Work and Energy

I. Work
- dot product
- varying force

II. Work-Energy Theorem
- Kinetic Energy

III. Potential Energy
- Conservative Forces

IV. Machines, Power, Efficiency

	The student will be able to:	HW:
1	Define and apply the concept of work (and the joule) for constant or varying force and solve related problems.	1 – 9
2	Define and apply kinetic energy. State and apply the work-energy theorem and solve related problems.	10 – 15
3	Solve problems using conservation of mechanical energy, including situations involving nonconservative forces.	16 – 23
4	Solve problems involving gravitational potential energy in which g is not taken to be constant.	24 – 26
5	Solve problems involving work and energy for a mass attached to a spring.	27 – 29
6	Define and apply the concepts of conservative force and potential energy and solve related problems.	30 – 32
7	Define and apply the concept of power (and the Watt) and solve related problems.	33 – 37
8	Solve problems involving machines and efficiency.	38 – 40

Work

A unique quantity

Work

The work done by a *constant* force acting on an object over a certain displacement:

$$W = \vec{F} \cdot \vec{d}$$

Note that this is a "dot product" of two vector quantities.

This can be shown to equal the following:

$$W = Fd\cos\phi$$

Work

The work done by a *constant* force acting on an object over a certain displacement:

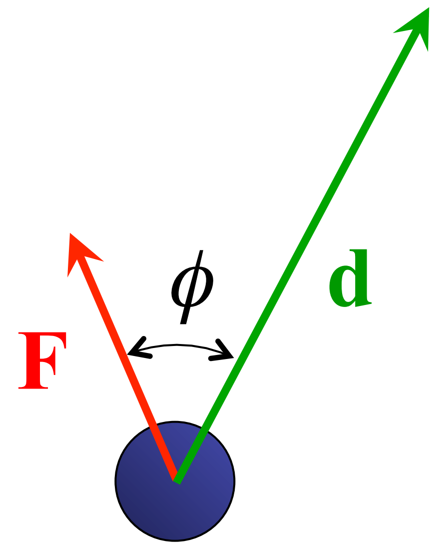
$$W = \vec{F} \cdot \Delta\vec{r}$$

Note that this is a "dot product" of two vector quantities.

This can be shown to equal the following:

$$W = F(\Delta r)\cos\phi$$

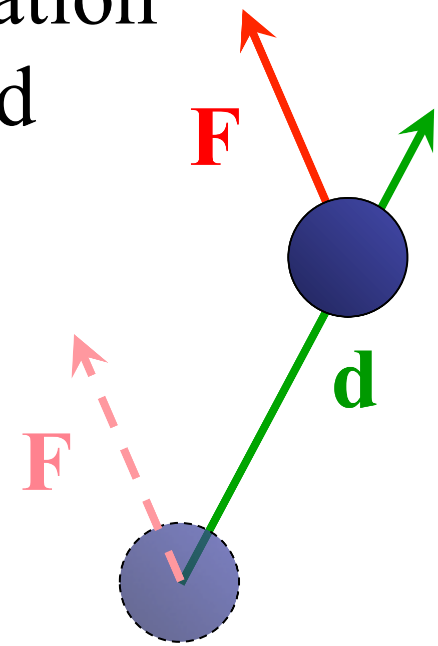
Work



$$W = Fd\cos\phi$$

where: F = magnitude of force
d = magnitude of displacement
ϕ = angle *between* **F** and **d**
(*i.e.* the *difference* in directions)

Note: when using this formulation to calculate work it is assumed that the force is constant in magnitude and direction as the motion occurs.



$$W = Fd\cos\phi$$

where: F = magnitude of force
d = magnitude of displacement
ϕ = angle *between* **F** and **d**
(*i.e.* the *difference* in directions)

Key Ideas About Work

- Work is a scalar quantity that may be positive or negative. Work does not have a direction and it is not a vector.
- Because energy is required to do work there will always be a corresponding change in energy when work is done.
- The SI unit of work is the joule, defined by: 1 joule = 1 newton × 1 meter (J = Nm)

Multiplying Vectors

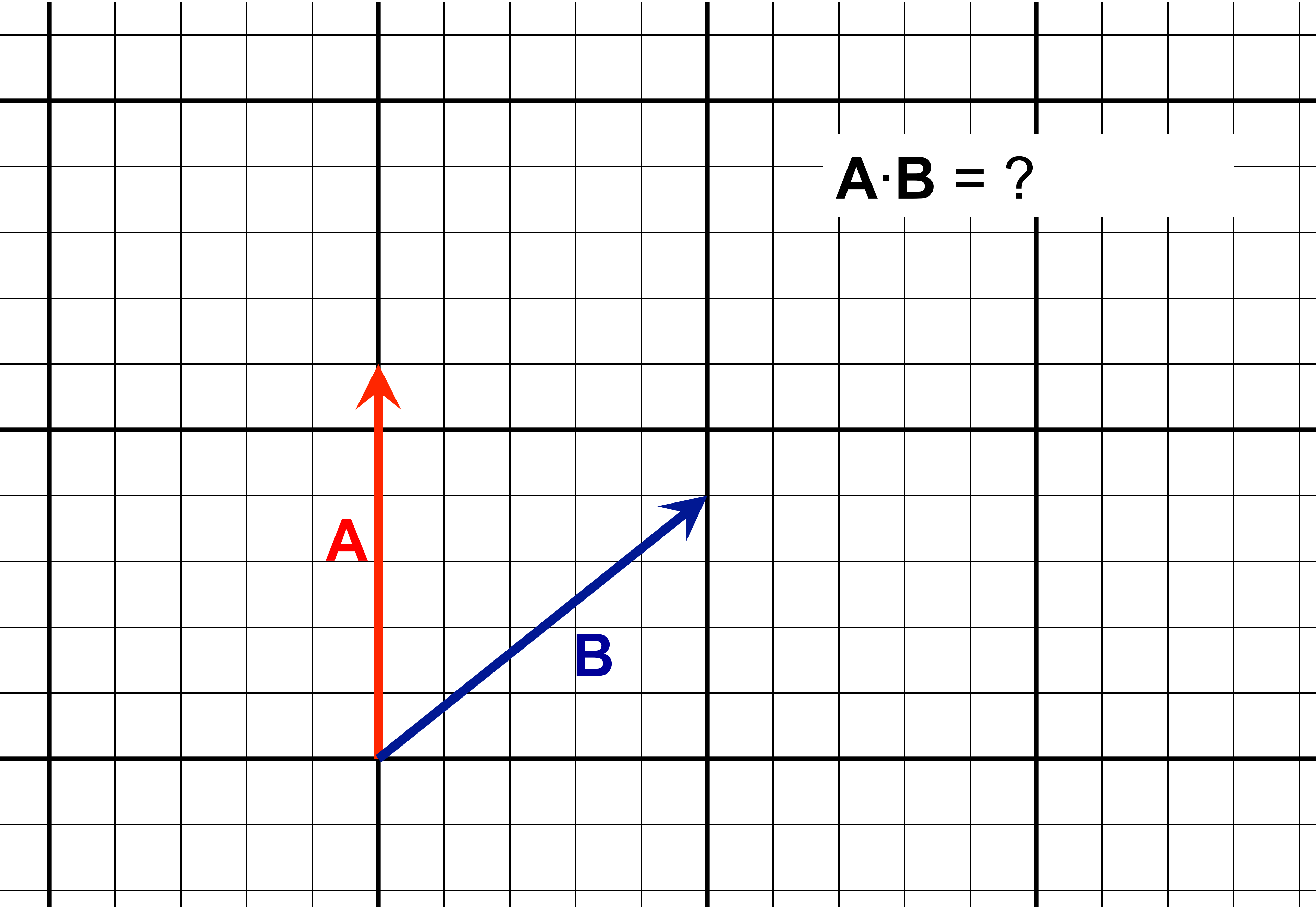
- There are two different ways to multiply vector quantities – dot product and cross product.
- The dot product is a vector operation that yields a scalar result (it is also sometimes called the scalar product). The result may be positive or negative.
- The cross product is a vector operation that yields a vector result (it is sometimes called the vector product).

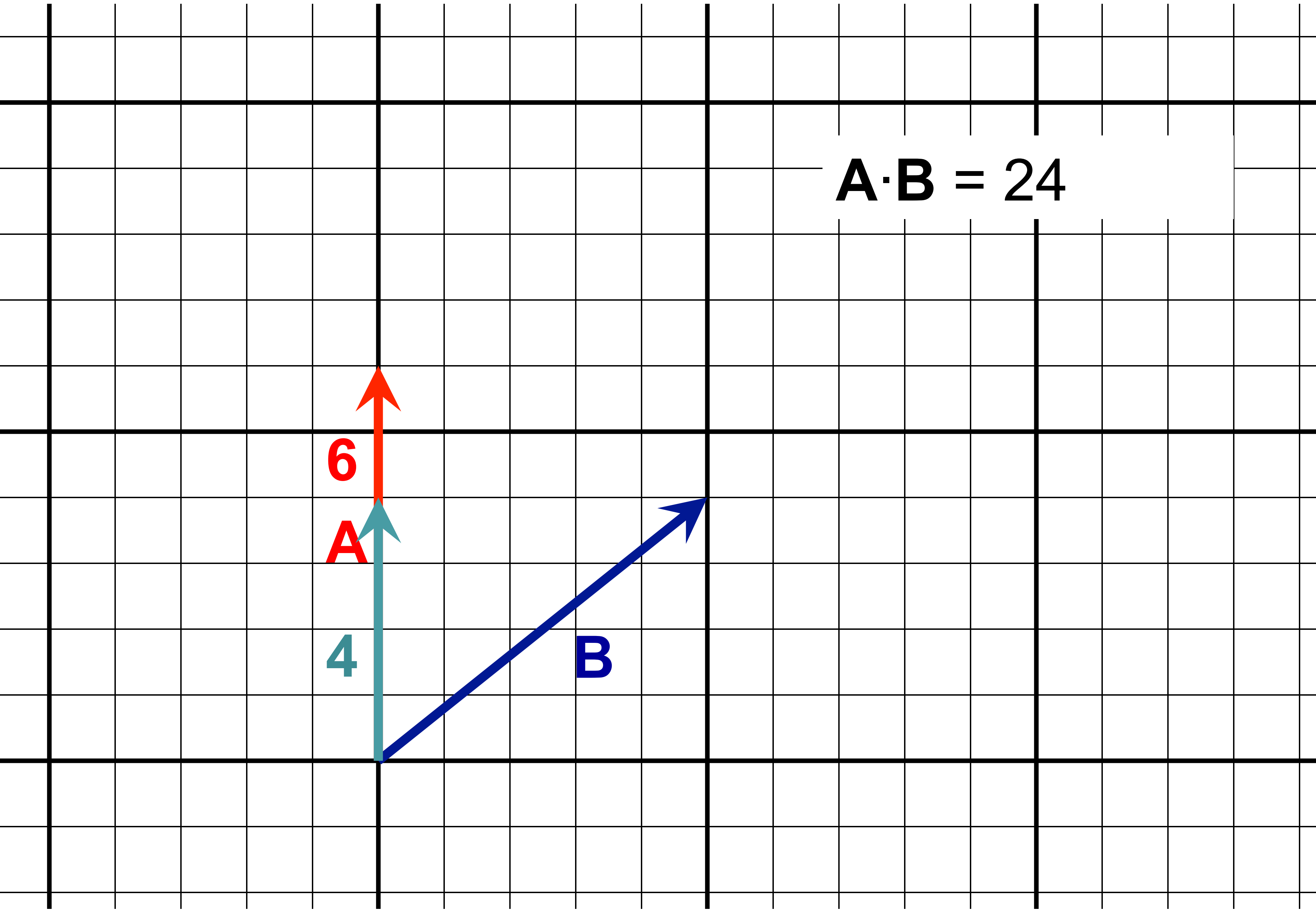
The dot product of two vectors is found by multiplying the collinear components:

$$\vec{\mathrm{A}} \cdot \vec{\mathrm{B}} = A_x B_x + A_y B_y$$

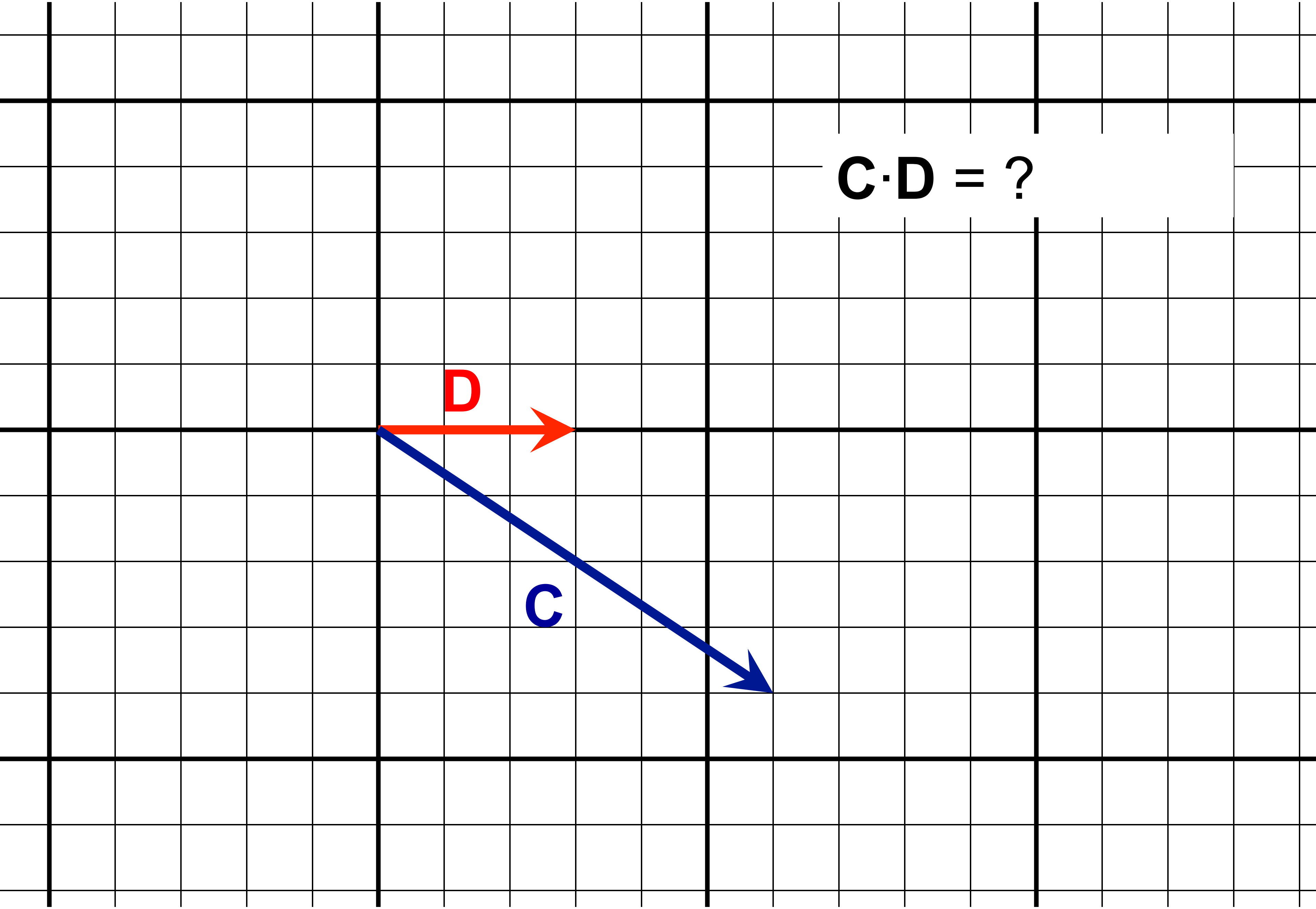
or

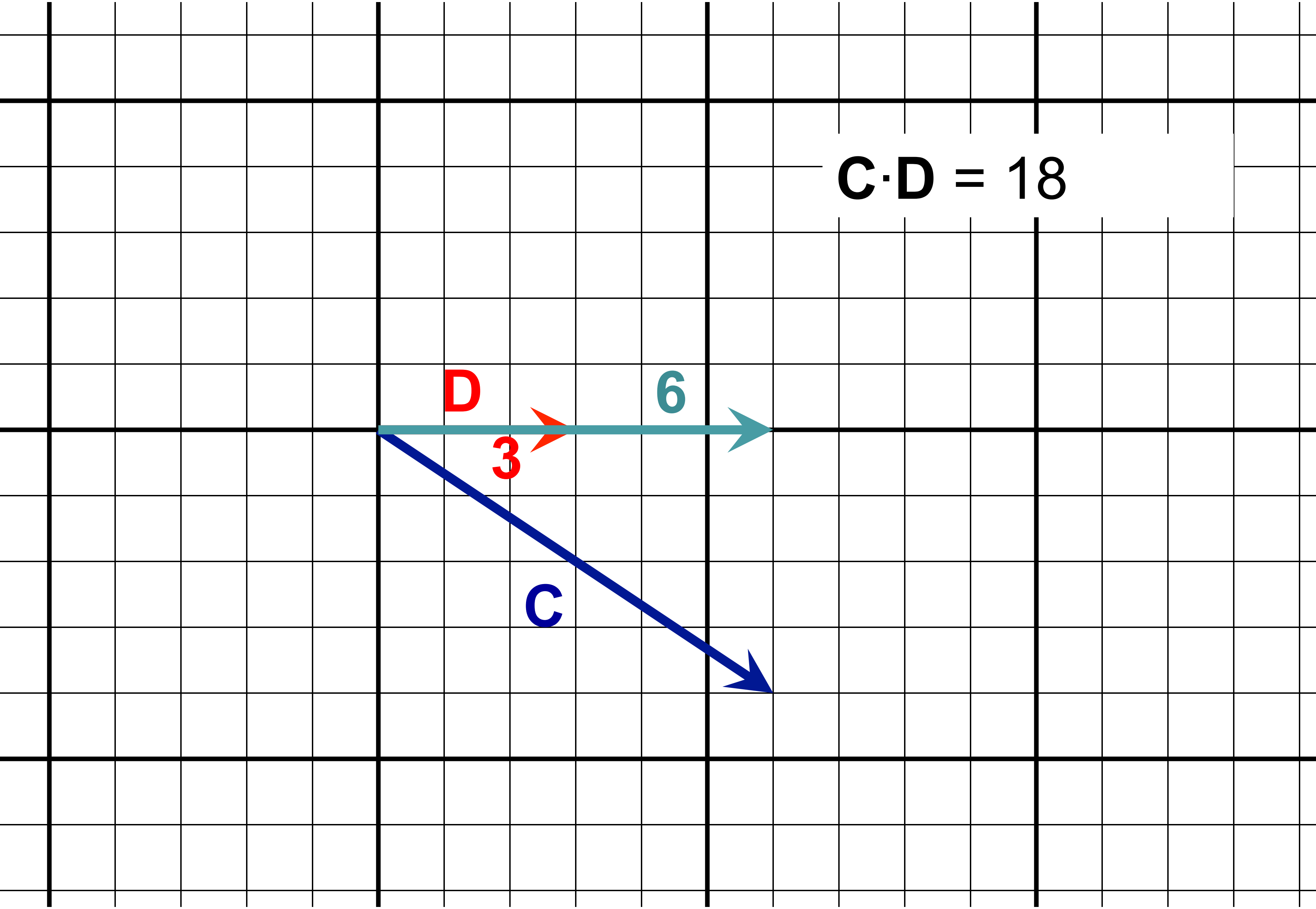
$$\vec{\mathrm{A}} \cdot \vec{\mathrm{B}} = AB\cos\theta$$

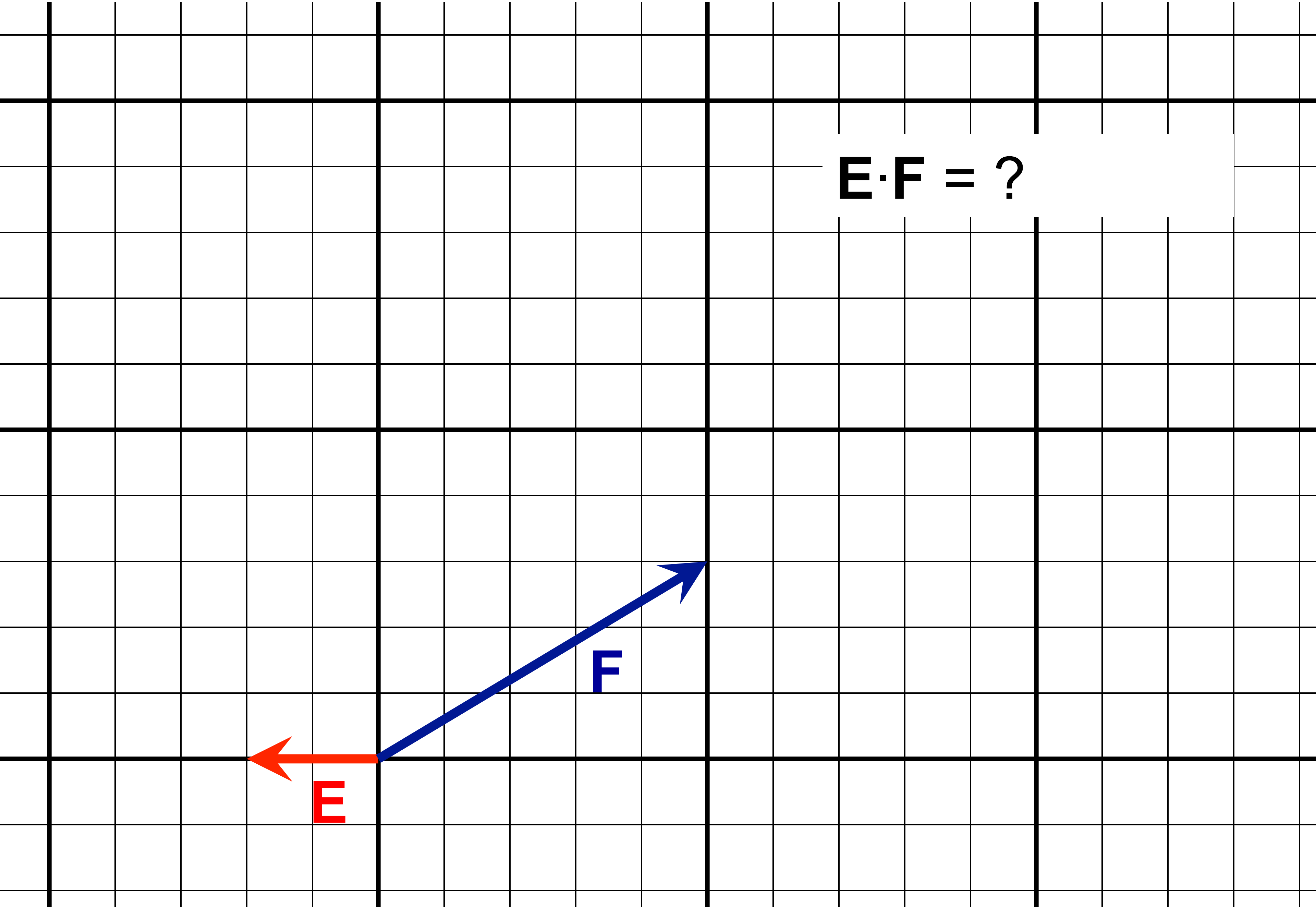

A·B = ?
A
B

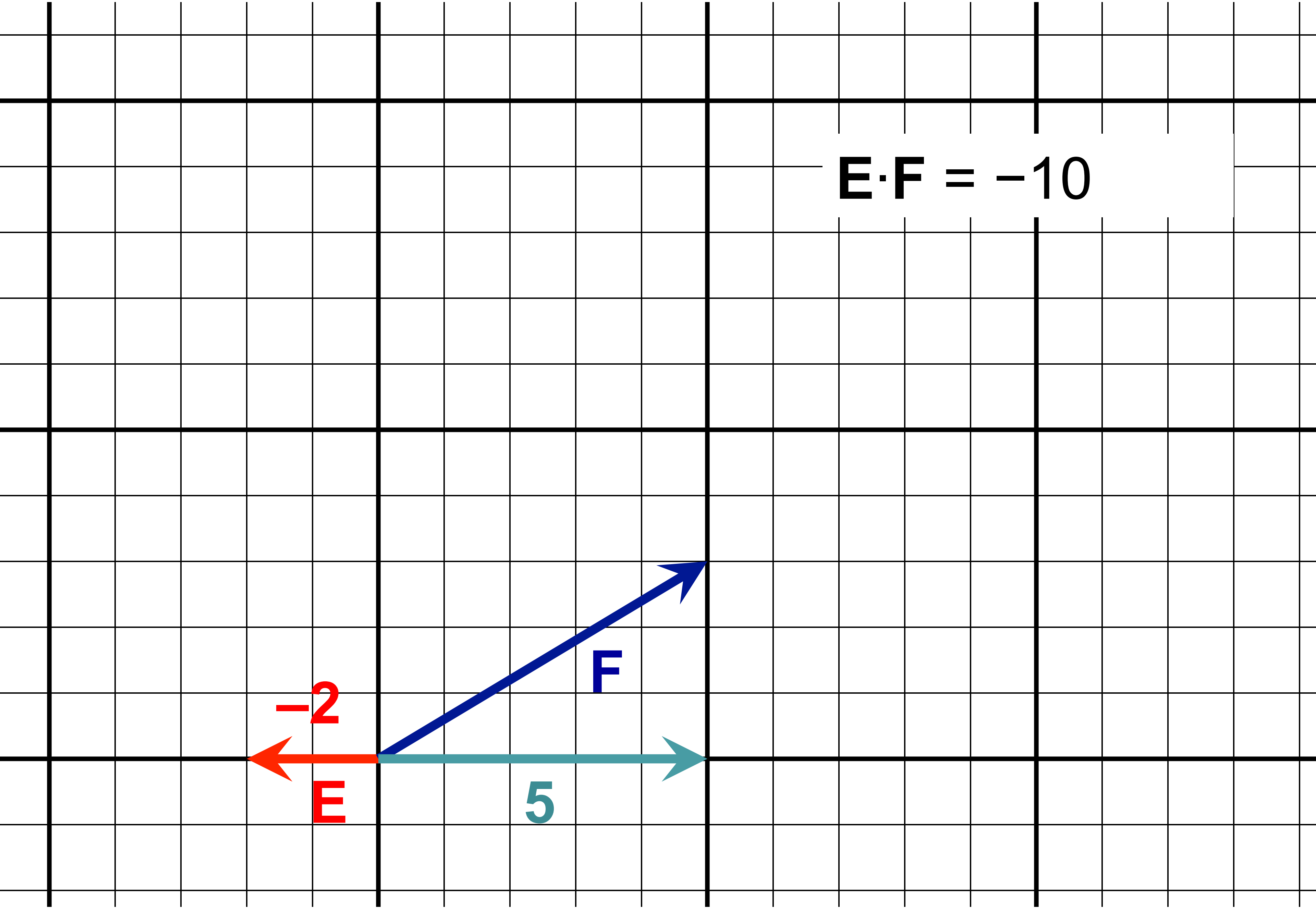


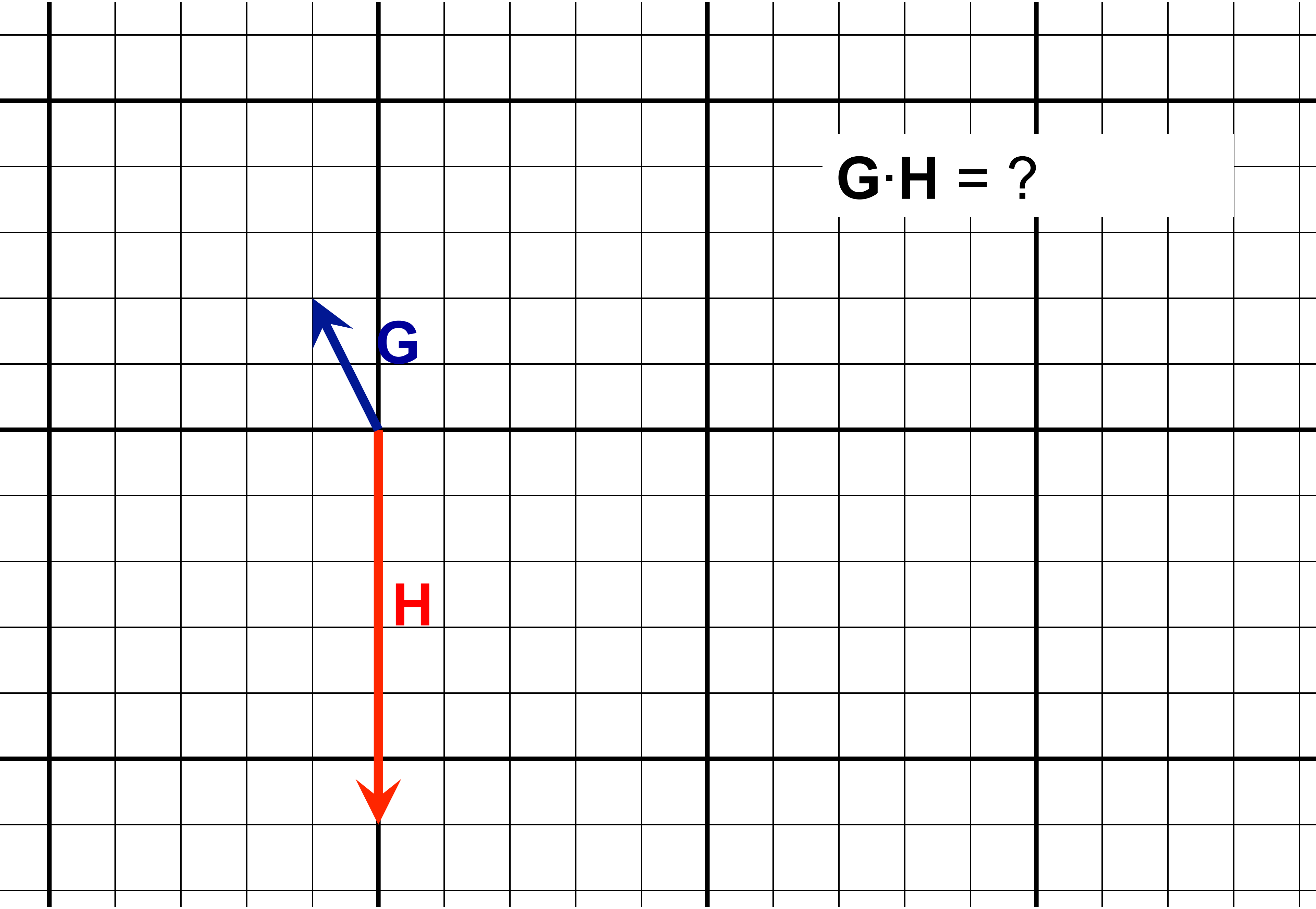
A·B = 24
6
A
4
B

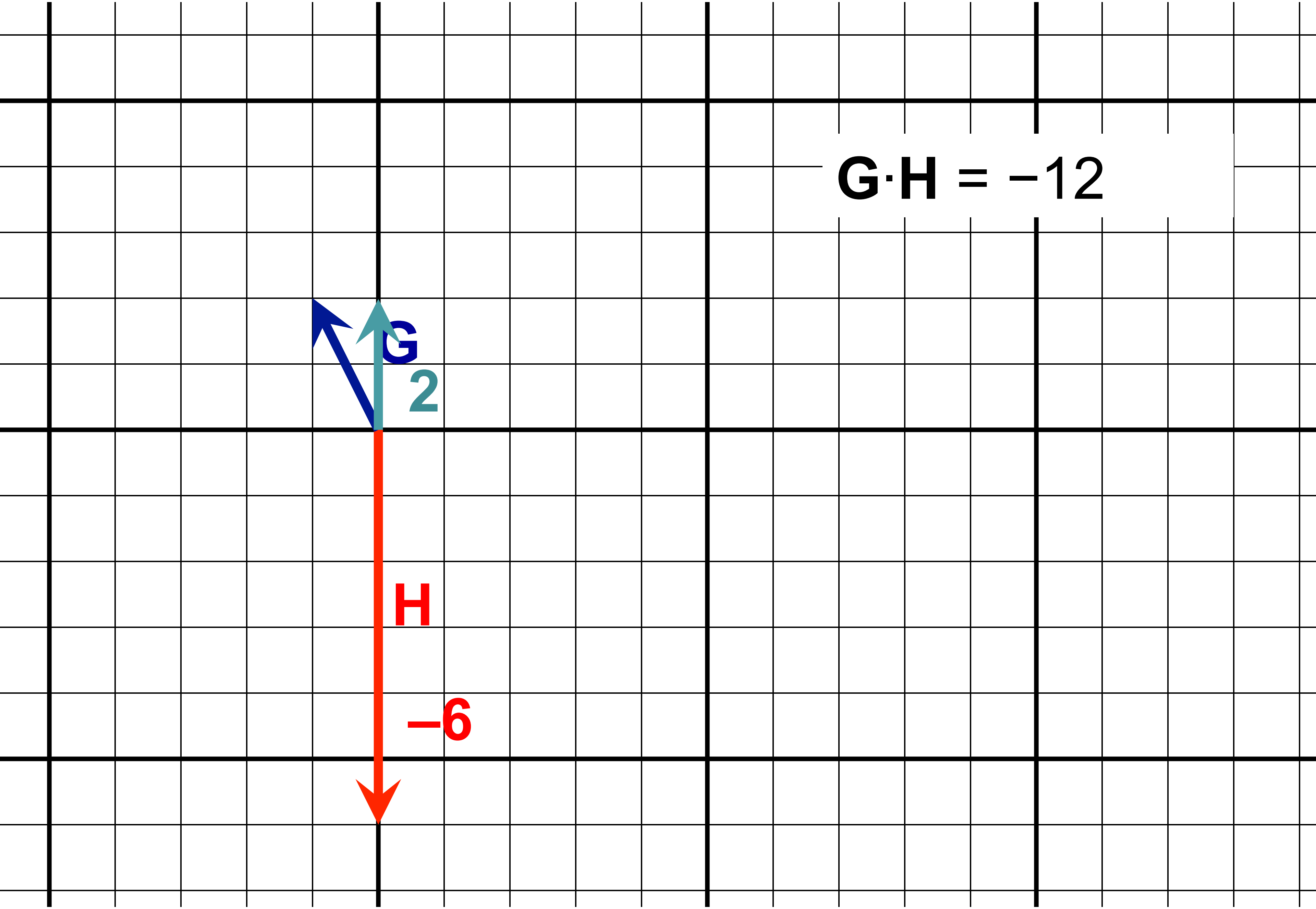

C·D = ?
D
C

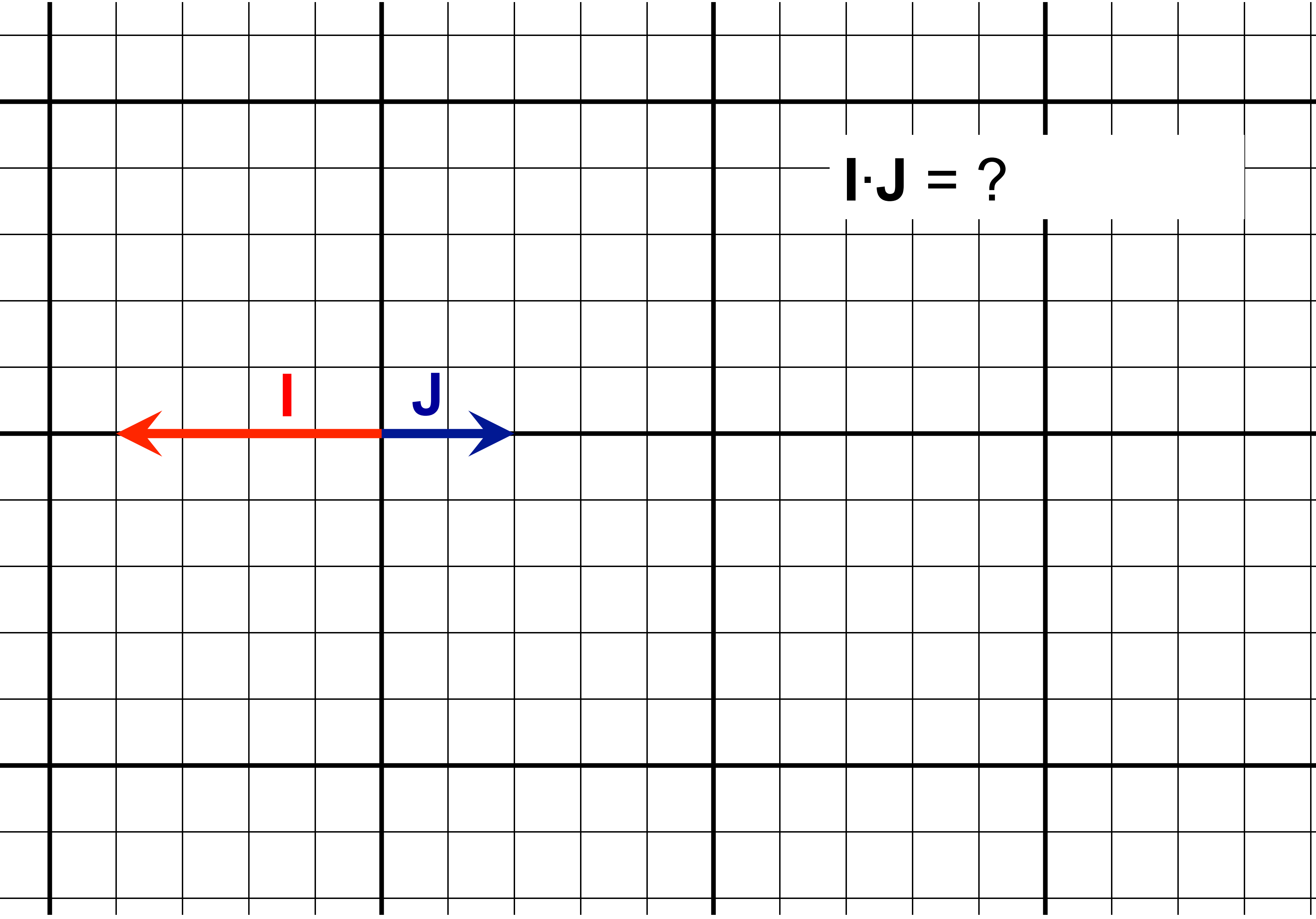

C·D = 18
D
6
3
C


E·F = ?
F
E

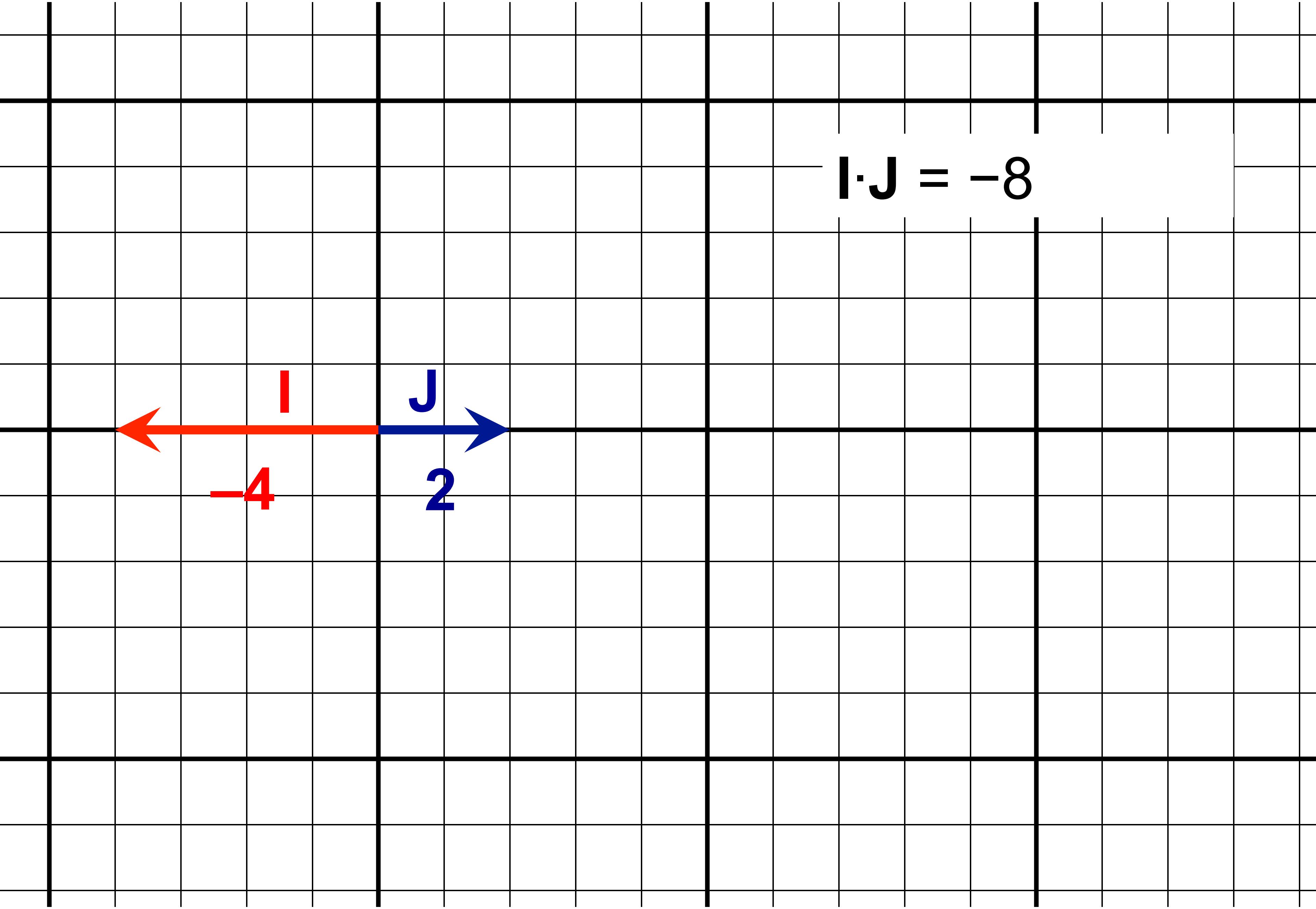

E·F = −10
F
−2
E
5

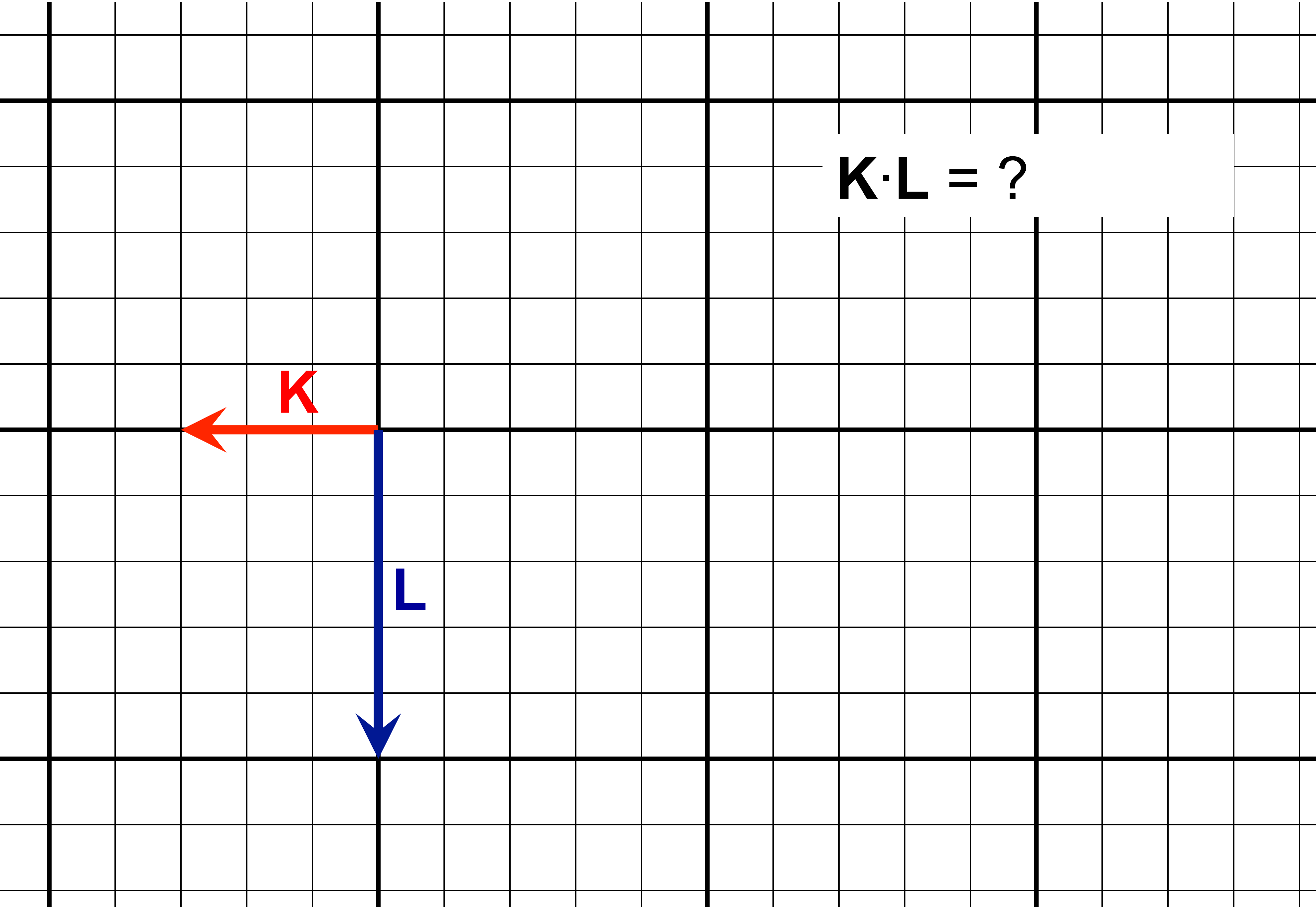

G·H = ?
G
H


G·H = −12
G
2
H
−6

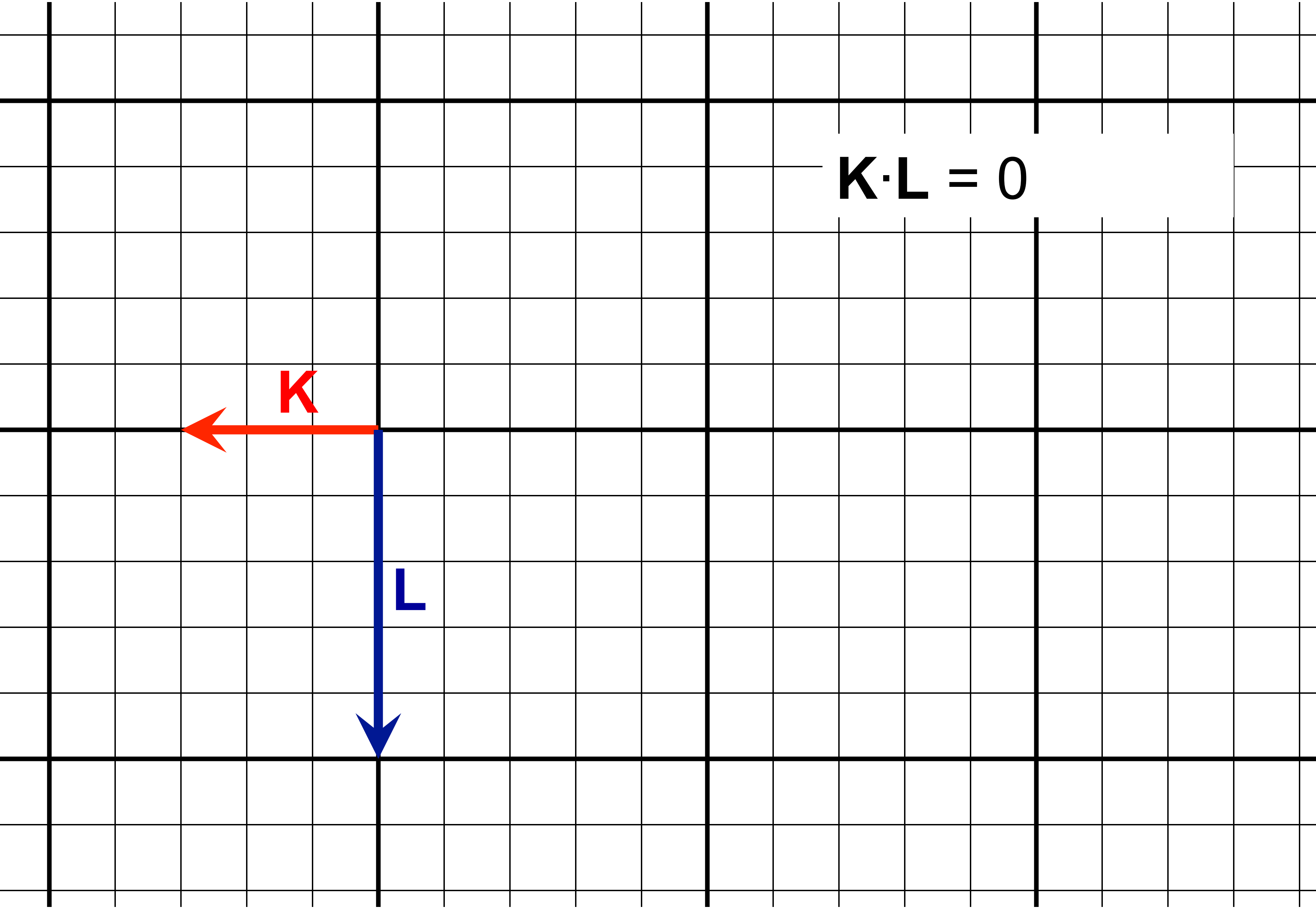


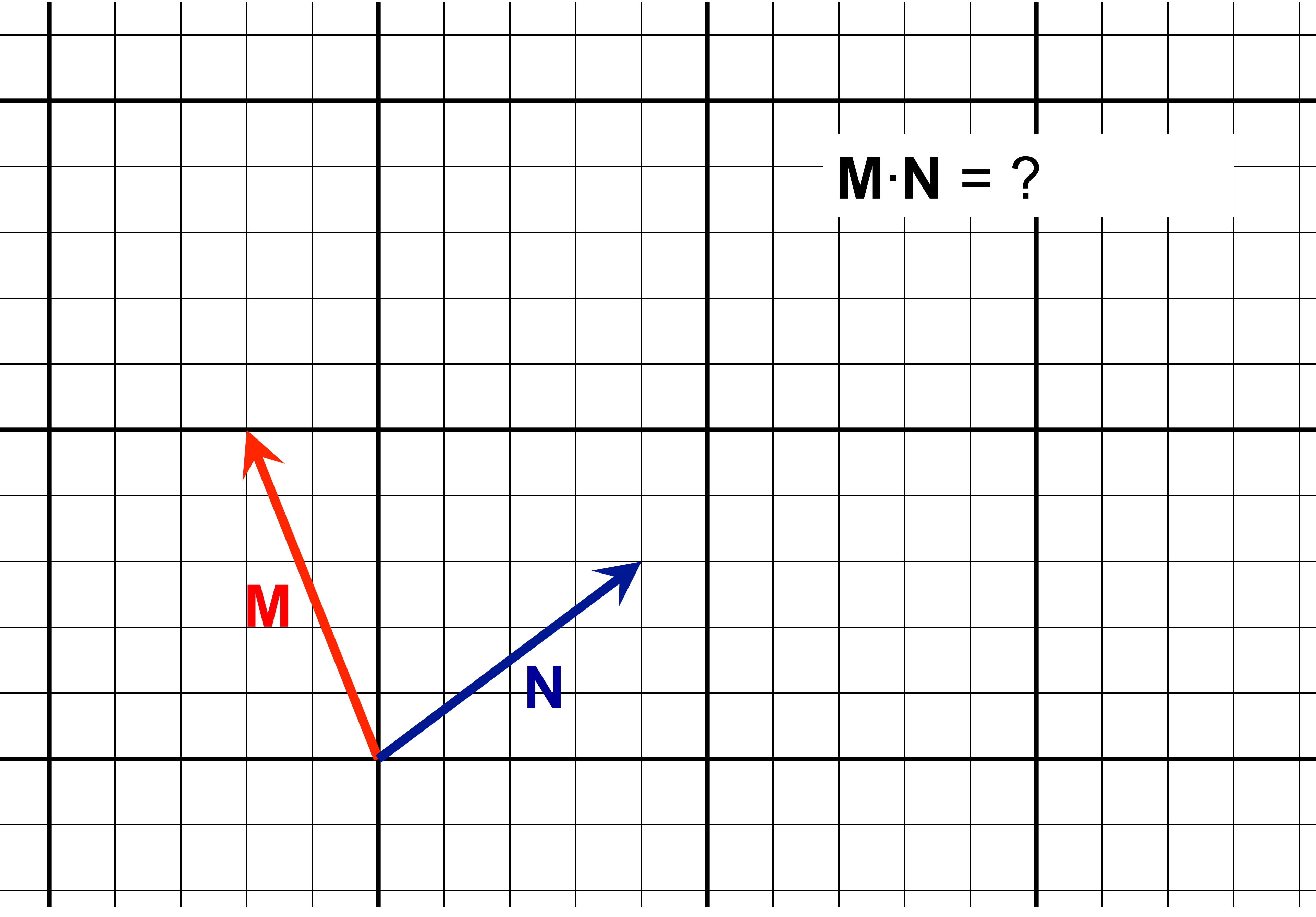
$\mathbf{I} \cdot \mathbf{J} = ?$

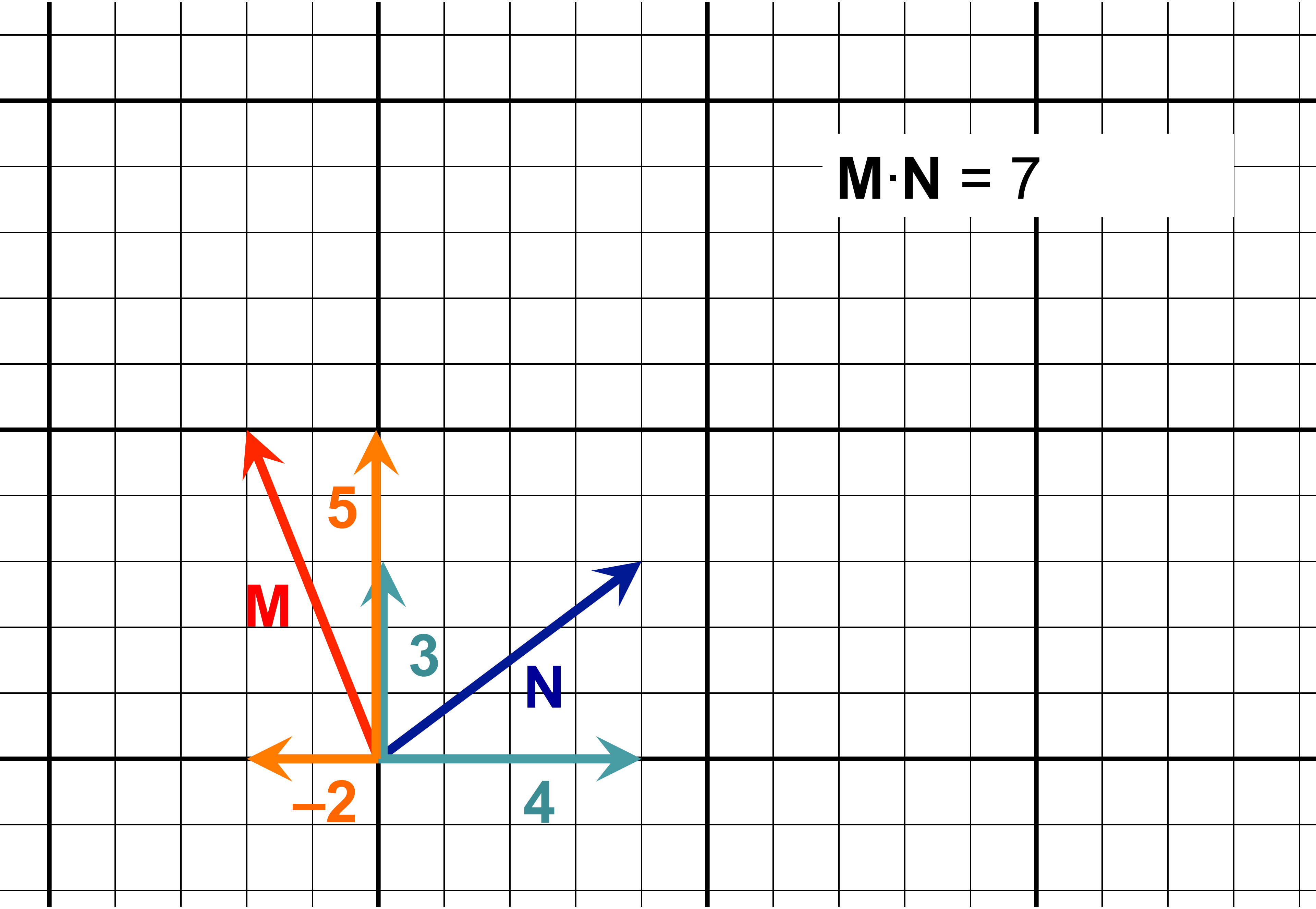

I·J = −8
I
J
−4
2


K·L = ?
K
L

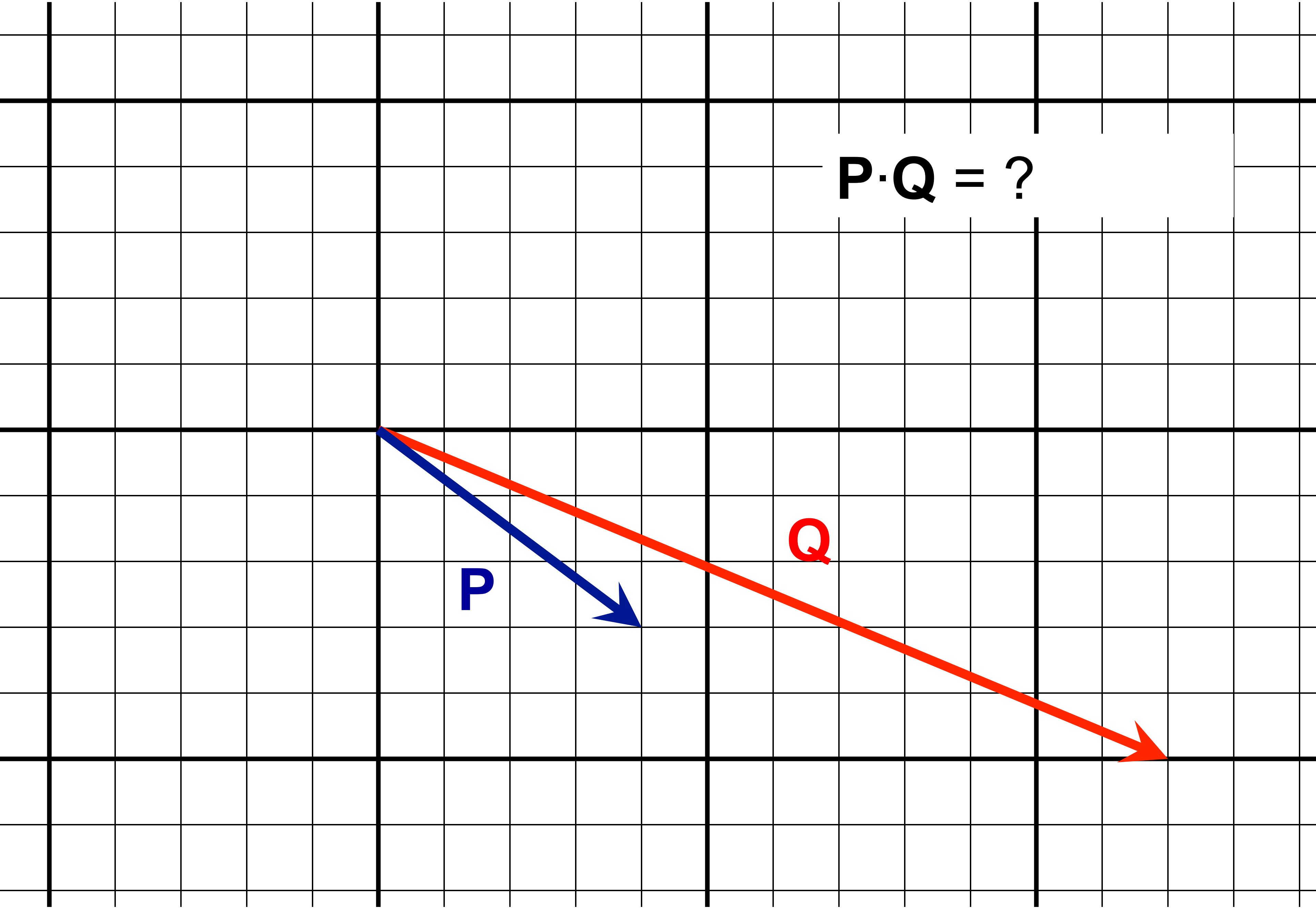
$$K \cdot L = 0$$

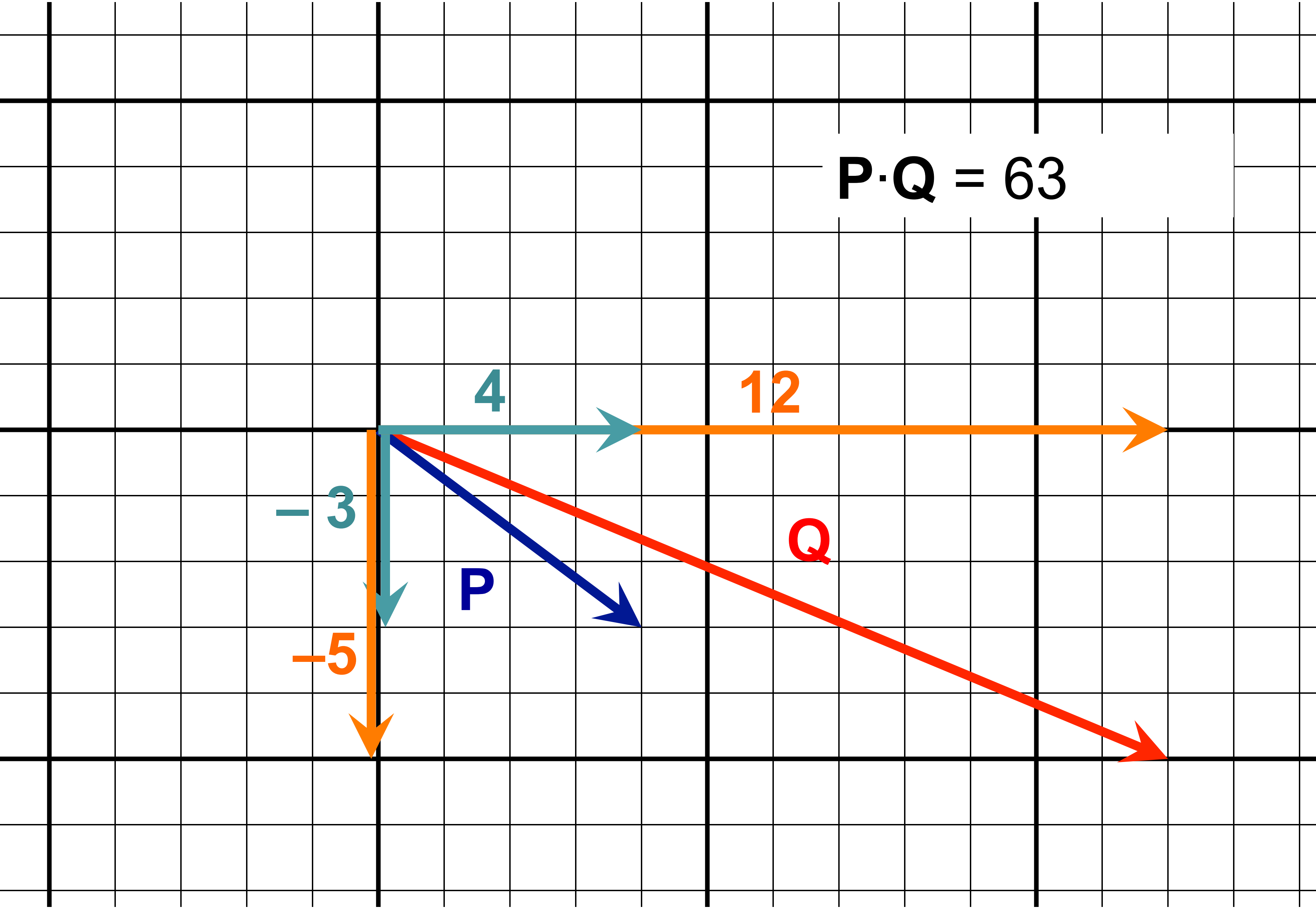



M·N = ?
M
N

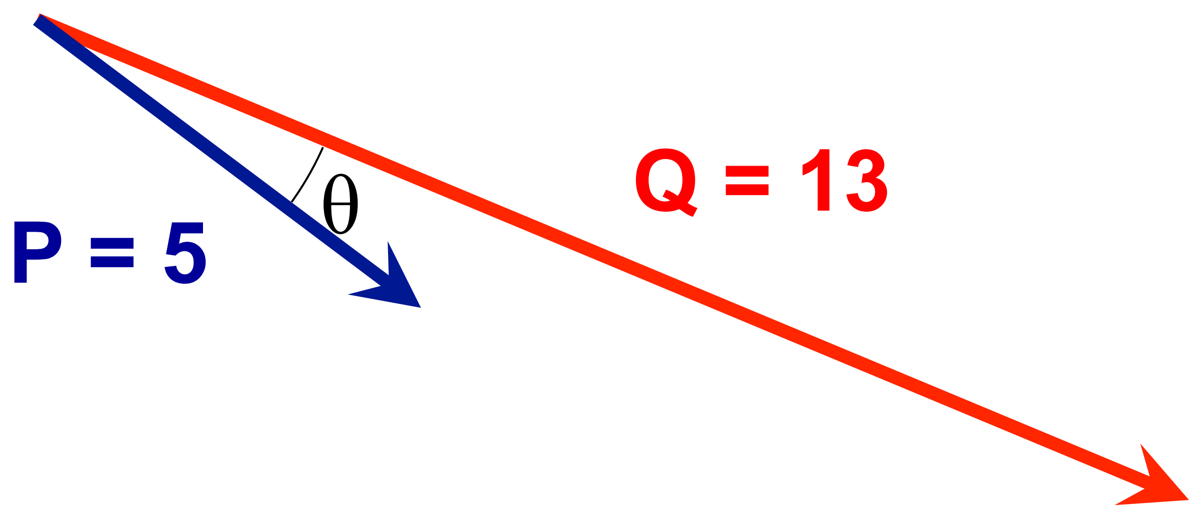


M·N = 7
5
M
3
N
–2
4


P·Q = ?
Q
P

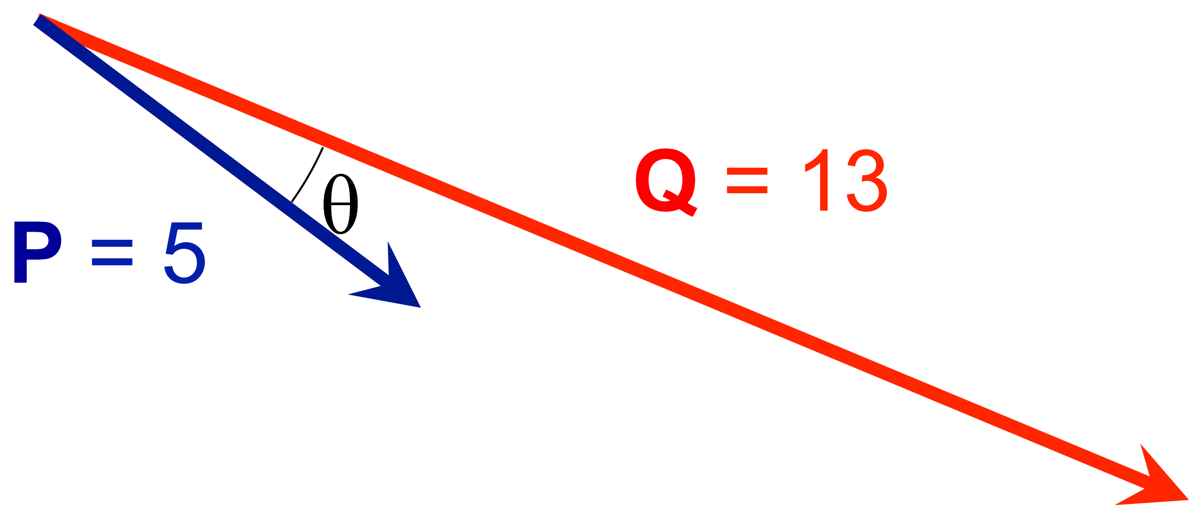

P·Q = 63
4
12
– 3
–5
P
Q

$\mathbf{P} \cdot \mathbf{Q} = ?$



$\theta = 14.25°$

$$\mathbf{P}\cdot\mathbf{Q} = 63$$



$$5\times13\times\cos(14.25°) = 63$$

Work Done by a Varying Force

- If the force acting on an object is not constant it is necessary to sum work values.
- The path followed by the object can be divided into segments during which force is constant.
- If the force varies continuously then this becomes an infinite sum or integral.

Work Done by a Varying Force

$$W = \int \vec{F} \cdot d\vec{r}$$

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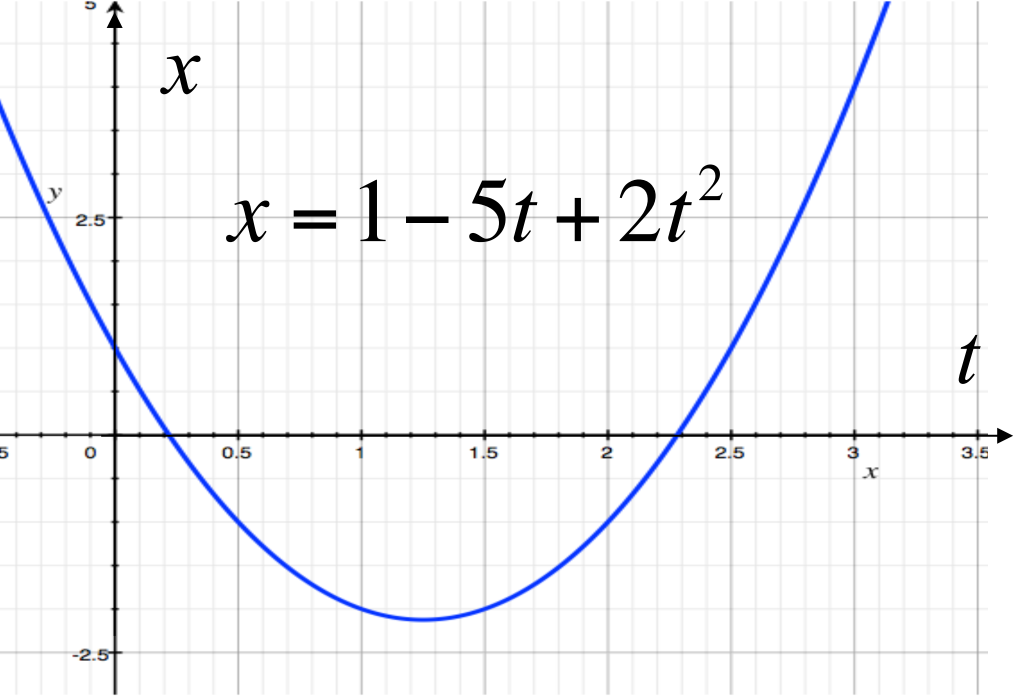
For one dimensional motion and force this simplifies to:

$$W = \int F(x)\,dx$$



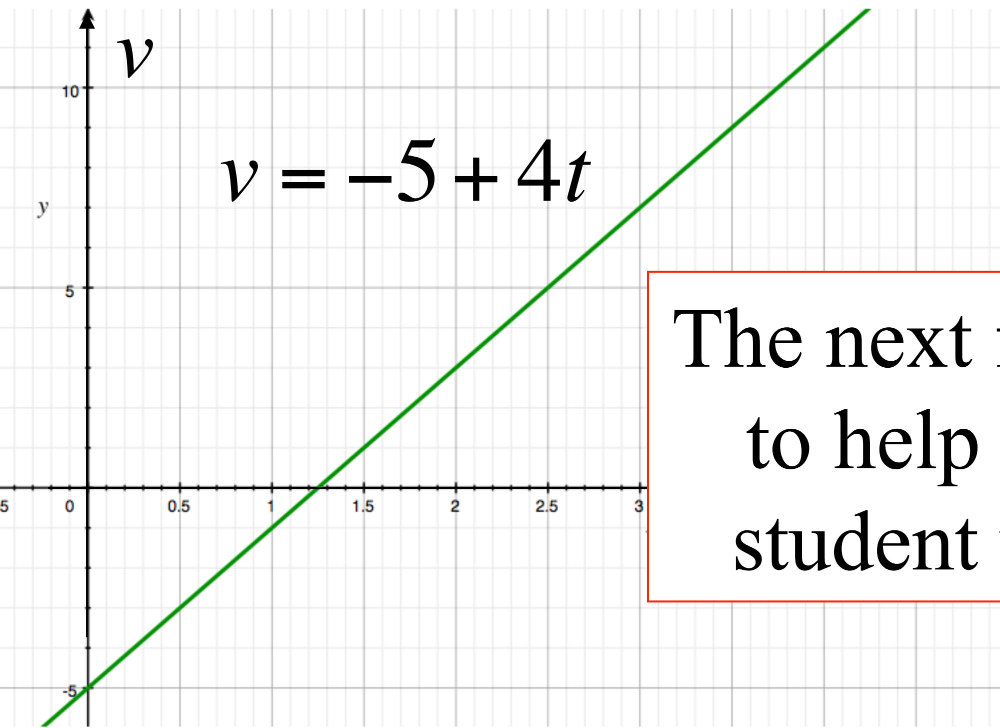
Understanding integrals:

Find change in position between t = 1 and t = 4 s:

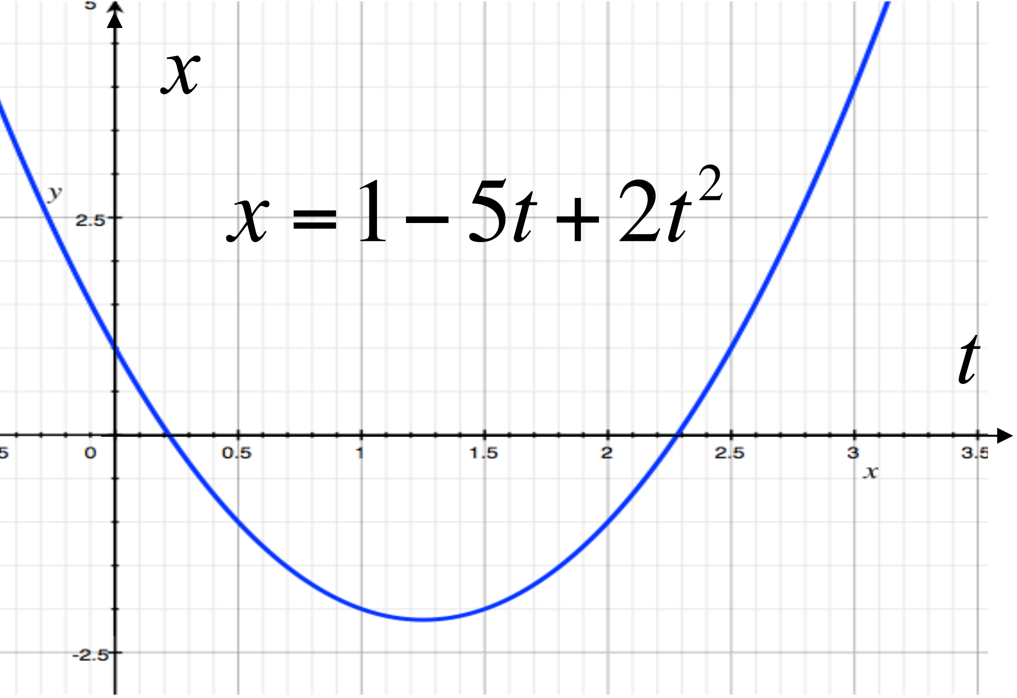
(a) Use $x(t)$

(b) Use $v(t)$

(c) Find area under curve.



The next few pages are intended to help a beginning calculus student understand integrals!



Understanding integrals:

Find change in position between $t = 1$ and $t = 4$ s:

(a) Use $x(t)$ 15 m

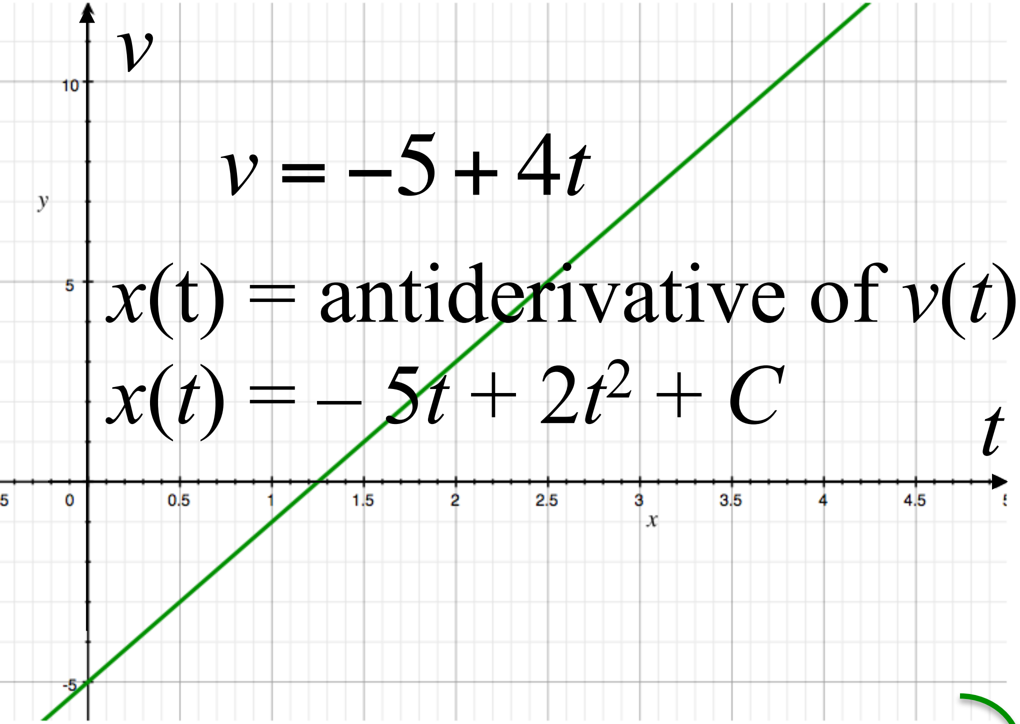
(b) Use $v(t)$

(c) Find area under curve.

$x(1) = 1 - 5(1) + 2(1)^2$
$x(1) = -2$

$x(4) = 1 - 5(4) + 2(4)^2$
$x(4) = 13$

$\Delta x = 13 - (-2)$
$\Delta x = 15$ m



$$v = -5 + 4t$$

$x(t)$ = antiderivative of $v(t)$

$$x(t) = -5t + 2t^2 + C$$

Understanding integrals:

Find change in position between $t = 1$ and $t = 4$ s:

(a) Use $x(t)$ 15 m

(b) Use $v(t)$ 15 m

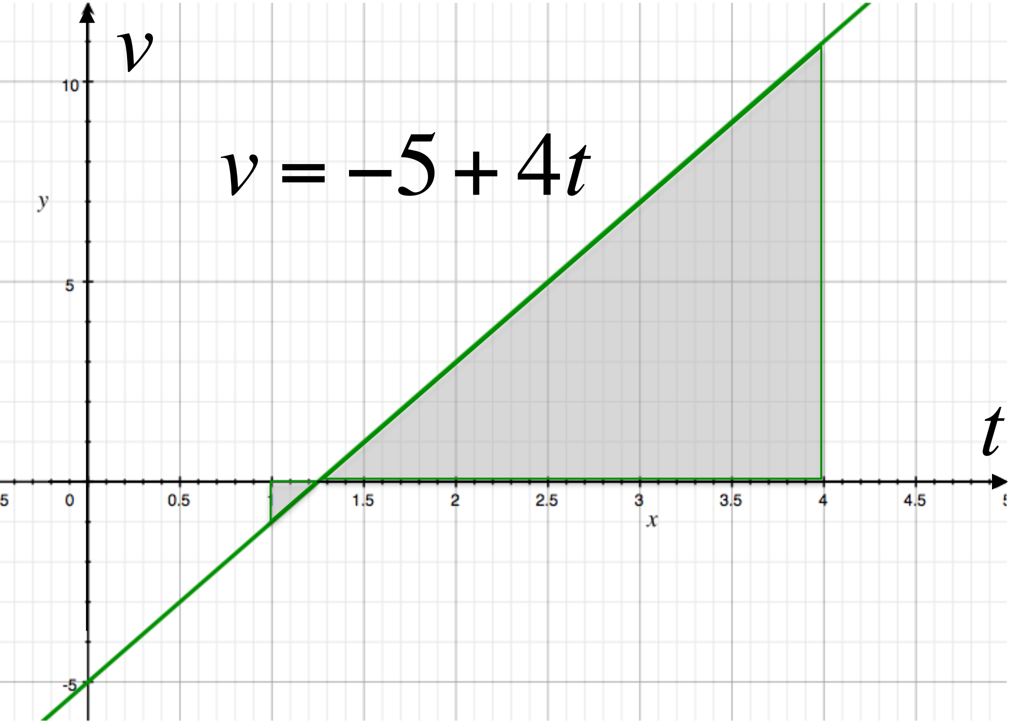
(c) Find area under curve.

$$x(1) = -5(1) + 2(1)^2 + C$$
$$x(1) = -3 + C$$

$$x(4) = -5(4) + 2(4)^2 + C$$
$$x(4) = 12 + C$$

$$\Delta x = (12 + C) - (-3 + C)$$
$$\Delta x = 15 \text{ m}$$

notice the value of C does not matter when finding Δx



Understanding integrals:

Find change in position between t = 1 and t = 4 s:

(a) Use $x(t)$ **15 m**

(b) Use $v(t)$ **15 m**

(c) Find area under curve. **15 m**

$A_1 = \frac{1}{2}bh = \frac{1}{2}(0.25)(-1)$

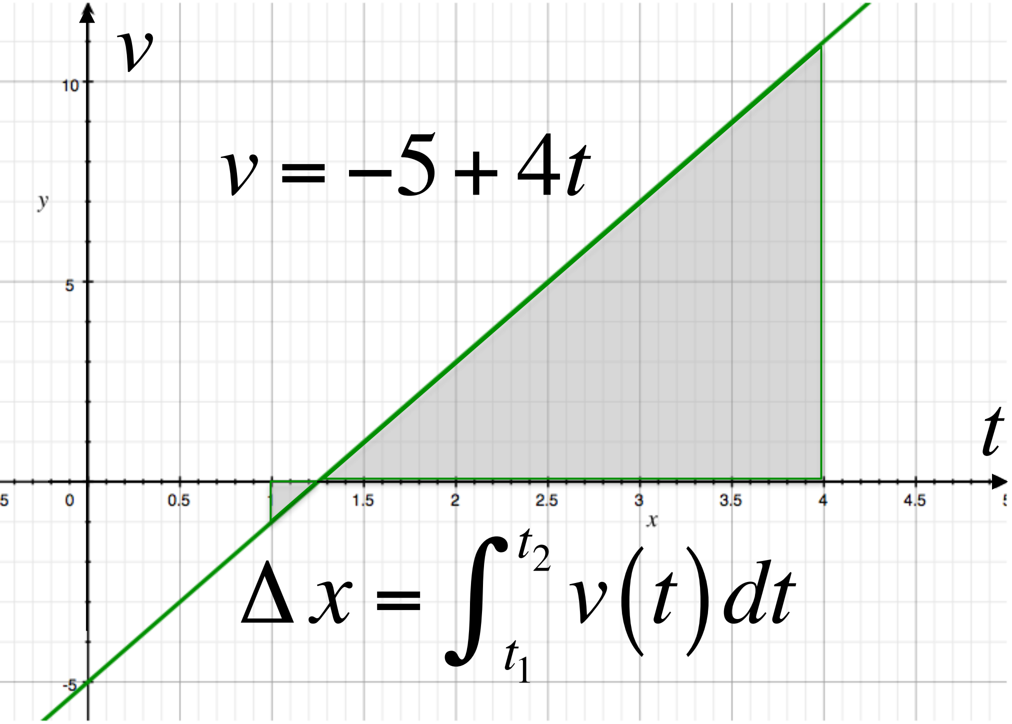
$A_1 = -0.125$

$A_2 = \frac{1}{2}bh = \frac{1}{2}(2.75)(11)$

$A_2 = 15.125$

$\Delta x = A_1 + A_2 = -0.125 + 15.125$

$\Delta x = 15$ m



Understanding integrals:

Find change in position between $t = 1$ and $t = 4$ s:

Speed times time is equivalent to distance, but because the speed varies it is necessary to integrate and find the area under the curve.

$$\Delta x = \int_{t_1}^{t_2} v(t)\,dt$$

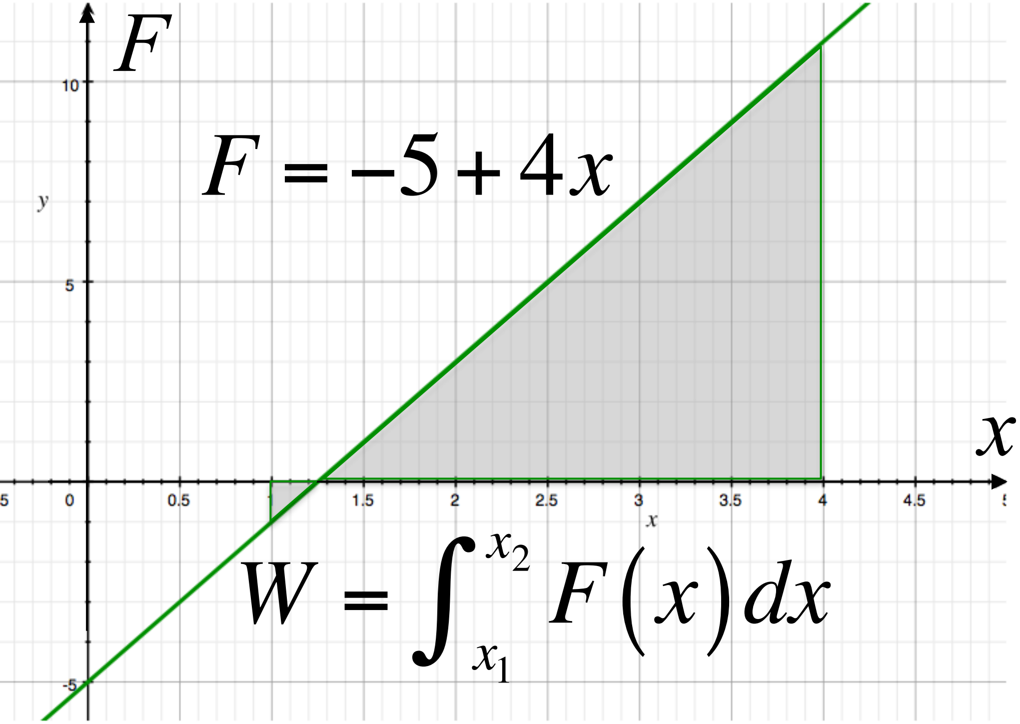
$$\Delta x = \int_{1}^{4} -5 + 4t\,dt$$

$$\Delta x = -5t + 2t^2 \Big|_1^4$$

$$\Delta x = \left(-5\cdot 4 + 2\cdot 4^2\right) - \left(-5\cdot 1 + 2\cdot 1^2\right)$$

$$\Delta x = 12 - (-3) = 15$$

$$\boxed{\Delta x = 15\text{ m}}$$



$$F = -5 + 4x$$

$$W = \int_{x_1}^{x_2} F(x)\,dx$$

$$W = \int_1^4 -5 + 4x\,dx$$

$$W = -5x + 2x^2 \Big|_1^4$$

$$W = \left(-5 \cdot 4 + 2 \cdot 4^2\right) - \left(-5 \cdot 1 + 2 \cdot 1^2\right)$$

$$W = 12 - (-3) = 15$$

Understanding integrals:

Find work done between $x = 1$ and $x = 4$ m

Force times distance is equivalent to work, but because the force varies it is necessary to integrate and find the area under the curve.

$$\boxed{W = 15\text{ J}}$$

$$W = \int_{x_1}^{x_2} F(x)\,dx$$

The integral notation can be interpreted to mean find the difference in the anitderivative of force as a function of position evaluated at two values of position. By the fundamental theorem of calculus this equals the area under the curve or the infinite sum of force times infinitesimal changes in position – the “*dx*” in the notation.

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When work is done there is always a corresponding energy transfer and/or transformation.

If a force does on an object:	The effect on the object will be:
Positive Work	Increase in Energy
Zero Work	No Change in Energy
Negative Work	Decrease in Energy

The amount of work will precisely equal the amount of change in energy!

Work-Energy Theorem

The total work done on an object is equal to the change in kinetic energy of the object:

$$\Sigma W = \Delta K$$

$$\Sigma W = K_2 - K_1$$

where: ΣW = the sum of work done by all forces acting on the object (may include some negative work values)

Mechanical Kinetic Energy

The kinetic energy that an object possesses due to its translational motion is given by:

$$K = \frac{1}{2}mv^2$$

where: m = mass of the object
v = speed of the object