Equilibrium

Statics Analyses

Equilibrium

- A rigid body is said to be in equilibrium if its linear and angular acceleration are both zero.
- An object at rest and not rotating is in equilibrium. But also motion with a constant velocity and/or constant angular velocity is considered to be equilibrium.

Conditions of Equilibrium:

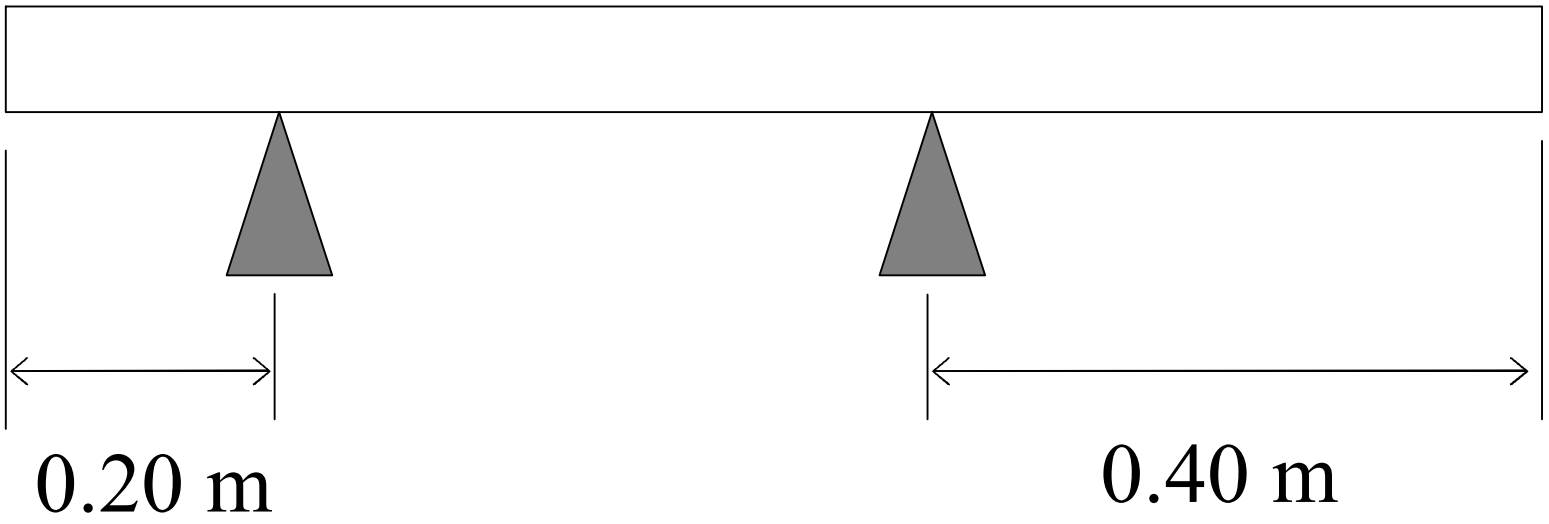
Any rigid body in equilibrium must have zero net force and zero net torque!

$$\Sigma \vec{F} = 0 \begin{cases} \Sigma F_x = 0 \\ \Sigma F_y = 0 \end{cases} \qquad \Sigma \vec{\tau} = 0$$

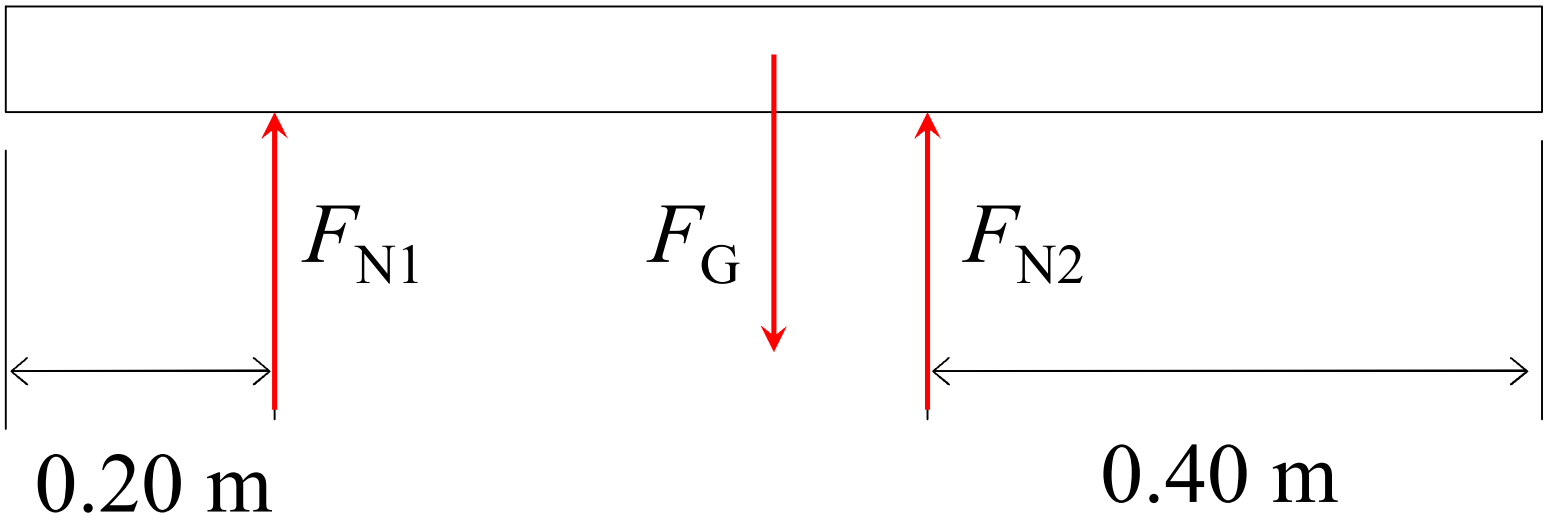
About Torque and Equilibrium . . .

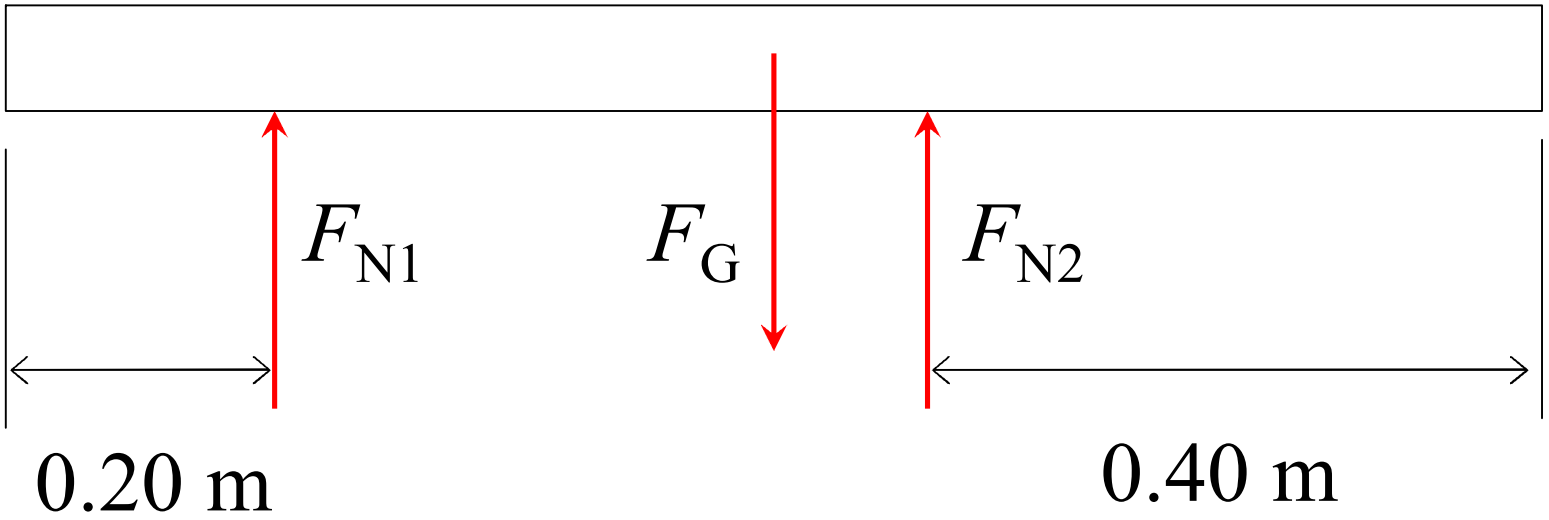
- In order to determine torque it is necessary to know the moment arm, **r**, relative to an axis of rotation. ($\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$)
- Because we are basically assuming there is *no* rotation, it is not important *where* the axis is located or *what* axis is used to determine **r**.
- Put another way, you can choose *any* convenient axis because the object is not rotating about *any* axis.

A uniform horizontal beam of mass 100 grams and length 1.00 m rests on two supports as shown. Determine the normal force at each support.



Draw a free body diagram and apply the conditions of equilibrium . . .



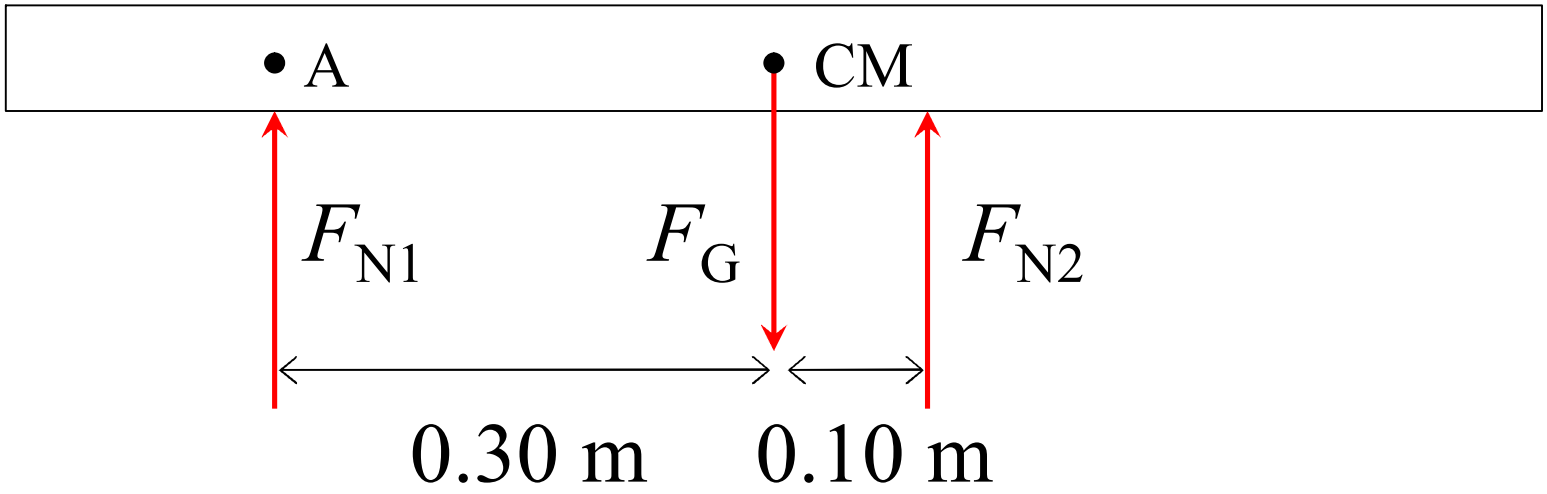


Net force equals zero:

$$\Sigma F_y = 0 = F_{N1} + F_{N2} - F_G$$

$$F_{N1} + F_{N2} - mg = 0$$

$$F_{N1} + F_{N2} = mg$$



Net torque about the center of mass equals zero:

$$\Sigma \tau_{CM} = 0 = -0.3F_{N1} + 0.1F_{N2}$$

OR, net torque about point A equals zero:

$$\Sigma \tau_{A} = 0 = -0.3F_{G} + 0.4F_{N2}$$

Either will work BUT, 2nd equation is *easier to use*!

Solving the equations for force and torque yields the following solution:

$$\mathbf{F}_{N1} = 0.25\, mg = 0.245 \text{ N, upward}$$

$$\mathbf{F}_{N2} = 0.75\, mg = 0.735 \text{ N, upward}$$

Note that these values satisfy all of the equations that were listed – if it is not moving and not rotating then there must be zero net force and zero net torque.

It is usually best to take the torque about a point through which an unknown force acts – thus eliminating a variable in the resulting equation.

Summarizing . . .

- To analyze a rigid body that is in equilibrium a series of equations involving force can be written.
- Set the sum of all x-forces equal to zero, set the sum of all y-forces equal to zero.
- Set the net torque about a convenient axis equal to zero.
- Solve the resulting system of equations.