Inductance

and Inductors

Magnetic Induction

I. Induction
   - Faraday's Law, Lenz's Law

II. Maxwell's equations

III. Inductance and Inductors
   - design and geometry

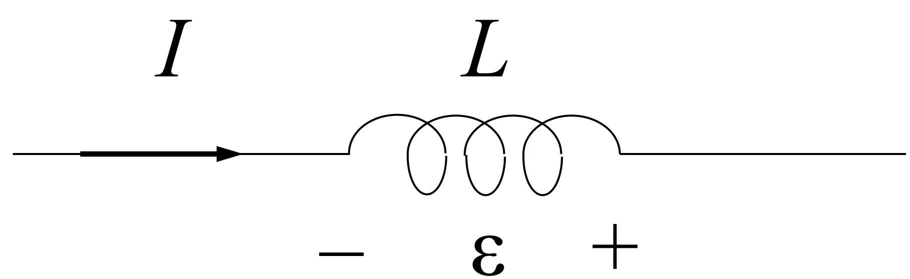
IV. RL Circuits
   - steady state, dynamic behavior

V. LC Circuits
   - oscillations

	The student will be able to:	HW:
1	State and apply Faraday's Law and Lenz's Law and solve magnetic induction problems involving changing magnetic flux, and induced emf or eddy currents.	1 – 16
2	Solve problems involving basic principles of generators, including production of back emf.	17 – 21
3	State and recognize Maxwell's equations and associate each equation with its implications.	22 – 23
4	Define and calculate inductance and solve related problems including those that involve parallel or series inductors.	24 – 31
5	Analyze RL circuits in terms of the appropriate differential equation and resulting exponential functions for charge, current, voltage, etc.	32 – 38
6	Analyze LC and RLC circuits in terms of the appropriate differential equation and resulting exponential functions for charge, current, voltage, etc.	39 – 41

Self-Inductance is the property of a coil such that an emf is induced by changes in its magnetic flux. The emf can be modeled by the following:

$$\mathcal{E} = -L\frac{dI}{dt}$$



where: $\mathcal{E}$ = induced emf

L = "inductance" – a constant particular to the coil

I = current through the coil

Inductance – SI Units

1 henry = 1 volt·second per ampere

$$1\,\text{H} = 1\,\frac{\text{V}\cdot\text{s}}{\text{A}}$$

$$1\,\text{H} = 1\,\frac{\text{Wb}}{\text{A}} = 1\,\frac{\text{T}\cdot\text{m}^2}{\text{A}}$$

$$1\,\text{H} = 1\,\Omega\cdot\text{s}$$

Because the magnetic field generated by a coil's current cannot decrease without inducing an emf, it may be said that energy is stored in the magnetic field of any inductor carrying a current:

$$U = \frac{1}{2} L I^2$$

where: U = potential energy
L = inductance
I = current

An inductor’s induced emf will always have a polarity that tends to maintain its previously held current when changes occur.

If current decreases, the induced emf will support the current.

If current increases, the induced emf will oppose the current.

An inductor can be thought of as a current *moderator* – it acts to oppose changes in current.

The greater the inductance the less rapid will be any changes in current.

Inductance can be defined as the ratio of magnetic flux to current and is a "geometric" property of any coil. This is because the magnetic field of the coil will be proportional to its current.

$$L = \frac{N\Phi_m}{I}$$

where: N = number of turns

Φ_m = magnetic flux for *each* turn

I = current

Multilayer air-core coil[23]	$$L = \frac{4}{5} \cdot \frac{r^2 N^2}{6r + 9l + 10d}$$ • L = inductance (μH) • r = mean radius of coil (in) • l = physical length of coil winding (in) • N = number of turns • d = depth of coil (outer radius minus inner radius) (in)

credit: Wikipedia

by this formula:

Pasco Field Coil $L = 16$ mH

UT Coil $L = 11$ mH

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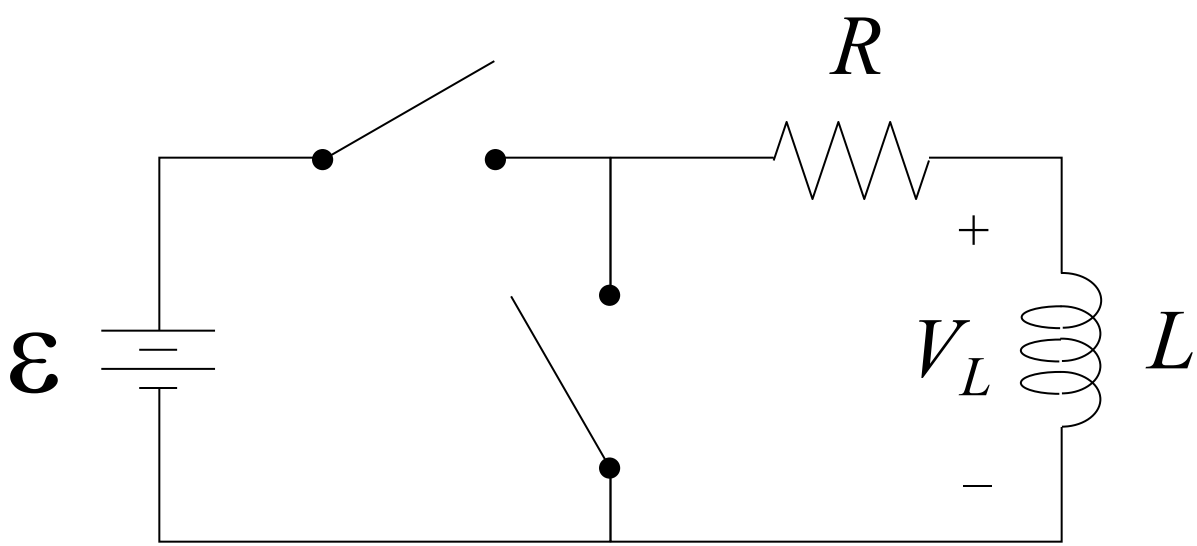
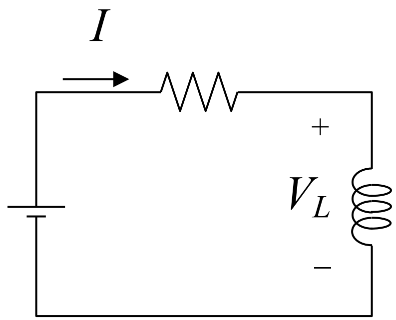
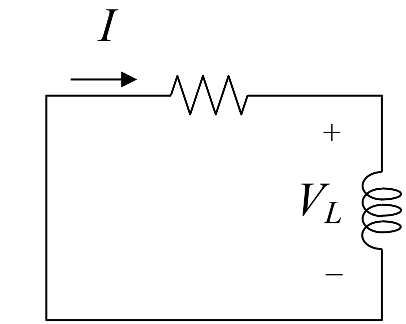
The upper switch is closed until a steady state is reached. Then the positions of the two switches are *simultaneously* reversed. Find functions of time for voltage and current.



$$\mathcal{E} - IR - L\frac{dI}{dt} = 0$$



$$\tau = \frac{L}{R}$$



"Charging"

(*i.e.* inductor is being energized)

$$I = \frac{\mathcal{E}}{R}\left(1 - e^{-\frac{t}{\tau}}\right)$$

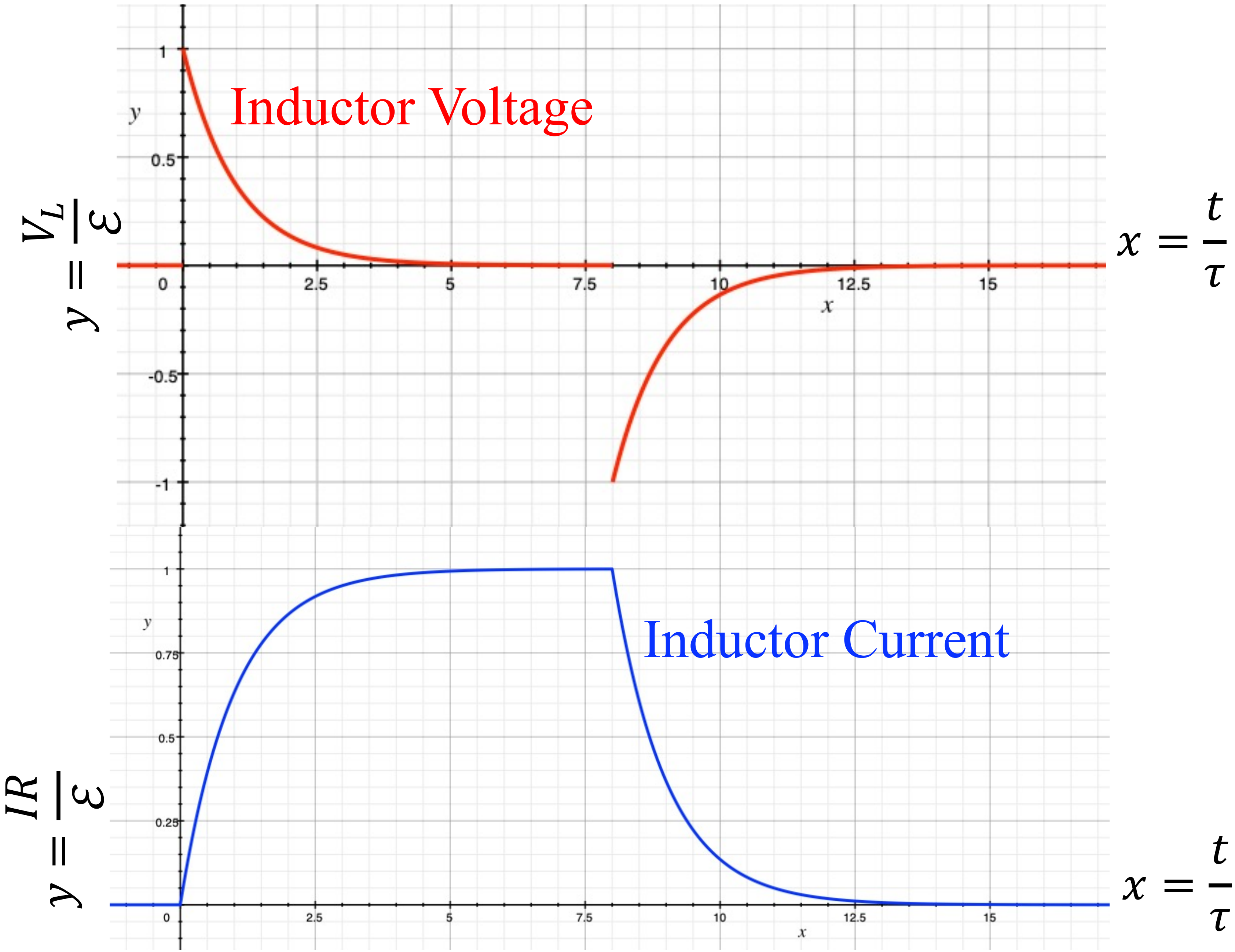
$$V_L = \mathcal{E}e^{-\frac{t}{\tau}}$$

"Discharging"

(*i.e.* inductor is being de-energized)

$$I = \mathcal{E}e^{-\frac{t}{\tau}}$$

$$V_L = -\mathcal{E}e^{-\frac{t}{\tau}}$$


Inductor Voltage
$y = \frac{V_L}{\varepsilon}$
$x = \frac{t}{\tau}$
Inductor Current
$y = \frac{IR}{\varepsilon}$
$x = \frac{t}{\tau}$

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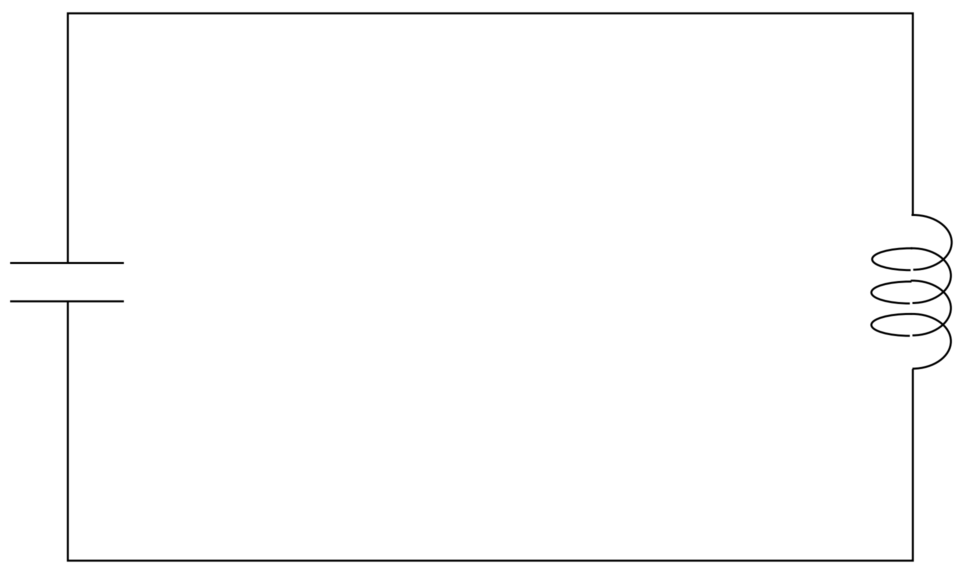
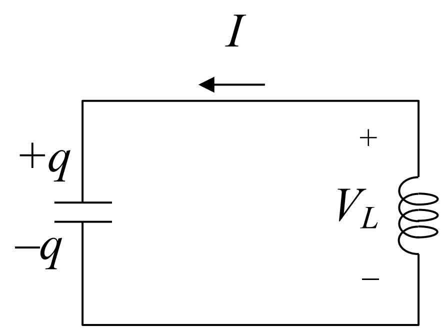
The capacitor shown is charged before it is connected to the inductor. Solve for current, charge, voltage, and energy as functions of time.



LC Circuit – charged capacitor connected to inductor, negligible circuit resistance

$$\frac{q}{C} = -L\frac{dI}{dt}$$

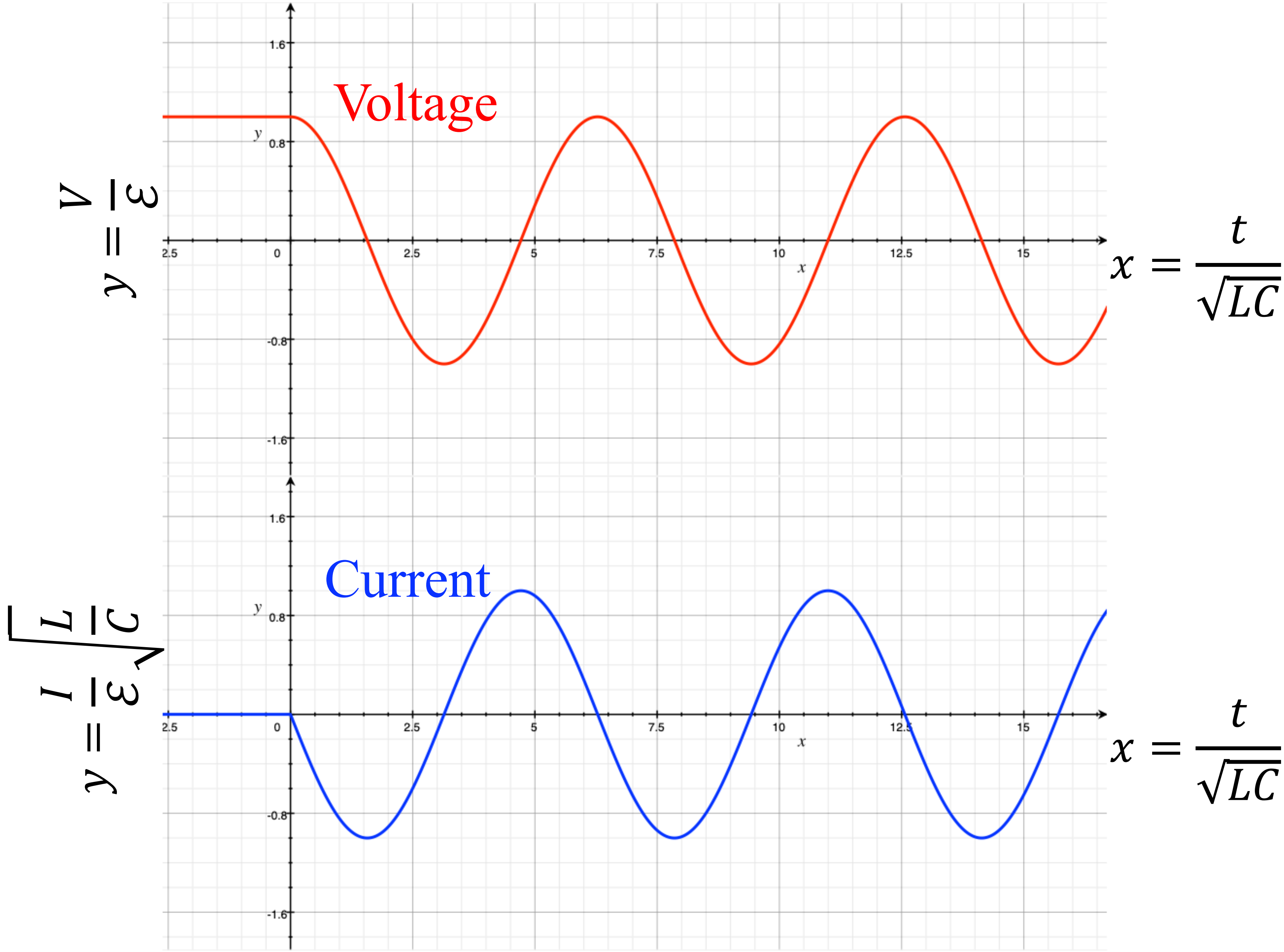


$$q = \mathcal{E}C\cos(\omega t)$$

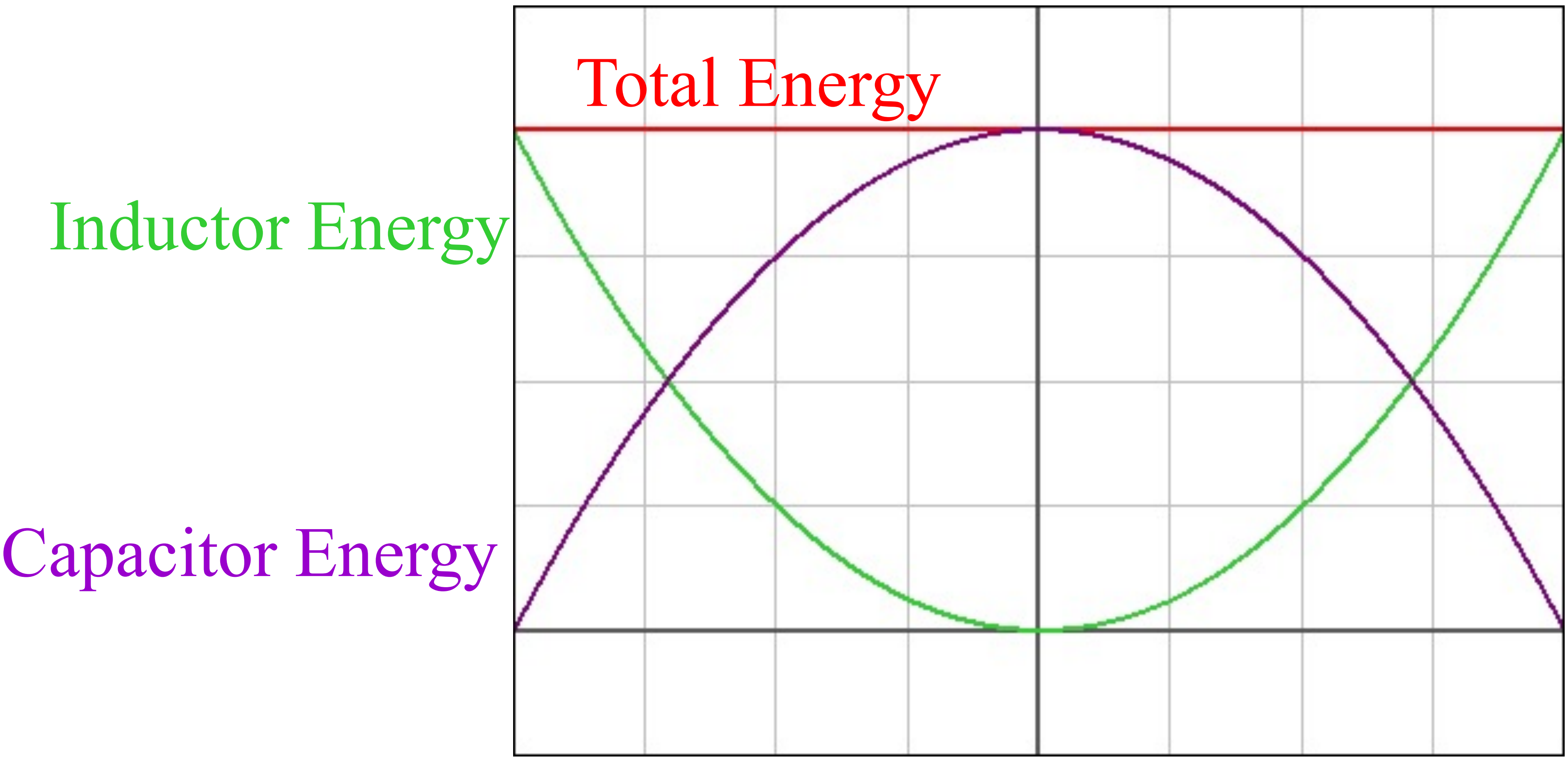
$$V = \mathcal{E}\cos(\omega t)$$

$$\omega = \frac{1}{\sqrt{LC}}$$

$$I = -\mathcal{E}\sqrt{\frac{C}{L}}\sin(\omega t)$$


Voltage
$y = \frac{V}{\varepsilon}$
$x = \frac{t}{\sqrt{LC}}$
Current
$y = \frac{I}{\varepsilon}\sqrt{\frac{L}{C}}$
$x = \frac{t}{\sqrt{LC}}$

Energy vs. Current



Energy vs. Voltage