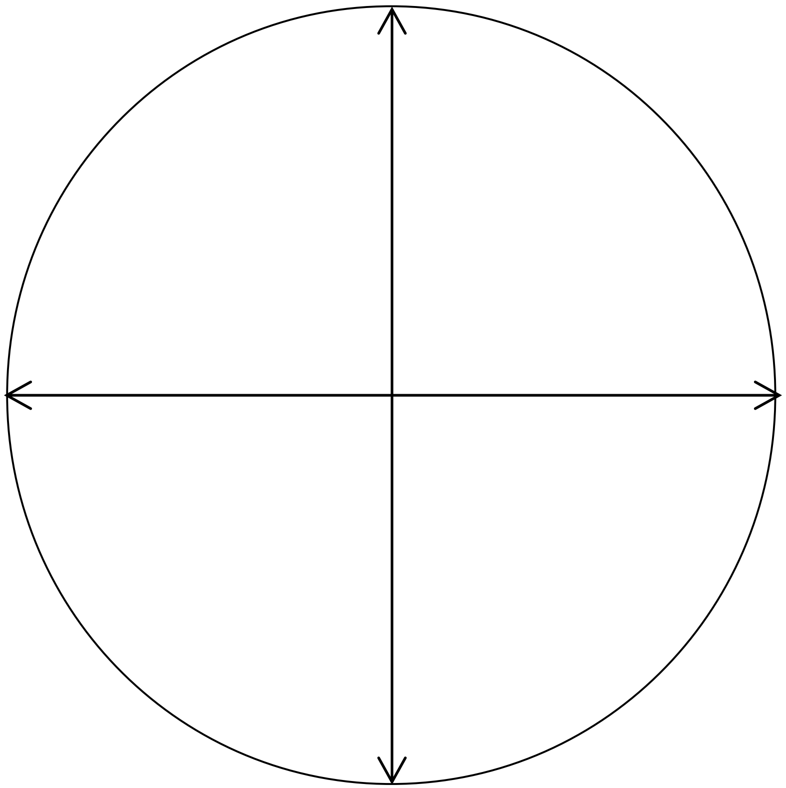
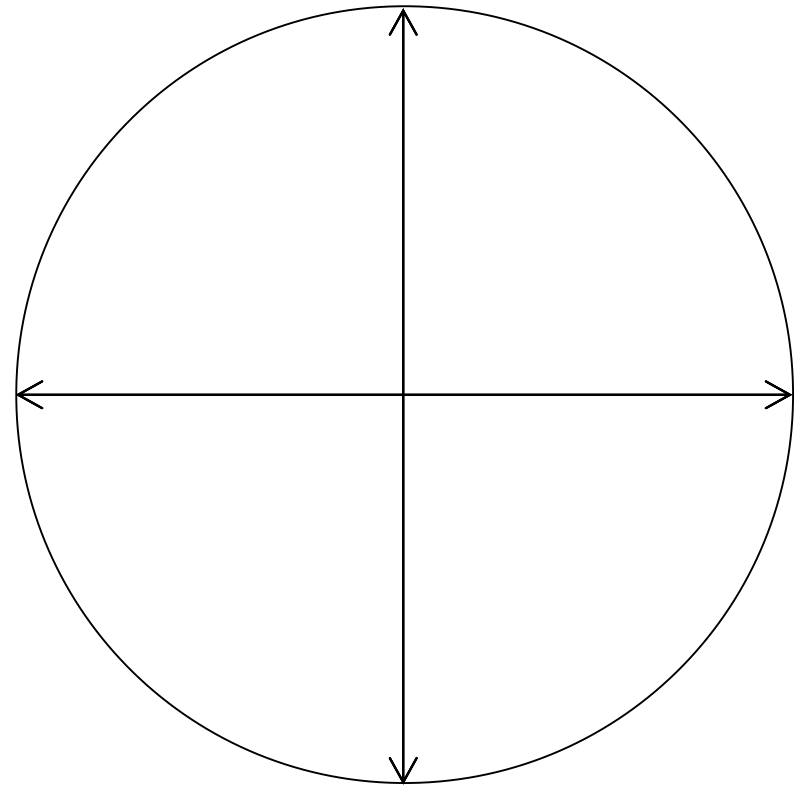
This one! (Barely!) Notice it is just a little taller than it is wide.

This is a perfect circle – not elliptical.





Amazingly Kepler was able to tell that orbits are not circular even though the discrepancy is about like what is shown here.

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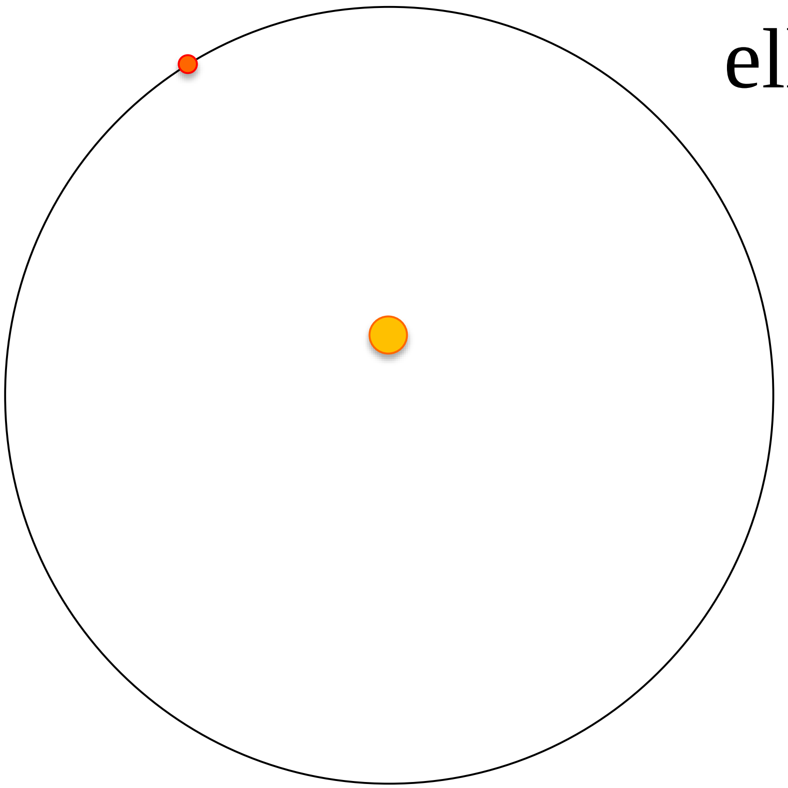
Kepler determined that Mars's orbit is not circular but rather elliptical – in spite of the very very small difference in shape!



The position of the Sun is noticeably off center when shown correct to scale.

Perhaps this helped Kepler to determine this orbit is not a circle but rather an ellipse.

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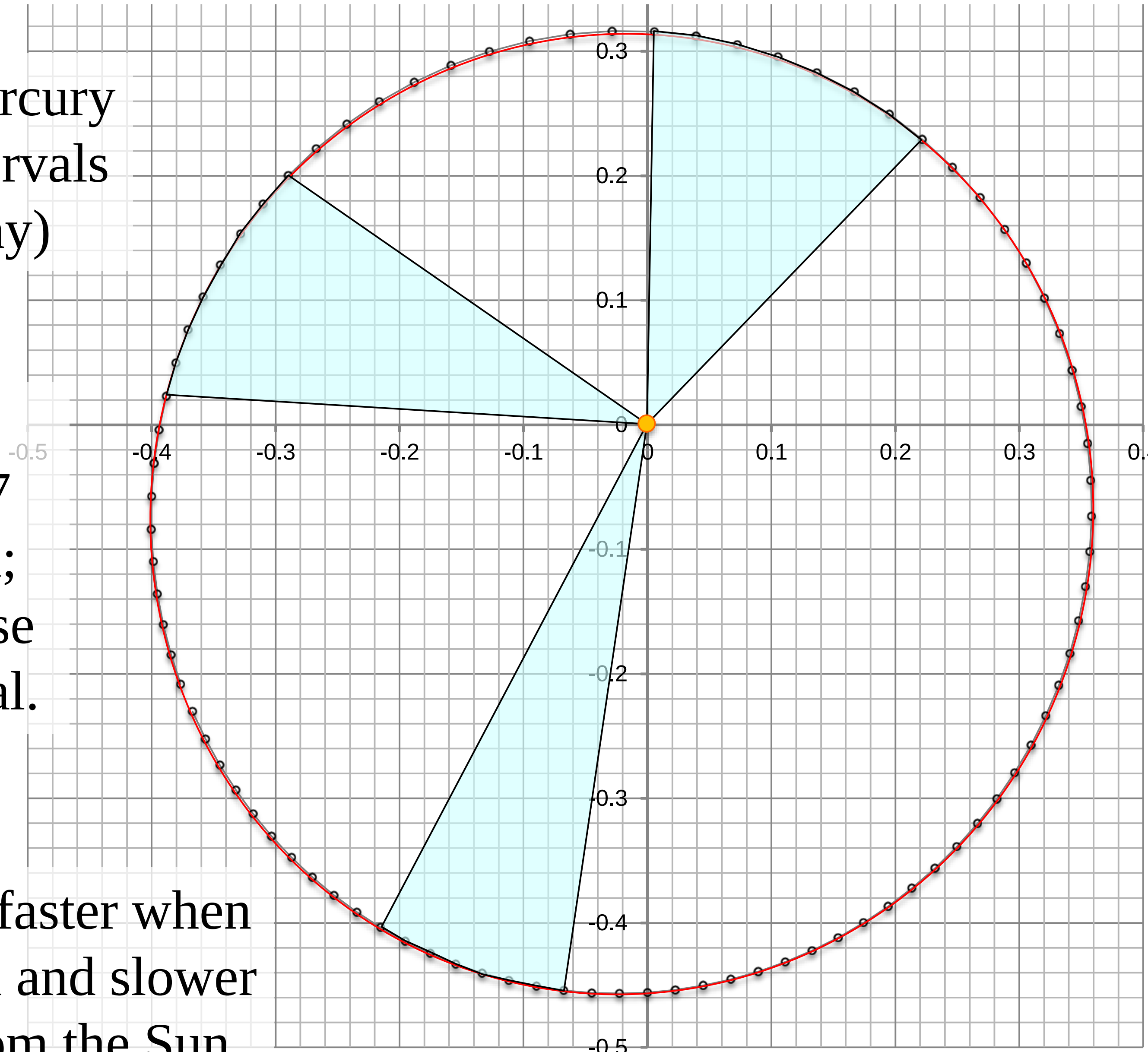
2nd Law – Sweeps Out Equal Area in Equal Time

Orbit of Mercury (dots at intervals of one day)



Each shaded region shows 7 days of motion; the areas of these regions are equal.

A planet moves faster when closer to the Sun and slower when farther from the Sun.

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3^{rd} Law – p^2 is proportional to a^3

	a (AU)	p (yr)	a^3	p^2
Mercury	0.387	0.241		
Venus	0.723	0.615		
Earth	1.00	1.00	1.00	1.00
Mars	1.52	1.88		
Jupiter	5.20	11.86		
Saturn	9.54	29.42		

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3^{rd} Law – p^2 is proportional to a^3

	a (AU)	p (yr)	a^3	p^2
Mercury	0.387	0.241	0.058	0.058
Venus	0.723	0.615	0.378	0.378
Earth	1.00	1.00	1.00	1.00
Mars	1.52	1.88	3.51	3.53
Jupiter	5.20	11.86	140.6	140.7
Saturn	9.54	29.42	868.3	865.5

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3^{rd} Law – p^2 is proportional to a^3

Note that this law is different than the first two because it deals with *all* of the planets, not the properties of an *individual* planet.

In simple terms, this law indicates that the farther from the Sun the more time it takes for an object to complete its orbit.

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More on Kepler

- Although Kepler worked out the *relative* sizes of the orbits, he was unable to determine the *absolute* sizes of the orbits.
- Taking the semi-major axis of Earth's orbit to be "one", he was able to determine other orbits as a multiple of this.
- This is the basis for the astronomical unit.
- The actual distance of 1 AU was first determined around 1761 by transit of Venus and then in 1964 by radar.

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More on Kepler

- As impressive and (still) useful as Kepler's Laws are, there is still something lacking...
- Kepler's Laws are *empirical*. This means that his laws are based purely on observation, with no underlying theory explaining *why*. (It is the *effect*, without the *cause*.)
- Kepler's Laws could be described as very technical and specific observations.

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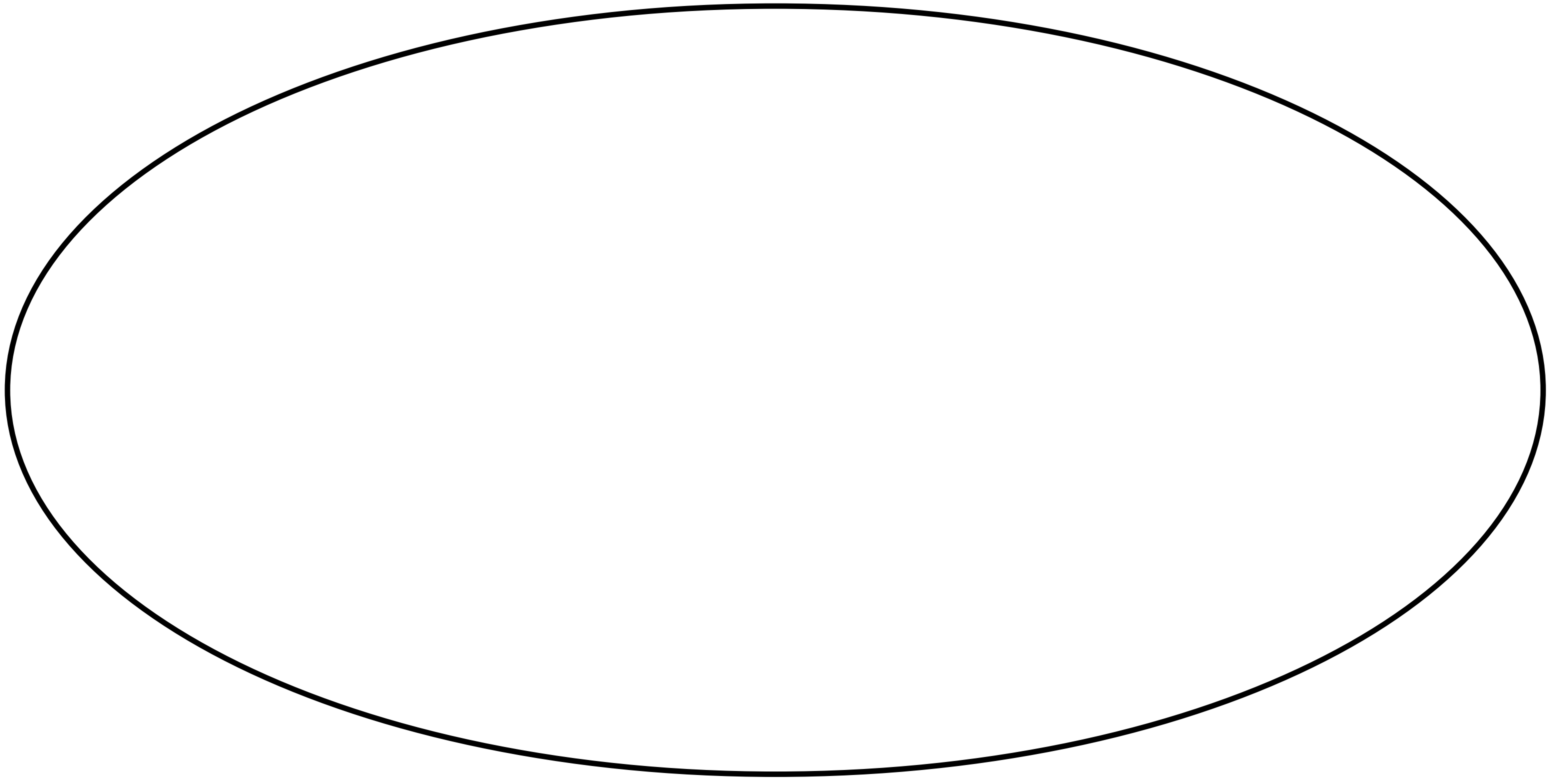

An ellipse is a unique oval shape
(not just *any* oval shape).

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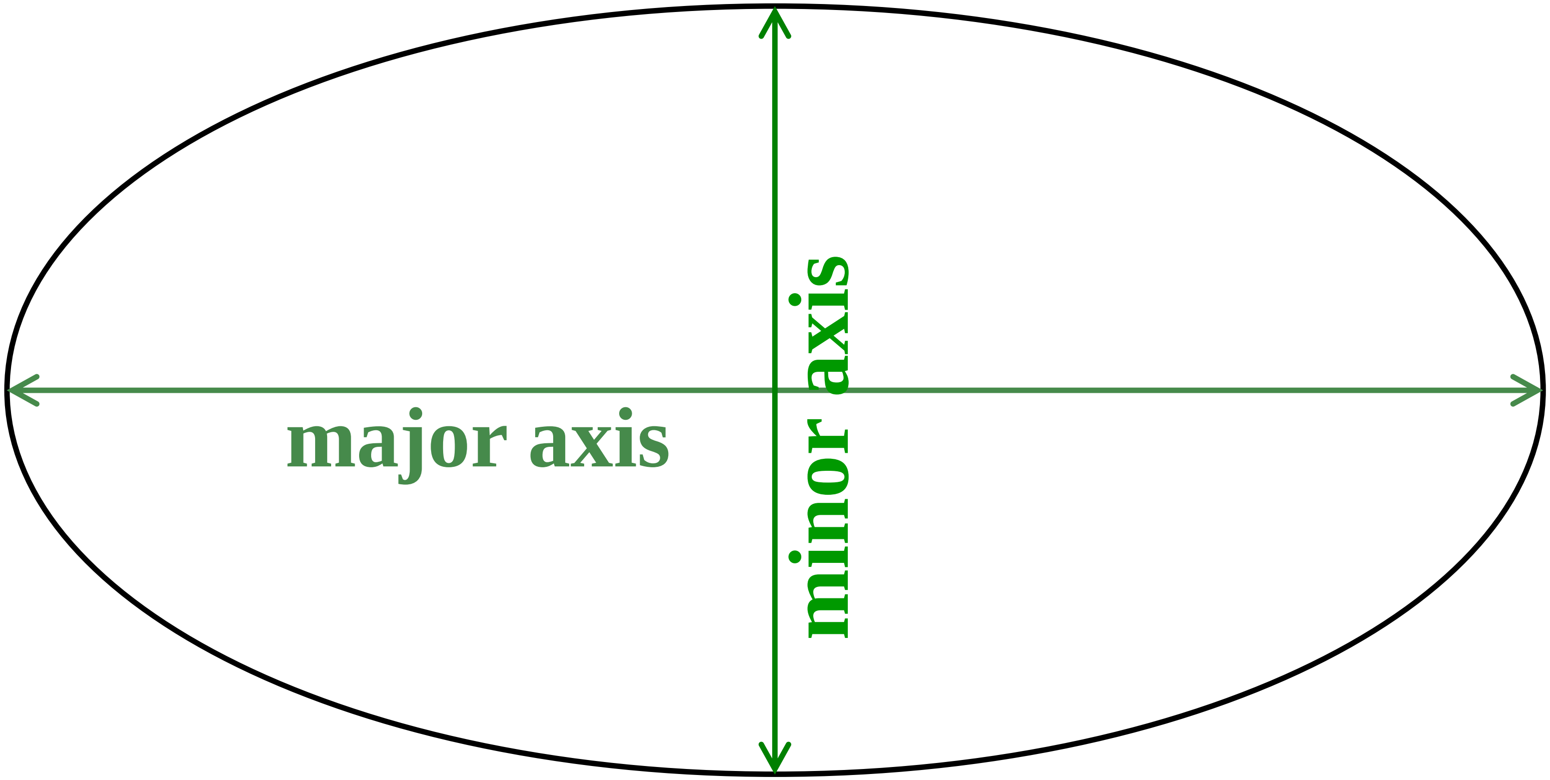

The “dimensions” of an ellipse are called the major axis and minor axis.

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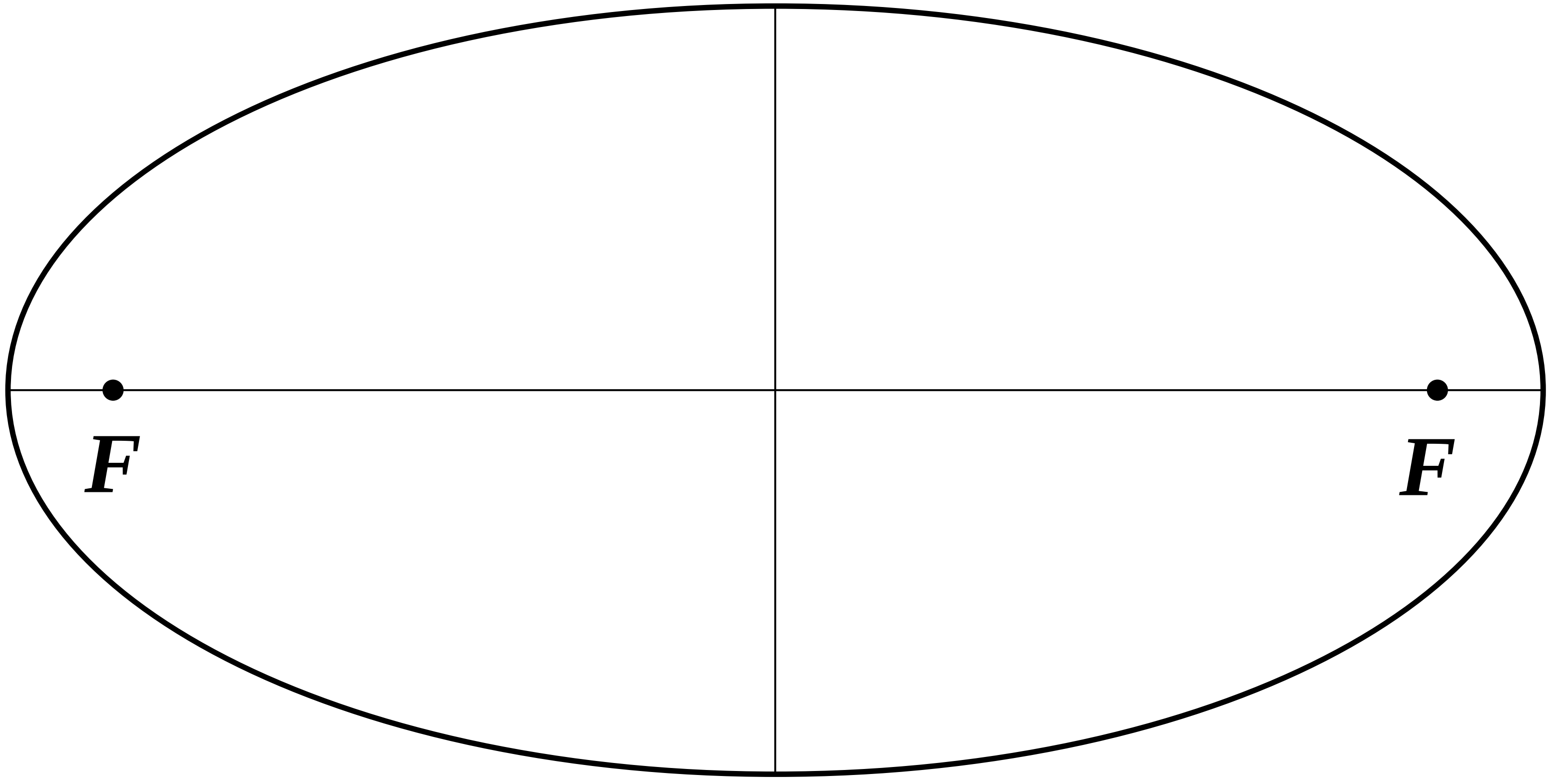

There are two unique points called foci.
Each focus is located on the major axis.

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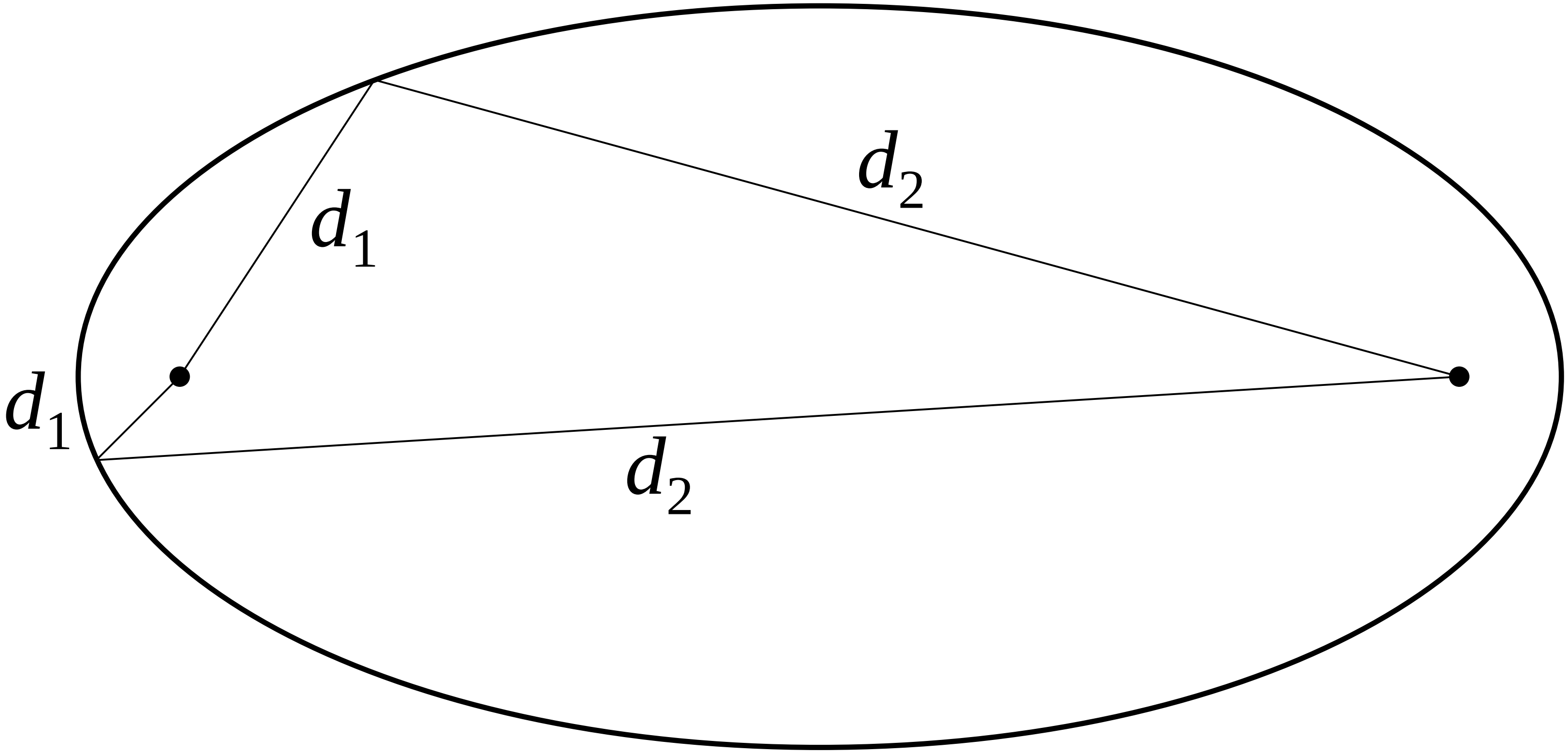

The sum of the distances to the foci is the same for any and every point on the ellipse.

$$d_1 + d_2 = 2a = \text{constant}$$

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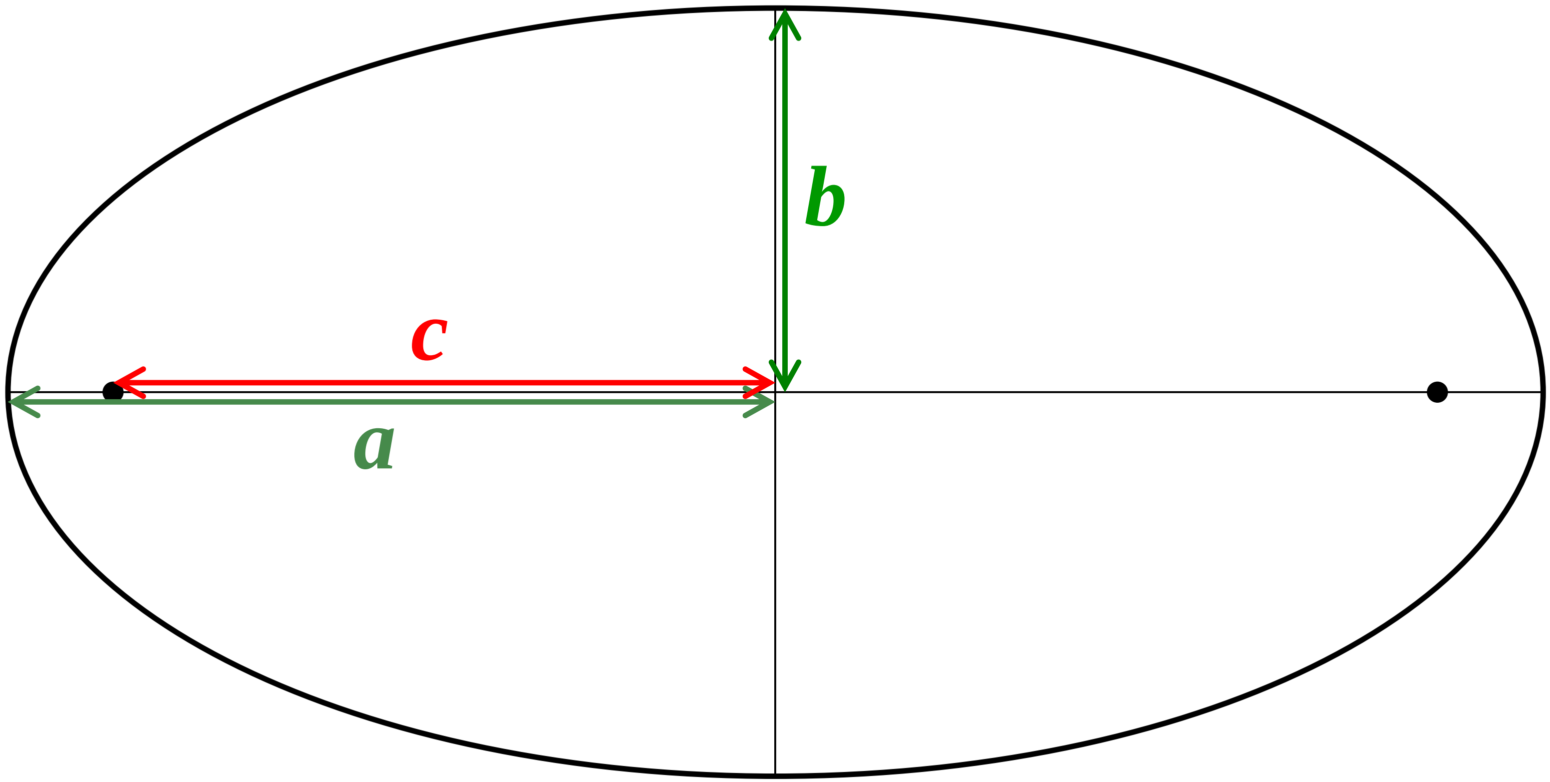

a = semi-major axis
b = semi-minor axis
c = distance from center to focus

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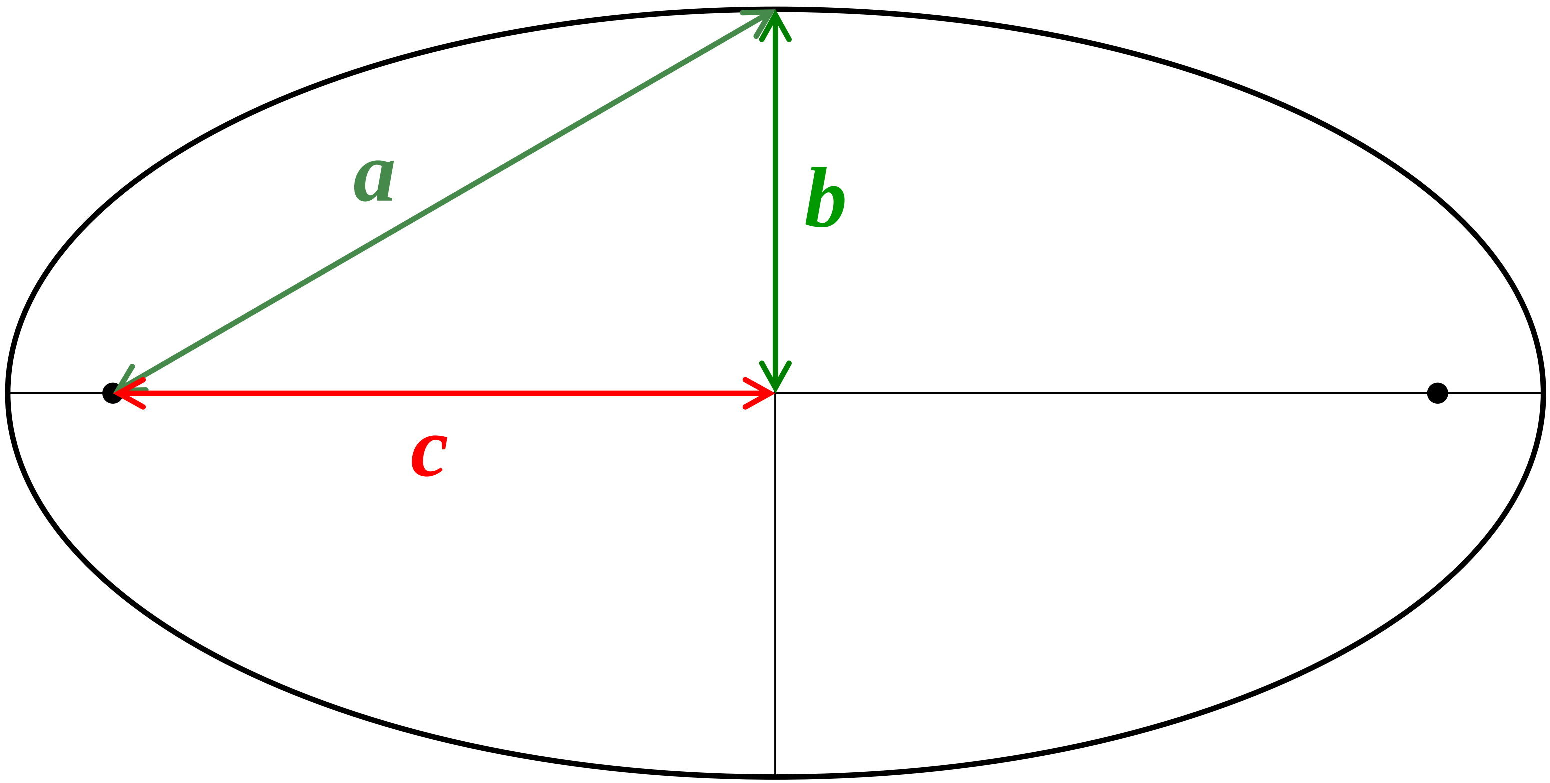

a = semi-major axis

b = semi-minor axis

c = distance from center to focus

note: $a^2 = b^2 + c^2$

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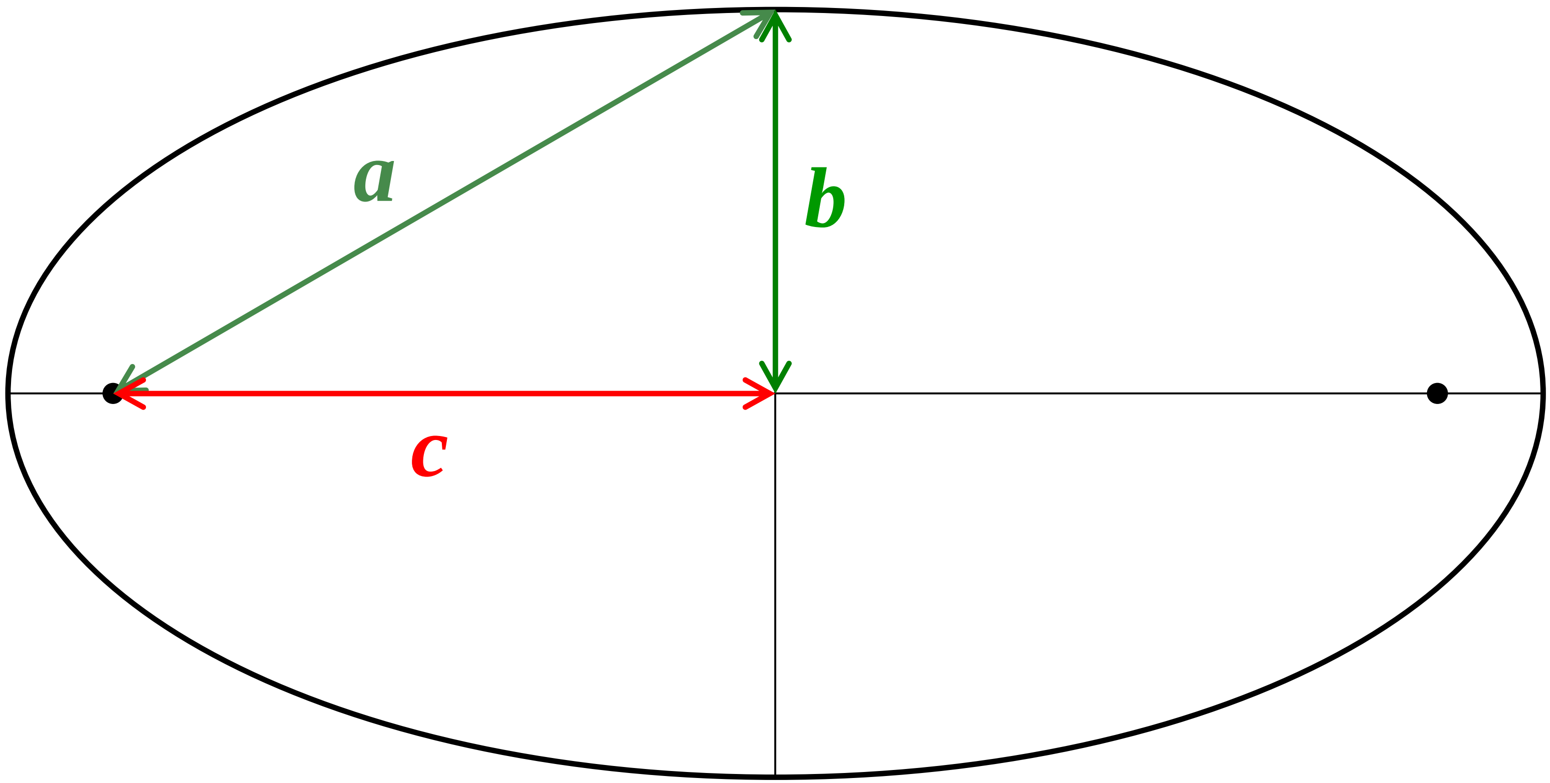

The eccentricity, *e*, of the ellipse: $e = \dfrac{c}{a}$

The value of *e* relates to how elongated the ellipse is and how far off-center the focii are located…

$0 < e < 1$

(closer to circular… …closer to parabolic)

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Example Eccentricities

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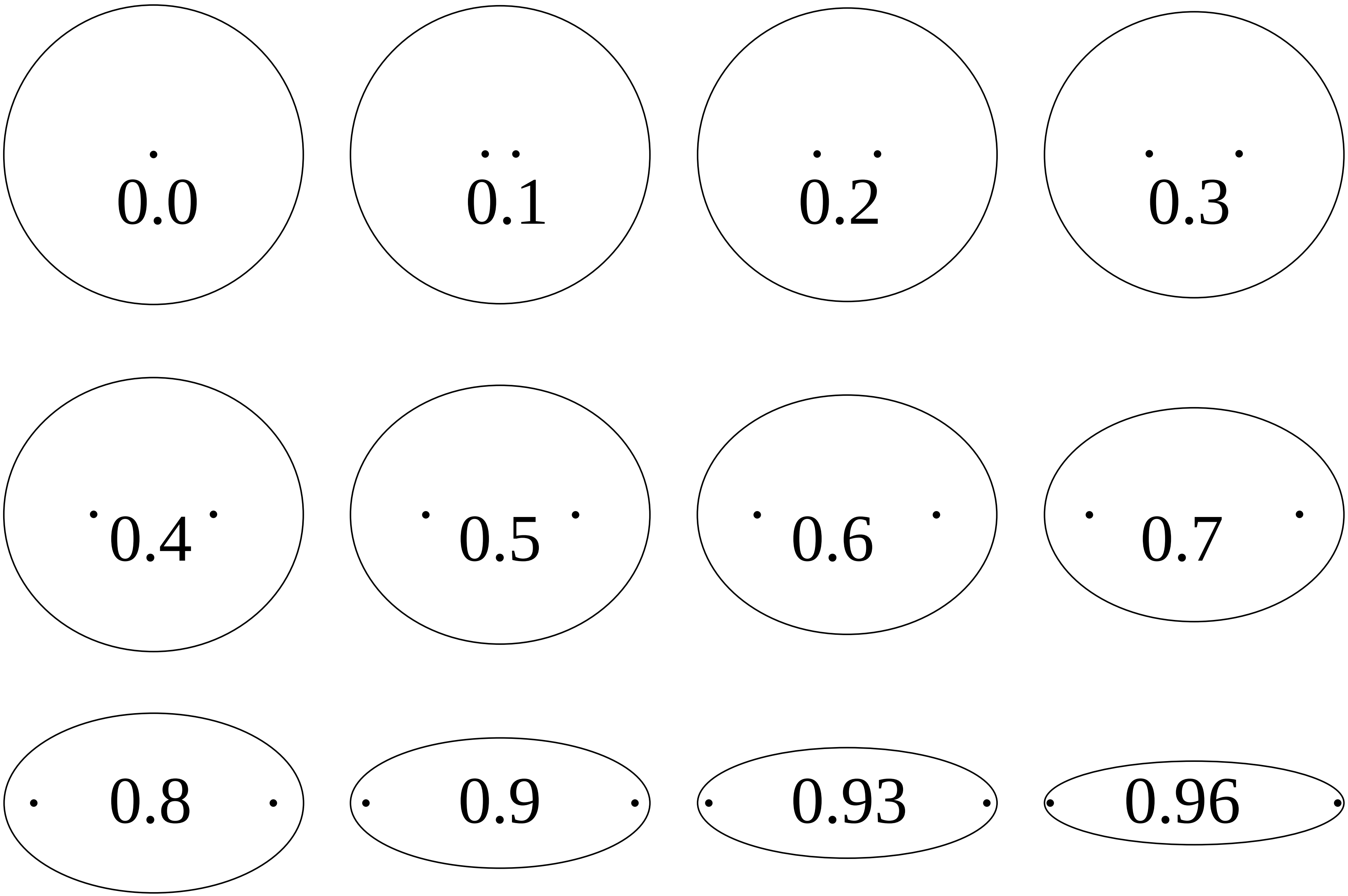
Example Eccentricities



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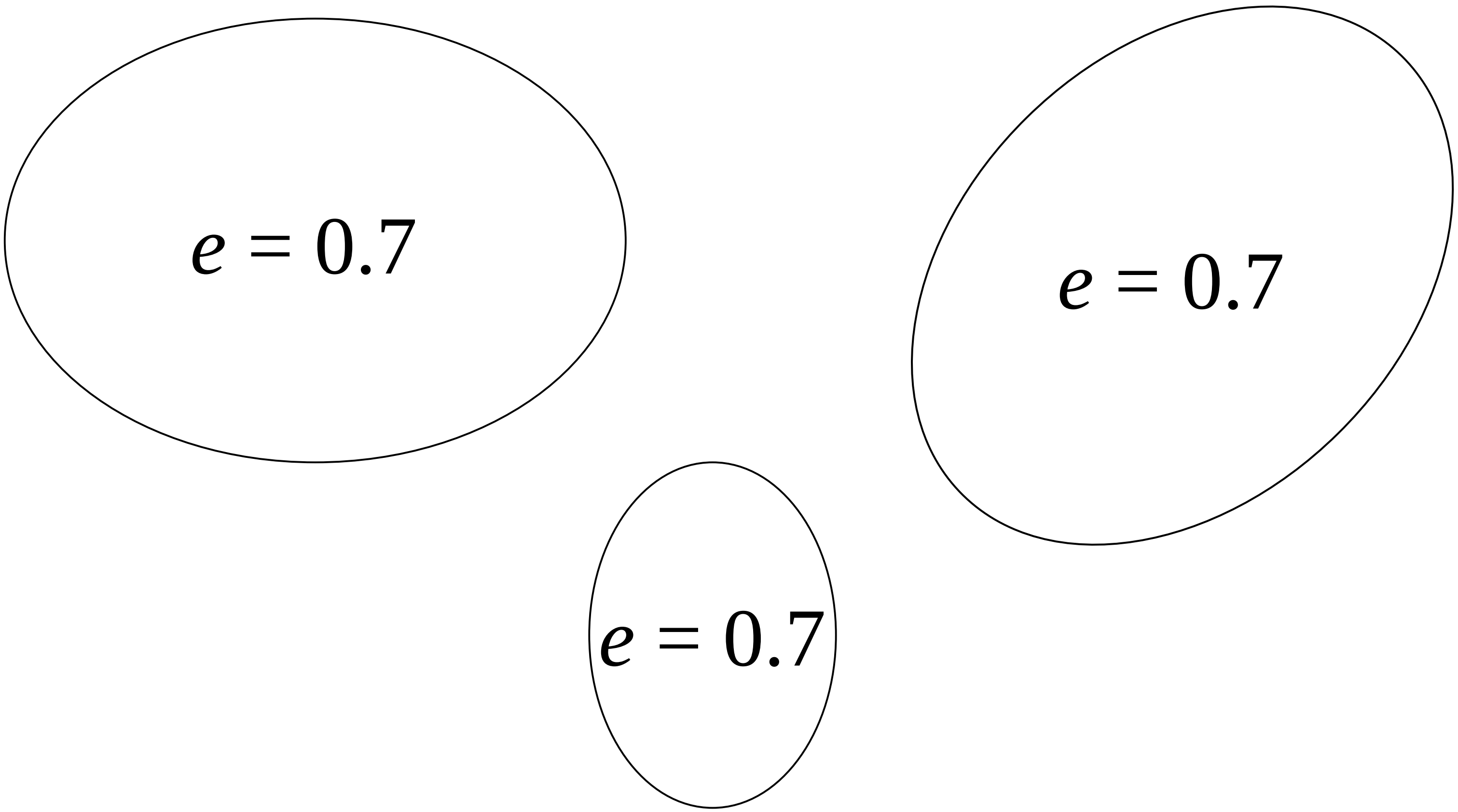
Eccentricity is not a measure of the *size* of an ellipse or its *orientation* – it depends only on the proportions of its dimensions.



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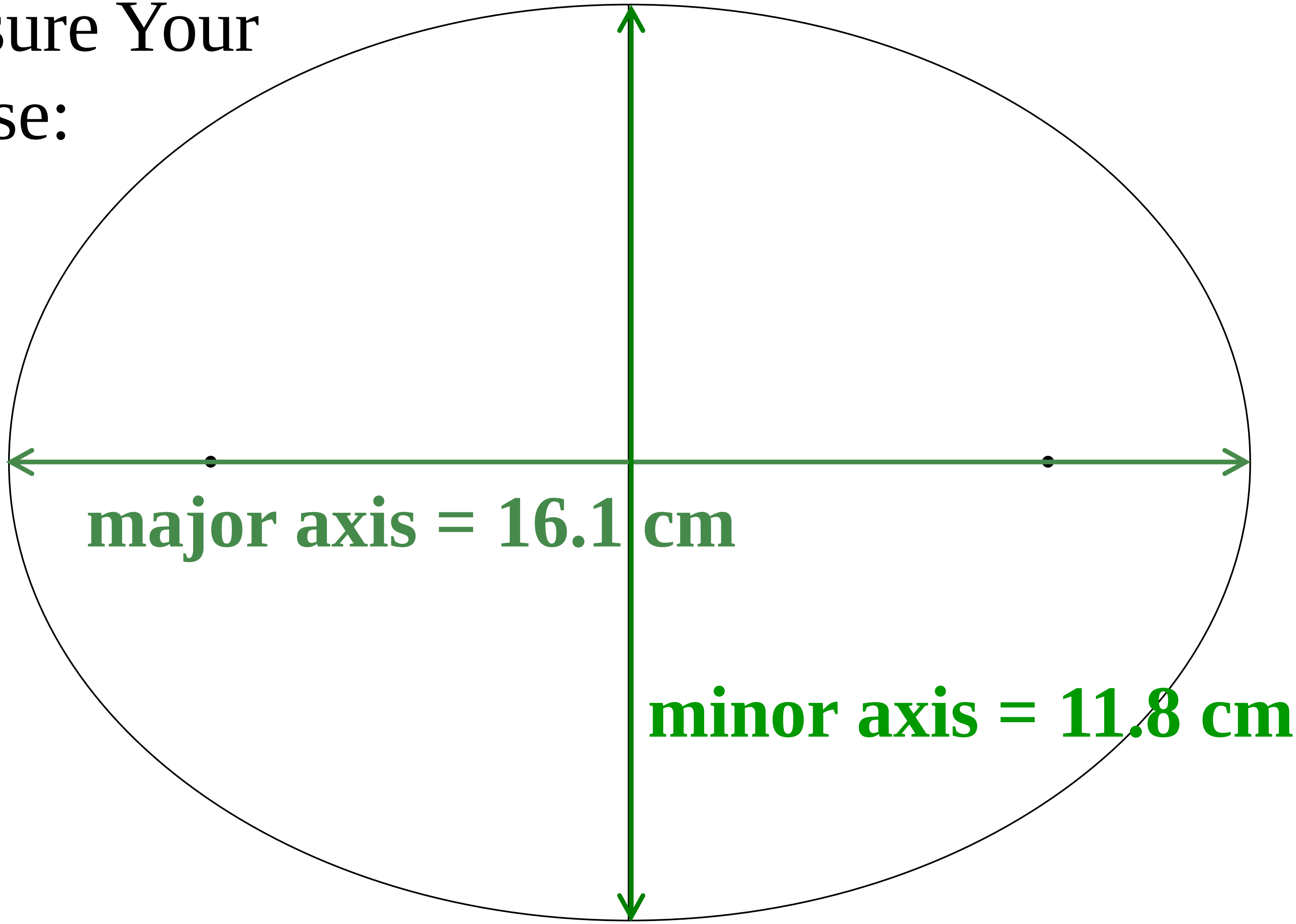
Measure Your Ellipse:



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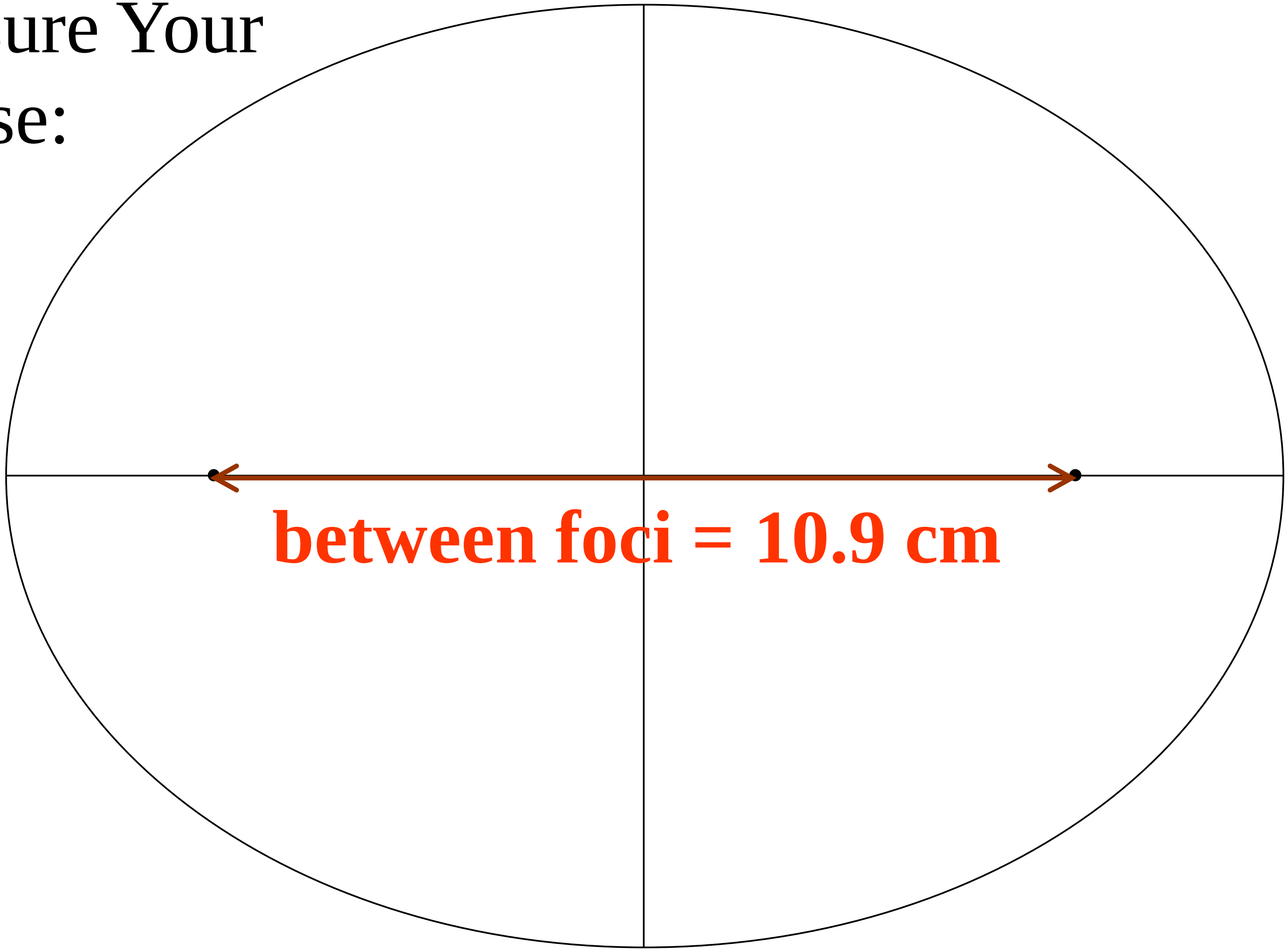
Measure Your Ellipse:



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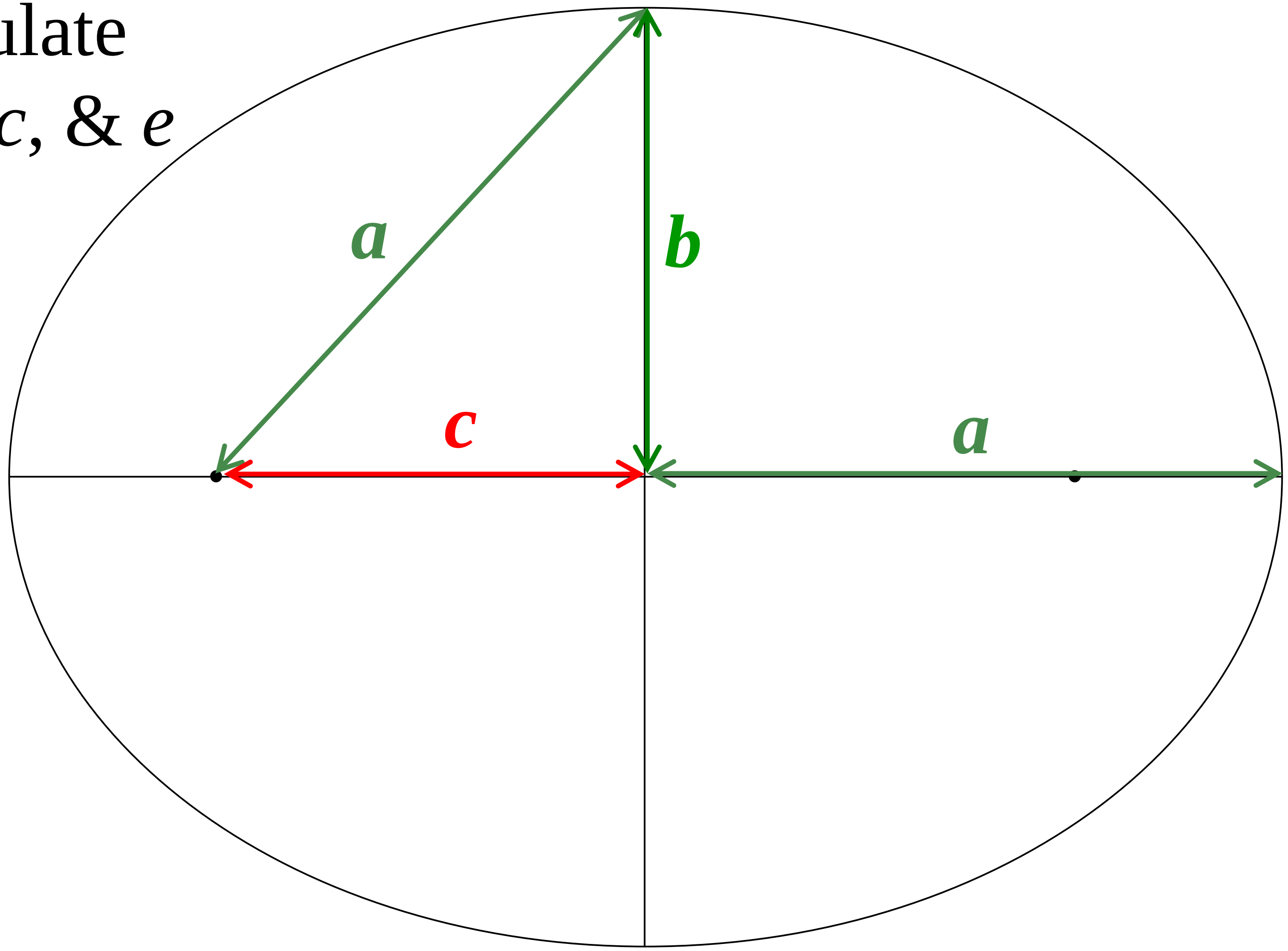
Calculate

a, *b*, *c*, & *e*



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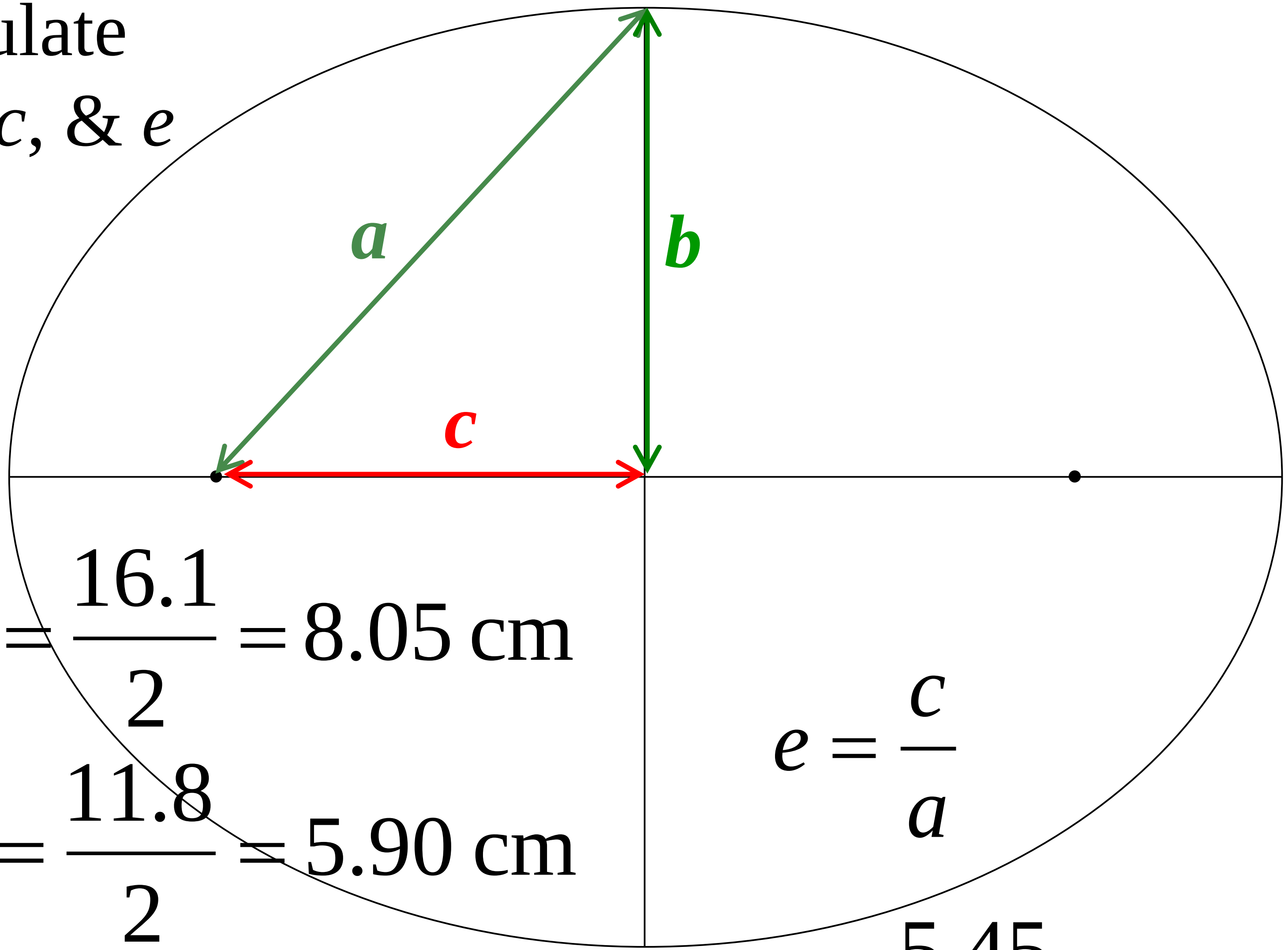
Calculate
a, *b*, *c*, & *e*



$$a = \frac{16.1}{2} = 8.05\ \text{cm}$$

$$b = \frac{11.8}{2} = 5.90\ \text{cm}$$

$$c = \frac{10.9}{2} = 5.45\ \text{cm}$$

$$e = \frac{c}{a}$$

$$e = \frac{5.45}{8.05} = 0.677$$

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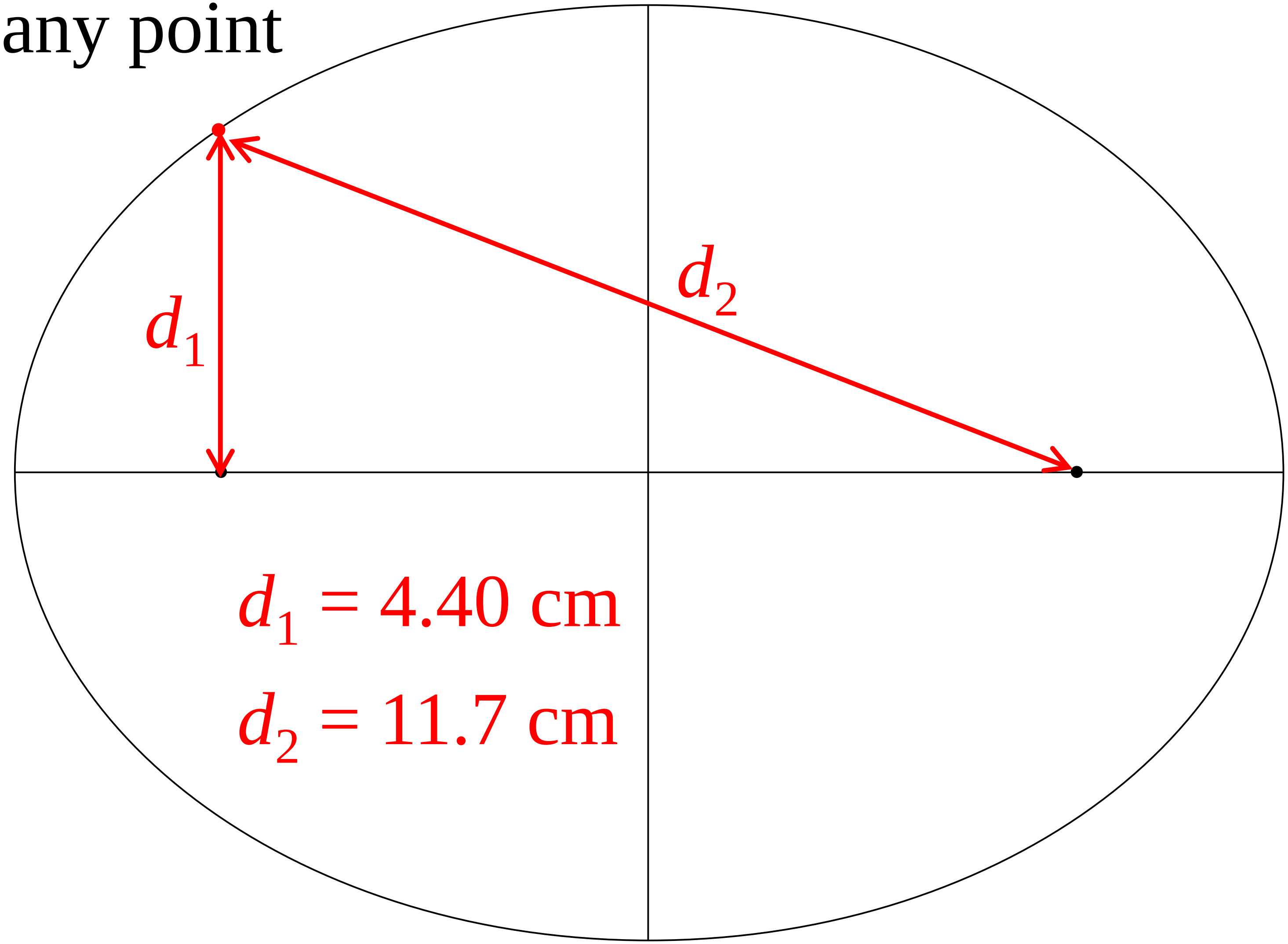
Pick any point



$d_1 = 4.40$ cm

$d_2 = 11.7$ cm

$d_1 + d_2 = 11.7 + 4.40 = 16.1$ cm

$d_1 + d_2 = 2a$ for any point on ellipse!

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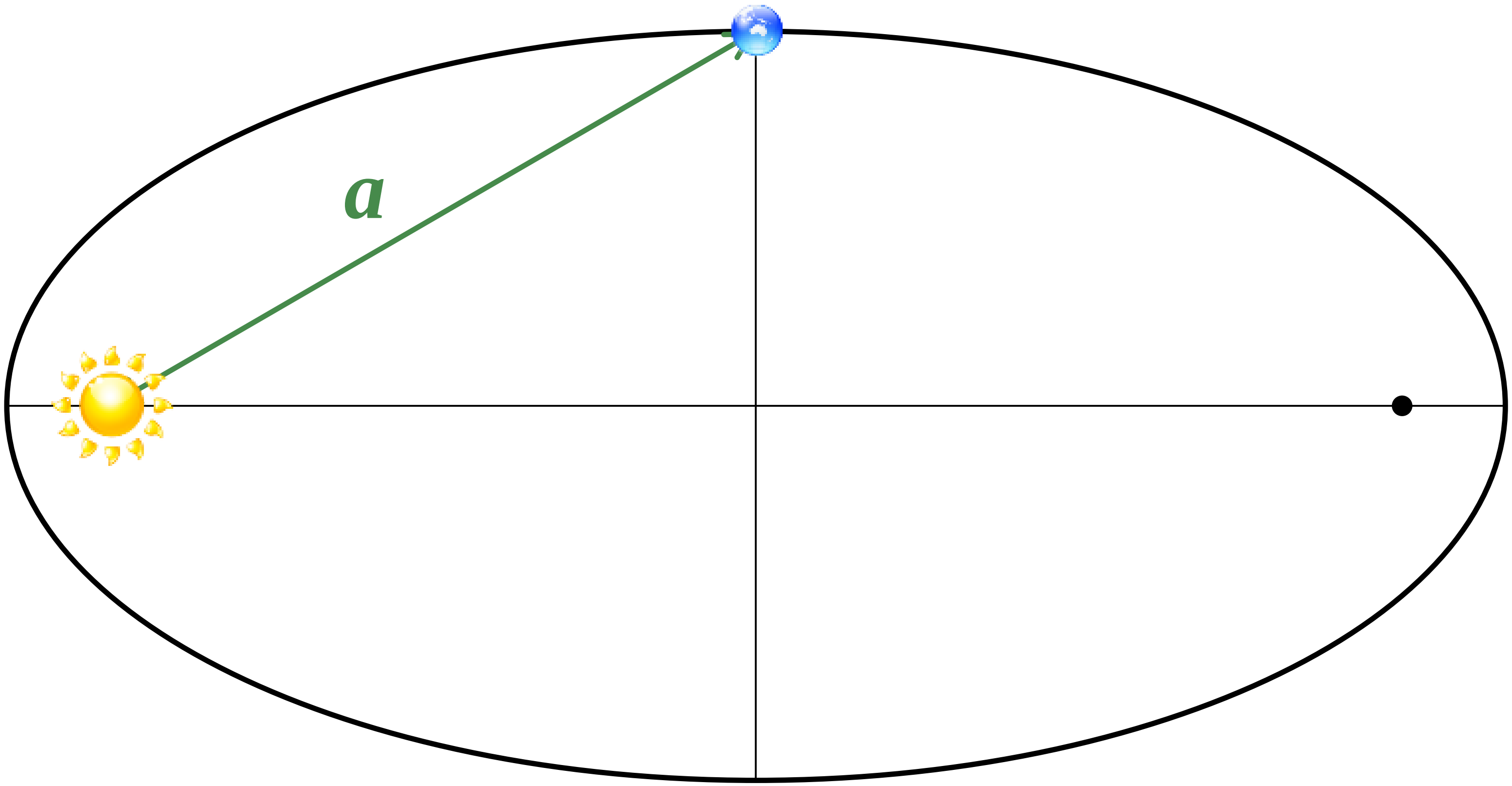

As applied to Kepler's Laws:
Sun is at one focus
Semi-major axis = average distance from Sun

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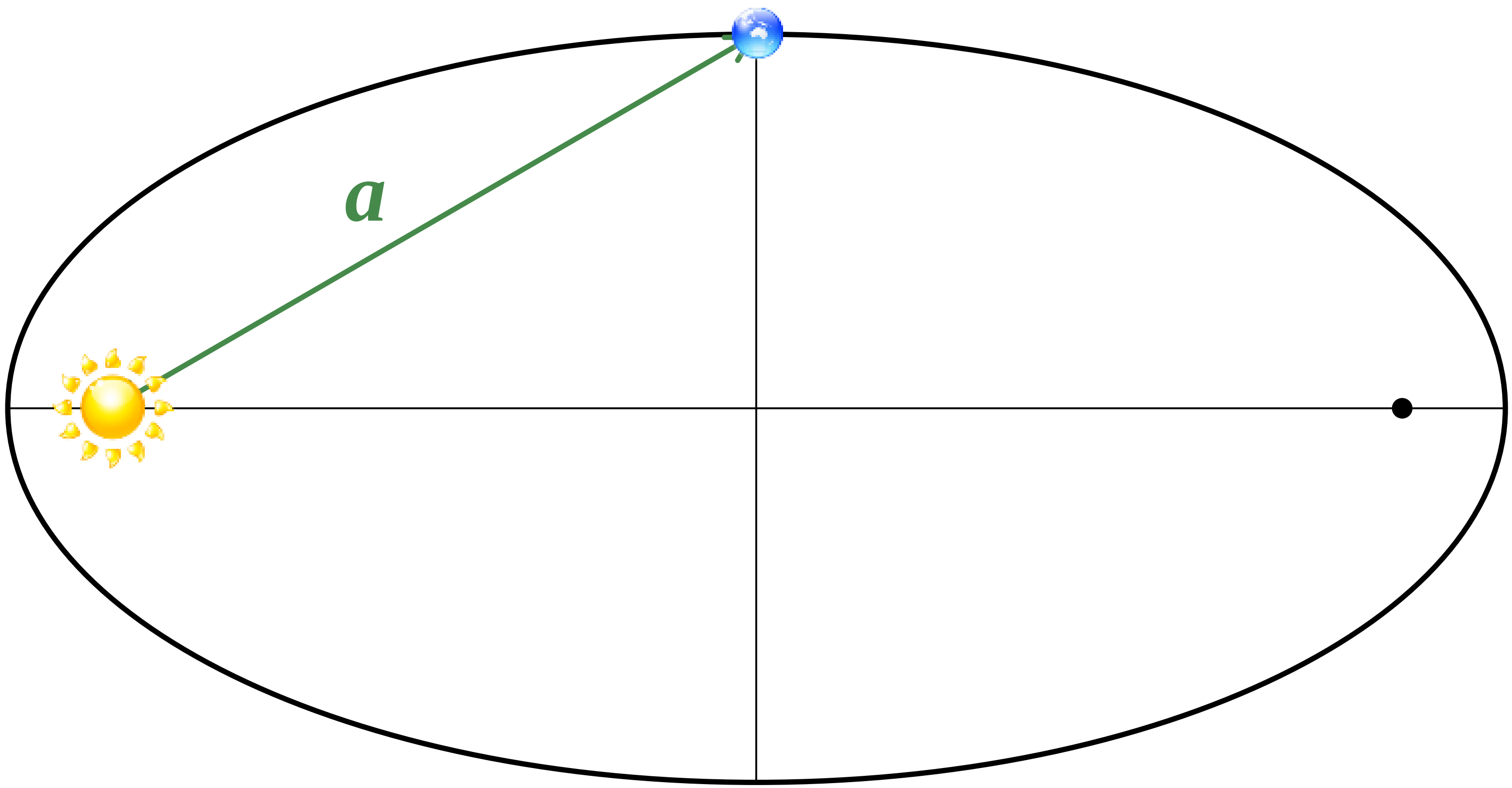

Note: the average distance from the Sun to the Earth is known as an “astronomical unit” or A.U.

For Earth, $a = 1$ A.U.

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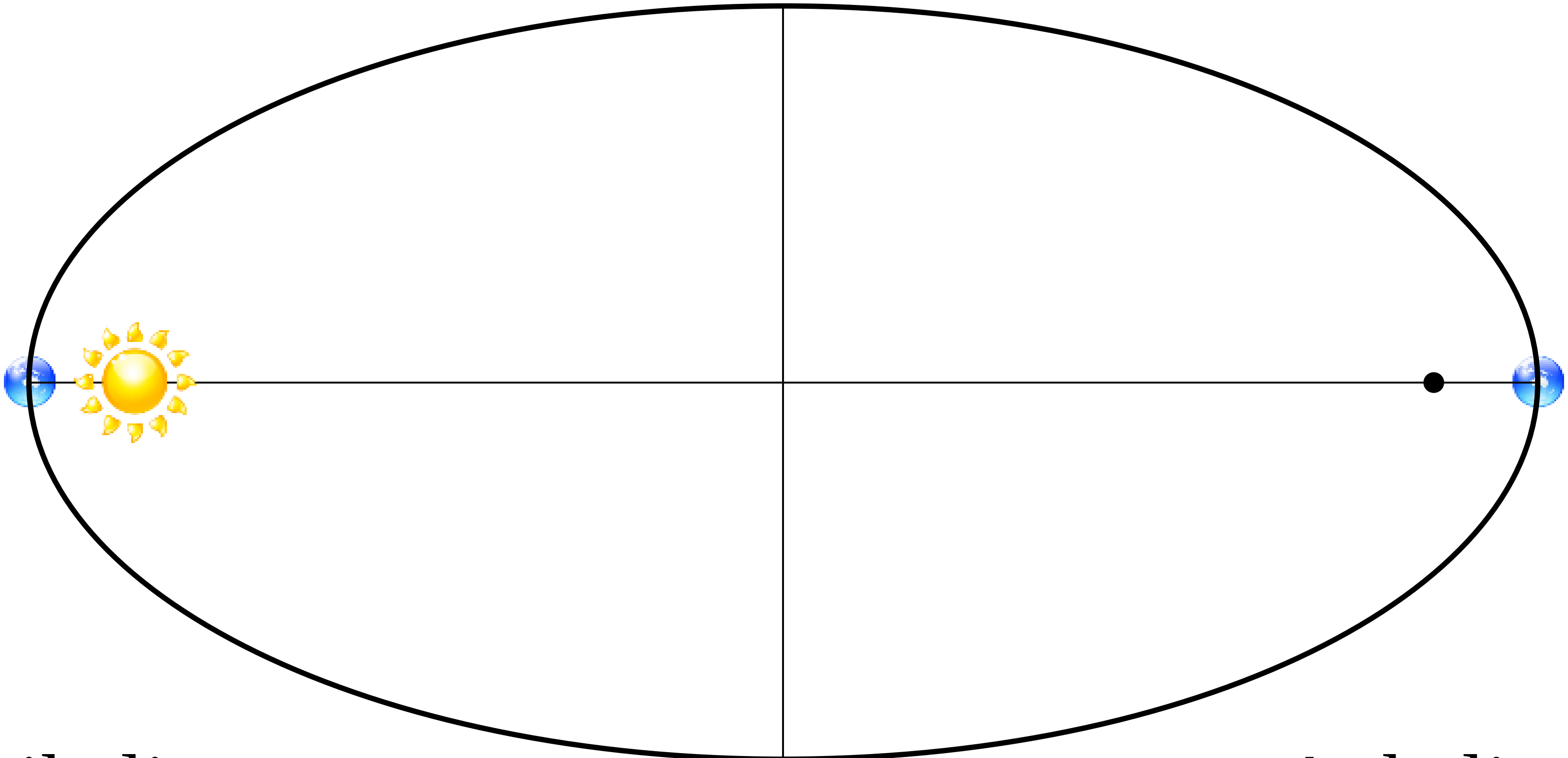

Perihelion = point in orbit closest to Sun

Aphelion = point in orbit farthest from Sun

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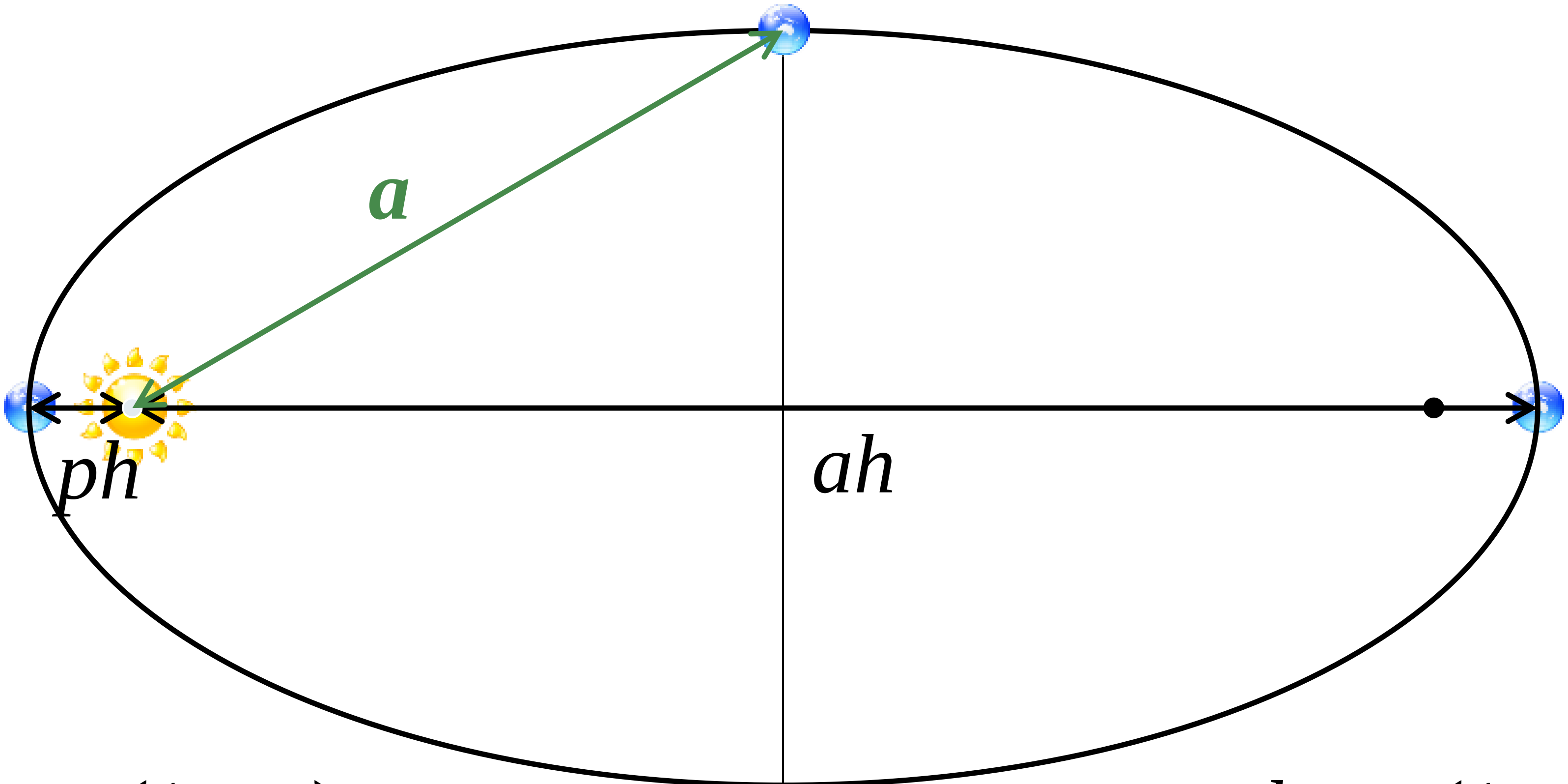

$ph = a(1 - e)$ $ah = a(1 + e)$

These formulas give the least and greatest distances from the Sun in terms of the average distance and the eccentricity.

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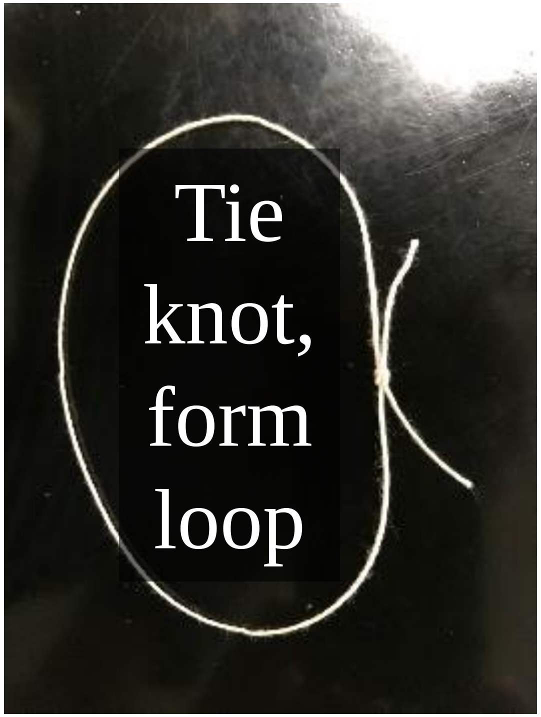

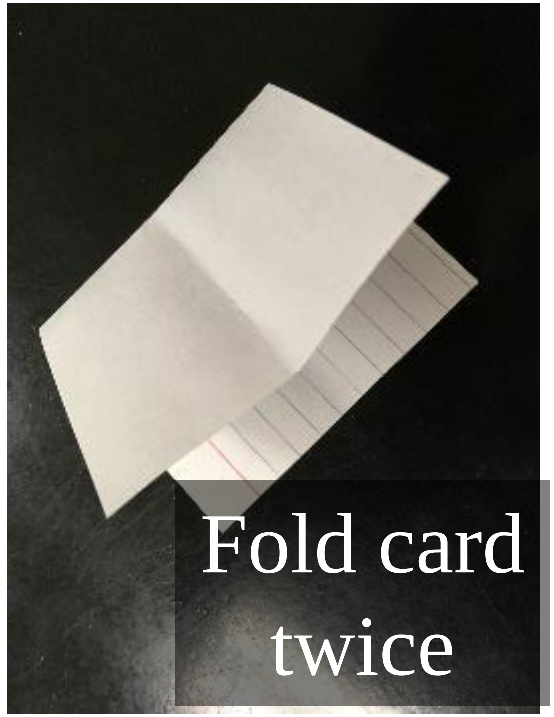

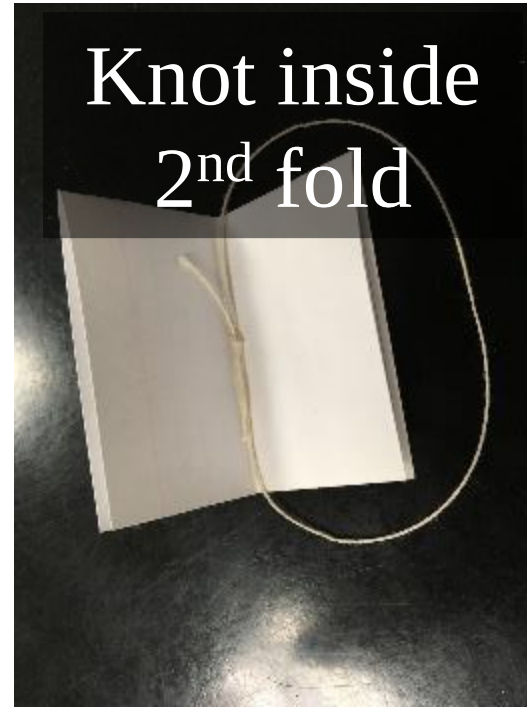

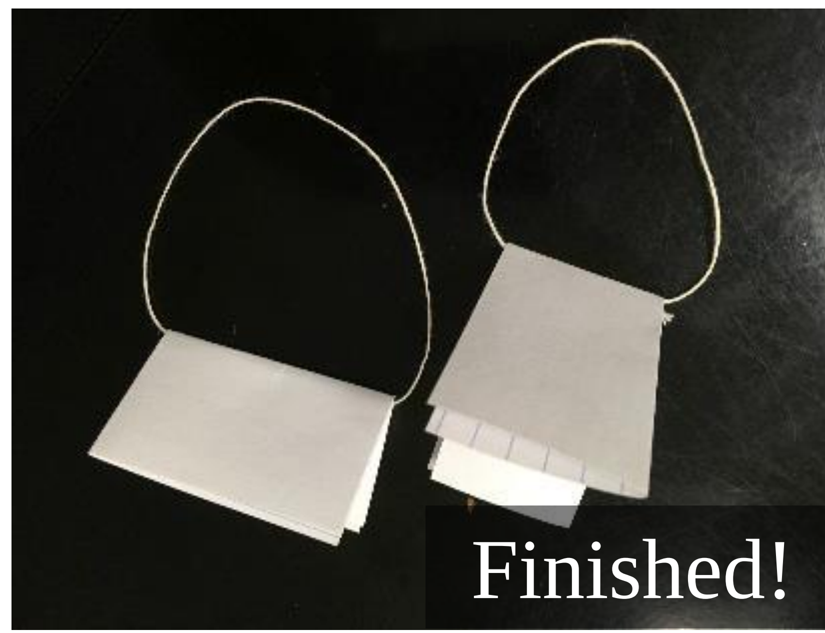

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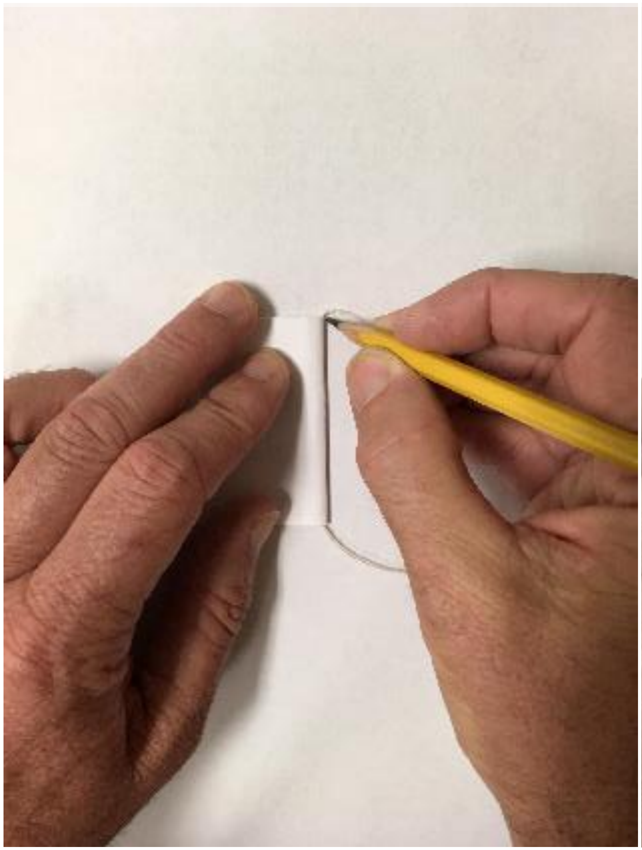
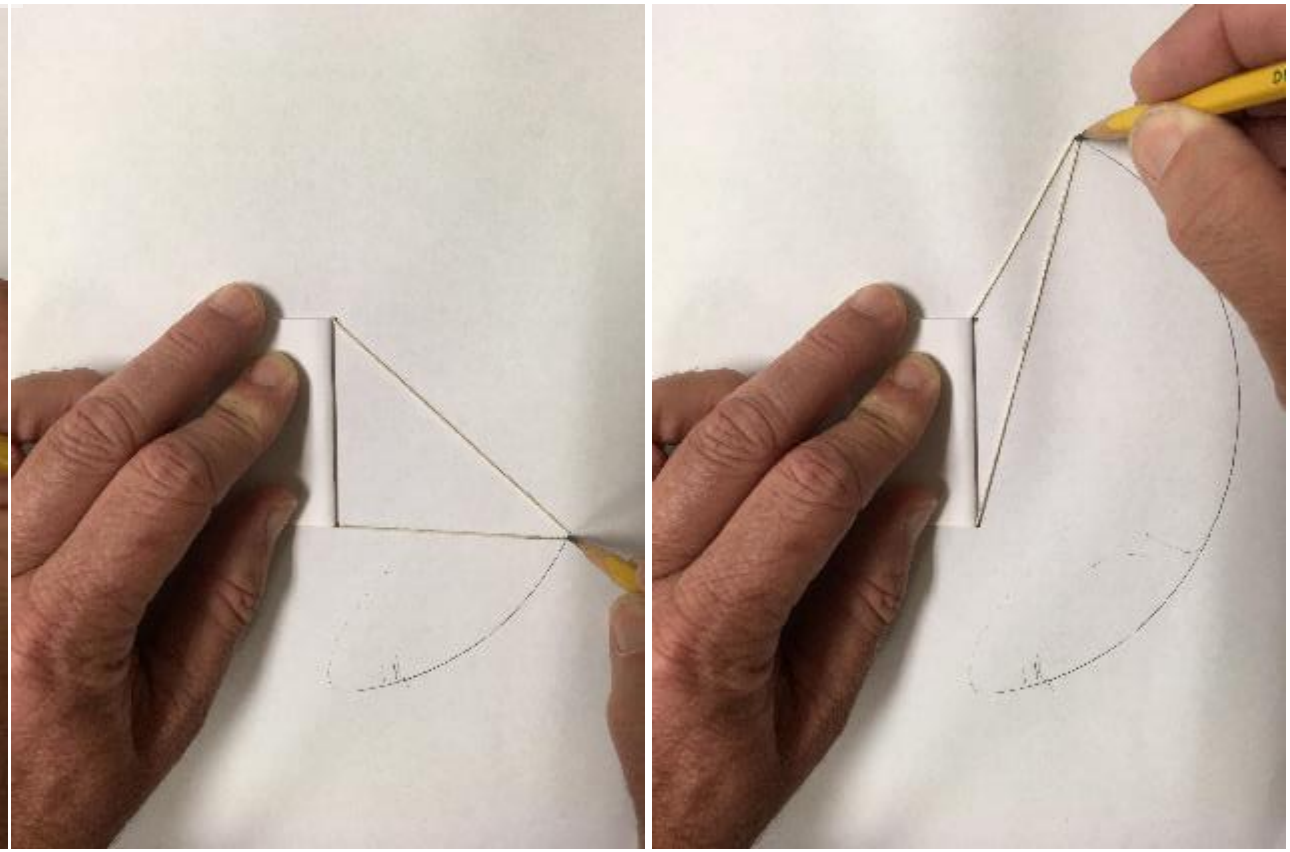
Hold folded card, mark each end.

Pencil inside loop, draw half of ellipse.





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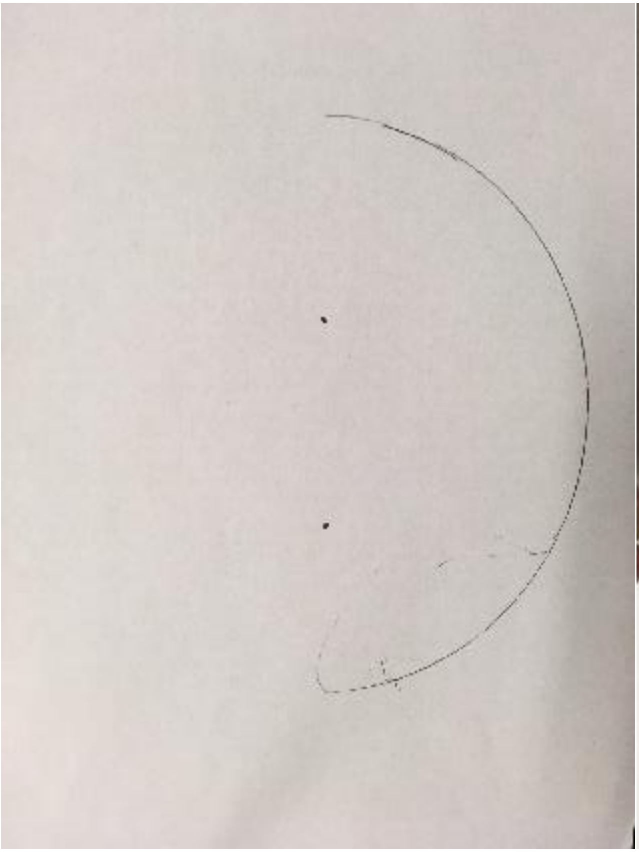
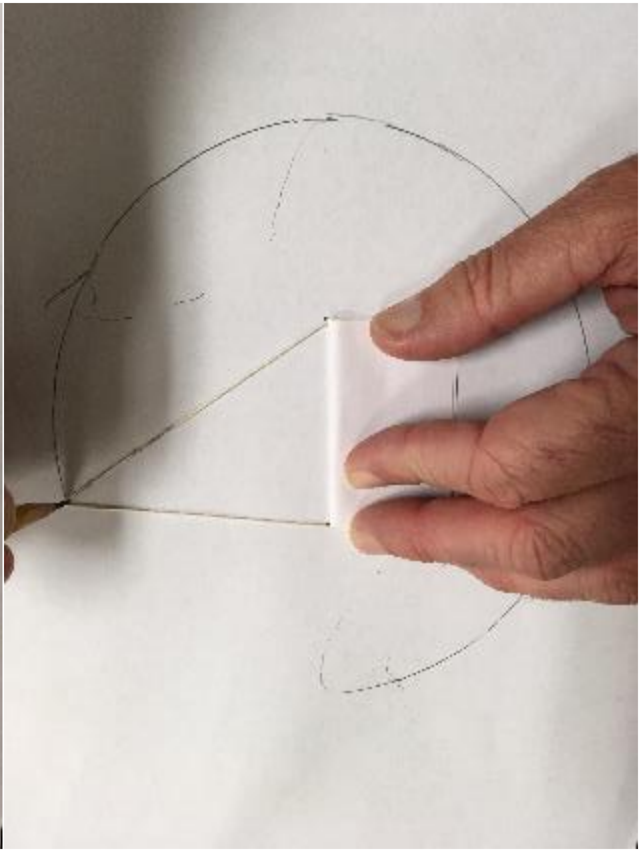
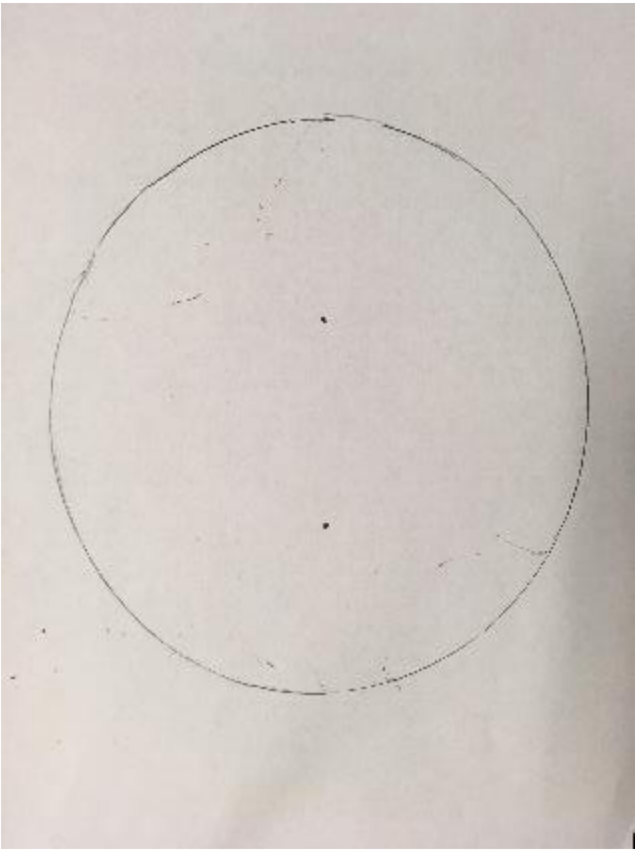
1st half done:

Reposition folded card against the two marks, draw 2nd half:







Finished!

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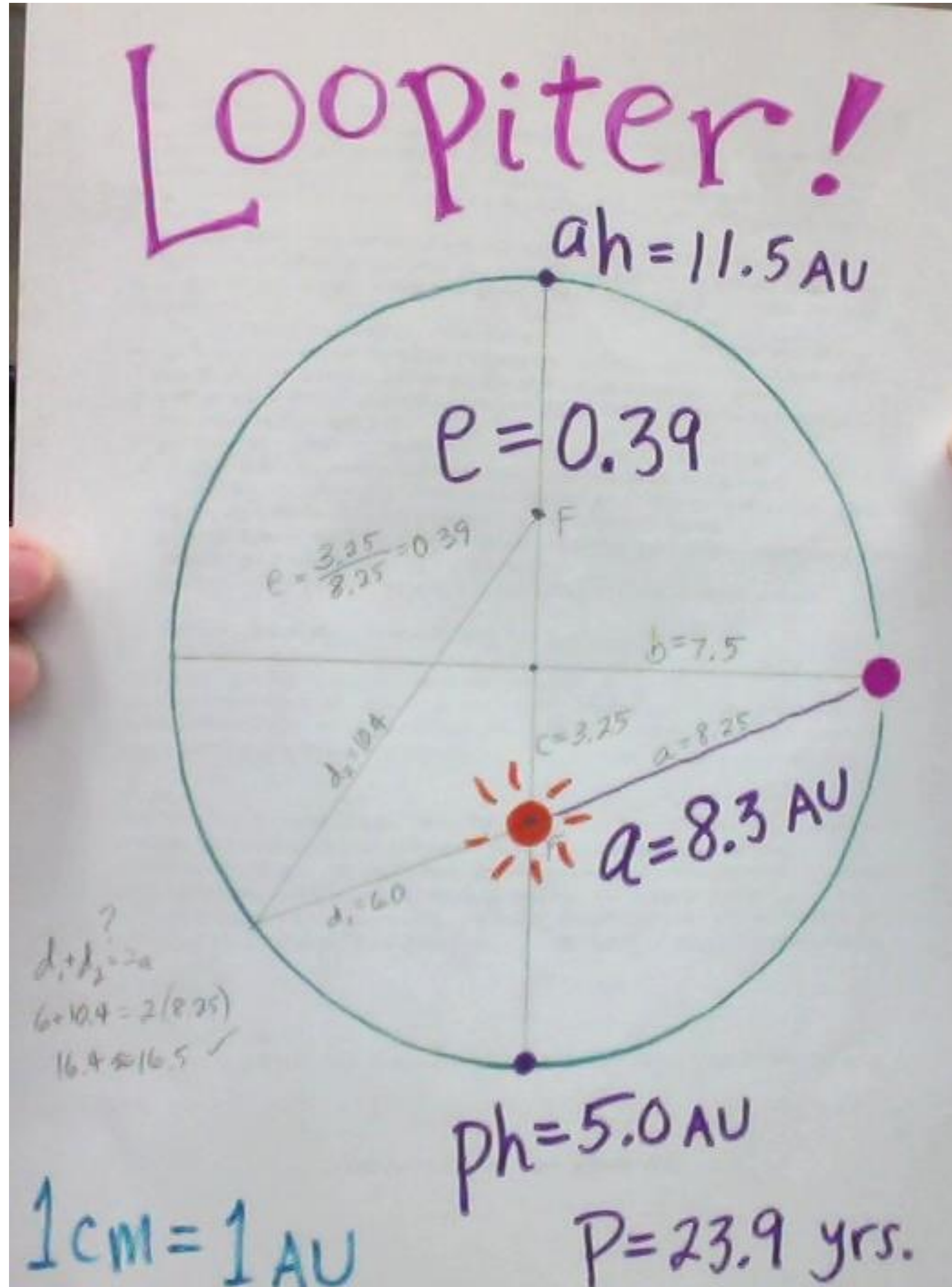
1. Name planet, be creative!
2. Measure *a*, *b*, *c*.
3. Confirm:
   $d_1 + d_2 = 2a$
   $b^2 + c^2 = a^2$
4. Calculate the eccentricity:
   $e = c/a$
5. Use 1 cm = 1 AU calculate ph and ah distances.
6. Determine period.



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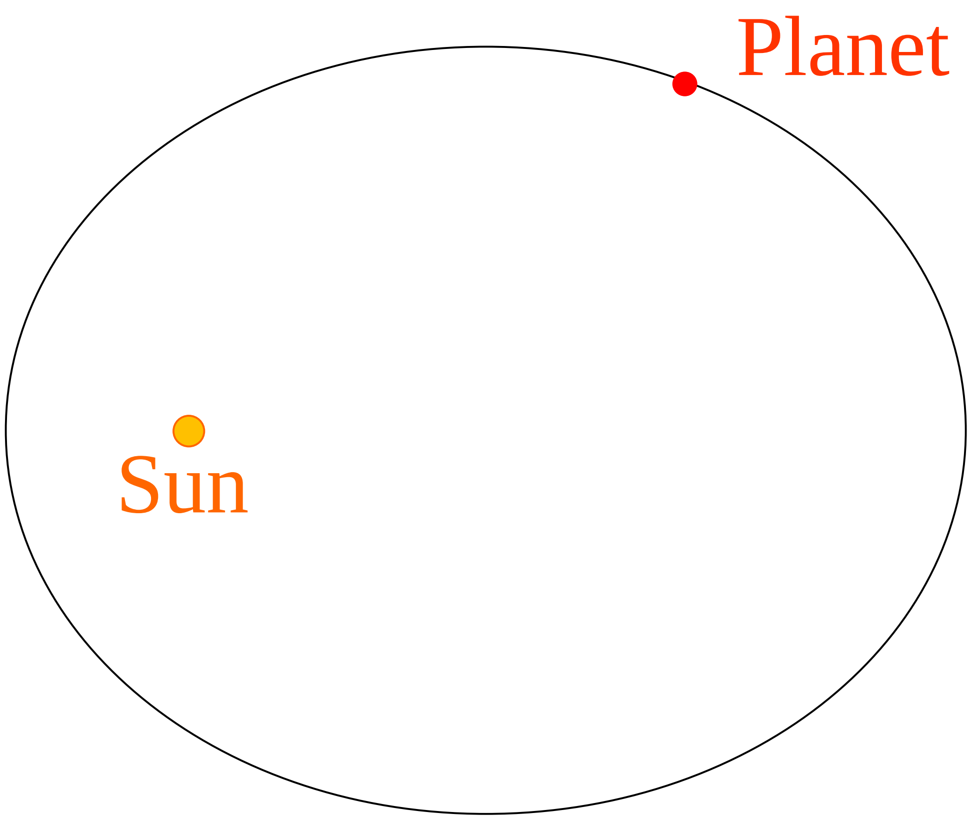
Newton's Modifications to Kepler's 1st Law

The Sun cannot simply be at rest at one focus as Kepler stated. Instead…



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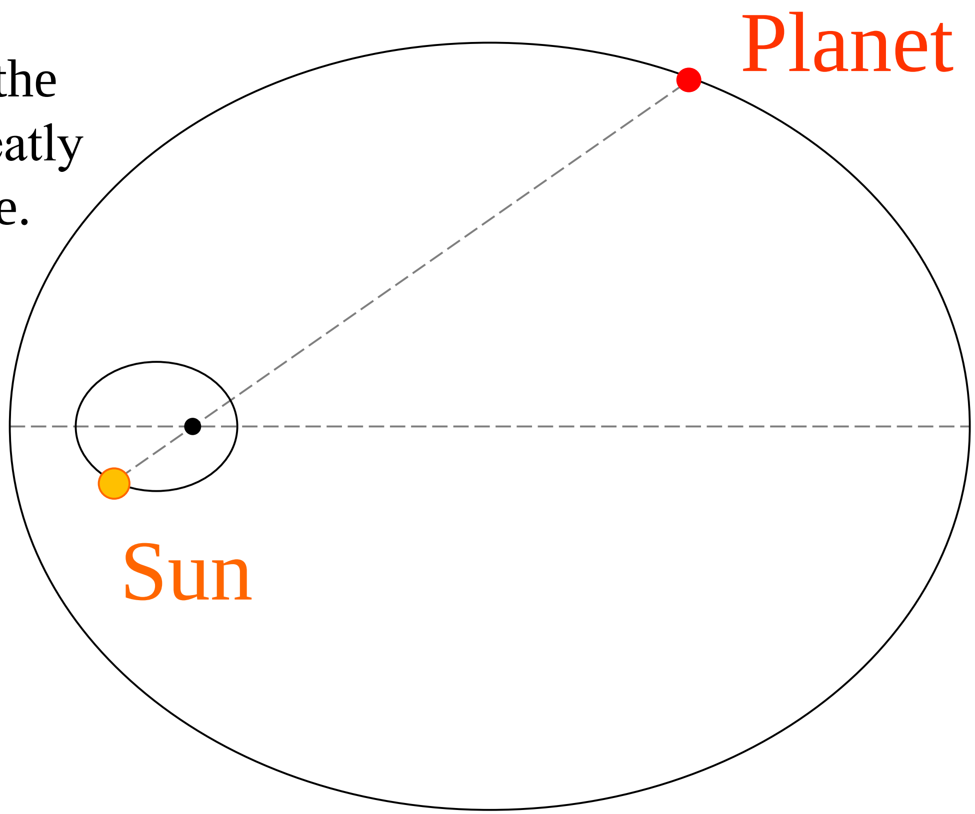
Newton’s Modifications to Kepler’s 1st Law

The Sun and planet *each* move in ellipses around a common focus.

Note: the size of the Sun’s ellipse is greatly exaggerated here.



The two ellipses have major axes that lie on the same line. The Sun and planet move in synchronization – always directly opposite one another on either side of the one focus that the two ellipses share.

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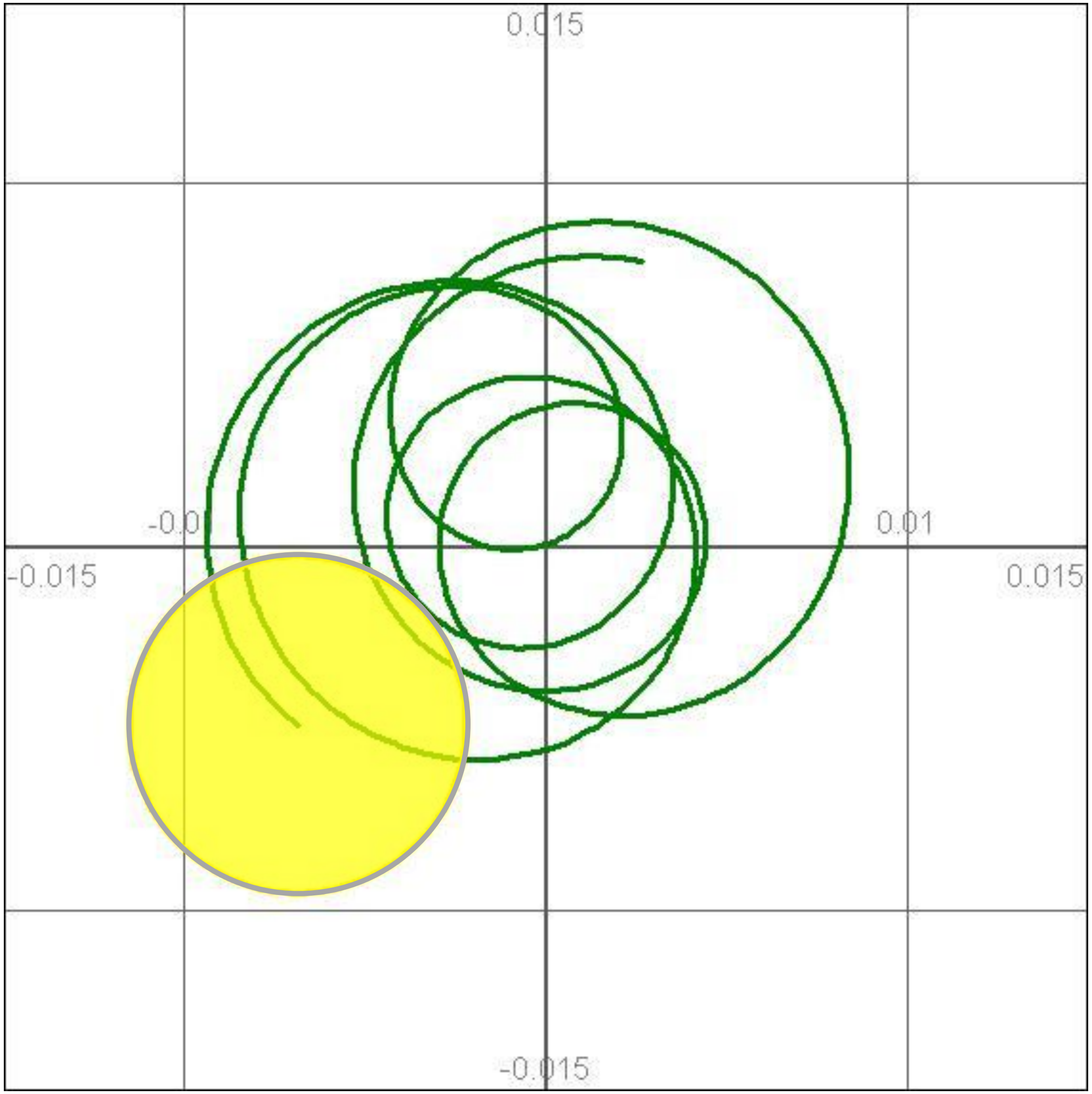
Sun's Actual Motion (to scale)



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Newton's Modifications to Kepler's 3rd Law

T^2 is proportional to a^3/M_{total}

In Newton's version, the total mass of the Sun and planet has an effect on the period of each.

Because the mass of any one planet is much less than that of the Sun, the total mass is nearly the same as the mass of the Sun itself.

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Newton's Modifications to Kepler's 3rd Law

T^2 is proportional to a^3/M_{total}

This is a minor discrepancy when applied to the solar system! This is why Kepler's 3rd Law in most cases can be used without worrying about the masses involved. It only amounts to a difference of at most 2 days when analyzing the solar system (in the case of Jupiter).

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Period vs. Orbit Size – Kepler vs. Newton

	a (AU)	T (yr)	M_{total}	T^2/a^3	T^2/Ma^3
Mercury	0.38709	0.24085	0.999997	1.00004	1.00004
Venus	0.72333	0.61520	0.999999	1.00004	1.00004
Earth	1.0000	1.0000	1.0000	1.0000	1.0000
Mars	1.5237	1.88085	0.999997	0.999996	0.999999
Jupiter	5.2044	11.8626	1.000952	0.99938	1.00033
Saturn	9.5415	29.4475	1.000283	0.99918	0.99922

Kepler's version

Newton's version

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