Magnetic Induction

I. Induction
   - Faraday's Law, Lenz's Law

II. Maxwell's equations

III. Inductance and Inductors
   - design and geometry

IV. RL Circuits
   - steady state, dynamic behavior

V. LC Circuits
   - oscillations

	The student will be able to:	HW:
1	State and apply Faraday's Law and Lenz's Law and solve magnetic induction problems involving changing magnetic flux, and induced emf or eddy currents. ✓	1 – 16
2	Solve problems involving basic principles of generators, including production of back emf. ✓	17 – 21
3	State and recognize Maxwell's equations and associate each equation with its implications.	22 – 23
4	Define and calculate inductance and solve related problems including those that involve parallel or series inductors.	24 – 31
5	Analyze RL circuits in terms of the appropriate differential equation and resulting exponential functions for charge, current, voltage, etc.	32 – 38
6	Analyze LC and RLC circuits in terms of the appropriate differential equation and resulting exponential functions for charge, current, voltage, etc.	39 – 41

Maxwell's Equations (c. 1873)

- Although Maxwell did not "discover" these equations or laws, he is credited with realizing that the four equations "summarize" electricity and magnetism.
- Maxwell *did* make an important "modification" to *one* of the equations.
- It is thought that all E & M phenomena can be related to these equations.
- Maxwell showed that these equations *allow* for electromagnetic waves and derived the speed of such, giving c in terms of μ_0 and ε_0.

Maxwell's Equations

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

Gauss's Law

$$\oint \vec{B} \cdot d\vec{A} = 0$$

Gauss's Law for Magnetic Fields

$$\oint \vec{E} \cdot d\vec{\ell} = -\frac{d}{dt}\int \vec{B} \cdot d\vec{A}$$

Faraday's Law

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I$$

Ampere's Law

Maxwell's Equations

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$\oint \vec{B} \cdot d\vec{A} = 0$$

$$\oint \vec{E} \cdot d\vec{\ell} = -\frac{d}{dt}\int \vec{B} \cdot d\vec{A}$$

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d}{dt}\int \vec{E} \cdot d\vec{A}$$

Maxwell's modification to Ampere's Law

Ampere-Maxwell Law

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d}{dt} \int \vec{E} \cdot d\vec{A}$$

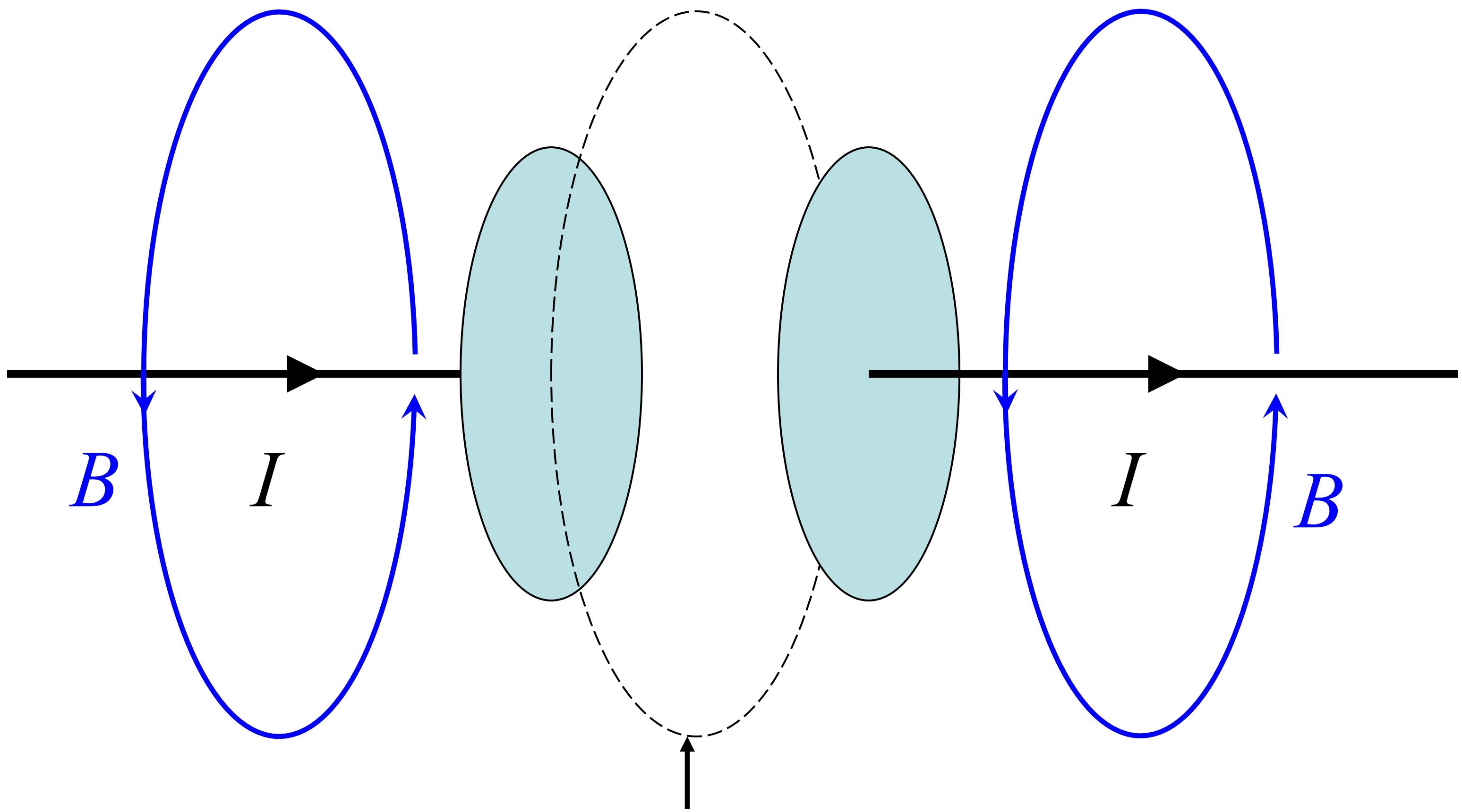
This quantity is known as the “displacement current”.

Ampere-Maxwell Law

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 \left(I + \varepsilon_0 \frac{d}{dt} \int \vec{E} \cdot d\vec{A} \right)$$

This quantity is dimensionally equivalent to a "current" measureable in amperes.

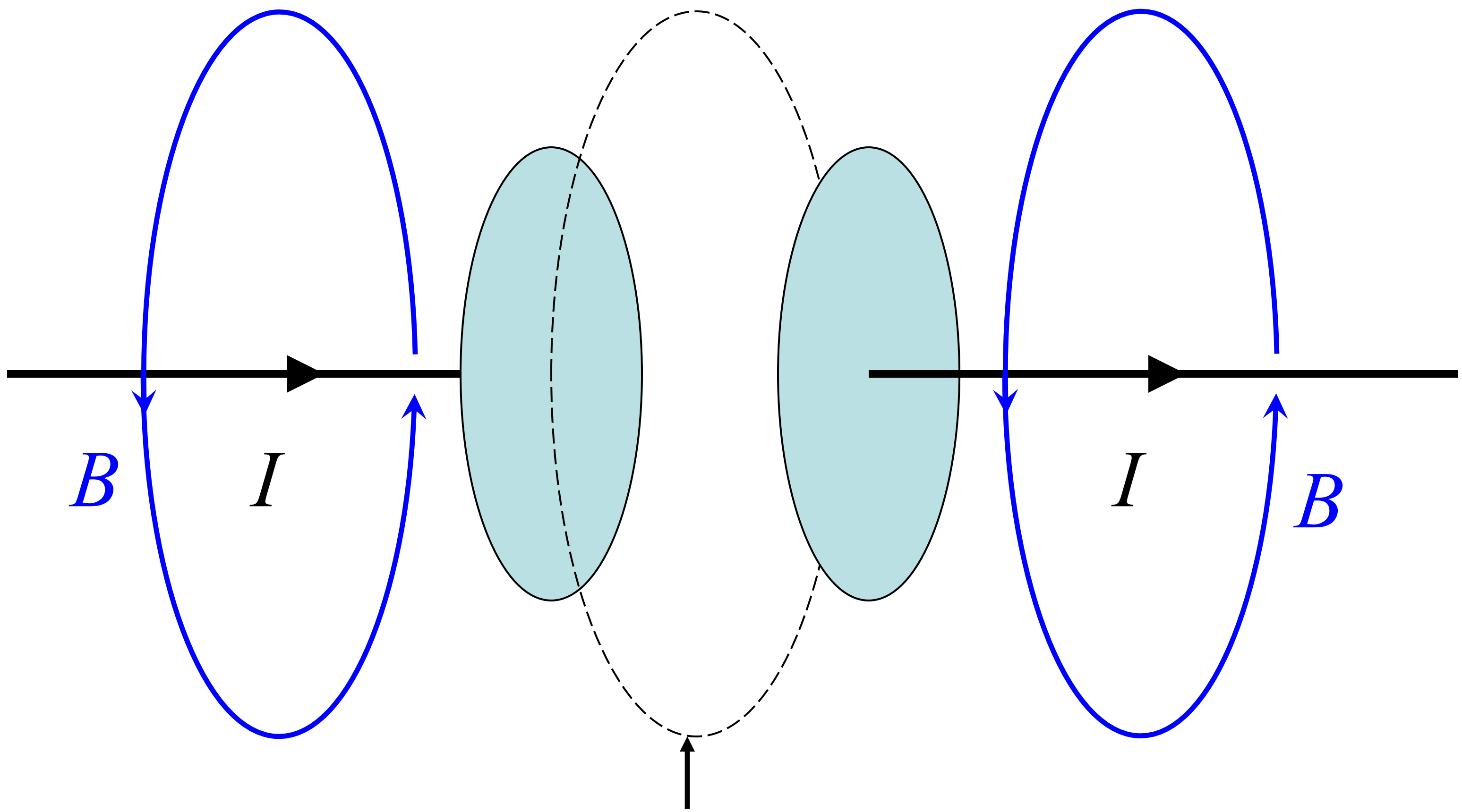
Thought Experiment: Charging a Capacitor



Wouldn't there have to be a magnetic field here?

Not according to Ampere's Law!

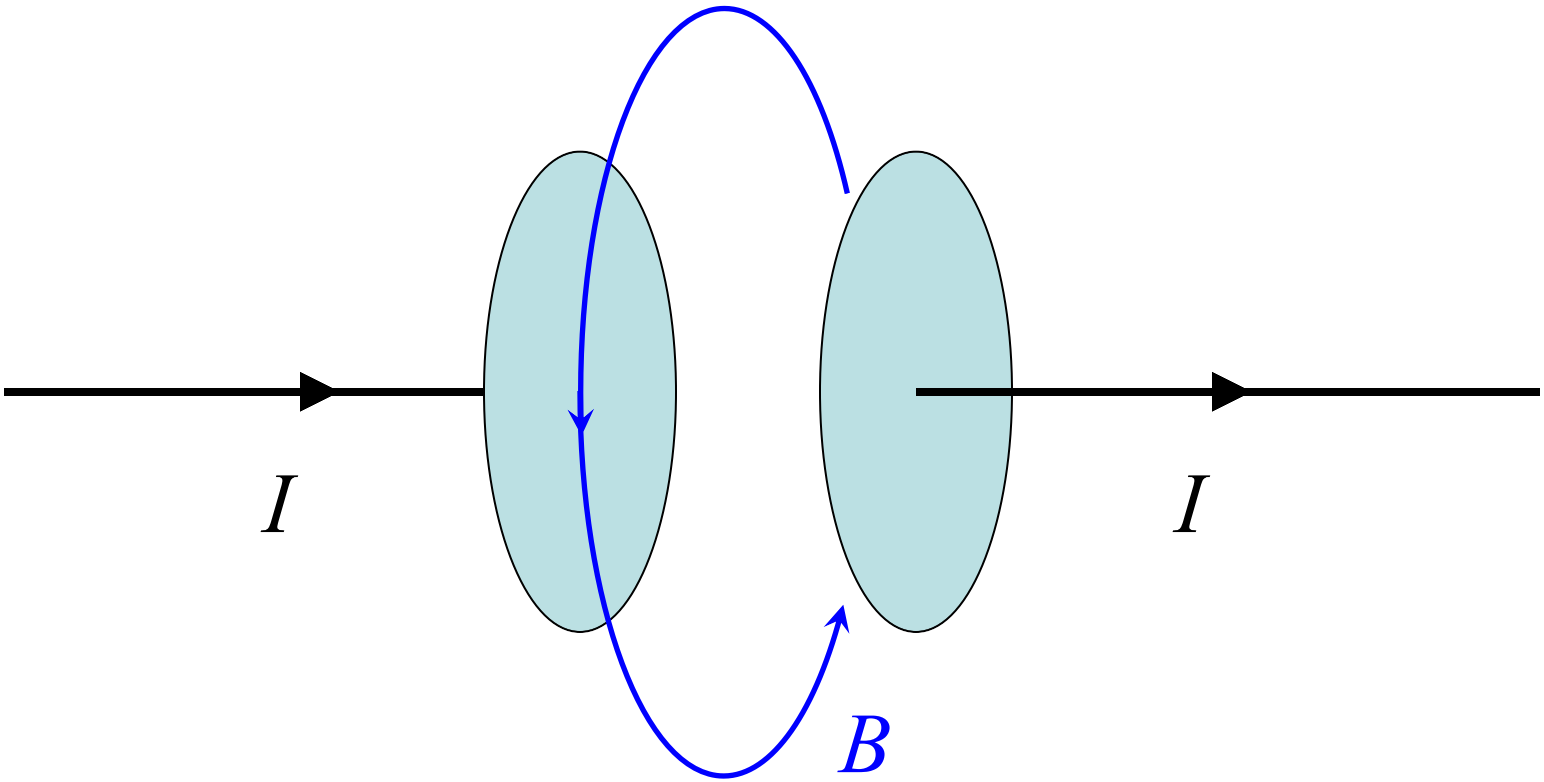
Thought Experiment: Charging a Capacitor



Wouldn't there have to be a magnetic field here?

Not according to Ampere's Law! $\oint \vec{B} \cdot d\vec{\ell} = \mathbf{0}$?!

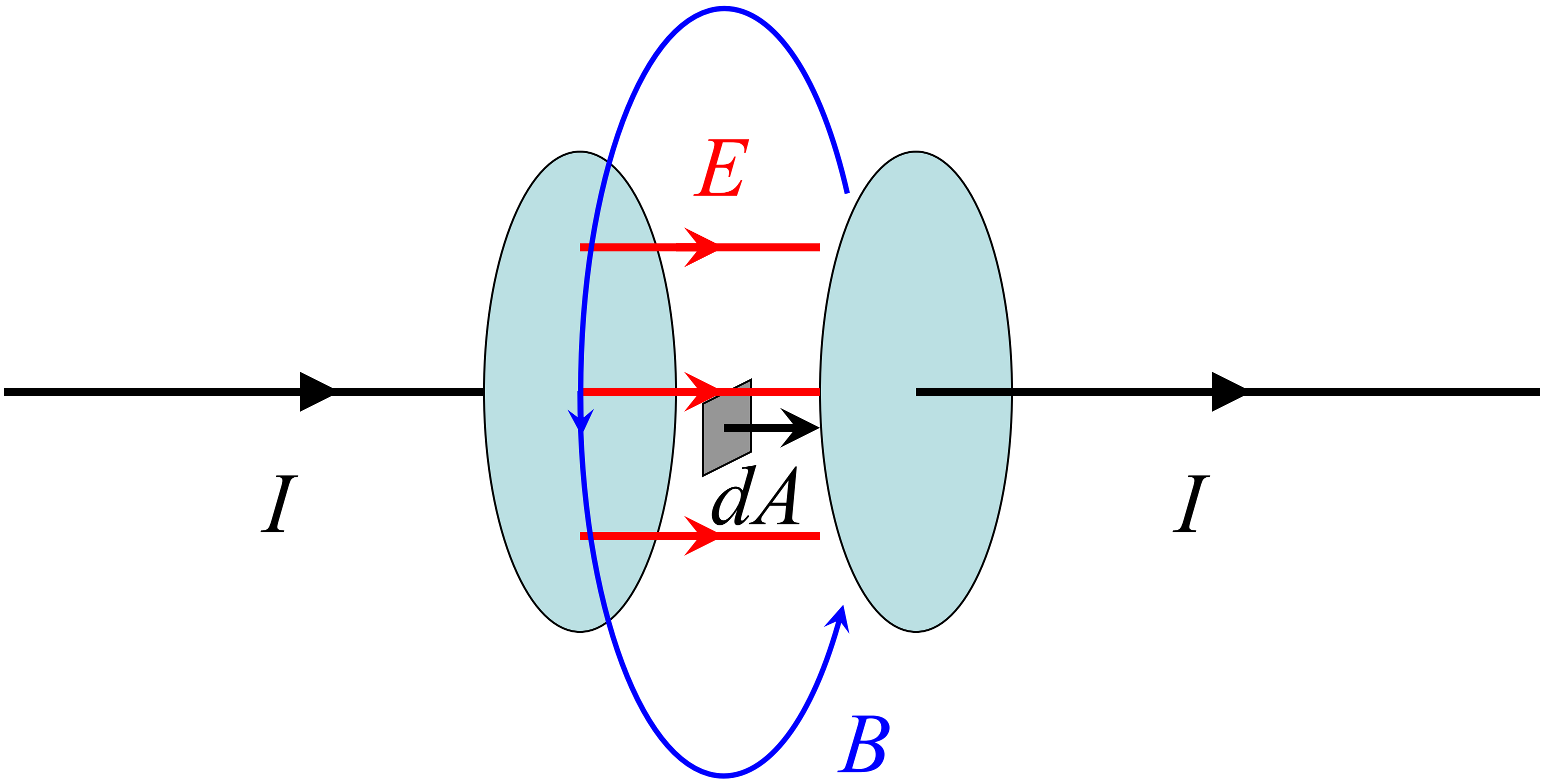
Thought Experiment: Charging a Capacitor



Maxwell resolved the problem:

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I$$

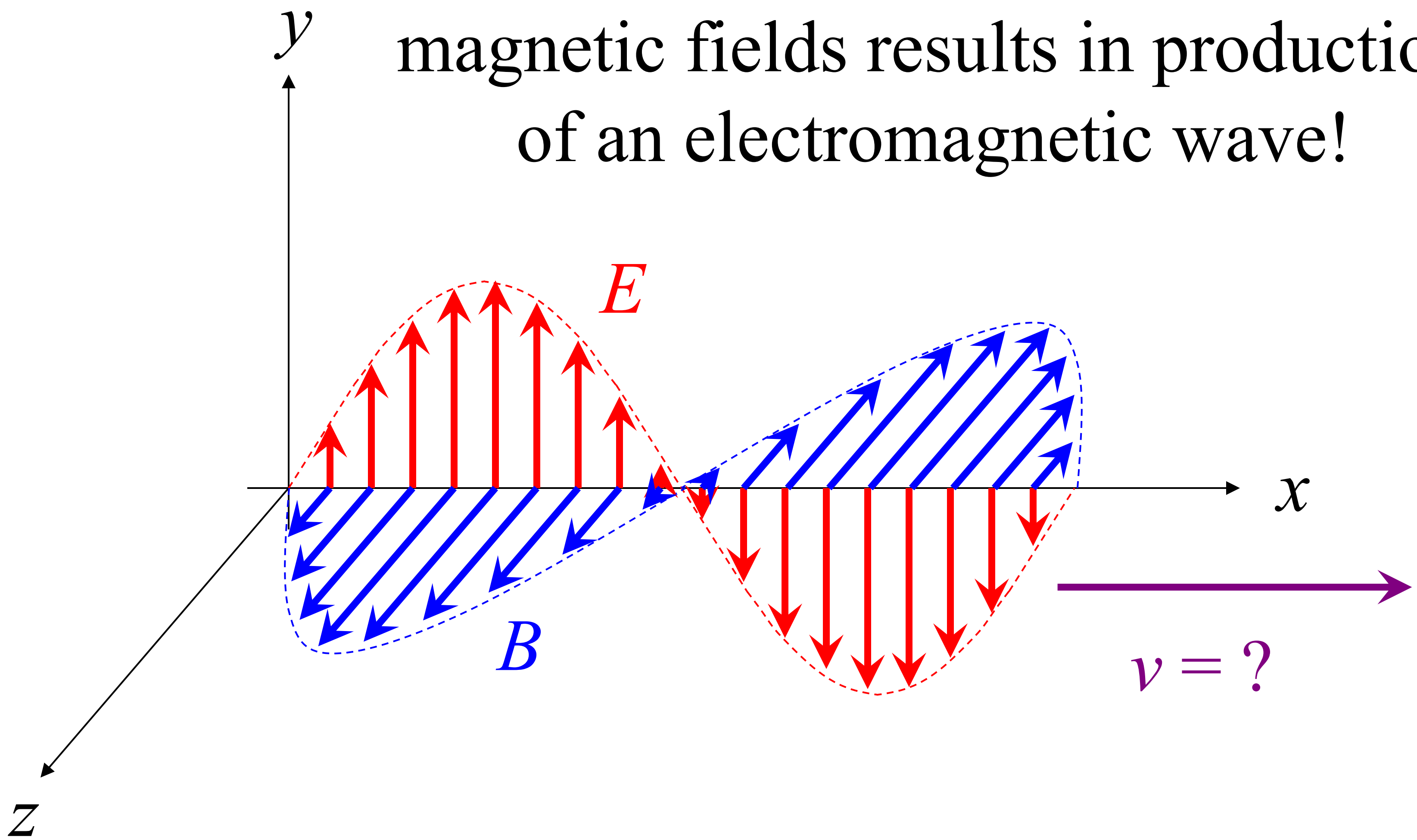
Thought Experiment: Charging a Capacitor



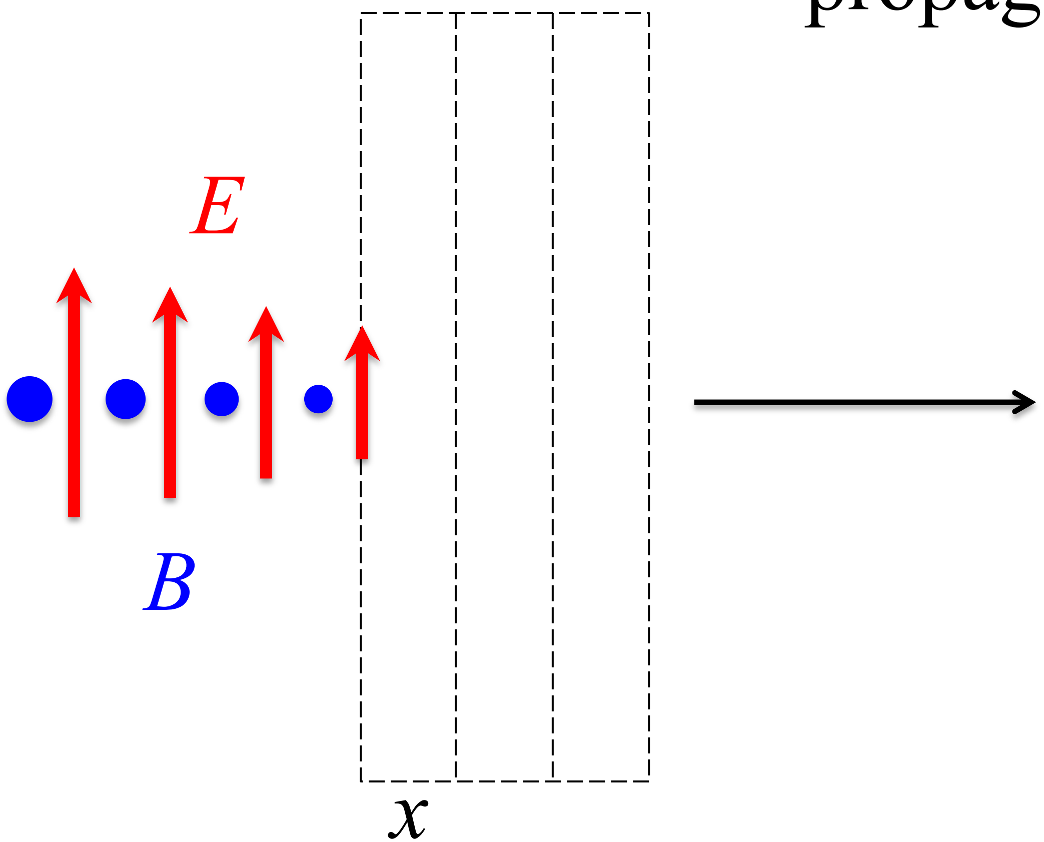
Maxwell resolved the problem:

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d}{dt} \int \vec{E} \cdot d\vec{A}$$

Mutual induction of electric and magnetic fields results in production of an electromagnetic wave!

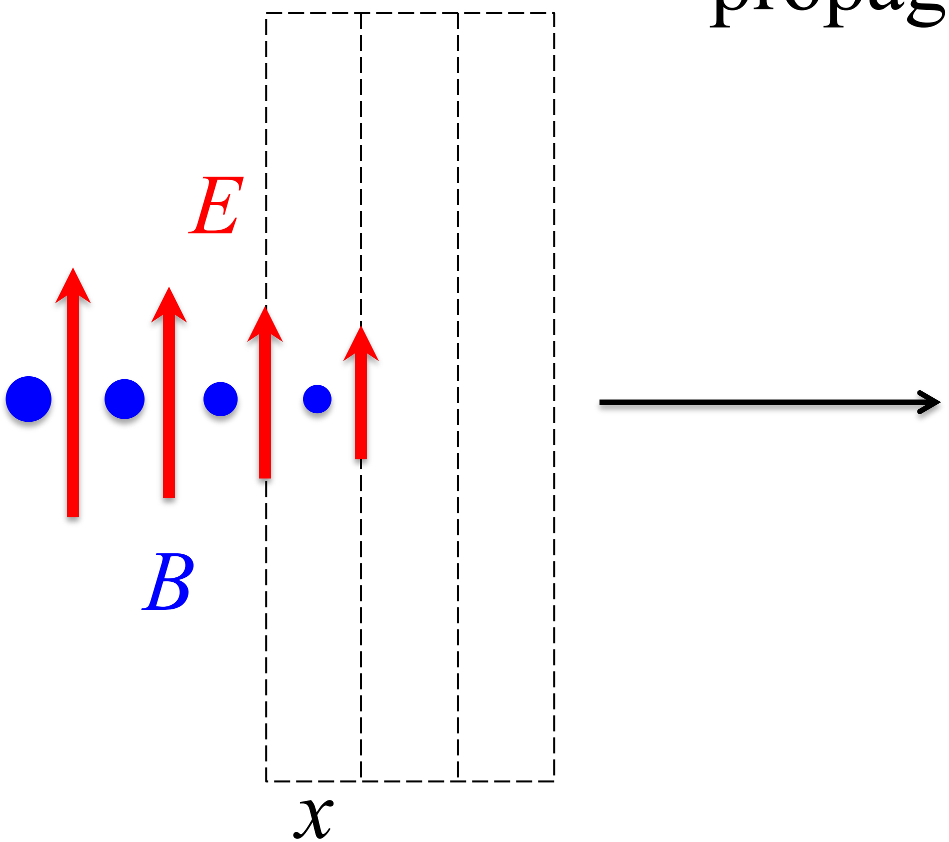


An electromagnetic disturbance propagates at speed v…



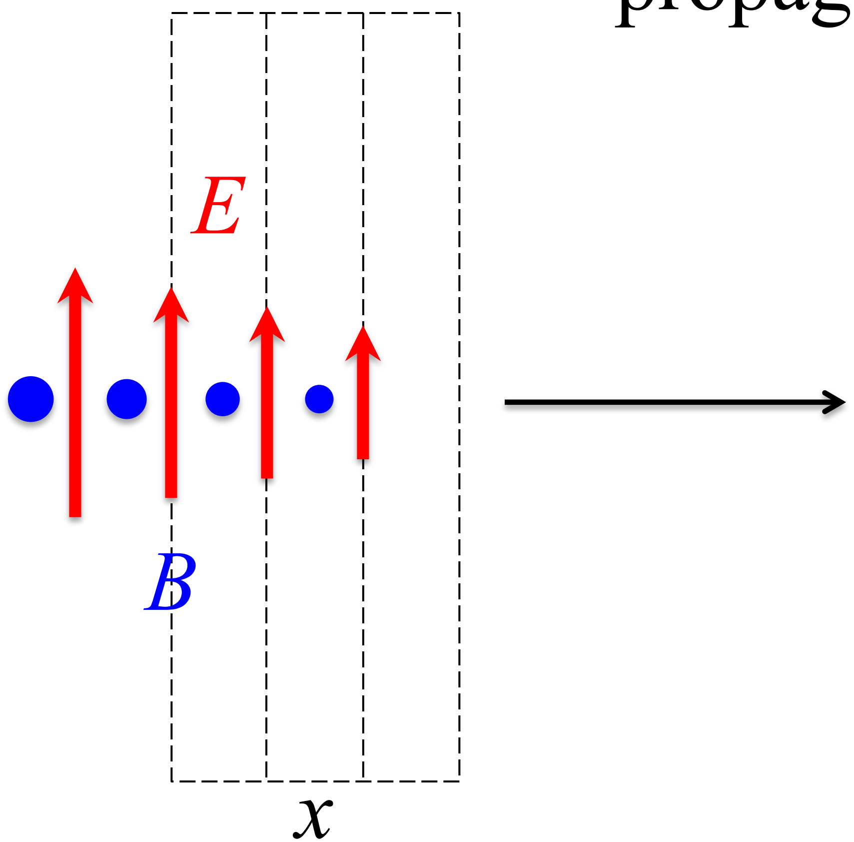
$$v = \frac{x}{t}$$

An electromagnetic disturbance propagates at speed v…



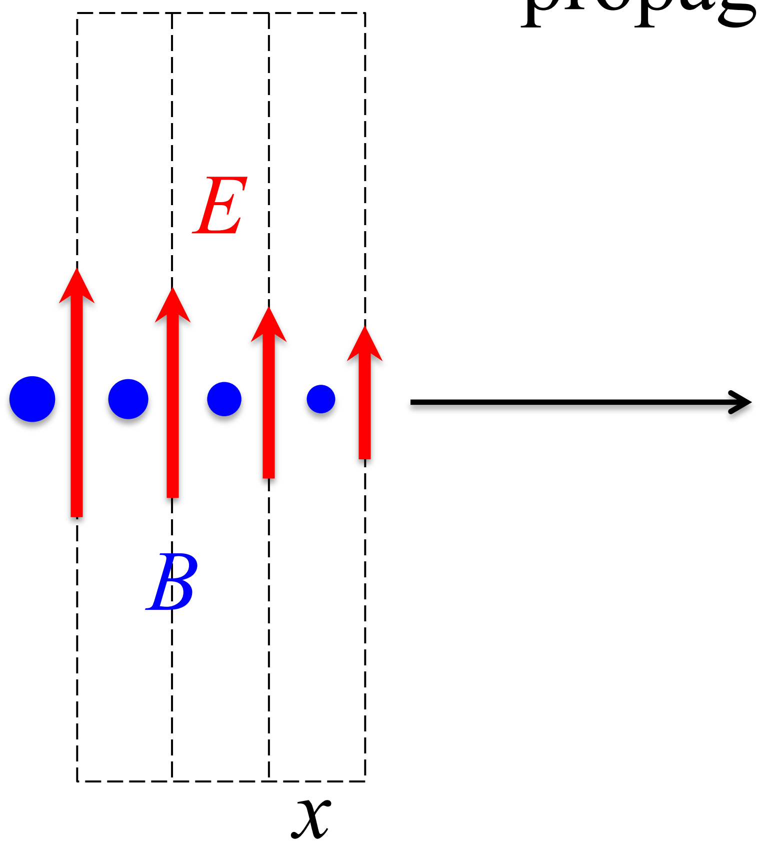
$$v = \frac{x}{t}$$

An electromagnetic disturbance propagates at speed v…



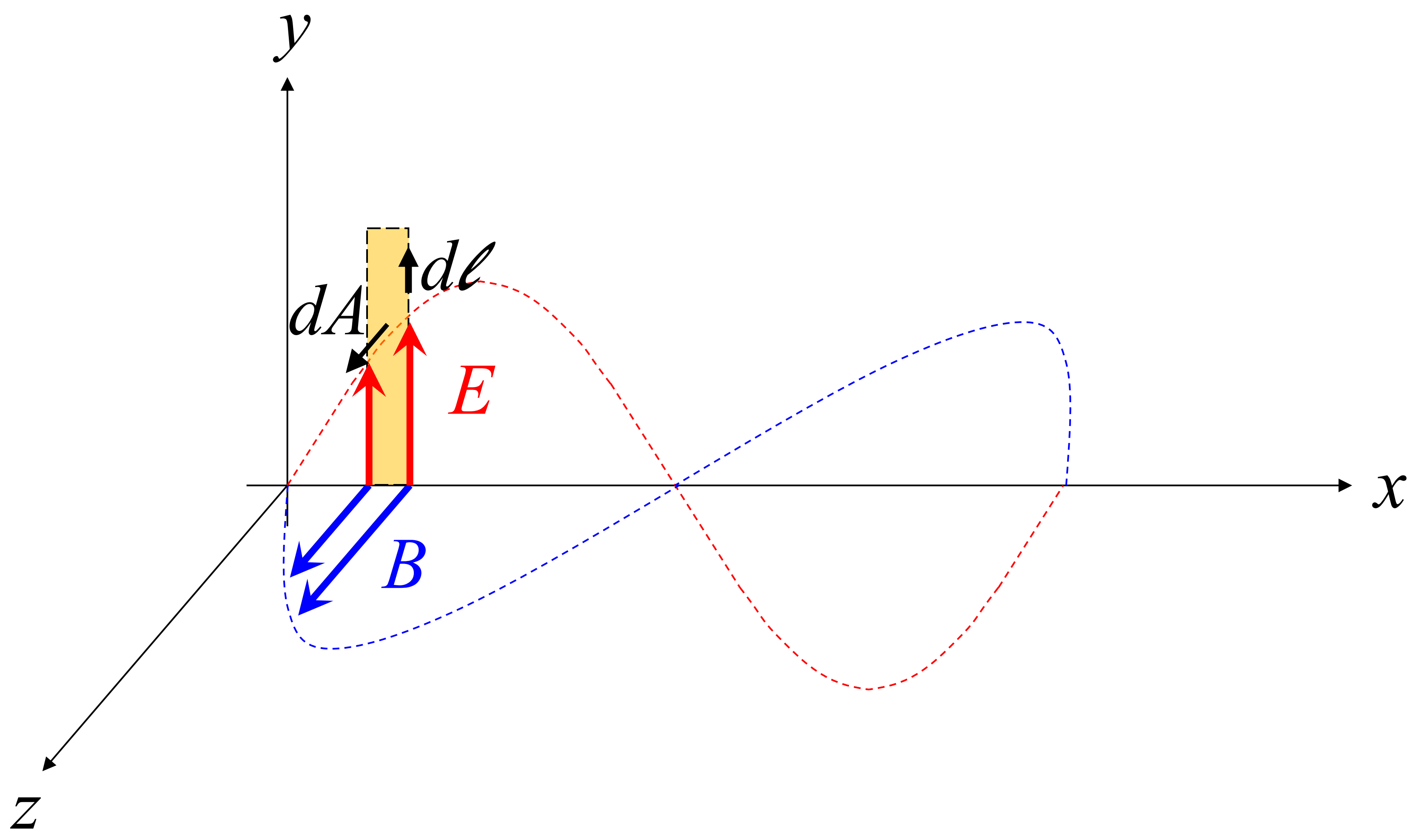
$$v = \frac{x}{t}$$

An electromagnetic disturbance propagates at speed *v*…



$$v = \frac{x}{t}$$

$$\oint \vec{E} \cdot d\vec{\ell} = -\frac{d}{dt}\int \vec{B} \cdot d\vec{A}$$

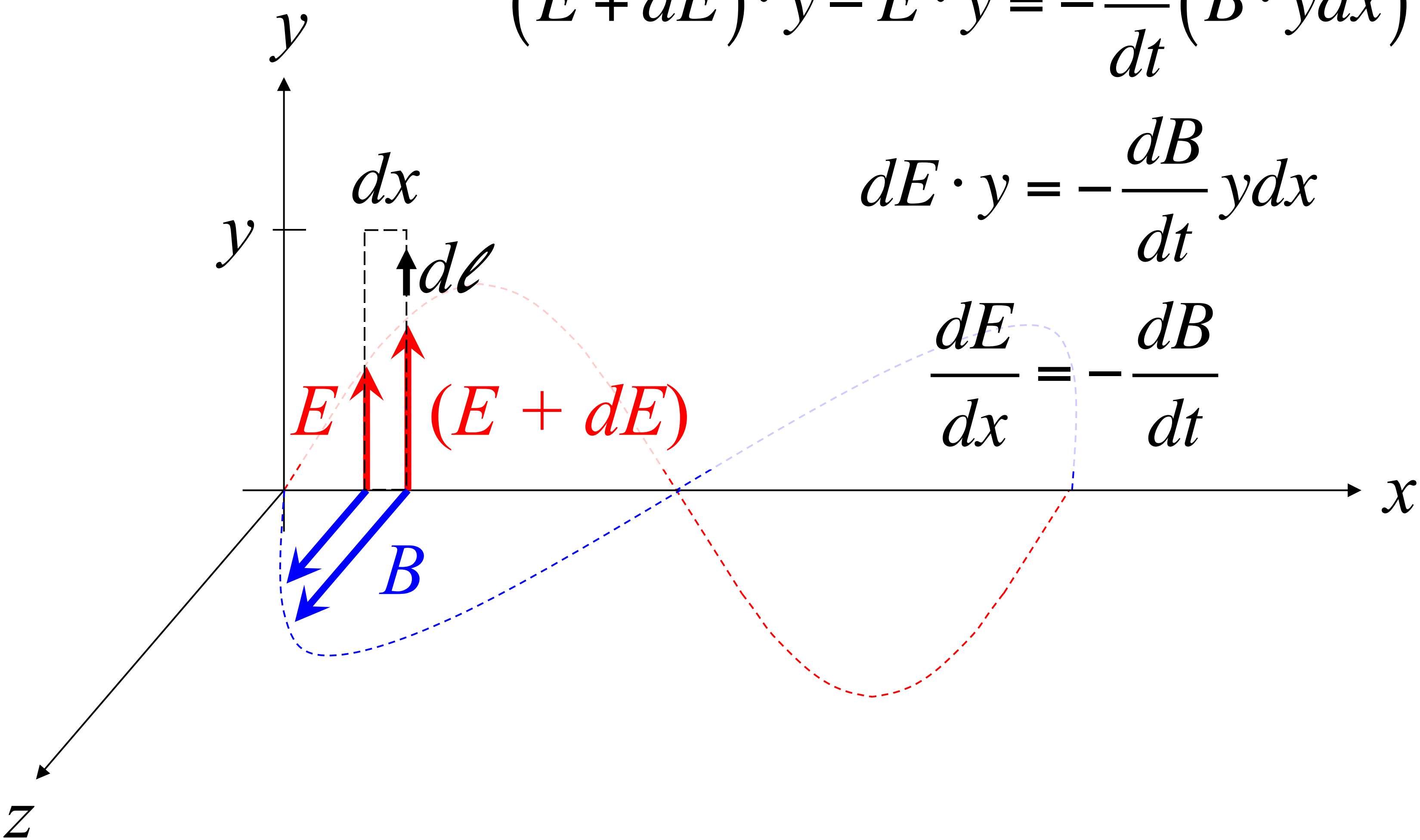


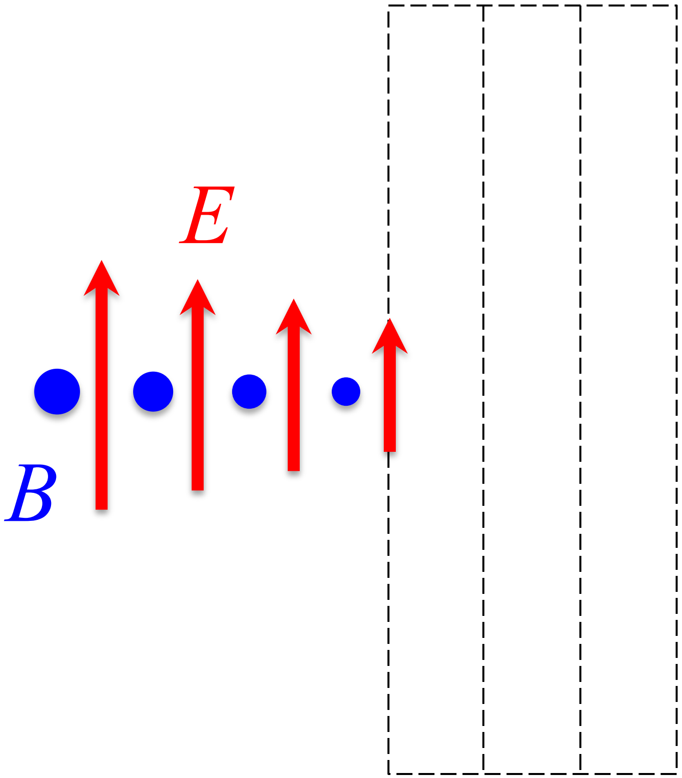
$$\oint \vec{E} \cdot d\vec{\ell} = -\frac{d}{dt}\int \vec{B} \cdot d\vec{A}$$

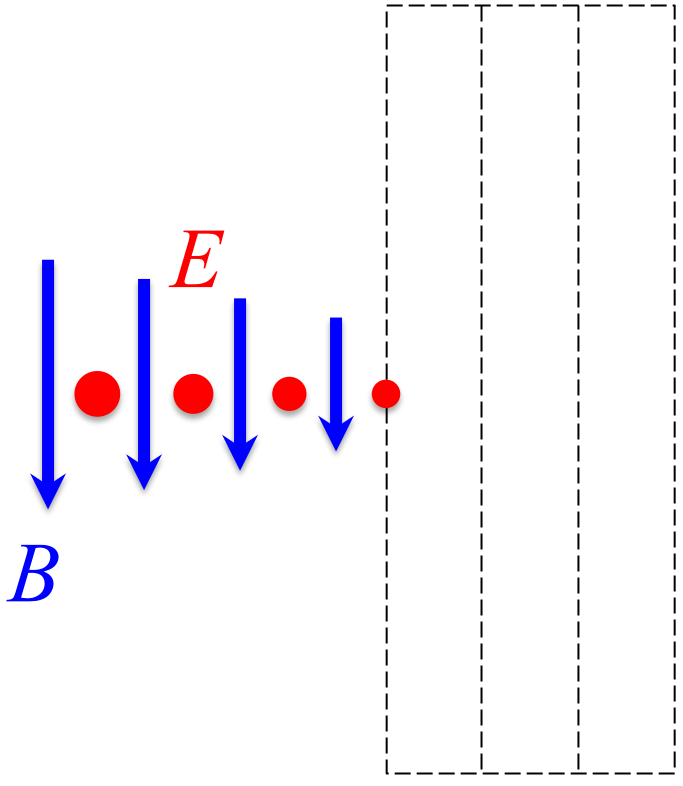
$$\left(E + dE\right) \cdot y - E \cdot y = -\frac{d}{dt}\left(B \cdot y dx\right)$$

$$dE \cdot y = -\frac{dB}{dt} y dx$$

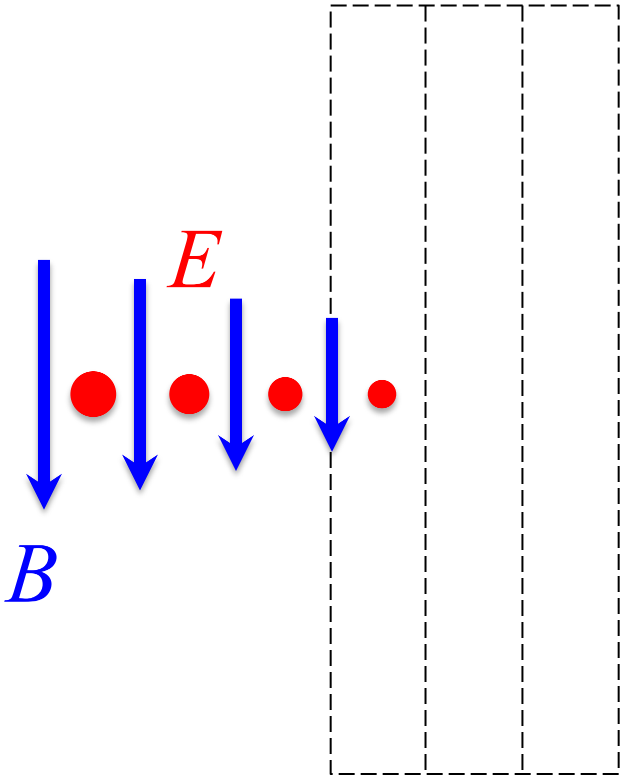
$$\frac{dE}{dx} = -\frac{dB}{dt}$$

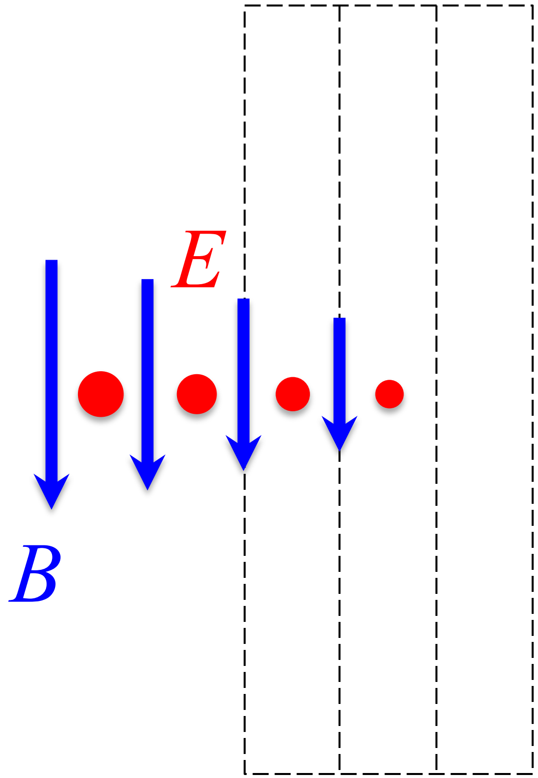


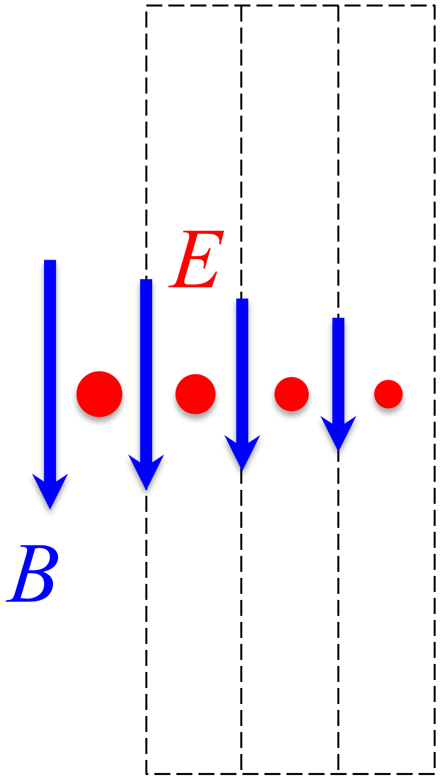

E
B



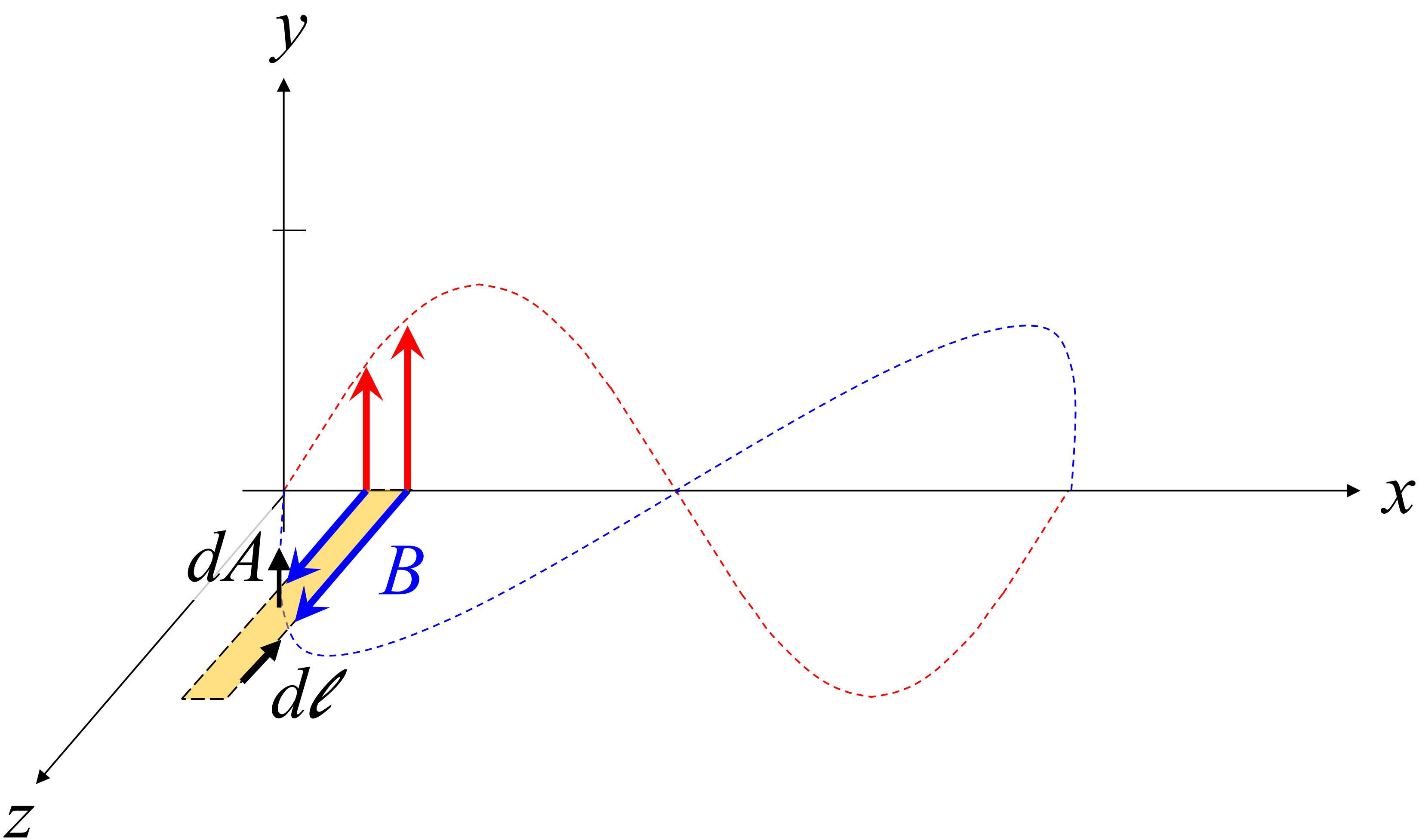
E
B


E
B


E
B


E
B

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 \varepsilon_0 \frac{d}{dt} \int \vec{E} \cdot d\vec{A}$$

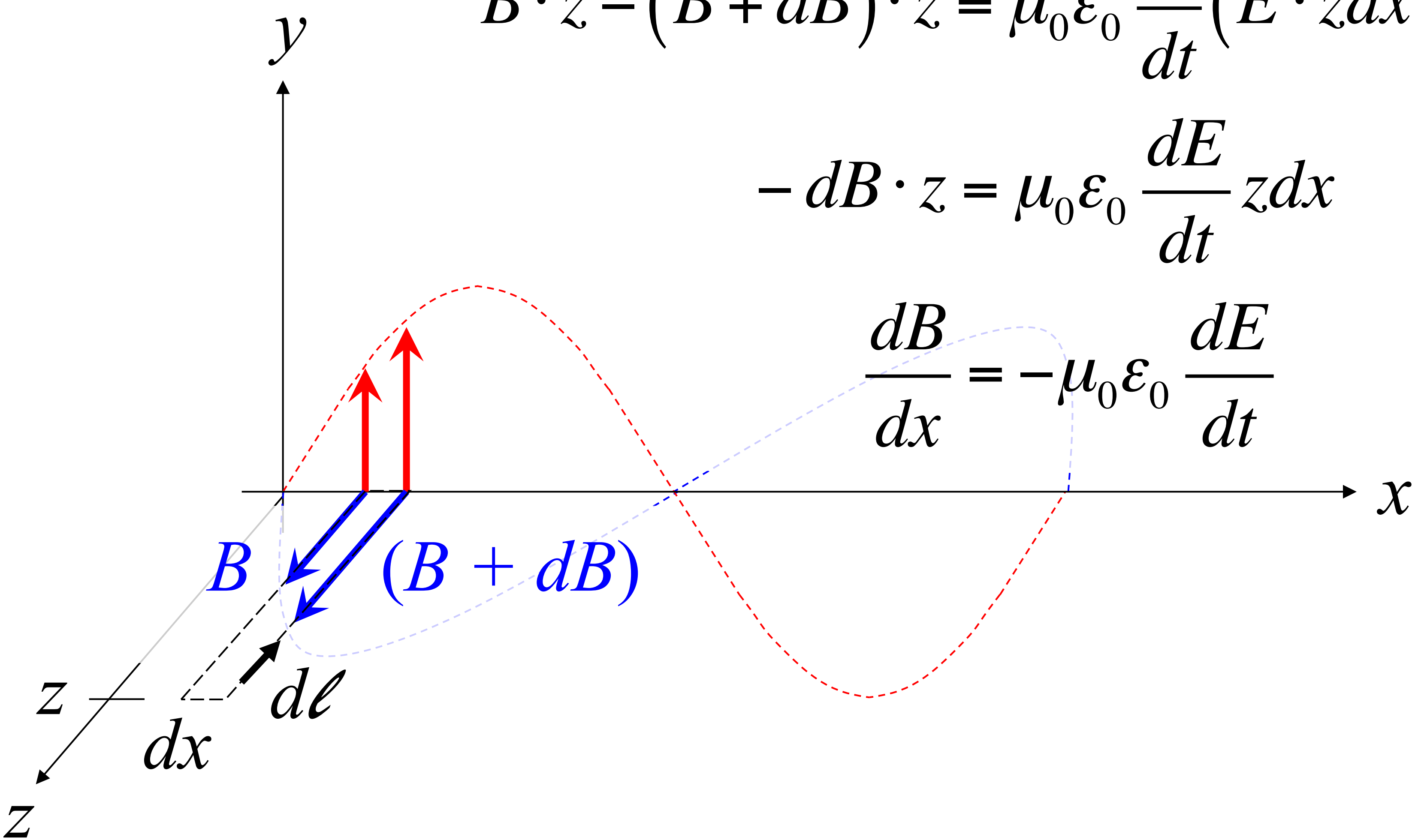


$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 \varepsilon_0 \frac{d}{dt} \int \vec{E} \cdot d\vec{A}$$

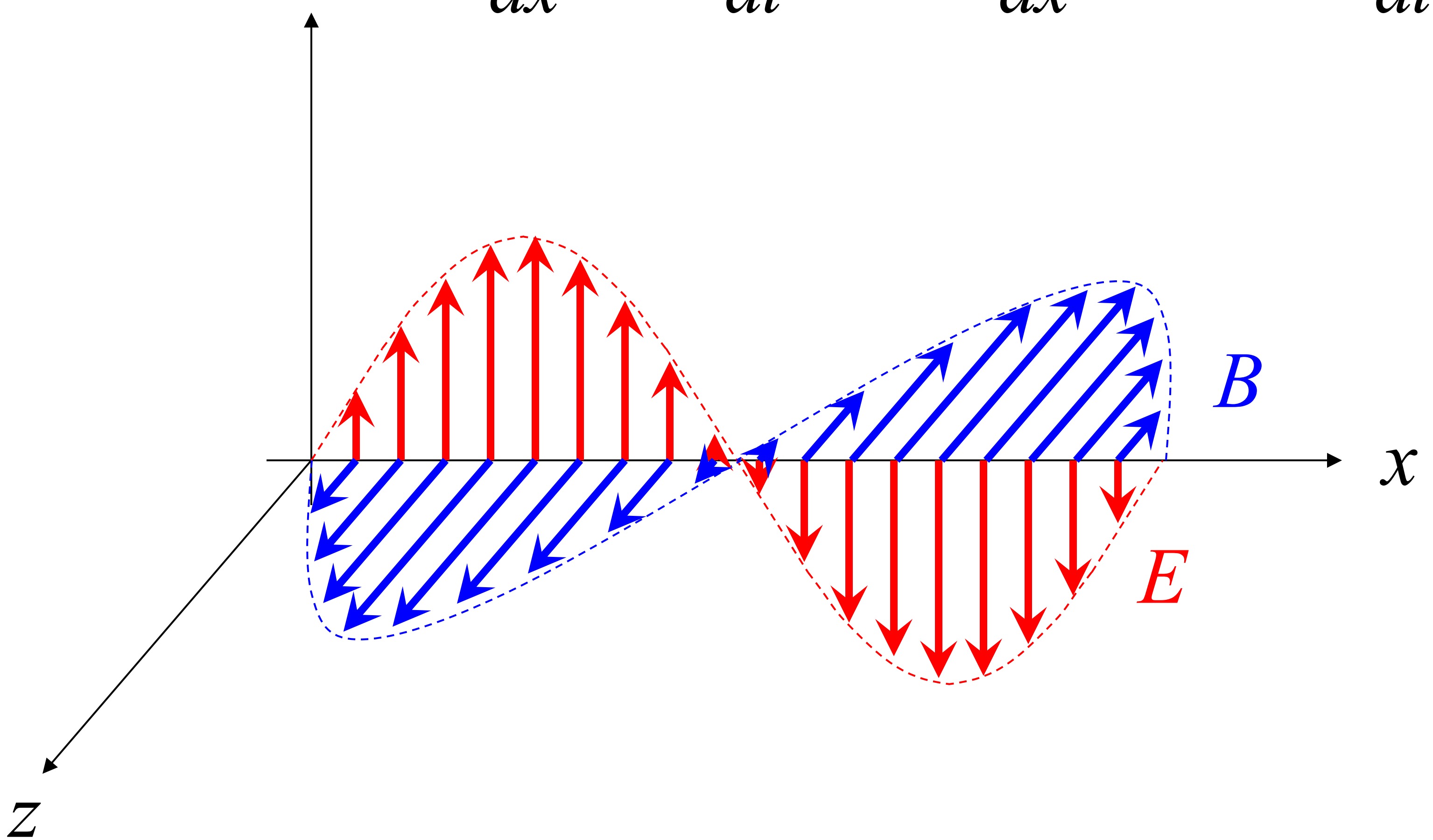
$$B \cdot z - \left(B + dB\right) \cdot z = \mu_0 \varepsilon_0 \frac{d}{dt}\left(E \cdot z dx\right)$$

$$-dB \cdot z = \mu_0 \varepsilon_0 \frac{dE}{dt} z dx$$

$$\frac{dB}{dx} = -\mu_0 \varepsilon_0 \frac{dE}{dt}$$

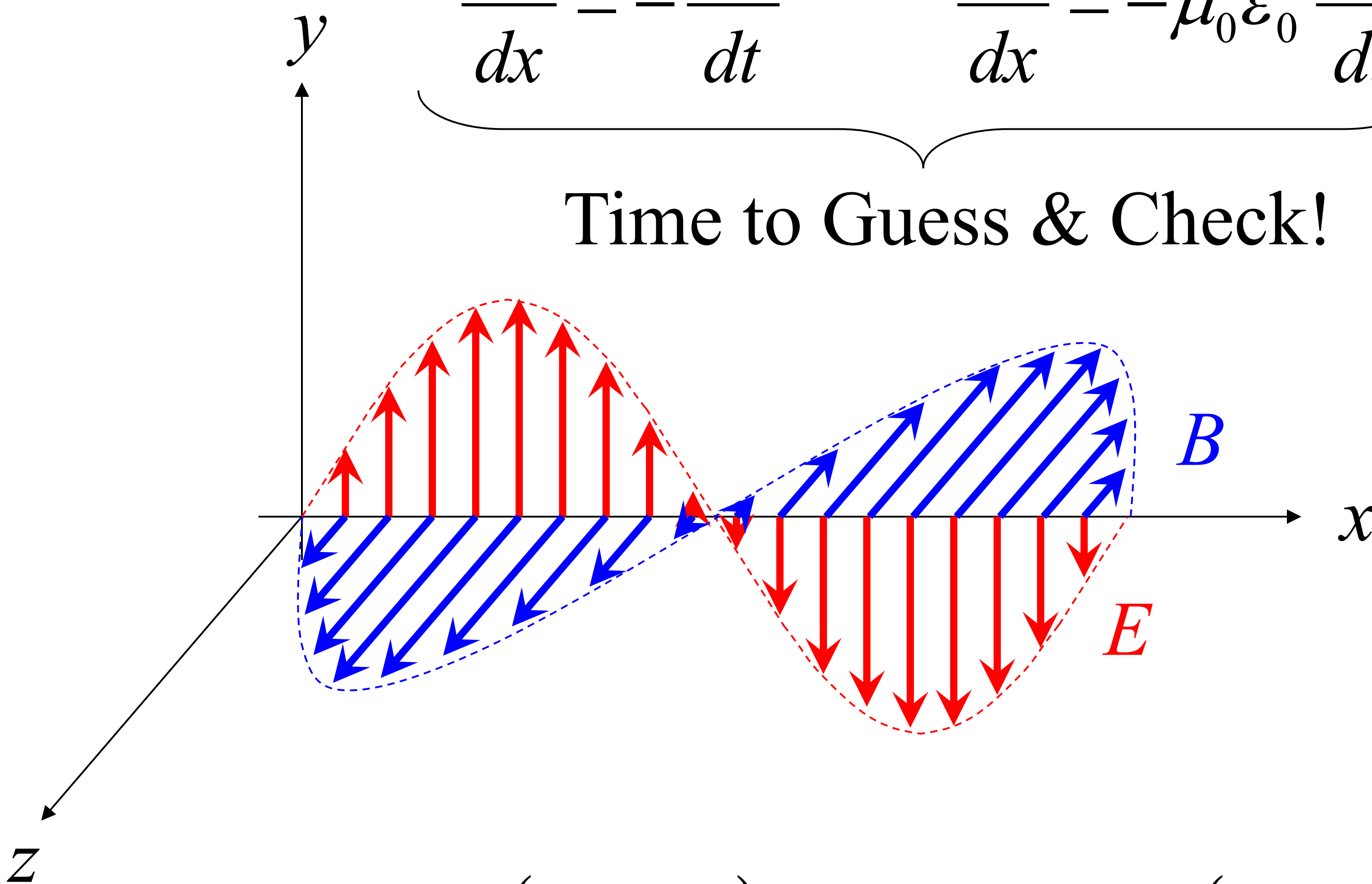


$$\frac{dE}{dx} = -\frac{dB}{dt} \qquad \frac{dB}{dx} = -\mu_0 \varepsilon_0 \frac{dE}{dt}$$



$$\frac{dE}{dx} = -\frac{dB}{dt} \qquad \frac{dB}{dx} = -\mu_0 \varepsilon_0 \frac{dE}{dt}$$

Time to Guess & Check!



$$E = A\sin(kx + \omega t) \qquad B = C\sin(kx + \omega t)$$

$$E = A\sin(kx + \omega t) \qquad B = C\sin(kx + \omega t)$$

$$\frac{dE}{dt} = A\omega\cos(kx + \omega t) \qquad \frac{dB}{dt} = C\omega\cos(kx + \omega t)$$

$$\frac{dE}{dx} = Ak\cos(kx + \omega t) \qquad \frac{dB}{dx} = Ck\cos(kx + \omega t)$$

$$\frac{dE}{dx} = -\frac{dB}{dt}$$

$$\frac{dB}{dx} = -\mu_0\varepsilon_0\frac{dE}{dt}$$

$$E = A\sin(kx + \omega t) \qquad B = C\sin(kx + \omega t)$$

$$\frac{dE}{dt} = A\omega\cos(kx + \omega t) \qquad \frac{dB}{dt} = C\omega\cos(kx + \omega t)$$

$$\frac{dE}{dx} = Ak\cos(kx + \omega t) \qquad \frac{dB}{dx} = Ck\cos(kx + \omega t)$$

$$\frac{dE}{dx} = -\frac{dB}{dt}$$

$$Ak\cos(kx + \omega t) = -C\omega\cos(kx + \omega t)$$

$$\frac{dB}{dx} = -\mu_0\varepsilon_0\frac{dE}{dt}$$

$$Ck\cos(kx + \omega t) = -\mu_0\varepsilon_0 A\omega\cos(kx + \omega t)$$

$$Ck\cos(kx+\omega t) = -\mu_0\varepsilon_0 A\omega\cos(kx+\omega t)$$

$$Ak\cos(kx+\omega t) = -C\omega\cos(kx+\omega t)$$

$$\frac{\cancel{C}k\,\cancel{\cos(kx+\omega t)}}{-\cancel{C}\omega\,\cancel{\cos(kx+\omega t)}} = \frac{-\mu_0\varepsilon_0\cancel{A}\omega\,\cancel{\cos(kx+\omega t)}}{\cancel{A}k\,\cancel{\cos(kx+\omega t)}}$$

$$\frac{\cancel{C}k\cancel{\cos(kx+\omega t)}}{-\cancel{C}\omega\cancel{\cos(kx+\omega t)}} = \frac{-\mu_0\varepsilon_0\cancel{A}\omega\cancel{\cos(kx+\omega t)}}{\cancel{A}k\cancel{\cos(kx+\omega t)}}$$

$$\frac{k}{\omega} = \mu_0\varepsilon_0\frac{\omega}{k}$$

$$\frac{1}{\mu_0\varepsilon_0} = \frac{\omega^2}{k^2}$$

$$\frac{\omega}{k} = \sqrt{\frac{1}{\mu_0\varepsilon_0}}$$

$$B = C\sin(kx + \omega t)$$

$$\frac{dB}{dt} = C\omega\cos(kx + \omega t)$$

$$\frac{dB}{dx} = Ck\cos(kx + \omega t)$$

$$v = \frac{dx}{dt} = \frac{dx}{dB} \cdot \frac{dB}{dt}$$

$$v = \frac{1}{Ck\cos(kx + \omega t)} \cdot C\omega\cos(kx + \omega t)$$

$$v = \frac{\omega}{k} = \sqrt{\frac{1}{\mu_0 \varepsilon_0}} = 299792458 \text{ m/s}$$

(the speed of light!)