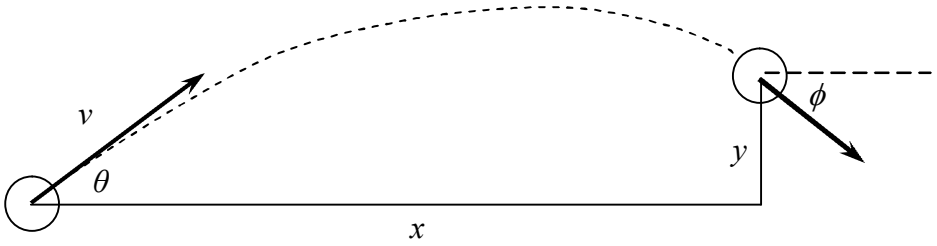
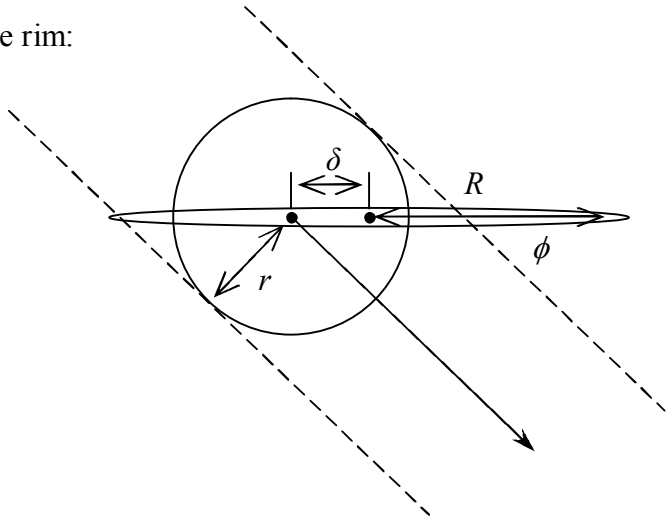
**Anatomy of a Free Throw**

Goal: Find an equation for making a free throw!

trajectory:




passing through the rim:




<u>variables:</u>
$v$ = initial speed
$\theta$ = initial angle
$x$ = range
$y$ = elevation
$\phi$ = final angle
$r$ = radius of ball
$R$ = radius of rim
$\delta$ = variation in $x$ (allowable for swish)

<u>components of displacement</u>:

$$x = (v\cos\theta)t$$

$$y = (v\sin\theta)t - \frac{gt^2}{2}$$

<u>components of final velocity</u>:

$$v_x = v\cos\theta$$

$$v_y = v\sin\theta - gt$$

variation in x:

$$\delta = R - \frac{r}{\sin\phi}$$

eliminate *t*:

$$y = x\tan\theta - \frac{gx^2}{2v^2\cos^2\theta}$$

$$v = \sqrt{\frac{gx^2}{2\left(x\sin\theta\cos\theta - y\cos^2\theta\right)}}$$

$$v_y = v\sin\theta - \frac{gx}{v\cos\theta}$$

solve for $\phi$:

$$\tan\phi = \frac{v_y}{v_x}$$

$$\tan\phi = \frac{gx}{v^2\cos^2\theta} - \tan\theta$$

$$\tan\phi = \tan\theta - \frac{2y}{x}$$

$$\phi = \arctan\left(\tan\theta - \frac{2y}{x}\right)$$

allowing for variation in *x*, the final equation for the initial velocity:

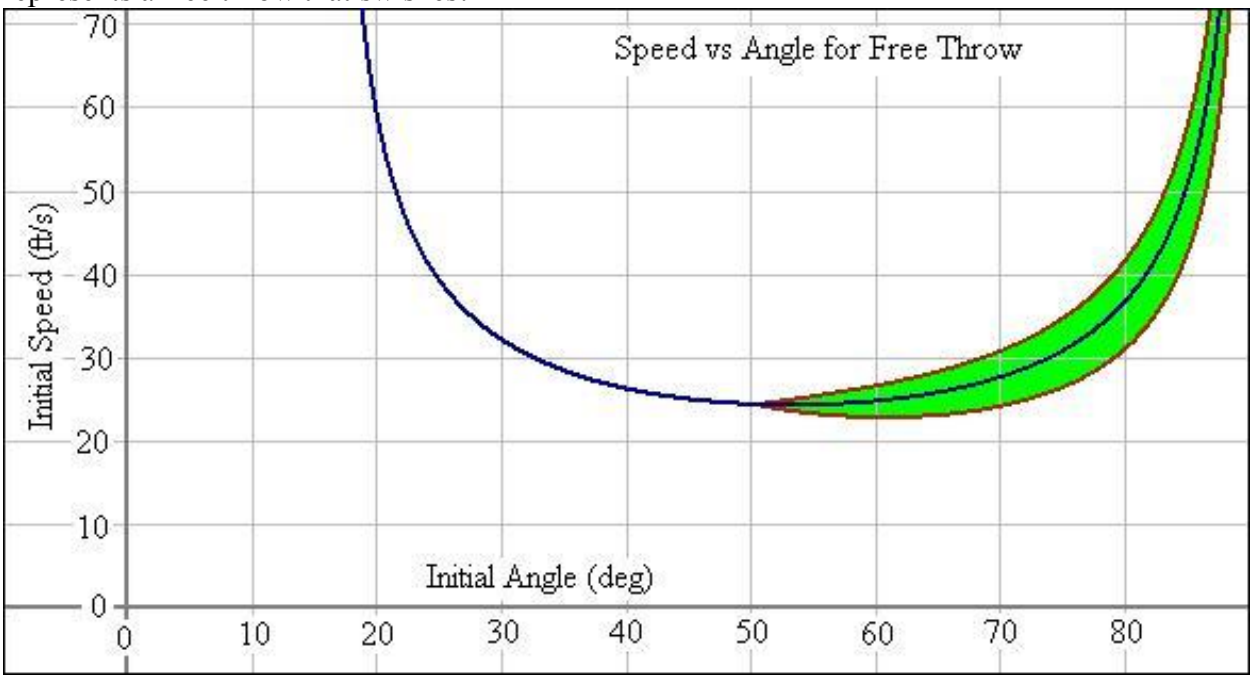
$$v = \sqrt{\frac{g\left(x \pm \delta\right)^2}{2\left(\left(x \pm \delta\right)\sin\theta\cos\theta - y\cos^2\theta\right)}}$$

where:

$$\delta = R - \frac{r}{\sin\phi}$$

$$\phi = \arctan\left(\tan\theta - \frac{2y}{x}\right)$$

Below is a graph showing $v$ versus $\theta$, assuming $x = 13.75$ ft, $y = 4$ ft, $g = 32.2$ ft/s$^2$, $R = 9$ in, $r = 4.77$ in. The blue curve represents getting the ball to the center of the rim ($\delta = 0$). The brown curves allow for $\delta$ with ball just missing the front or back of the rim, and the shaded region represents a free throw that swishes!




Compare to the same graph adjusted to reflect $y = 2$ ft (for a taller player):