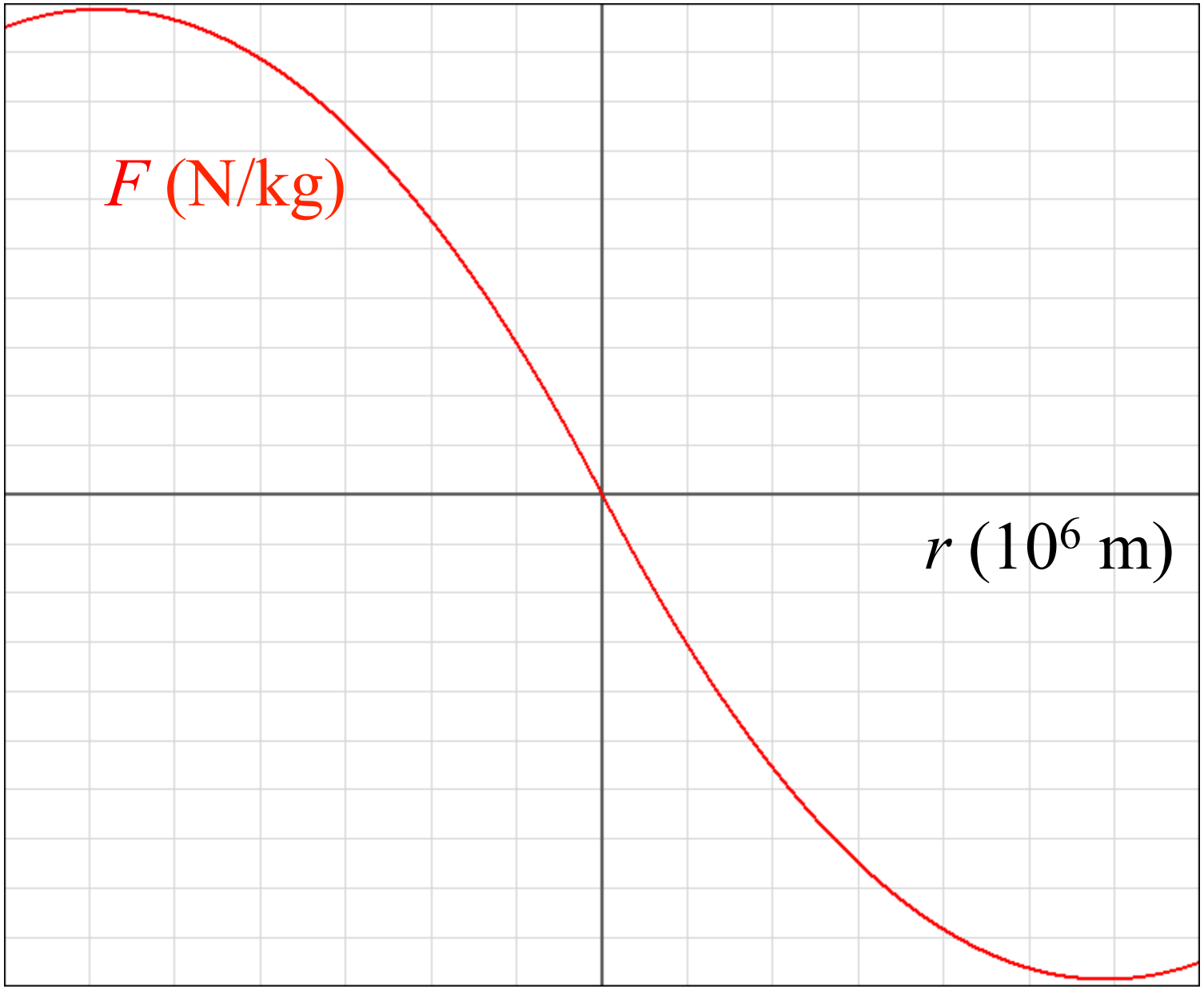
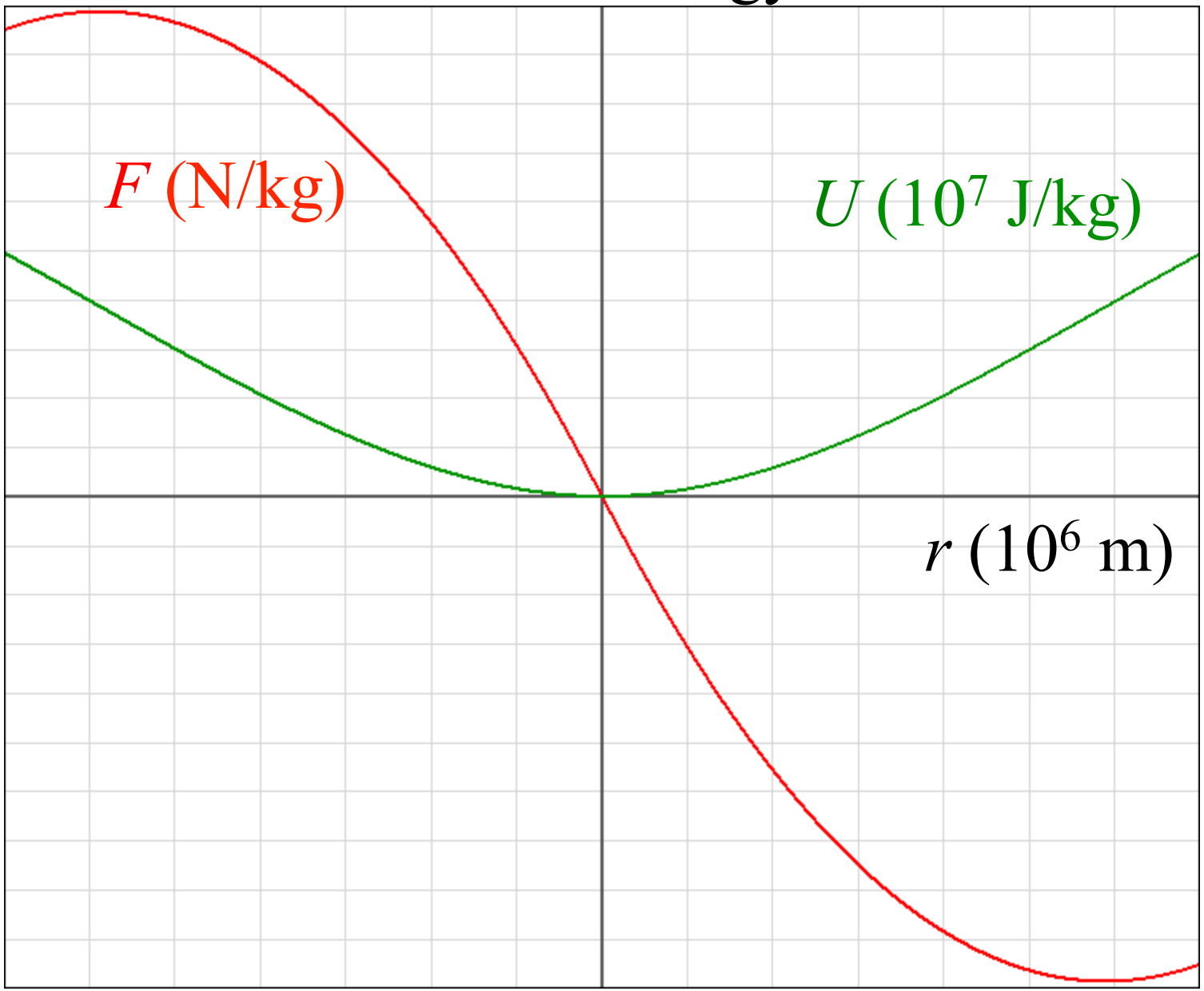
The gravitational field inside the earth may be modeled by $g = \alpha r^2 + \beta r$, where r = distance from center, $\alpha = -2.851 \times 10^{-13}$ m^{-1}s^{-2}, and $\beta = 3.354 \times 10^{-6}$ s^{-2}. (This model allows for the increasing density towards the earth's core using $\rho(r) = (-0.00136 \text{ kg/m}^4)\, r + 12000 \text{ kg/m}^3$). (a) Use the function $g(r)$ to determine the potential energy function for an object located inside the earth. Use the earth's center as a reference point. (b) Supposing an object could fall through a tunnel from the surface to the center of the earth what would be its final speed?

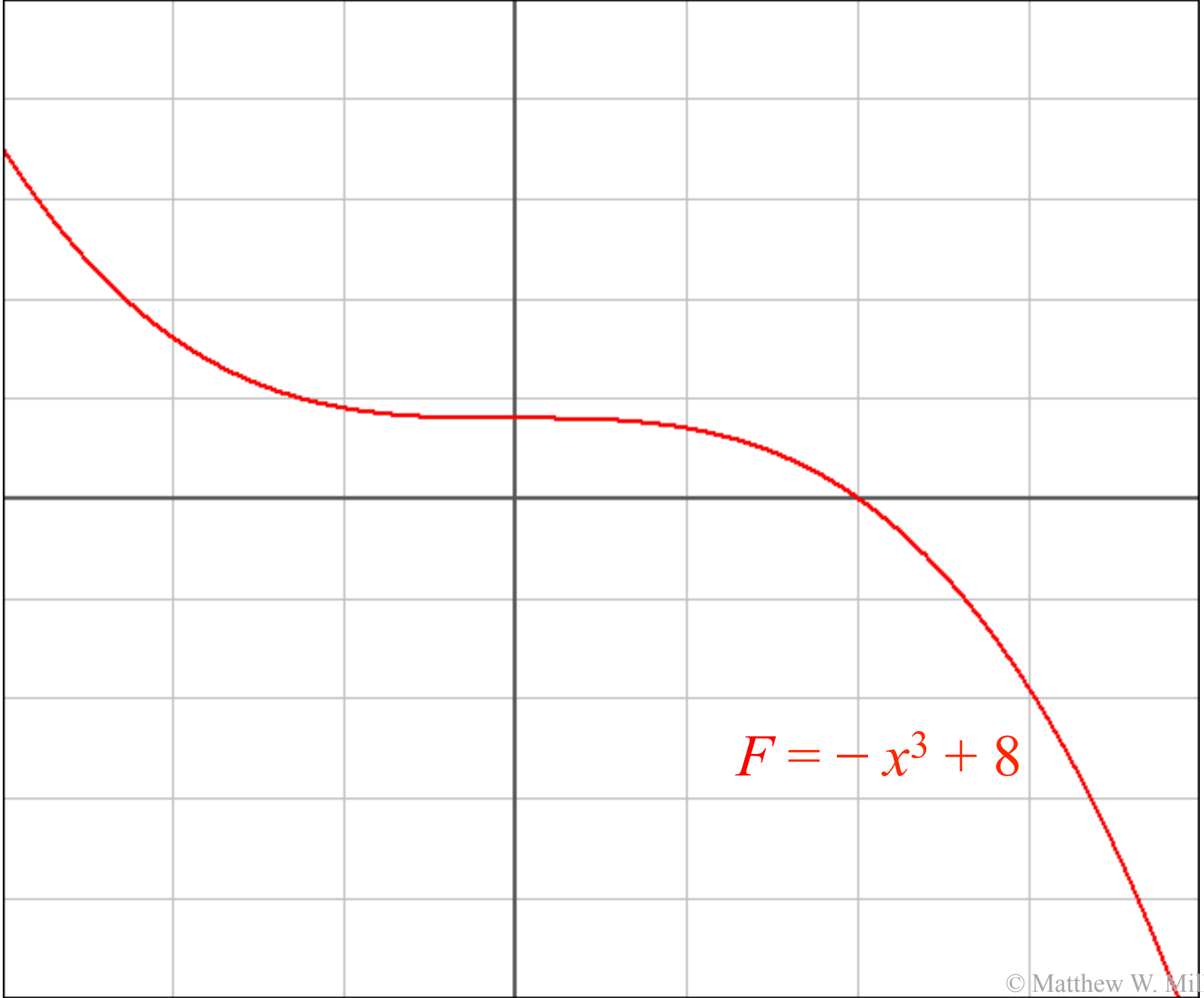
Gravitational Force Inside Earth

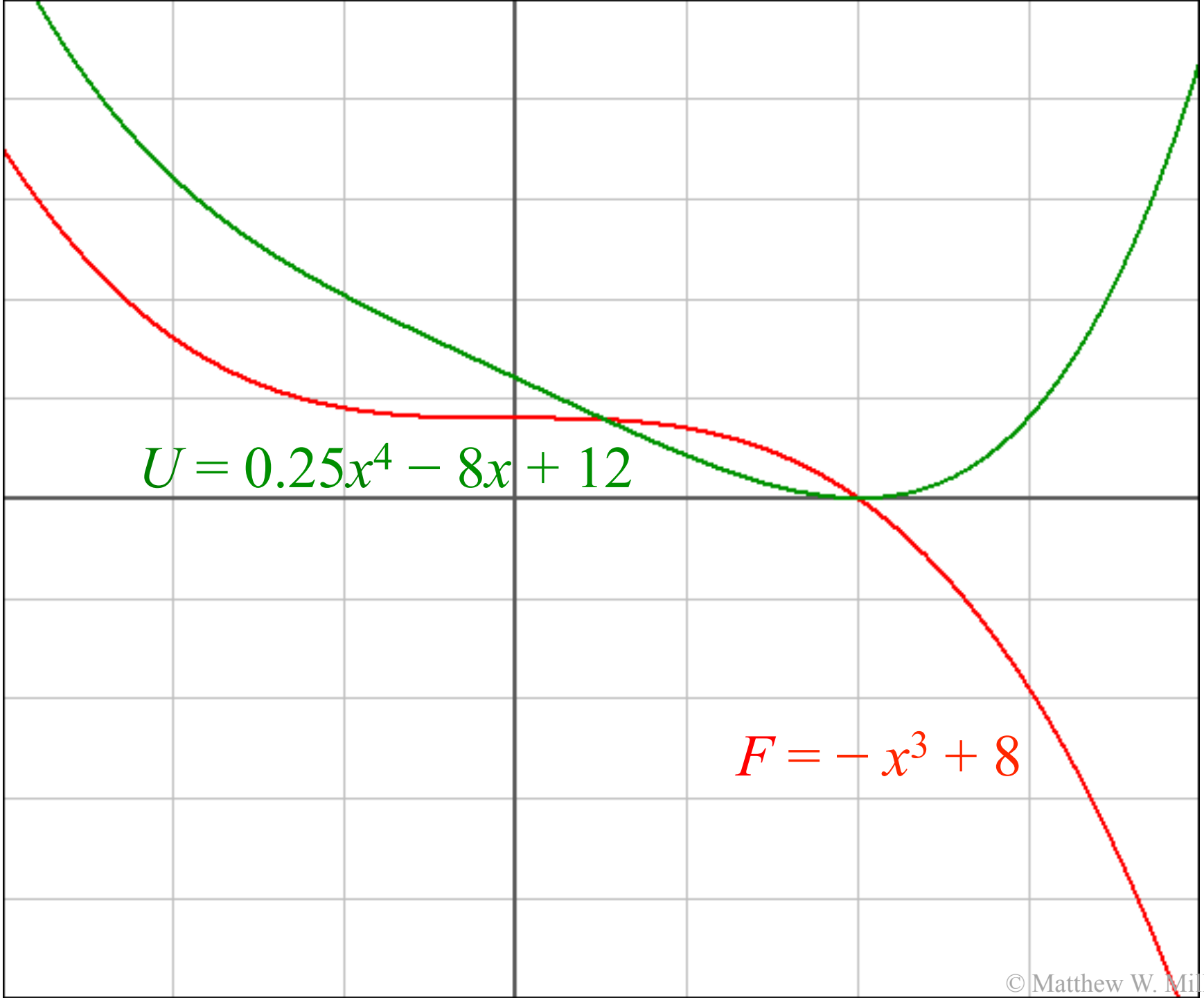


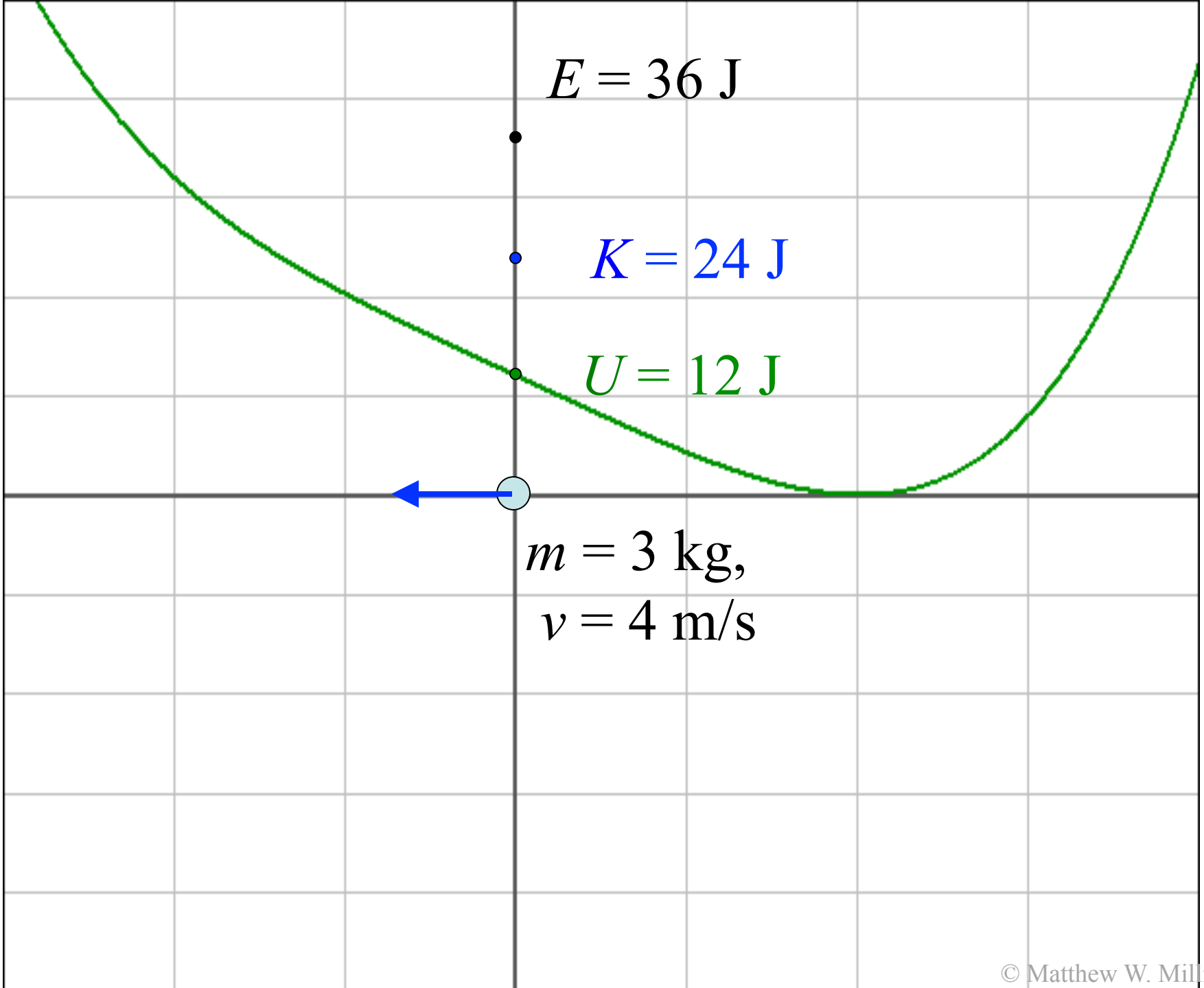
Force and Potential Energy Inside Earth

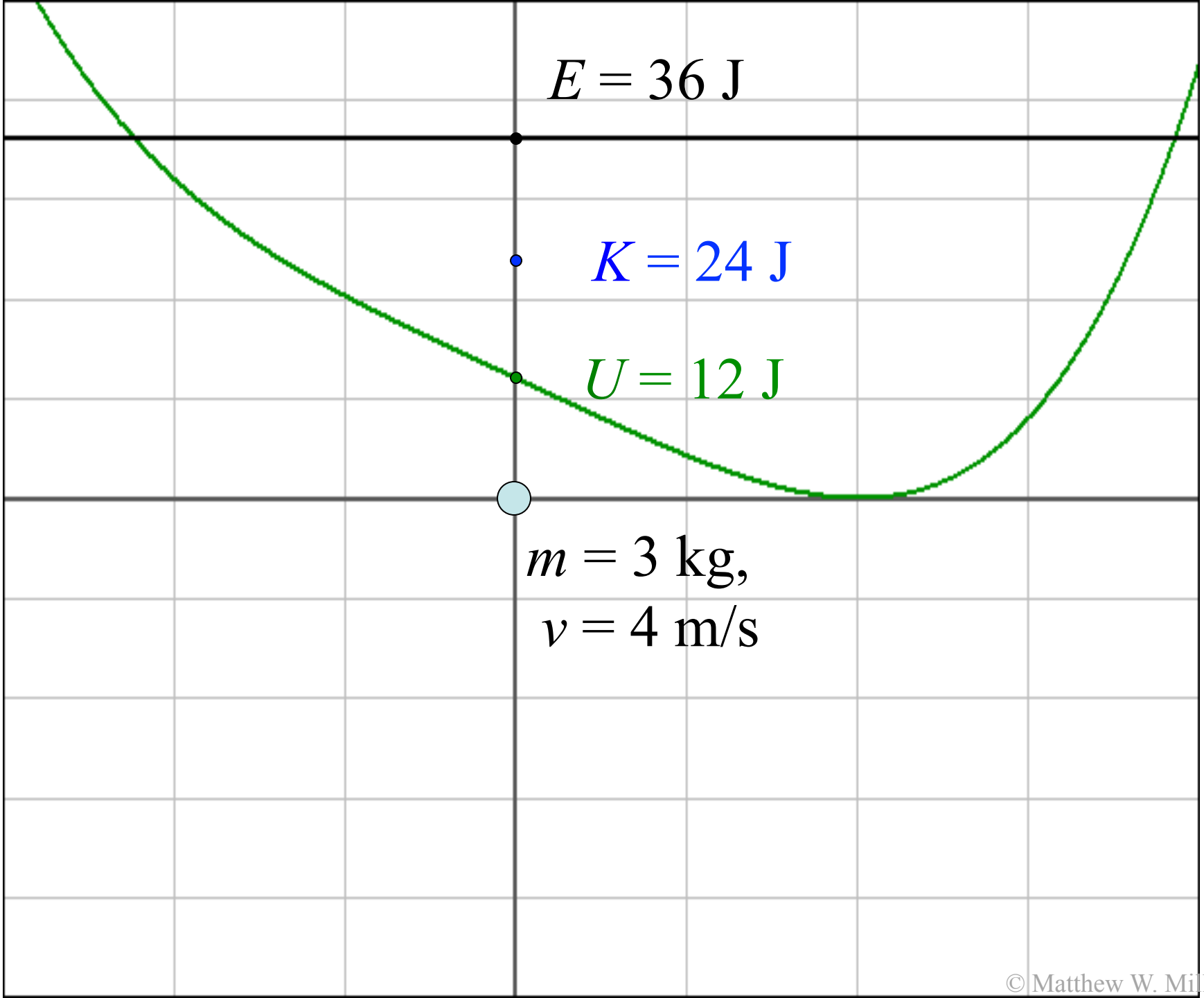


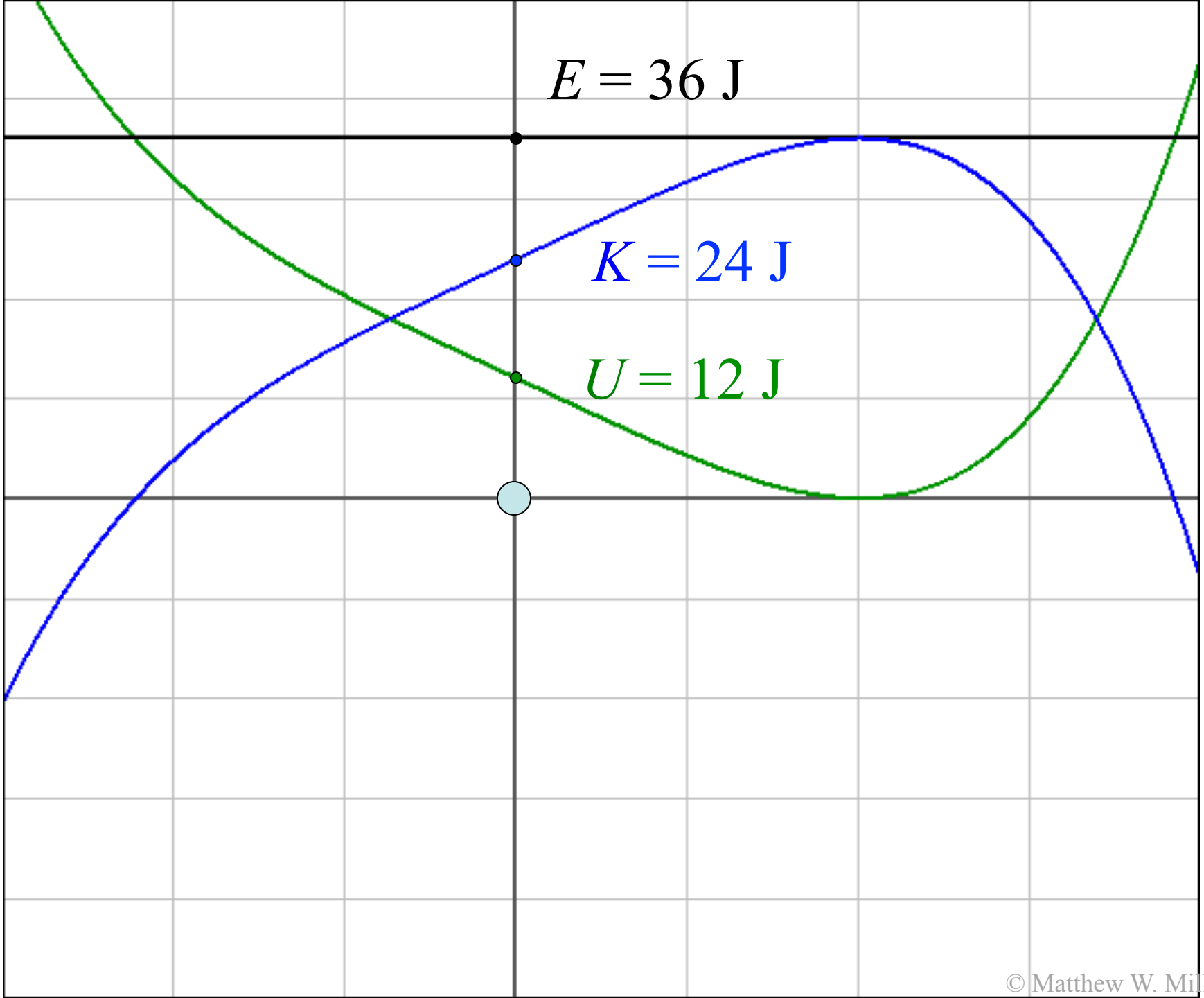
A particle of mass 3.0 kg moves along the x-axis under the influence of a conservative force given by $F = -x^3 + 8$, (force in newtons, position in meters). The particle begins at the origin with initial velocity 4.0 m/s, left. (a) Determine a potential energy function relative to position at which $F = 0$. (b) Find the total energy of the particle. (c) Find the maximum speed it attains. (d) What is the greatest distance from the origin reached by the particle?

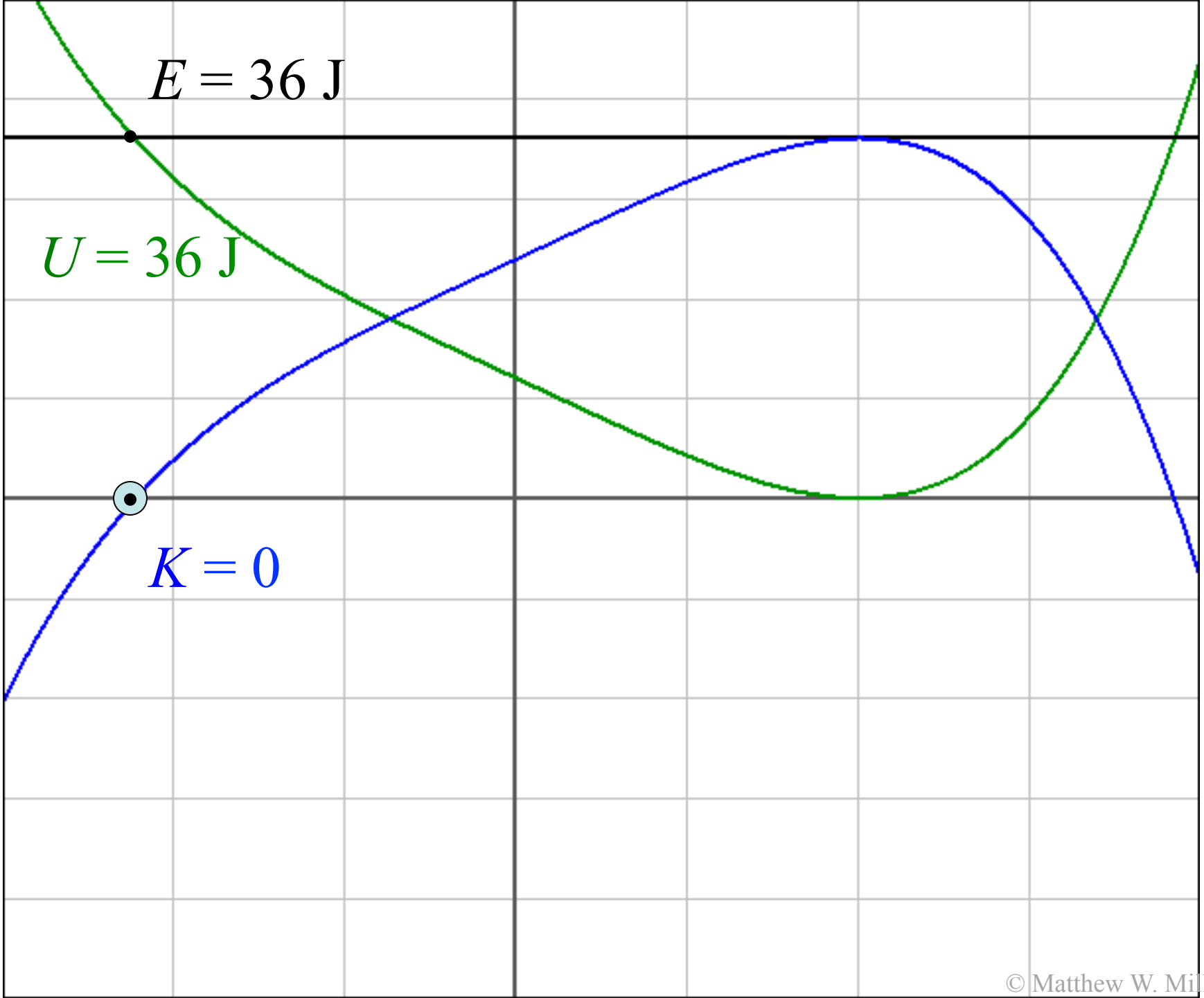


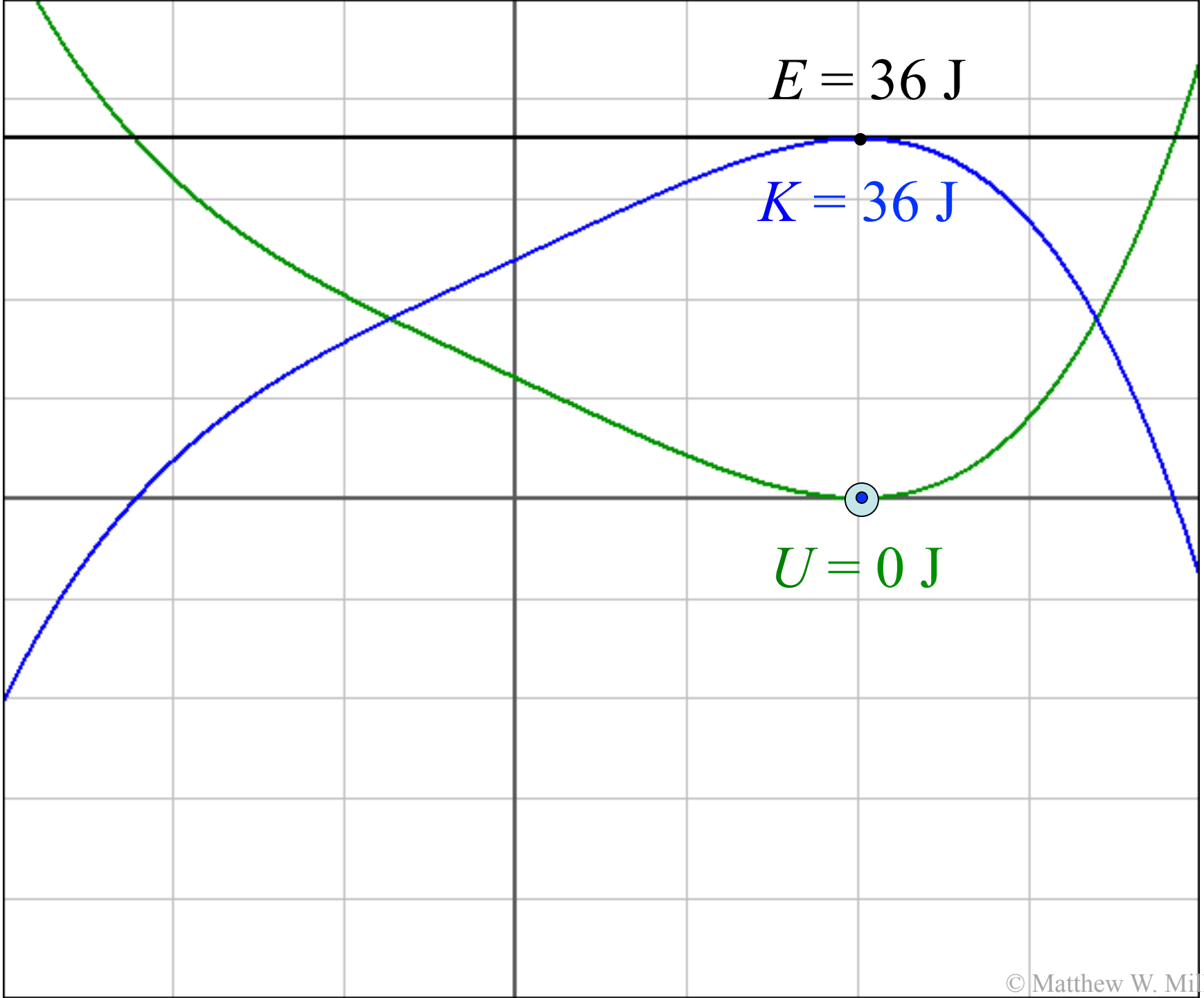


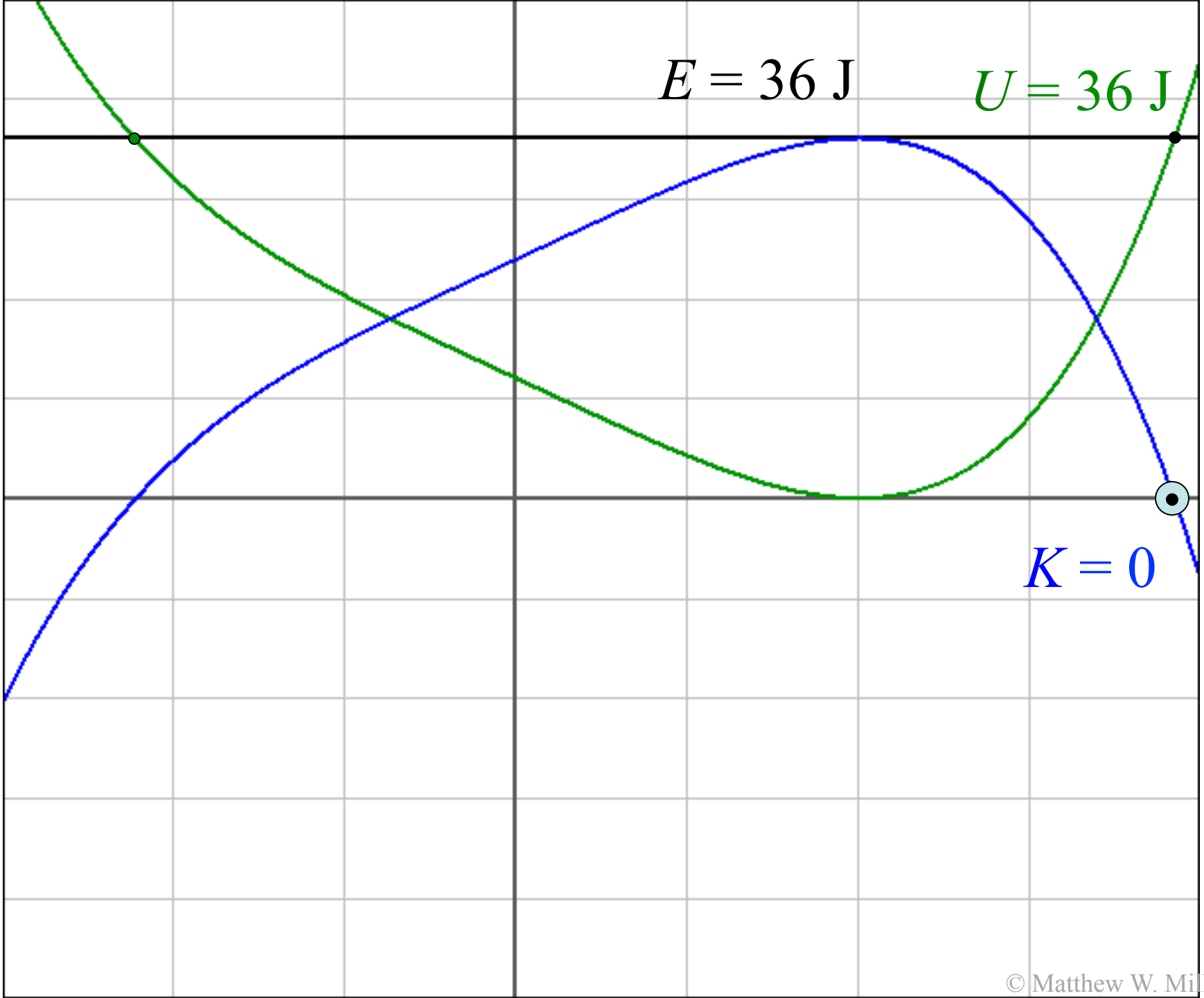












A particle of mass 2.00 kg is free to move along the x-axis. The particle's potential energy is given by $U(x) = x^3 - 3x + 3$, where x is position in meters and U is energy in joules. The particle begins at rest at the origin. (a) Determine the particle's maximum speed. (b) Determine the particle's greatest departure from the origin. (c) Determine the particle's maximum rate of acceleration (in either direction). (d) Describe in words the overall motion of the object. (e) What initial speed would be required to prevent the particle from returning to the origin? (f) If the particle is given speed 3.00 m/s at the origin what will be its speed at $x = -3$ m?