Kinematics

What do you remember?

Kinematics Unit Outline

I. Six Definitions:
Distance, Position, Displacement, Speed, Velocity, Acceleration

II. Graphical Interpretations

III. Constant acceleration model

IV. Freefall

	The student will be able to:	HW:
1	Define, distinguish, and apply the concepts: distance, displacement, position.	1, 2
2	Define, distinguish, and apply the concepts: average speed, instantaneous speed, constant speed, average velocity, instantaneous velocity, constant velocity.	3 – 9
3	Define, distinguish, and apply the concepts: average acceleration and instantaneous acceleration, and constant acceleration.	10, 11
4	Analyze a graph of distance, position, or displacement as a function of time in order to determine speed and/or velocity.	12 – 14
5	Analyze a graph of speed or velocity as a function of time in order to determine distance, position, displacement, and/or acceleration.	15 – 19
6	State the displacement and velocity relations for cases of constant acceleration and use these to solve problems given appropriate initial conditions and values.	20 – 34
7	State and use the conditions of freefall, including the value of g, to solve associated problems.	35 – 42
8	Measure and analyze data for a moving object and produce appropriate graphs including line or curve of best fit.	lab
9	Evaluate error, deviation, accuracy, and precision in experimental results.	lab

Kinematics

- Kinematics is the mathematical description used to describe motion.
- There are six main concepts: position, displacement, distance, speed, velocity, and acceleration.
- Three of these concepts are *rates*.
- Any rate may be described as average, constant, or instantaneous.

- **Position** is a vector indicating an object's location; linear distance and direction from a reference point. Symbols:
- **Displacement** is the net change in position. Symbols: **d**, **Δr**, **Δs**, or **Δx** $\vec{d}, \Delta\vec{r}, \Delta\vec{s}, \Delta\vec{x}$
- **Distance** is length of the path traveled. Symbols: *d*, *s*, or *x*
- **Speed** is the time rate of change in distance. Symbol: *v*
- **Velocity** is the time rate of change in position. Symbols: **v** $\vec{v}$
- **Acceleration** is the time rate of change in velocity. Symbols: **a** $\vec{a}$

Displacement:

Average Speed:

Average Velocity:

Average Acceleration:

Displacement: $$\vec{d} = \Delta\vec{r} = \vec{r} - \vec{r}_0$$

Average Speed: $$v_{avg} = \frac{d}{t} \qquad v_{avg} = \frac{\Delta d}{\Delta t}$$

Average Velocity: $$\vec{v}_{avg} = \frac{\vec{d}}{t} \qquad \vec{v}_{avg} = \frac{\Delta \vec{r}}{\Delta t}$$

Average Acceleration: $$\vec{a}_{avg} = \frac{\Delta \vec{v}}{\Delta t}$$

Kinematics Unit Outline

I. Six Definitions:
Distance, Position, Displacement,
Speed, Velocity, Acceleration

II. Graphical Interpretations

III. Constant acceleration model

IV. Freefall

	The student will be able to:	HW:
1	Define, distinguish, and apply the concepts: distance, displacement, position.	✓ 1, 2
2	Define, distinguish, and apply the concepts: average speed, instantaneous speed, constant speed, average velocity, instantaneous velocity, constant velocity.	3 – 9 ✓
3	Define, distinguish, and apply the concepts: average acceleration and instantaneous acceleration, and constant acceleration.	✓ 10, 11
4	Analyze a graph of distance, position, or displacement as a function of time in order to determine speed and/or velocity.	12 – 14
5	Analyze a graph of speed or velocity as a function of time in order to determine distance, position, displacement, and/or acceleration.	15 – 19
6	State the displacement and velocity relations for cases of constant acceleration and use these to solve problems given appropriate initial conditions and values.	20 – 34
7	State and use the conditions of freefall, including the value of *g*, to solve associated problems.	35 – 42
8	Measure and analyze data for a moving object and produce appropriate graphs including line or curve of best fit.	lab
9	Evaluate error, deviation, accuracy, and precision in experimental results.	lab

Slope

- Slope indicates rate of change in *y* relative to *x*.
- A linear graph has a unique slope, which indicates a constant rate of change.
- For a curvilinear graph the slope at a point is equal to that of a tangent line drawn at that point.

What is “instantaneous”?

- **Instantaneous** means at an instant or at a particular *point* in time.
- The duration of an instant is zero seconds. It is not an interval of time.
- The formulas for average speed or velocity are undefined for an instant!
- Although any moving object travels zero distance in zero amount of time, the distance can be in the process of change at a particular point in time.

Instantaneous Speed and Velocity

$$v = \lim_{\Delta t \to 0}\left(\frac{\Delta d}{\Delta t}\right)$$

$$\vec{v} = \lim_{\Delta t \to 0}\left(\frac{\Delta \vec{r}}{\Delta t}\right)$$

Instantaneous Acceleration

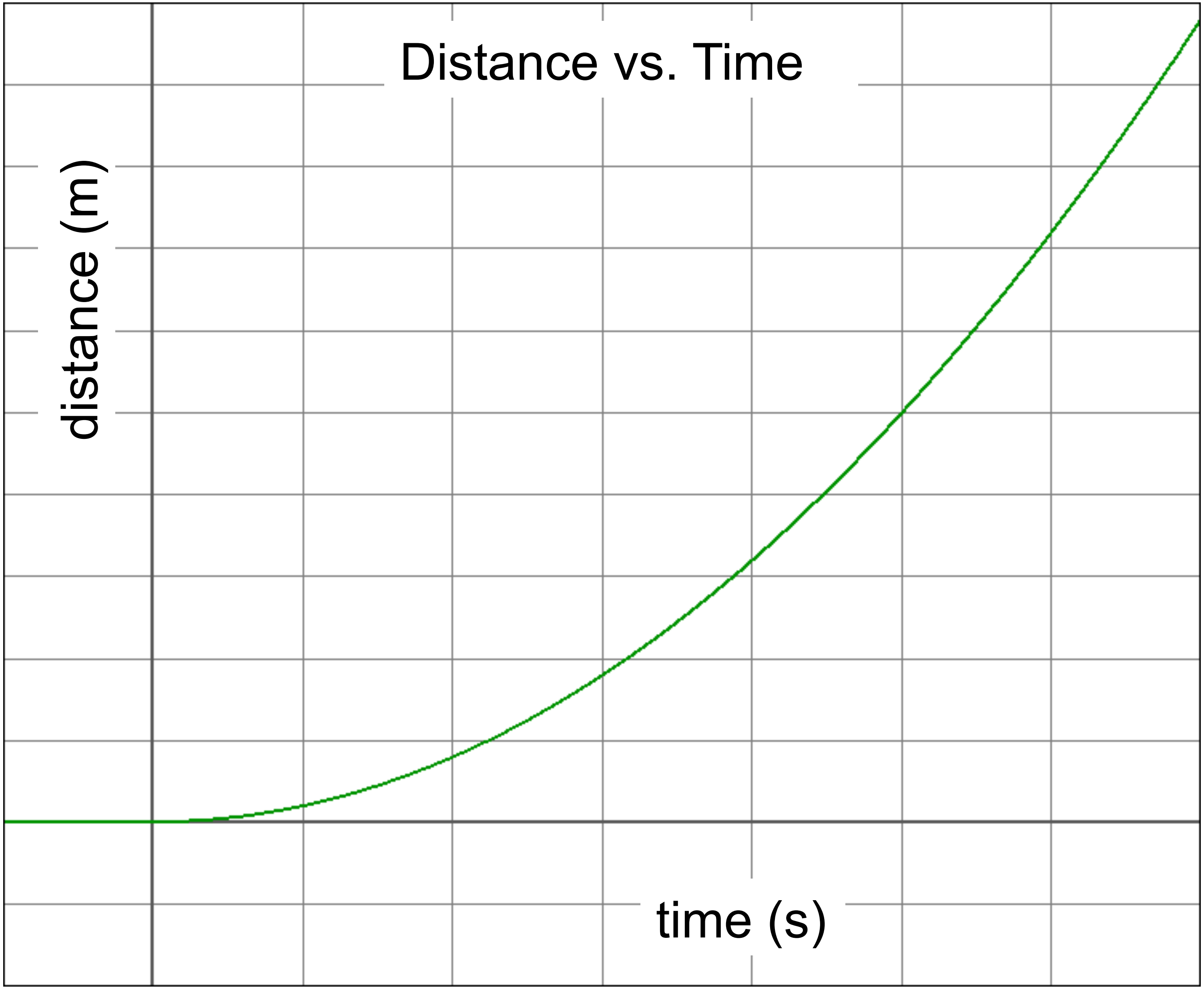
$$\vec{a} = \lim_{\Delta t \to 0} \left(\frac{\Delta \vec{v}}{\Delta t} \right)$$

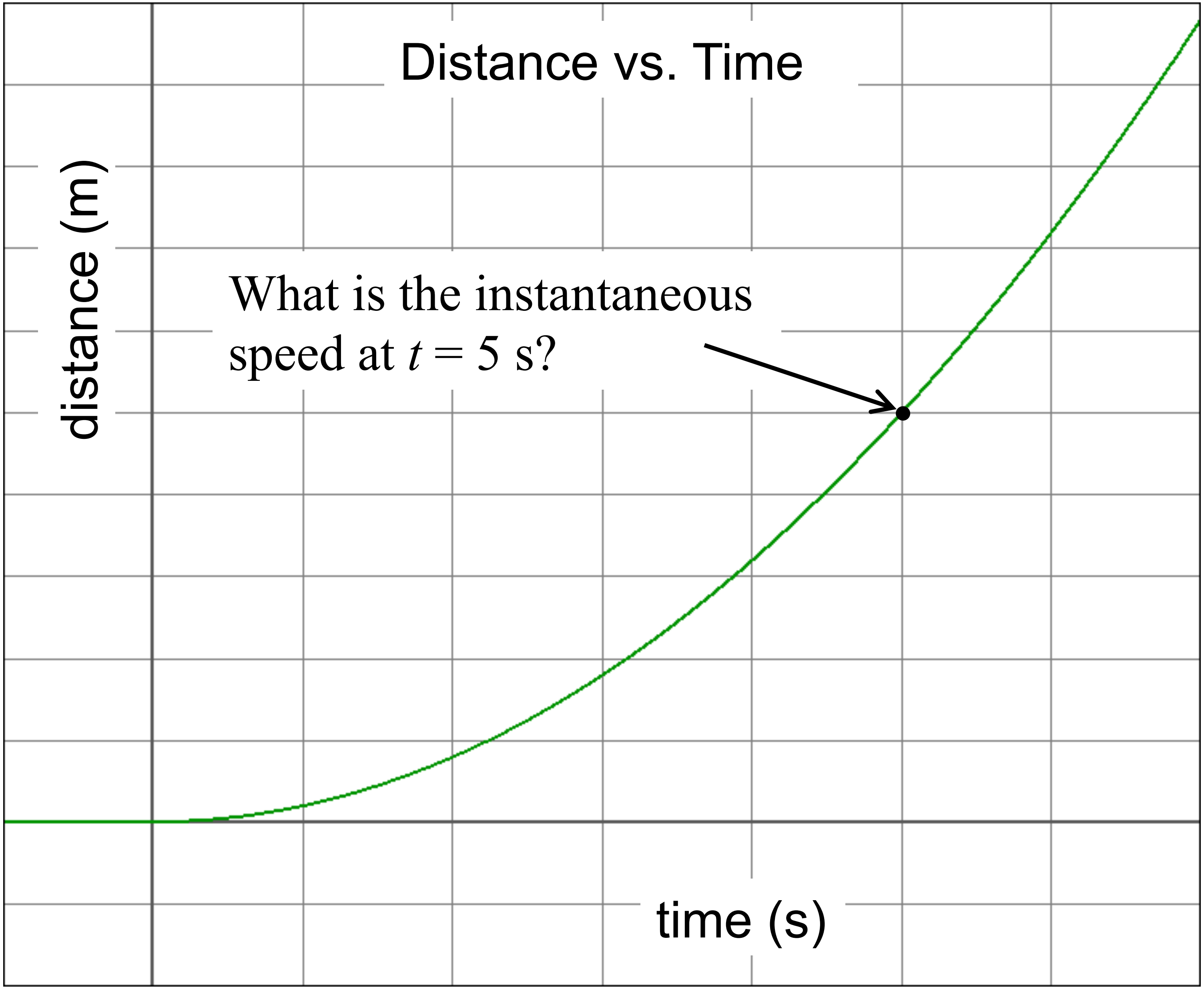
Understanding Limits

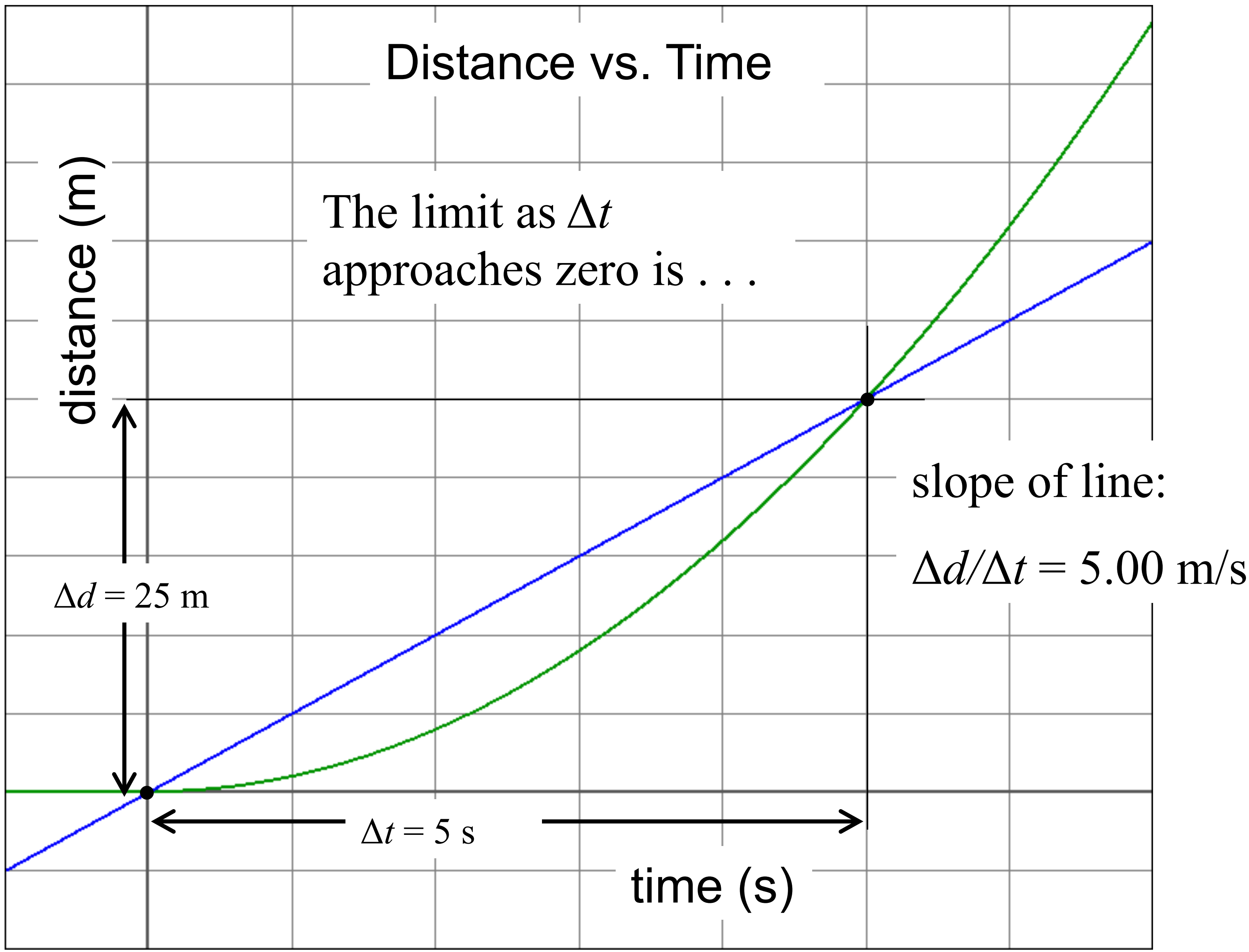
- Although dividing by zero is not defined there is a limit to which the quotient of distance over time will approach as the amount of time drops.
- If the interval of time gets less, *so does the distance traveled*. Meanwhile the *change* in the ratio of distance over time gets less and less as the ratio *settles* on a particular value. This value is called the limit.

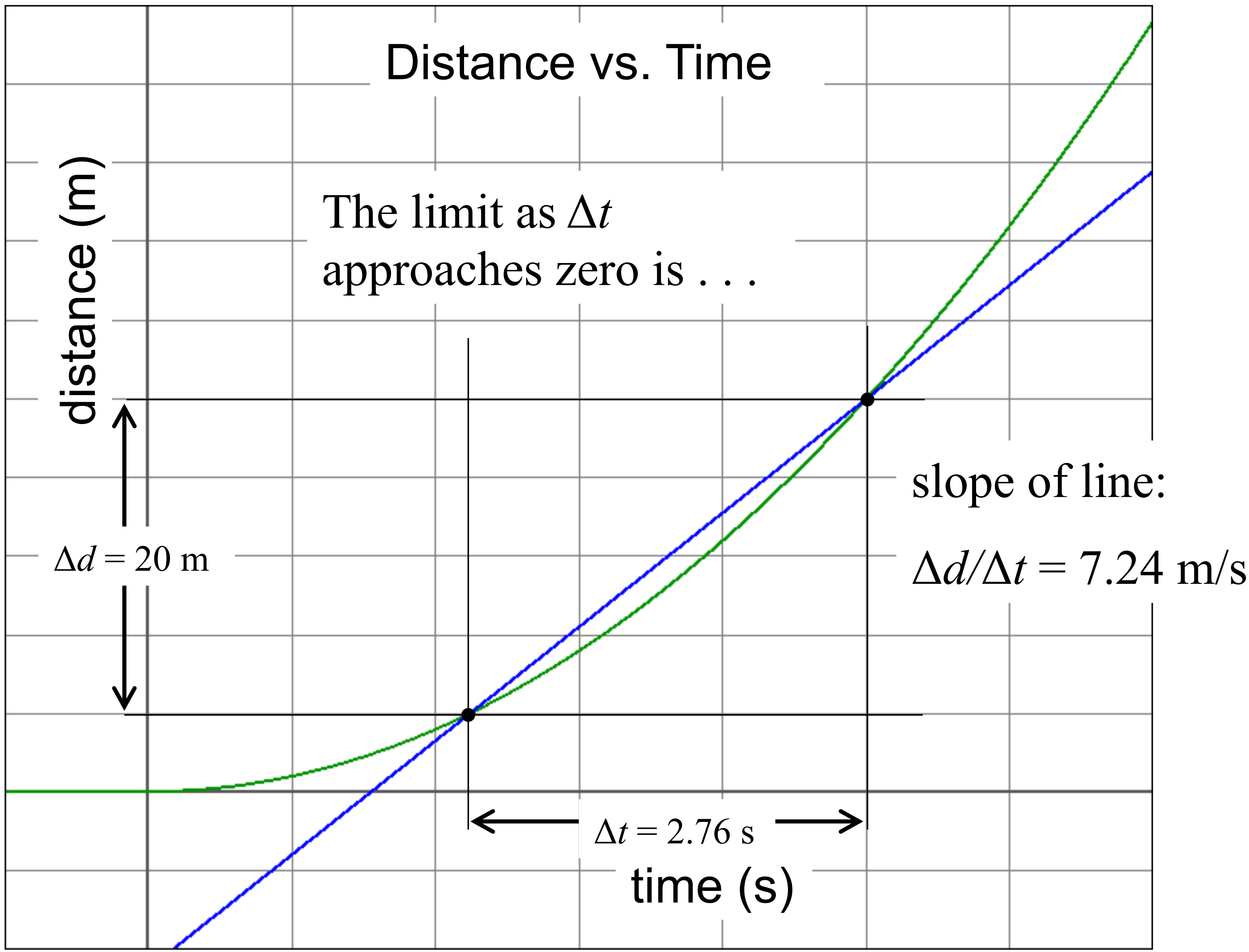
Instantaneous Rates - Graphical

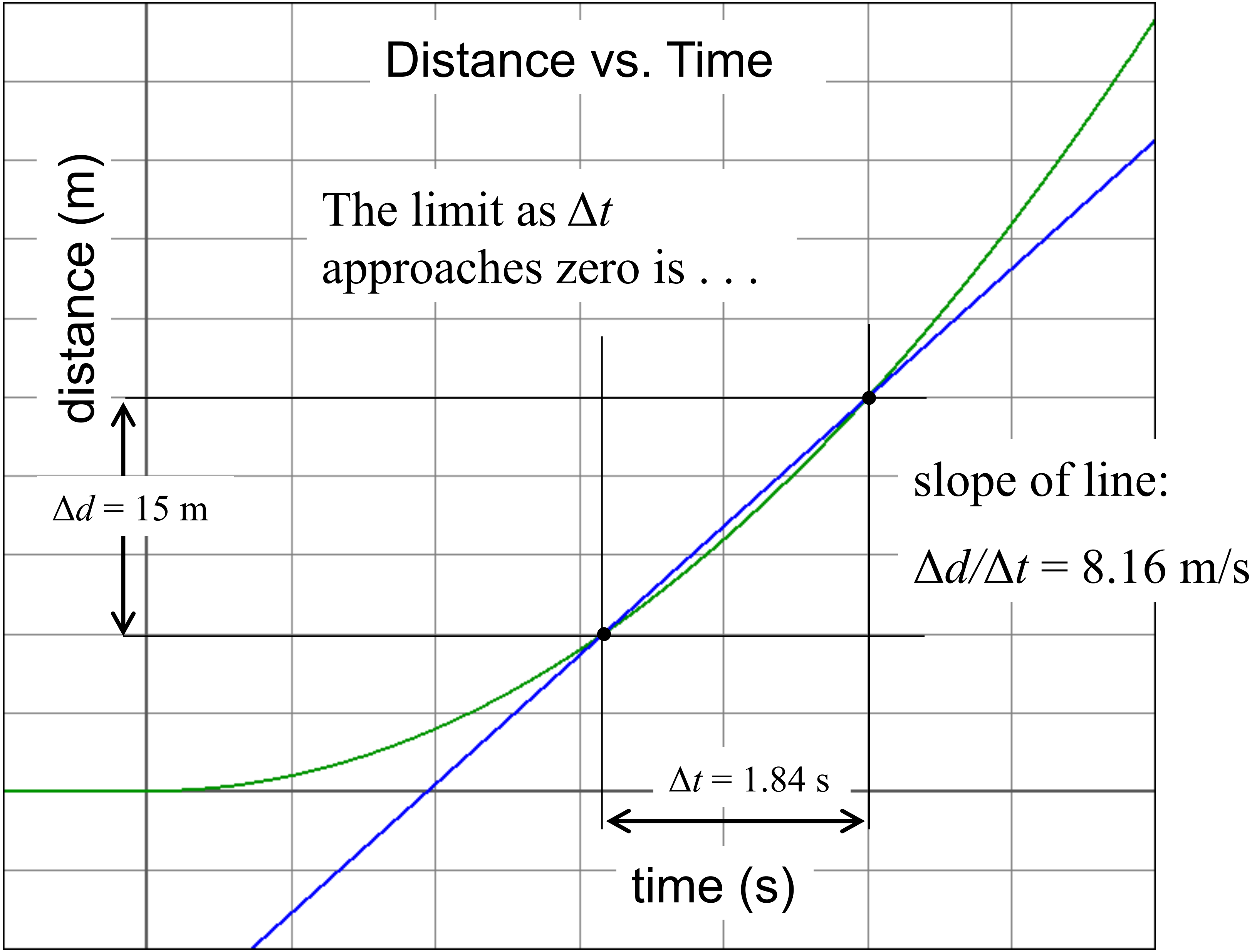
- On a graph of distance vs. time the instantaneous speed is equal to the slope at any particular point.
- If the graph is curved the slope is equal to that of a tangent line drawn at the point in question.
- Same concepts apply to position vs. time and velocity.


Distance vs. Time
distance (m)
time (s)

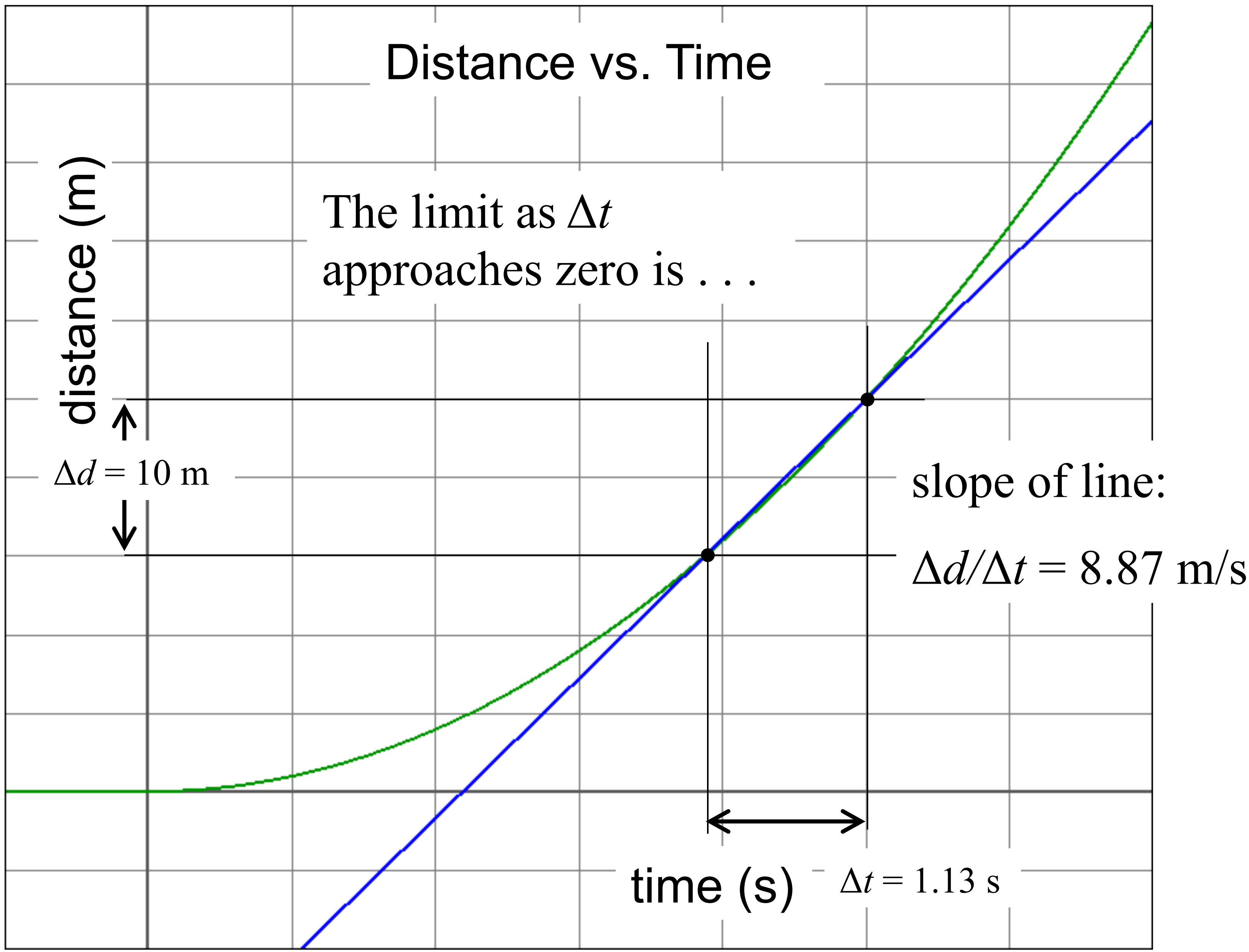

Distance vs. Time
distance (m)
What is the instantaneous
speed at t = 5 s?
time (s)


Distance vs. Time
distance (m)
The limit as Δt
approaches zero is . . .
slope of line:
Δd/Δt = 5.00 m/s
Δd = 25 m
Δt = 5 s
time (s)

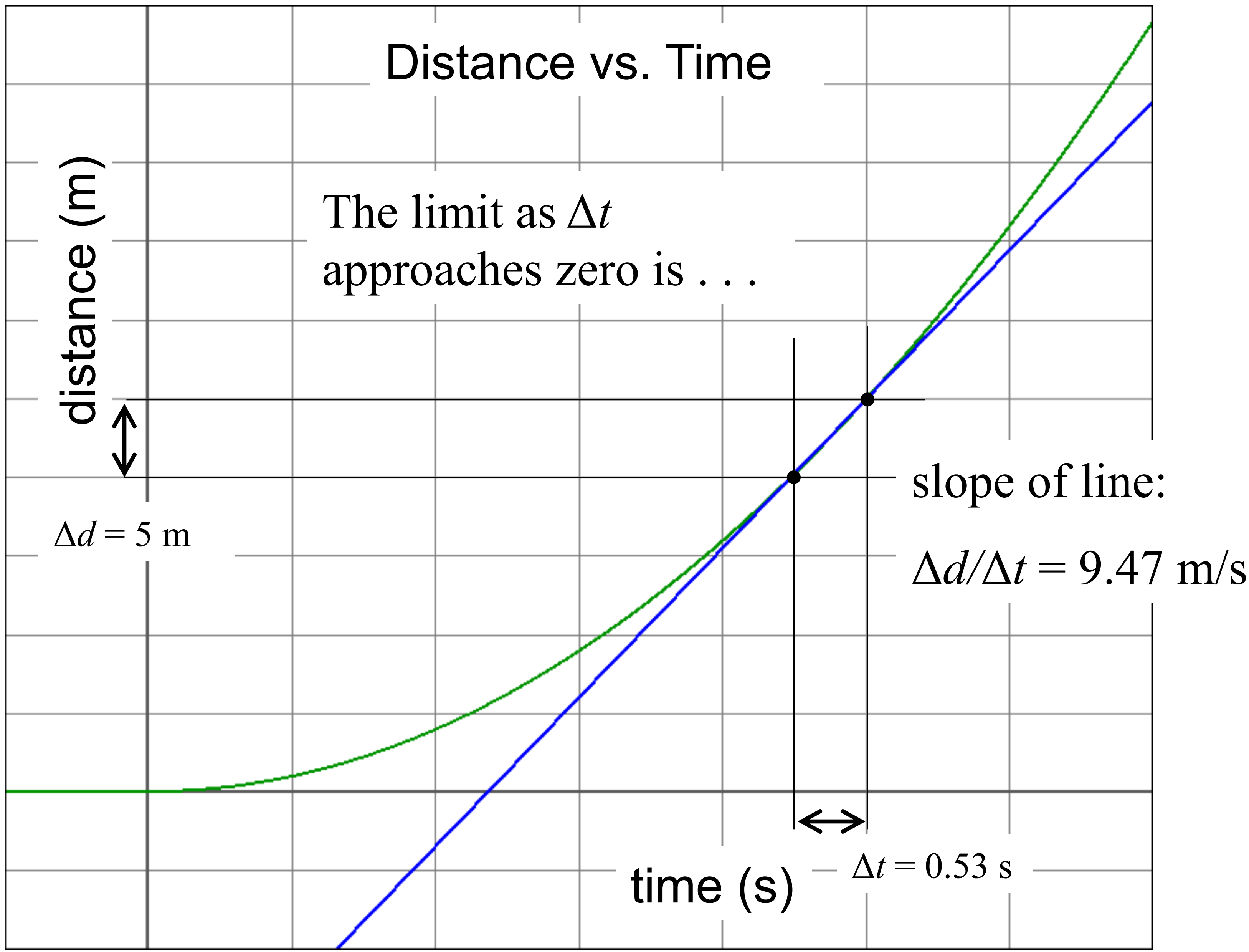

Distance vs. Time
distance (m)
The limit as Δt
approaches zero is . . .
slope of line:
Δd/Δt = 7.24 m/s
Δd = 20 m
Δt = 2.76 s
time (s)



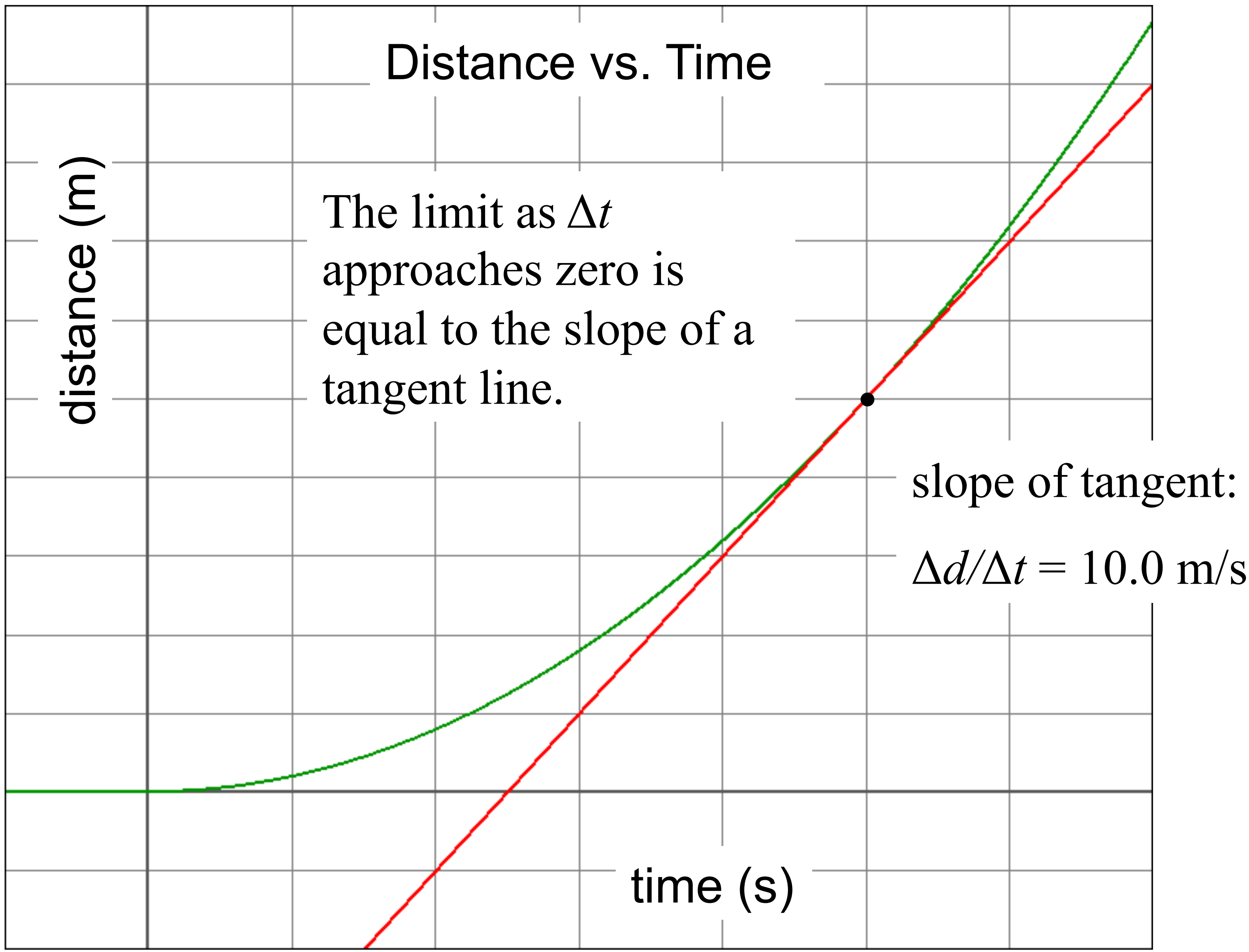
Distance vs. Time
The limit as Δt approaches zero is . . .
distance (m)
Δd = 15 m
slope of line:
Δd/Δt = 8.16 m/s
Δt = 1.84 s
time (s)

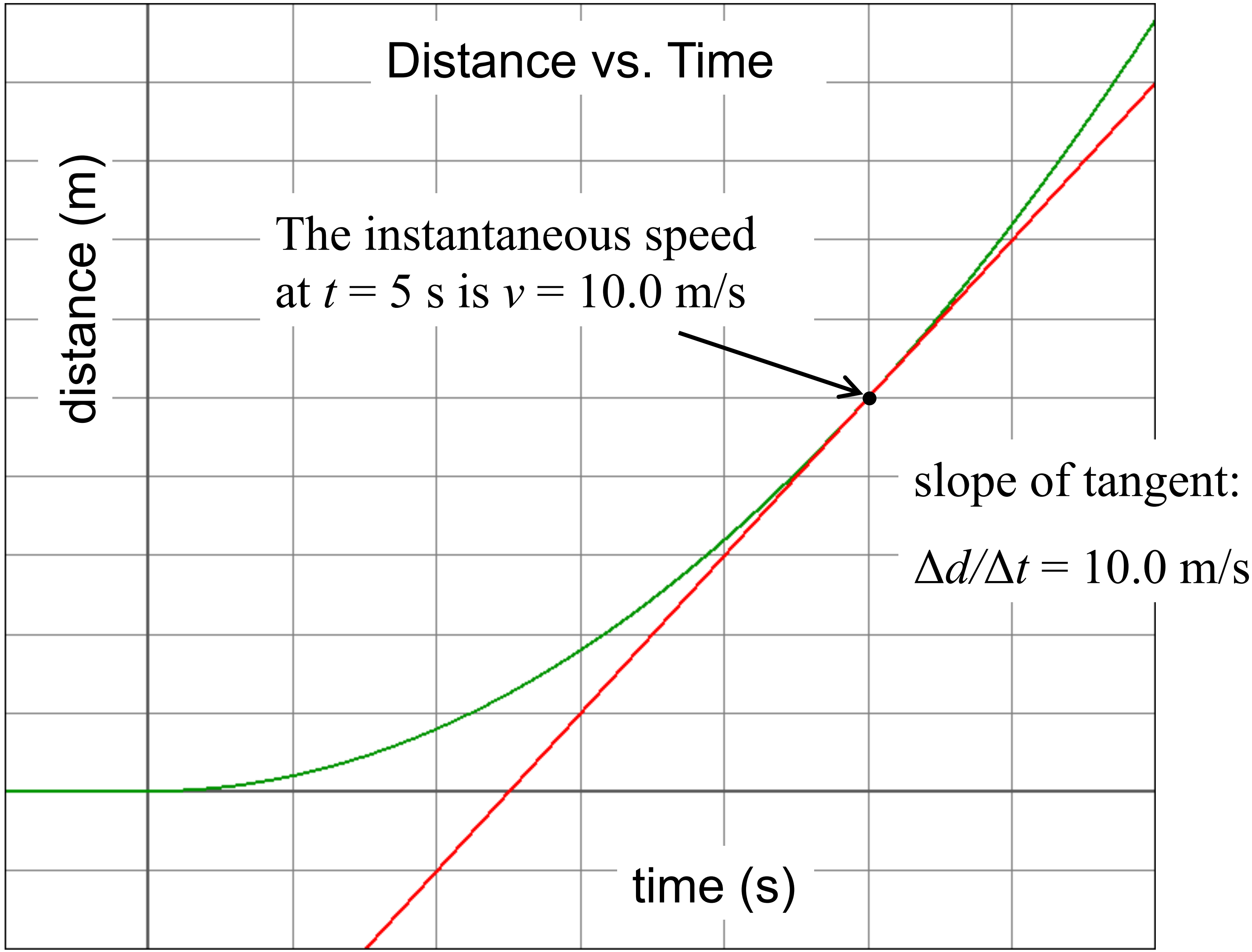


Distance vs. Time
distance (m)
The limit as Δt
approaches zero is . . .
Δd = 10 m
slope of line:
Δd/Δt = 8.87 m/s
time (s)
Δt = 1.13 s

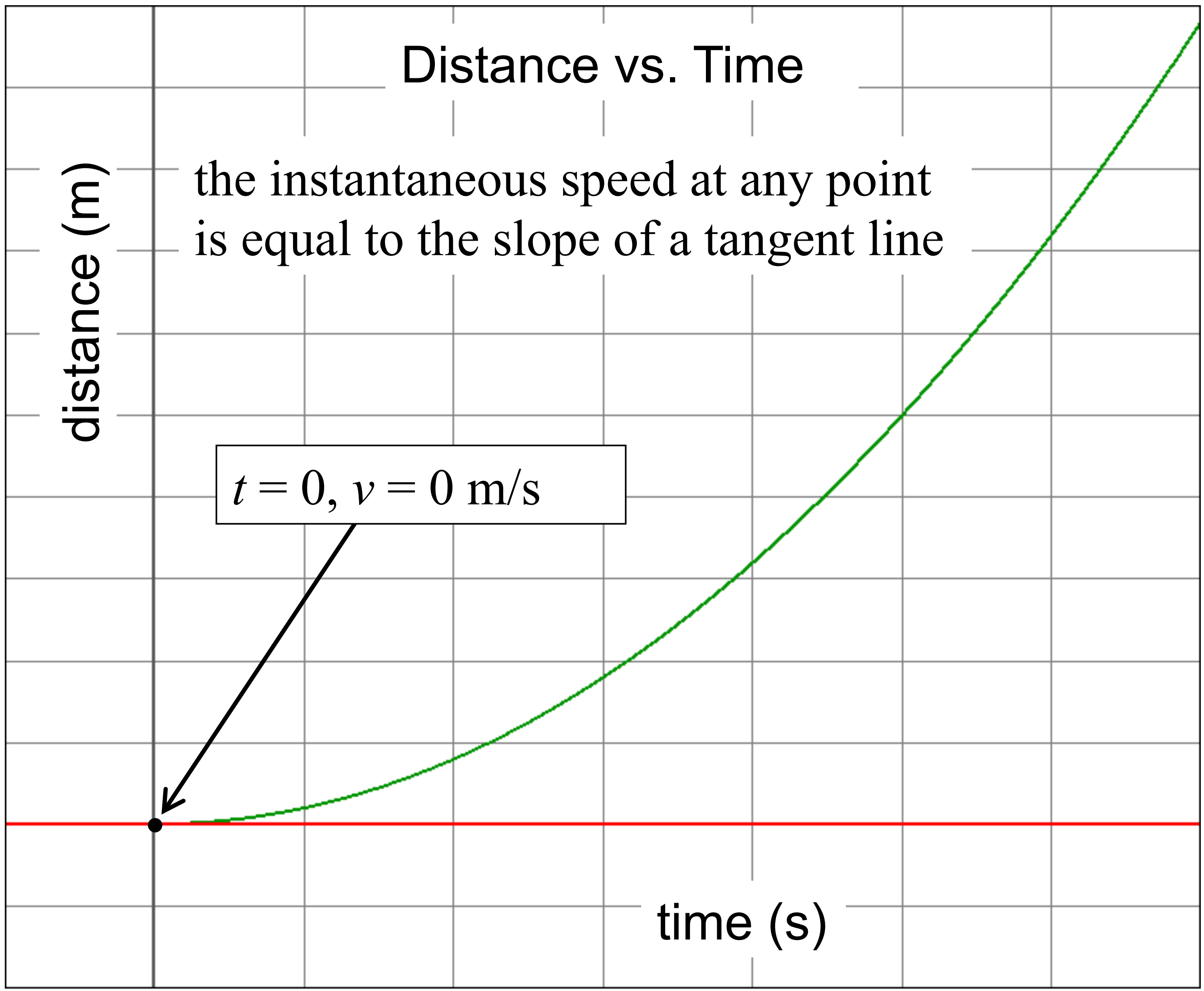


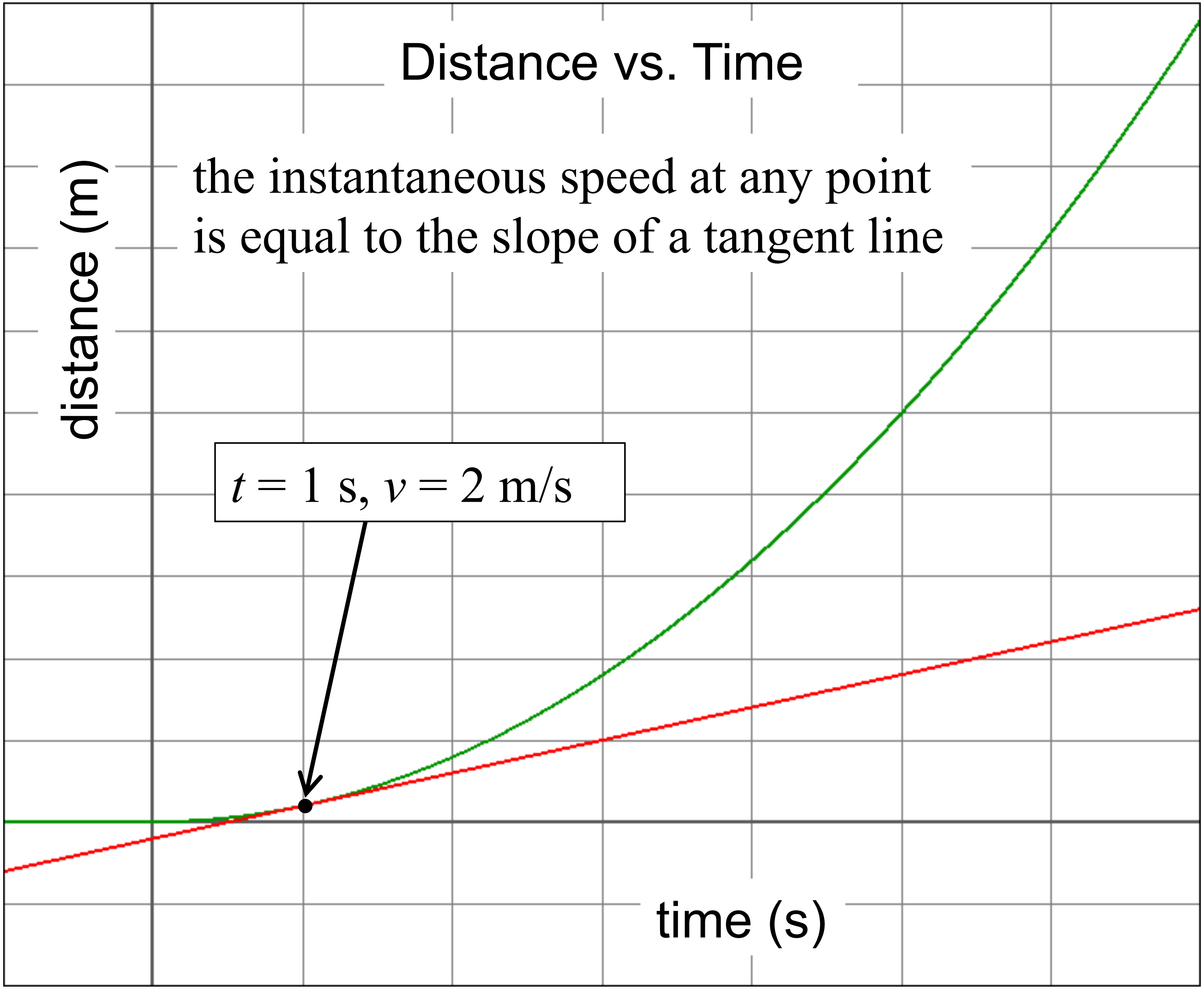
Distance vs. Time
distance (m)
The limit as Δt
approaches zero is . . .
Δd = 5 m
slope of line:
Δd/Δt = 9.47 m/s
time (s)
Δt = 0.53 s

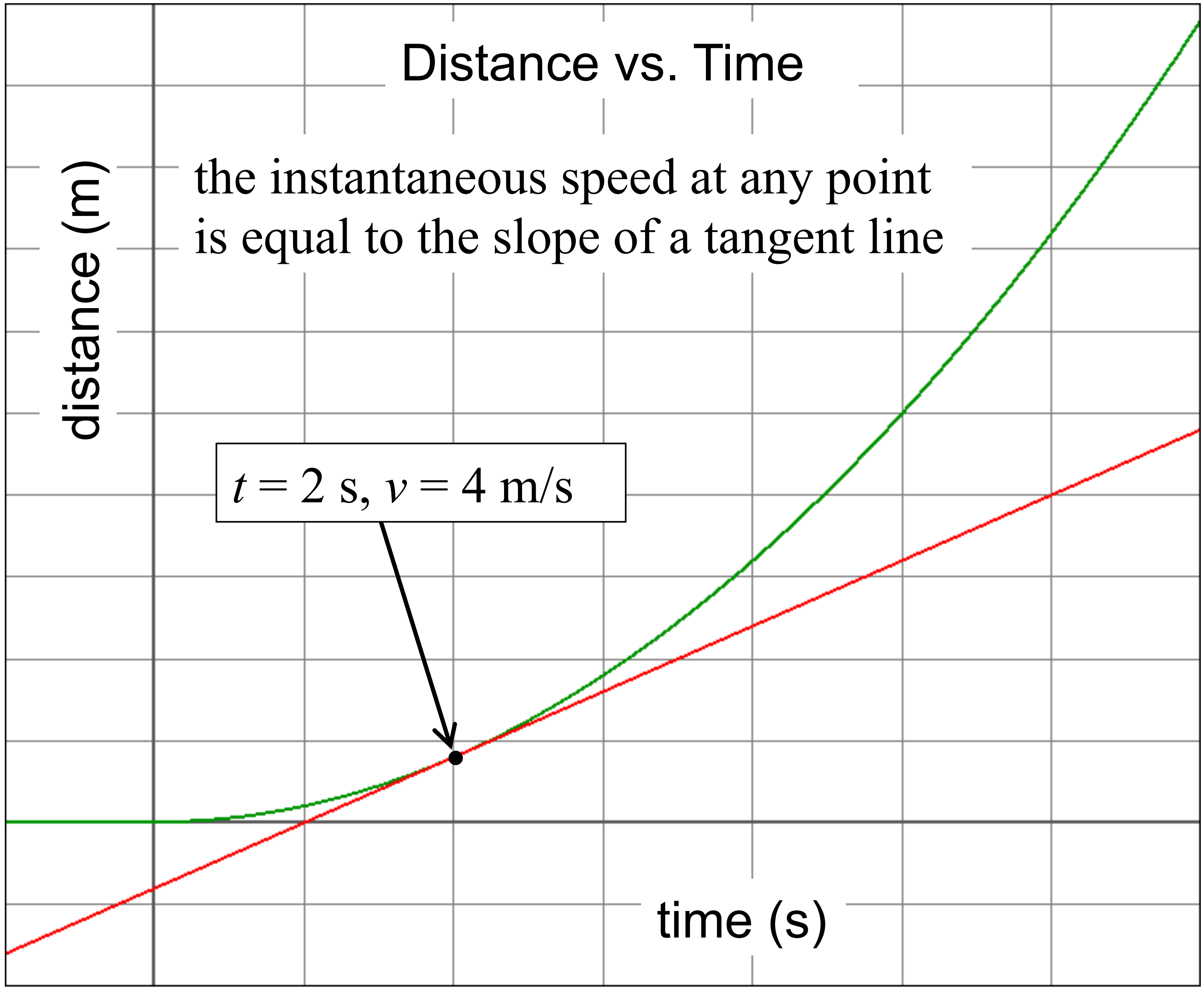

Distance vs. Time
distance (m)
The limit as Δt approaches zero is equal to the slope of a tangent line.
slope of tangent:
Δd/Δt = 10.0 m/s
time (s)

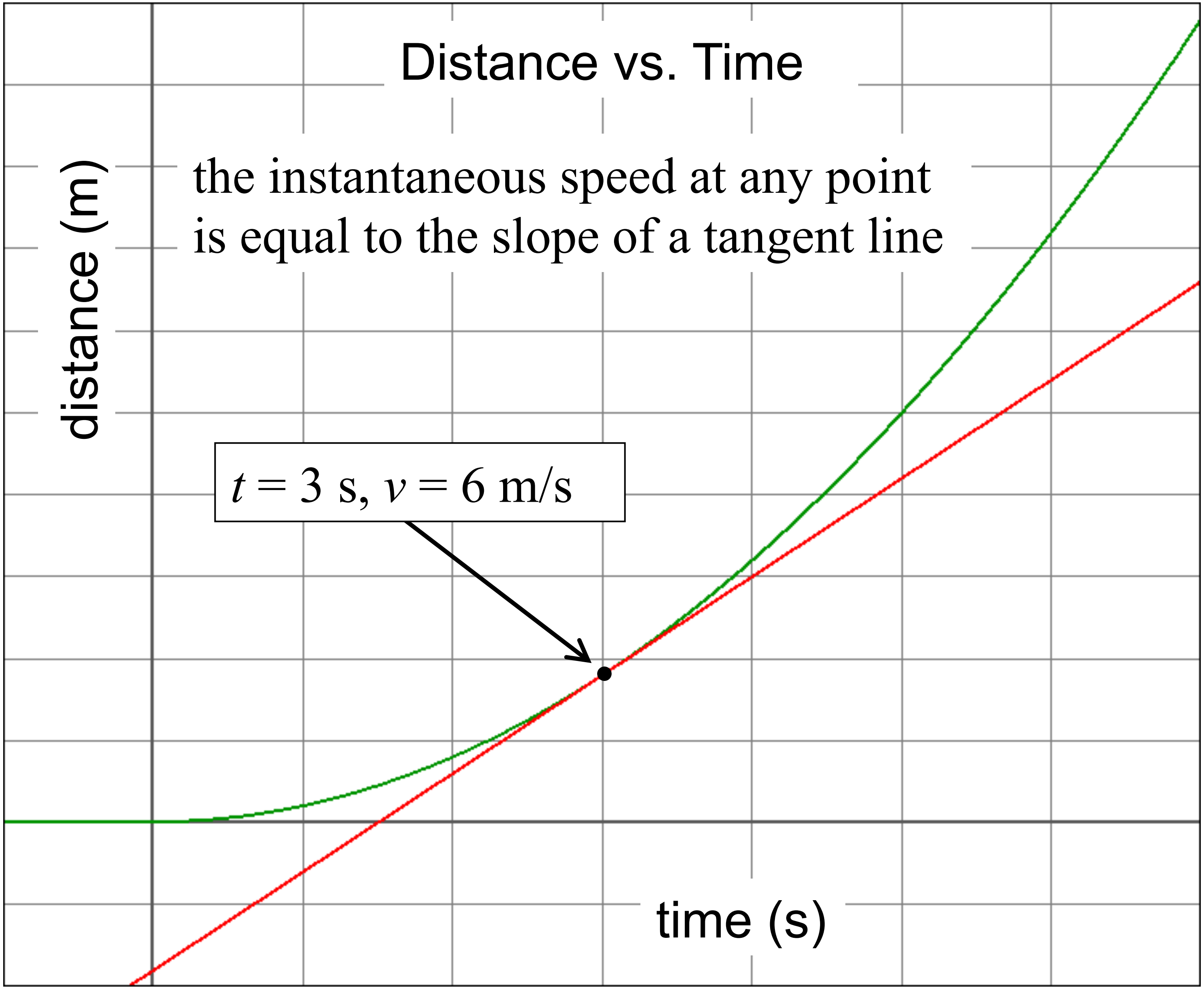


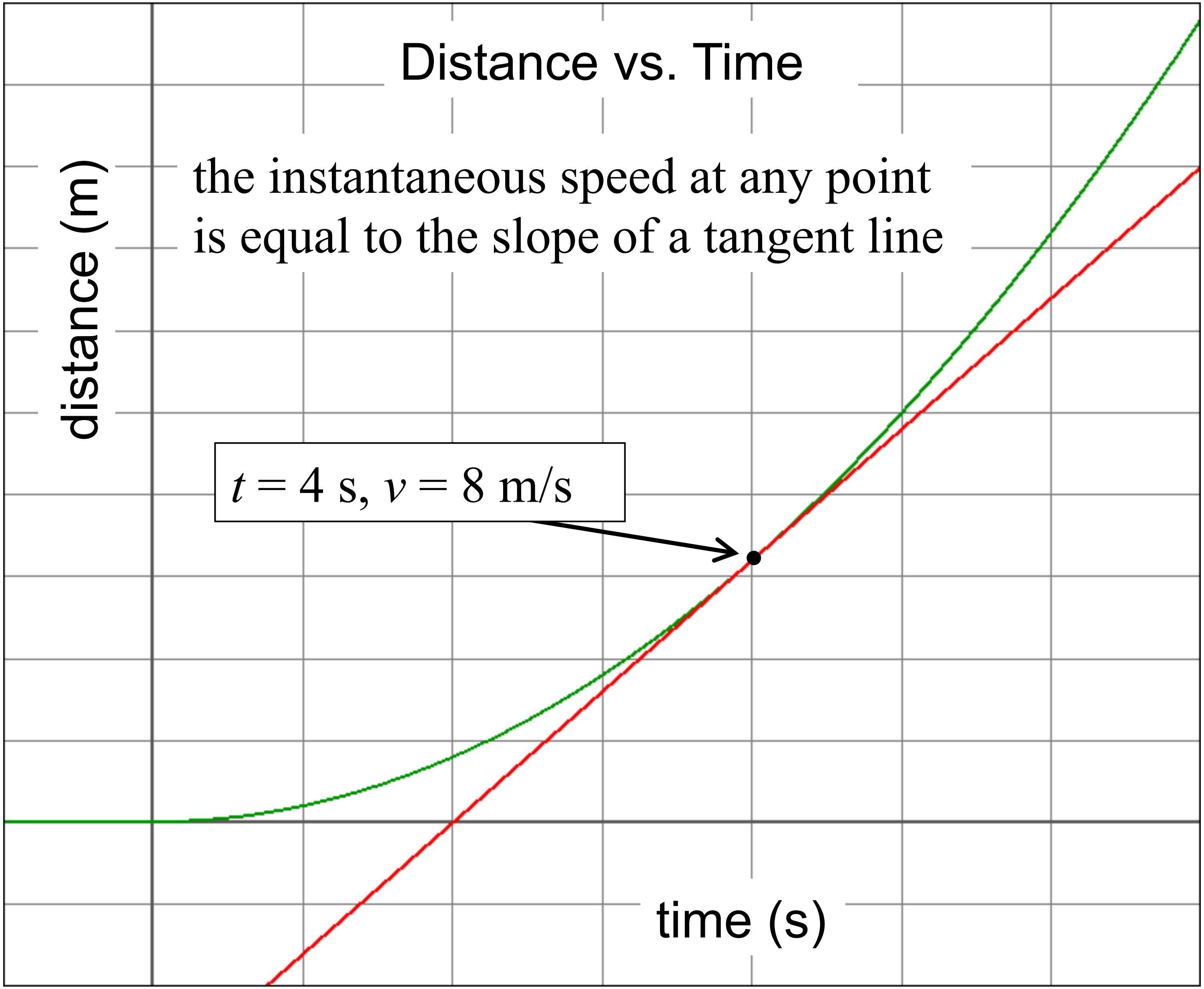
Distance vs. Time
distance (m)
The instantaneous speed
at t = 5 s is v = 10.0 m/s
slope of tangent:
$\Delta d/\Delta t$ = 10.0 m/s
time (s)

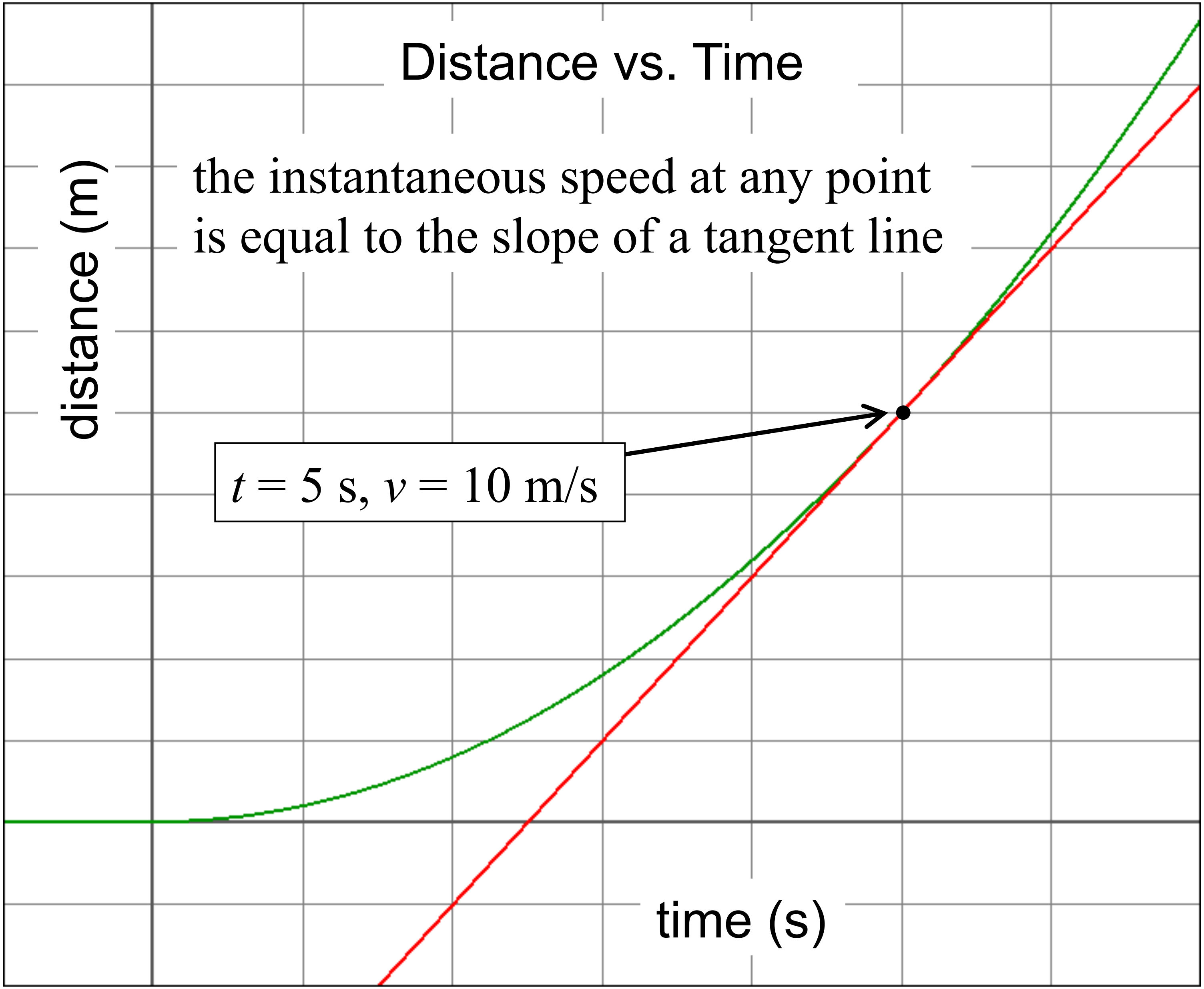

Distance vs. Time
the instantaneous speed at any point
is equal to the slope of a tangent line
distance (m)
t = 0, v = 0 m/s
time (s)

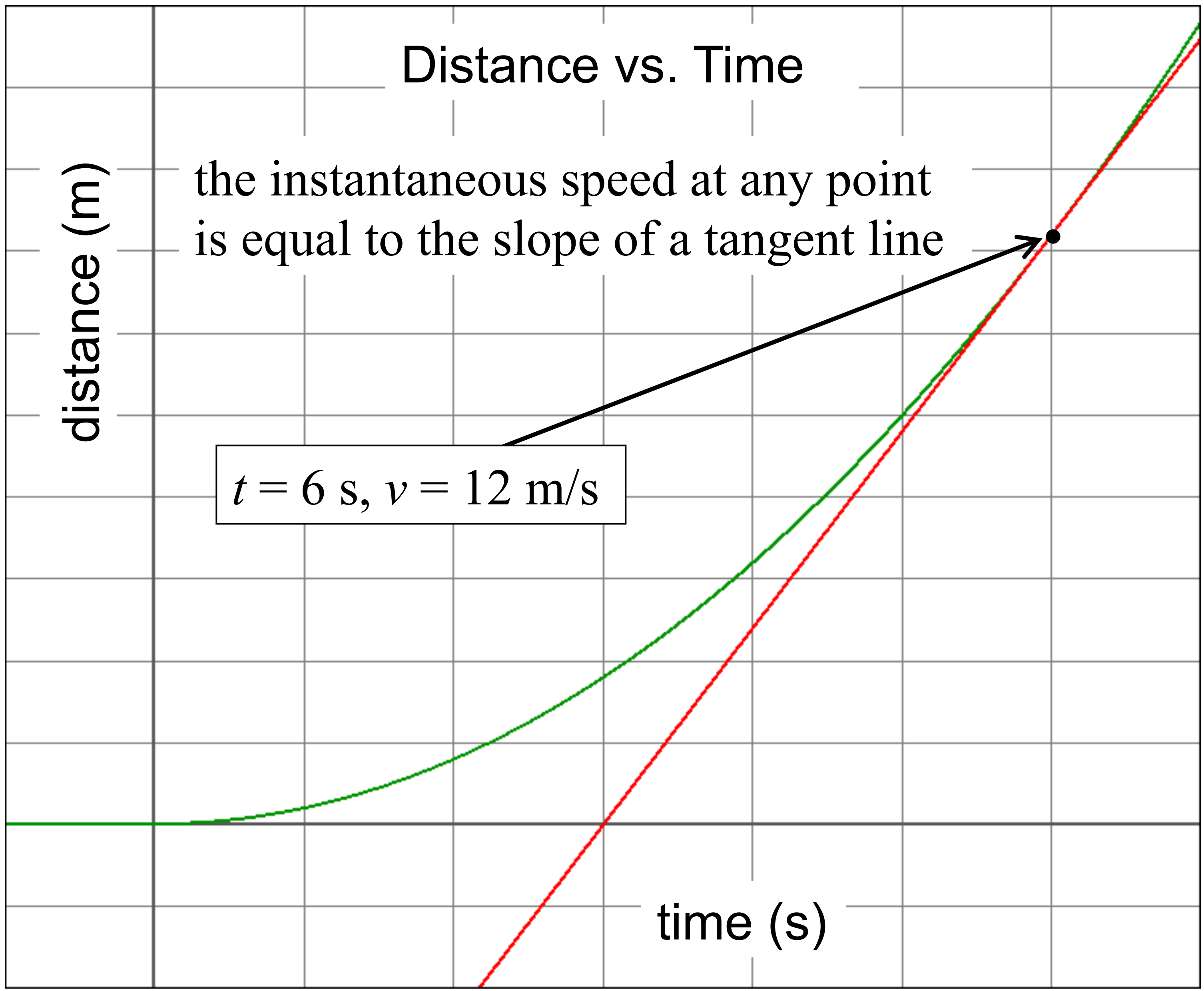

Distance vs. Time
the instantaneous speed at any point
is equal to the slope of a tangent line
distance (m)
$t = 1$ s, $v = 2$ m/s
time (s)

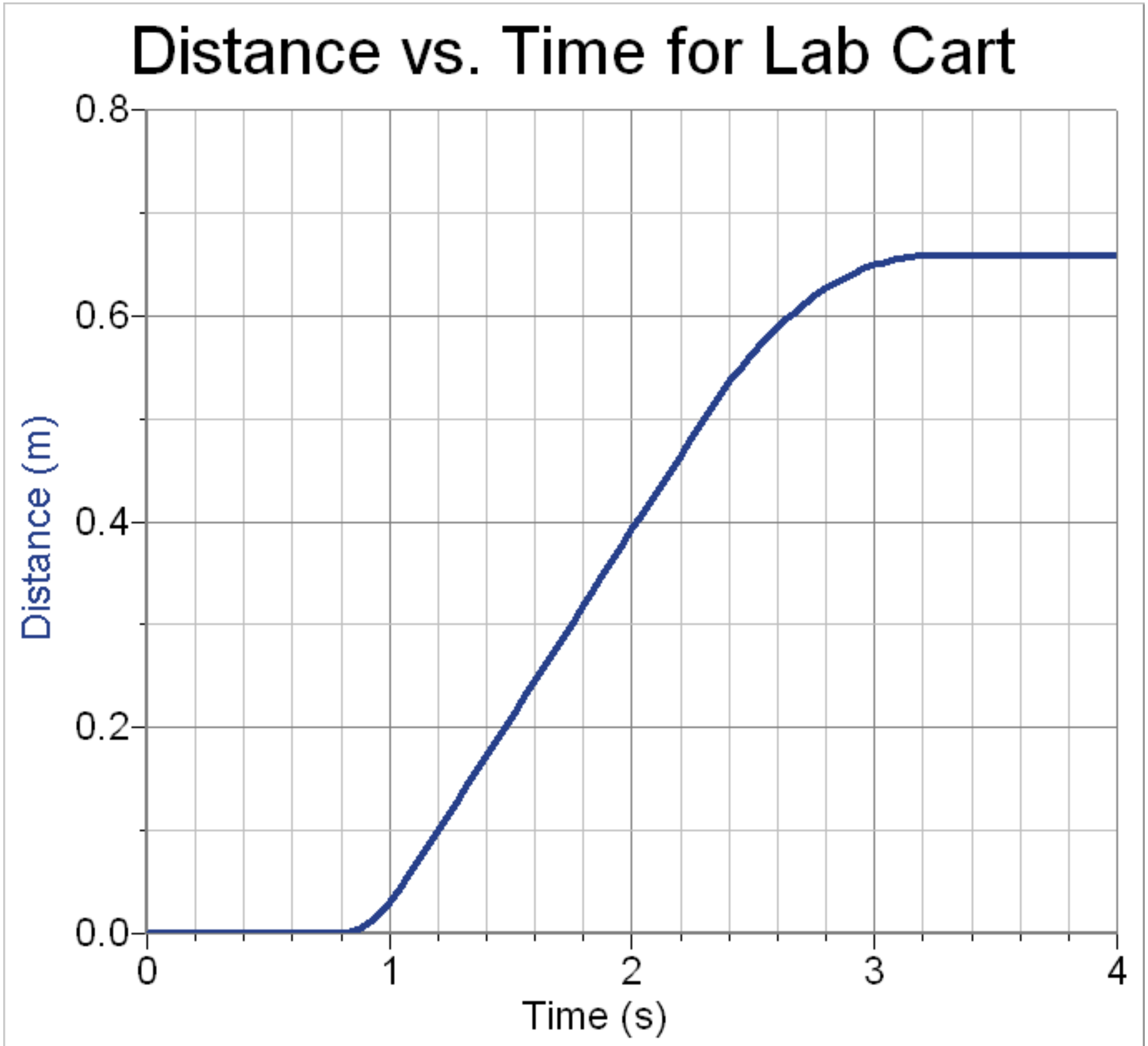

Distance vs. Time
the instantaneous speed at any point
is equal to the slope of a tangent line
distance (m)
t = 2 s, v = 4 m/s
time (s)


Distance vs. Time
distance (m)
the instantaneous speed at any point
is equal to the slope of a tangent line
t = 3 s, v = 6 m/s
time (s)

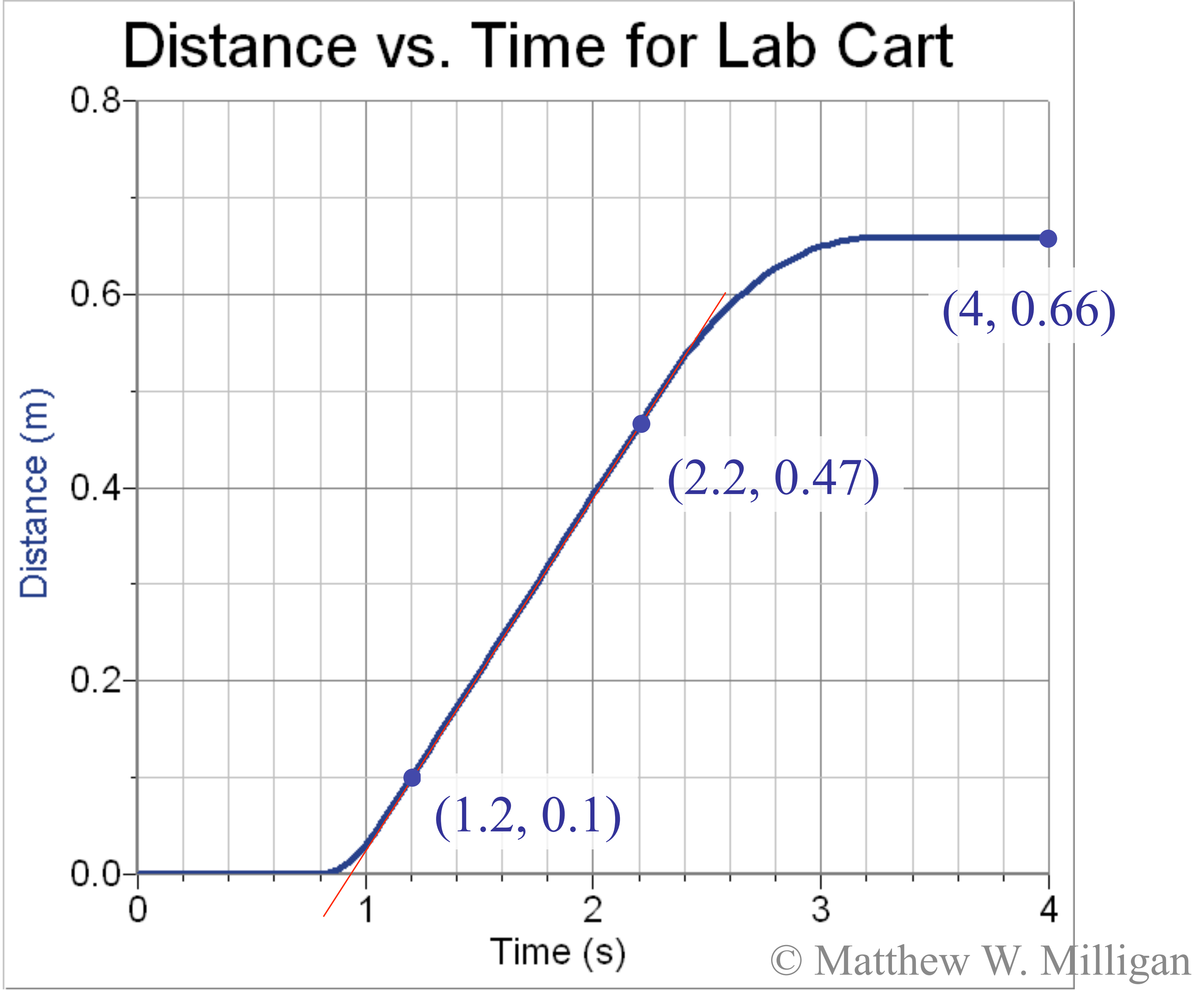

Distance vs. Time
the instantaneous speed at any point
is equal to the slope of a tangent line
distance (m)
t = 4 s, v = 8 m/s
time (s)


Distance vs. Time
the instantaneous speed at any point
is equal to the slope of a tangent line
distance (m)
t = 5 s, v = 10 m/s
time (s)

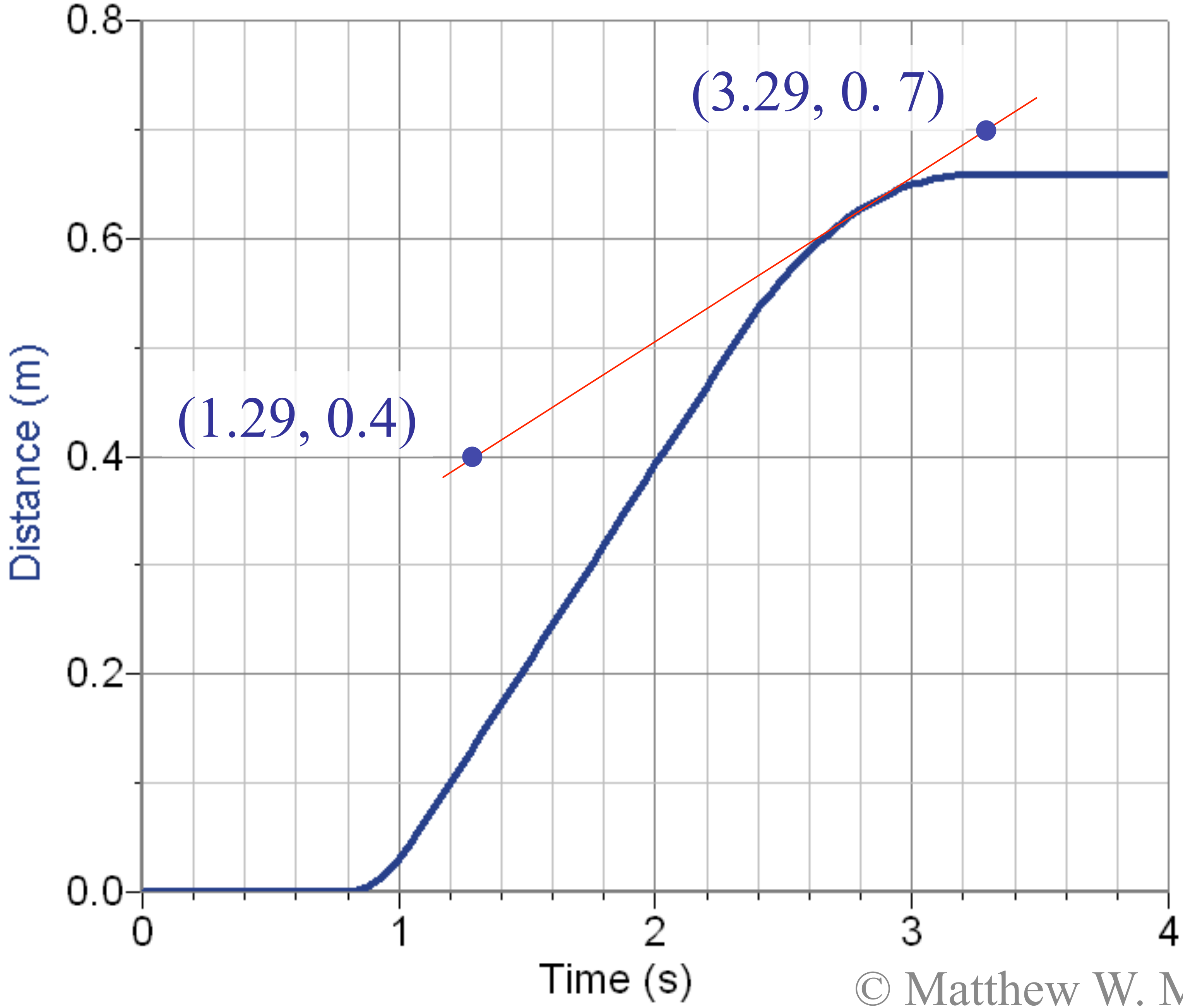

Distance vs. Time
distance (m)
the instantaneous speed at any point
is equal to the slope of a tangent line
t = 6 s, v = 12 m/s
time (s)

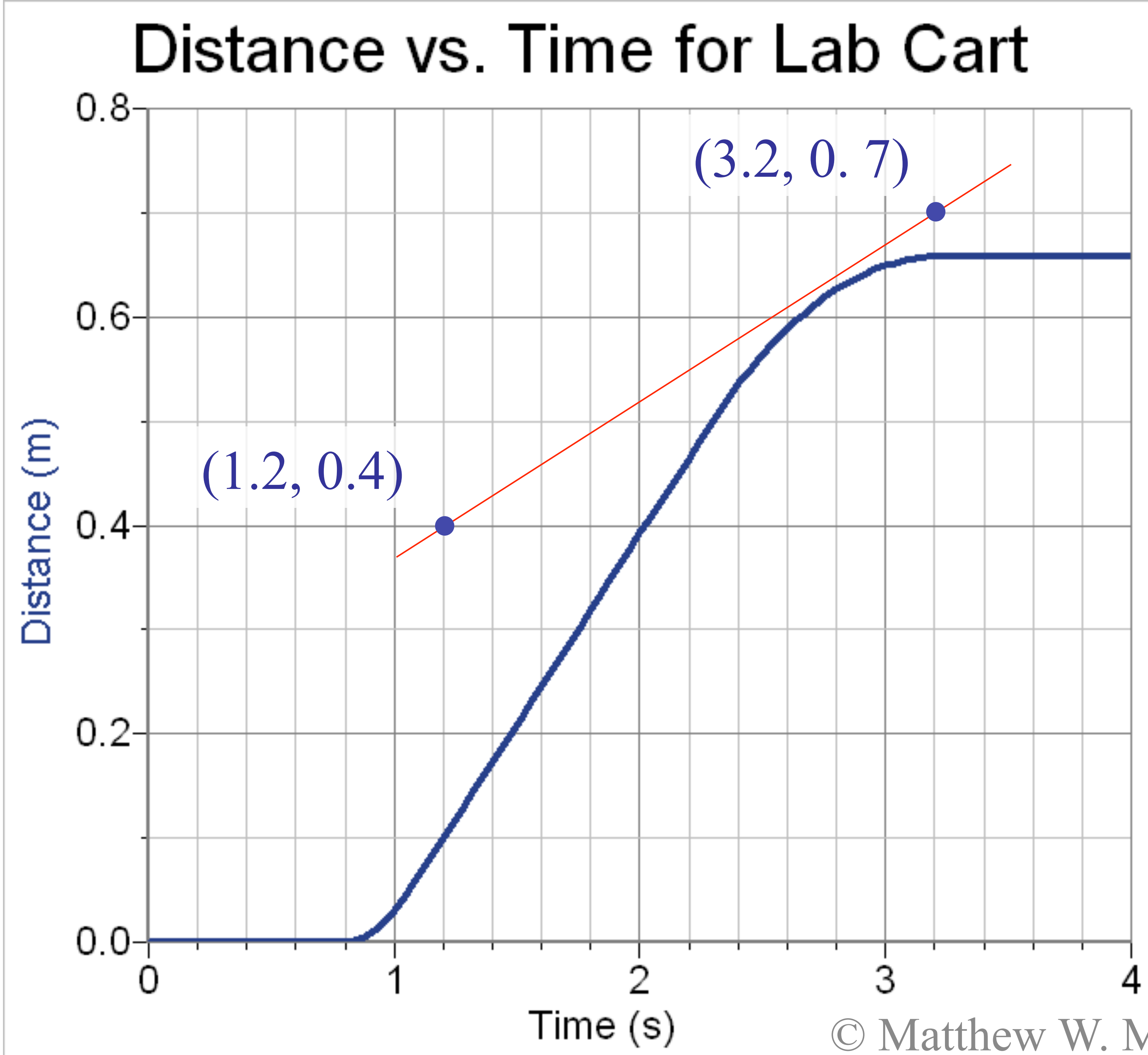

Distance vs. Time for Lab Cart
0.8
0.6
0.4
0.2
0.0
Distance (m)
0
1
2
3
4
Time (s)

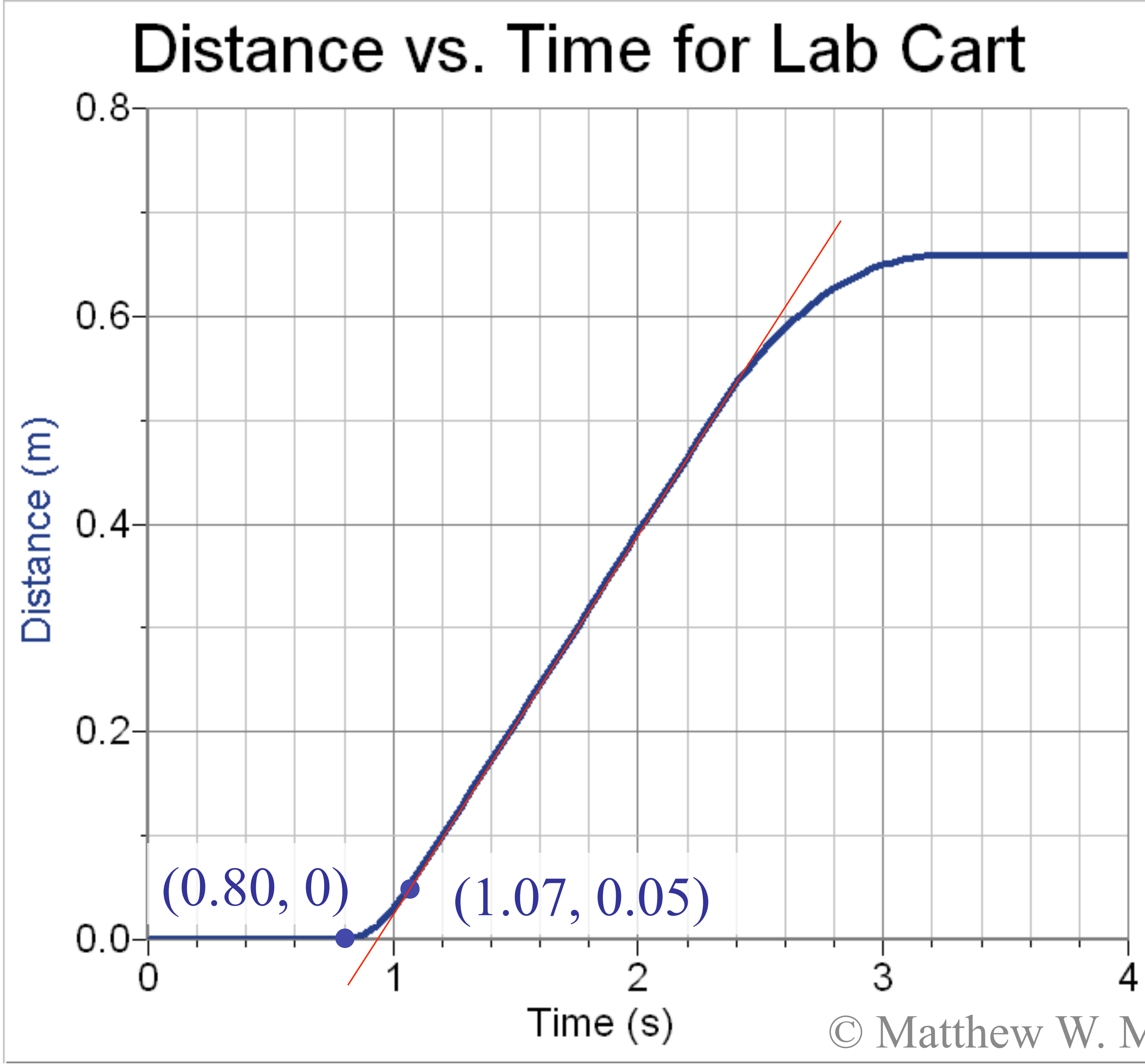
1. Find the average speed of the cart during the 4 seconds shown.
2. Find the maximum speed of the cart.
3. Find the speed at $t = 2.8$ s.
4. Find the maximum rate of acceleration.
5. Make careful sketches of speed vs. time and acceleration vs. time for this cart.


Distance vs. Time for Lab Cart
Distance (m)
Time (s)
0.0
0.2
0.4
0.6
0.8
0
1
2
3
4
(1.2, 0.1)
(2.2, 0.47)
(4, 0.66)
© Matthew W. Milligan

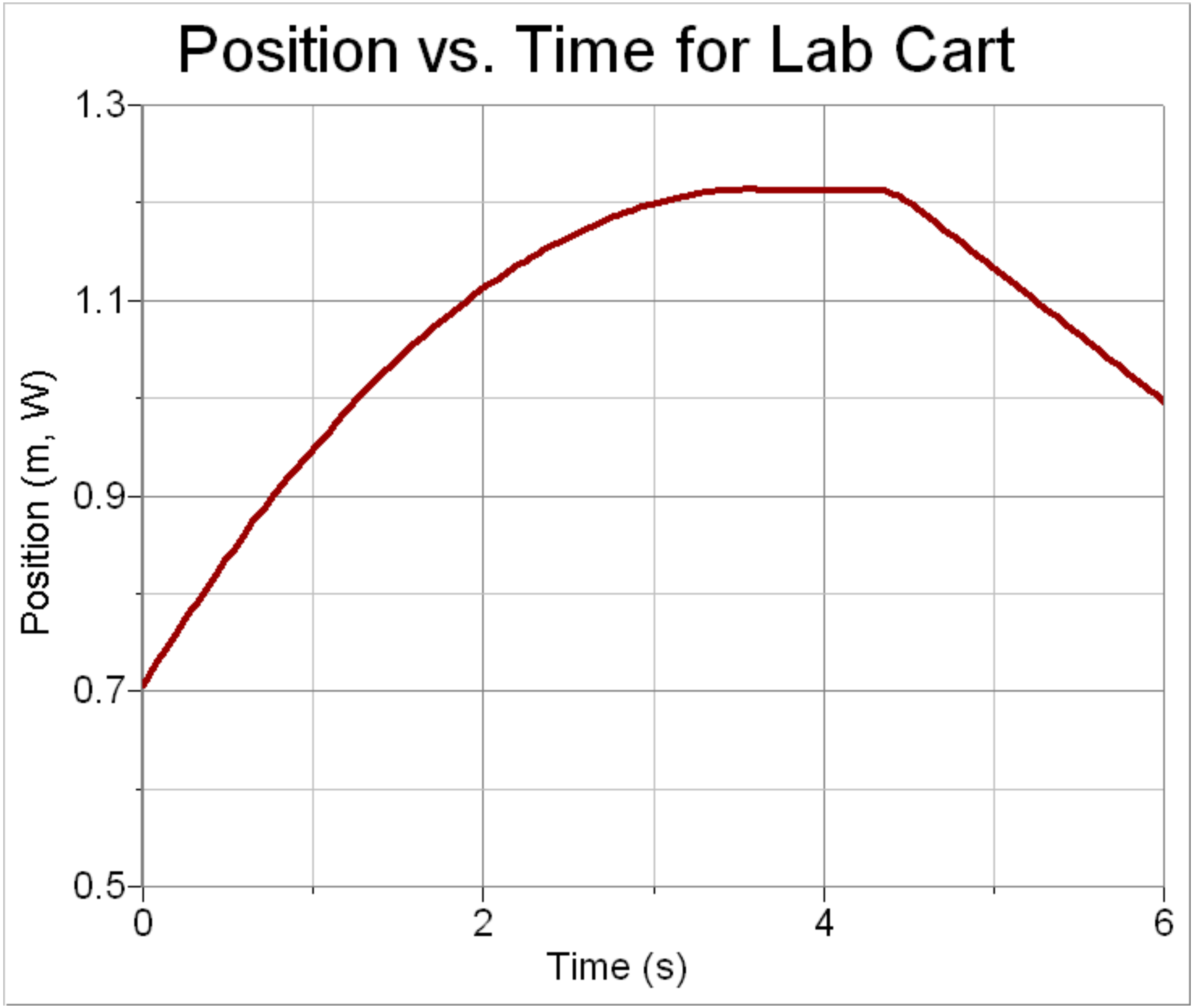
Distance vs. Time for Lab Cart




Distance vs. Time for Lab Cart
(3.2, 0. 7)
(1.2, 0.4)
Distance (m)
Time (s)
0.0
0.2
0.4
0.6
0.8
0
1
2
3
4


Distance vs. Time for Lab Cart
0.8
0.6
0.4
0.2
0.0
Distance (m)
0
1
2
3
4
Time (s)
(0.80, 0)
(1.07, 0.05)

1. $v_{avg} = 0.17$ m/s
2. $v_{max} = 0.37$ m/s
3. $v_{t\,=\,2.8\text{ s}} = 0.15$ m/s
4. $a_{max} = 1.4$ m/s^2
5.


Position vs. Time for Lab Cart
1.3
1.1
0.9
0.7
0.5
Position (m, W)
0
2
4
6
Time (s)

1. Find the average velocity and average speed for the six seconds shown.

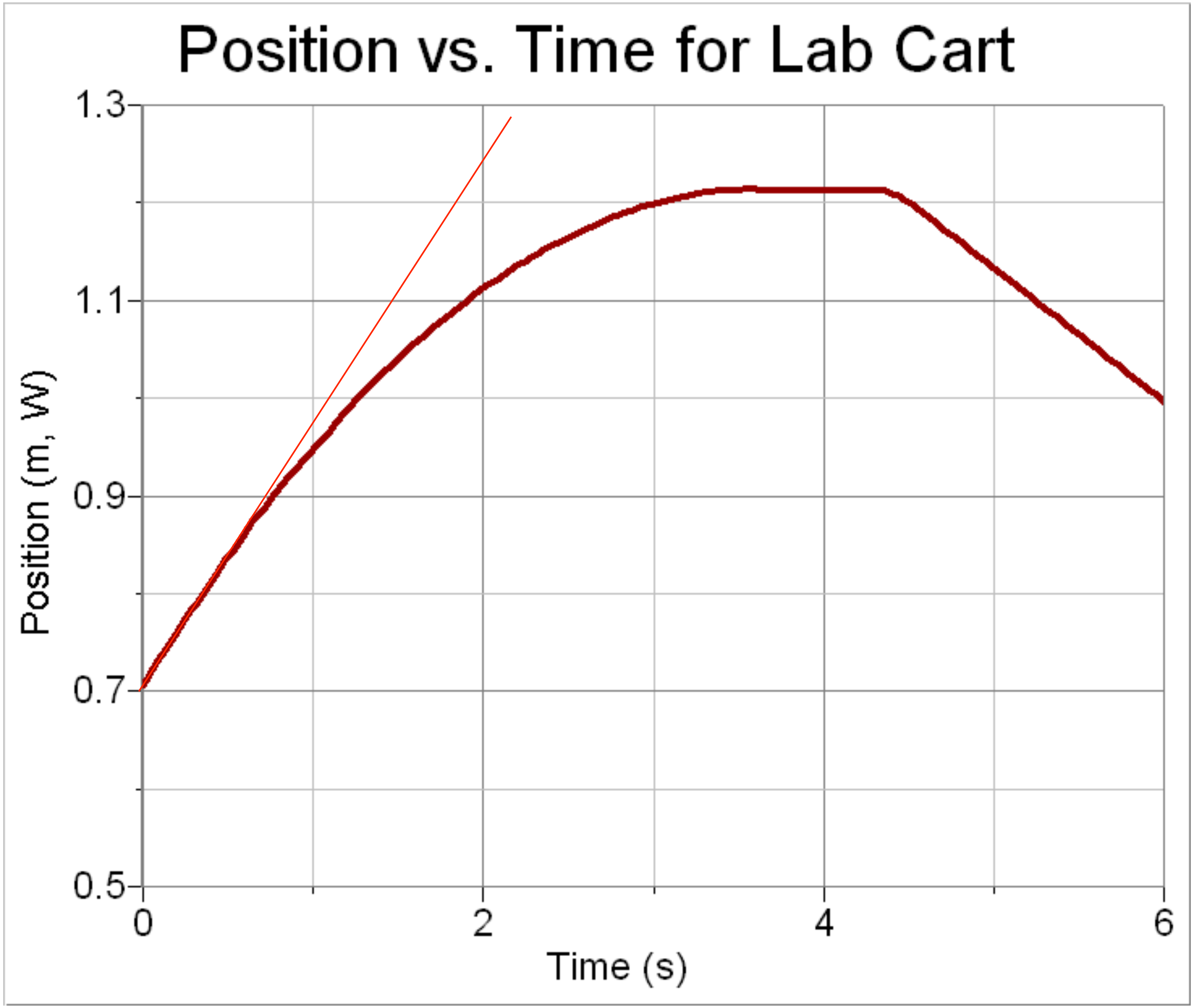
2. Find the velocity at $t = 5$ s.

3. In what interval(s) of time is speed increasing?

4. Find the maximum speed of the cart.

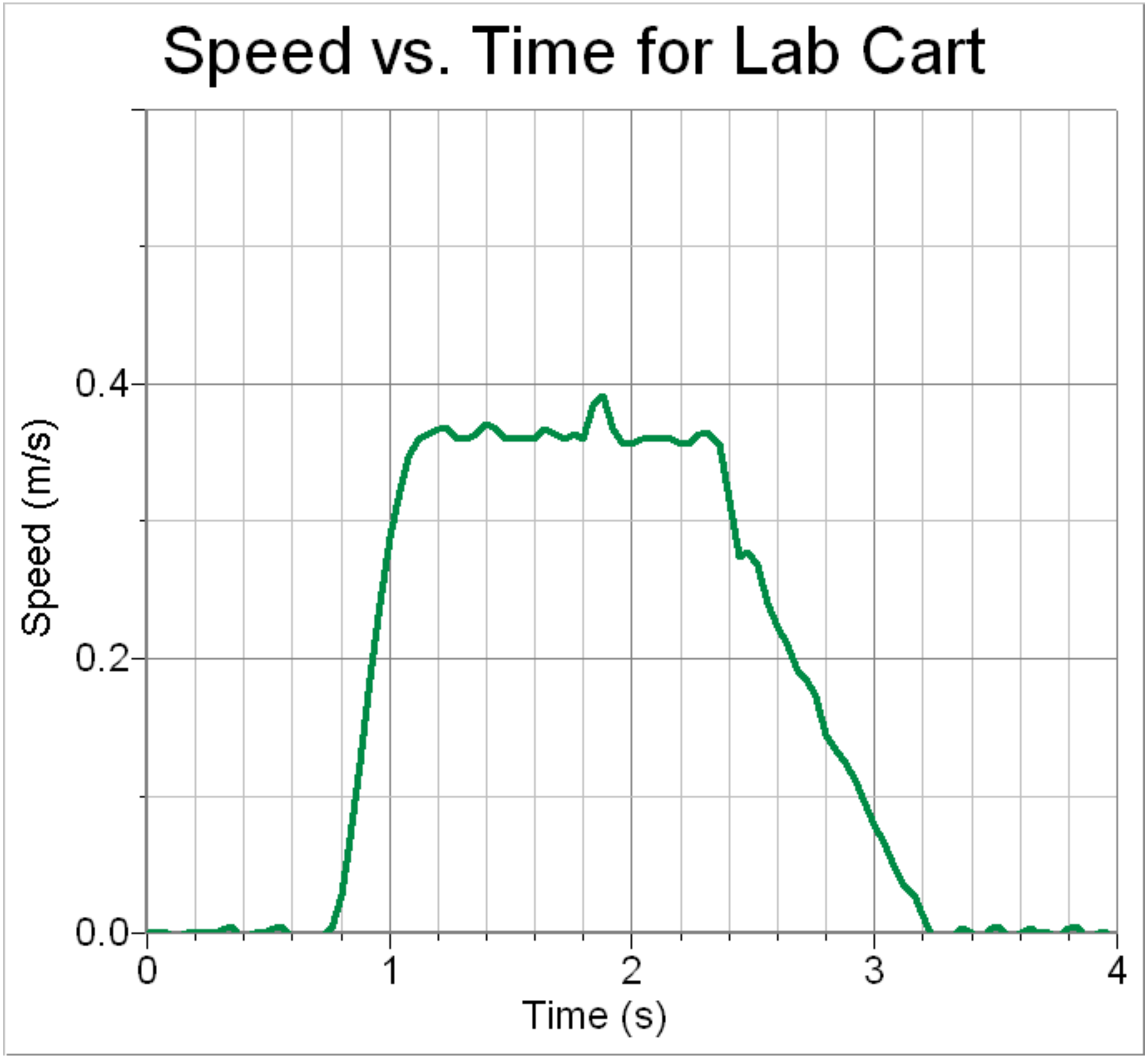
5. Make careful sketches of velocity vs. time and acceleration vs. time for the cart.

1. $\mathbf{v}_{avg} = 0.048$ m/s, W, $v_{avg} = 0.12$ m/s
2. $\mathbf{v}_{t\,=\,5\text{ s}} = 0.14$ m/s, E
3. Speed increases: 4.3 s $< t <$ 4.5 s
4. 0.26 m/s
5.


Position vs. Time for Lab Cart
1.3
1.1
0.9
0.7
0.5
Position (m, W)
0
2
4
6
Time (s)

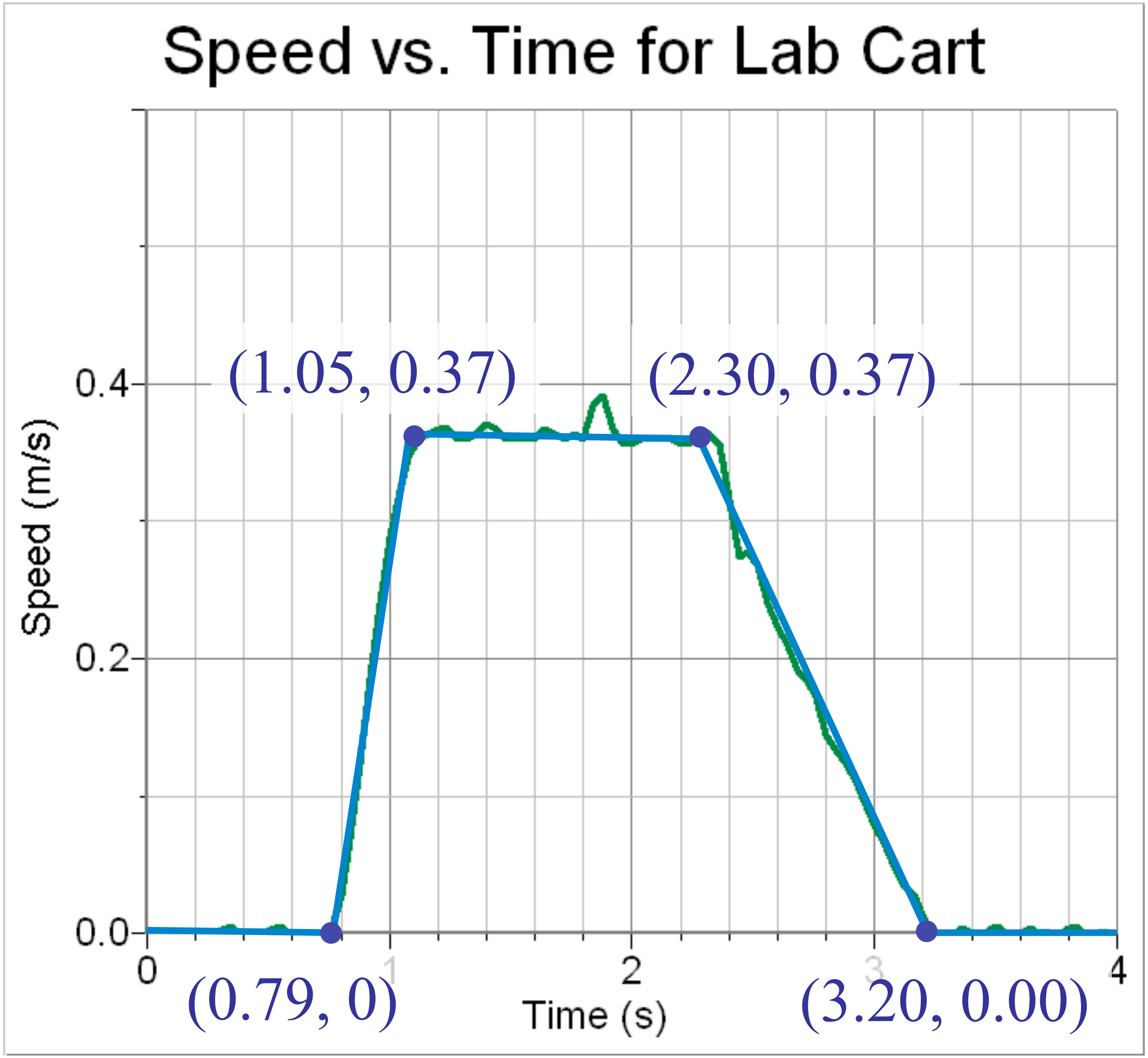
Area Under the Curve

- Area under the curve is more technically an "area-like" calculation for the region between the function and the x-axis.
- This area represents the product of x and y, and allows for variance in y.
- Unlike a true area, this result can be negative if y is negative.



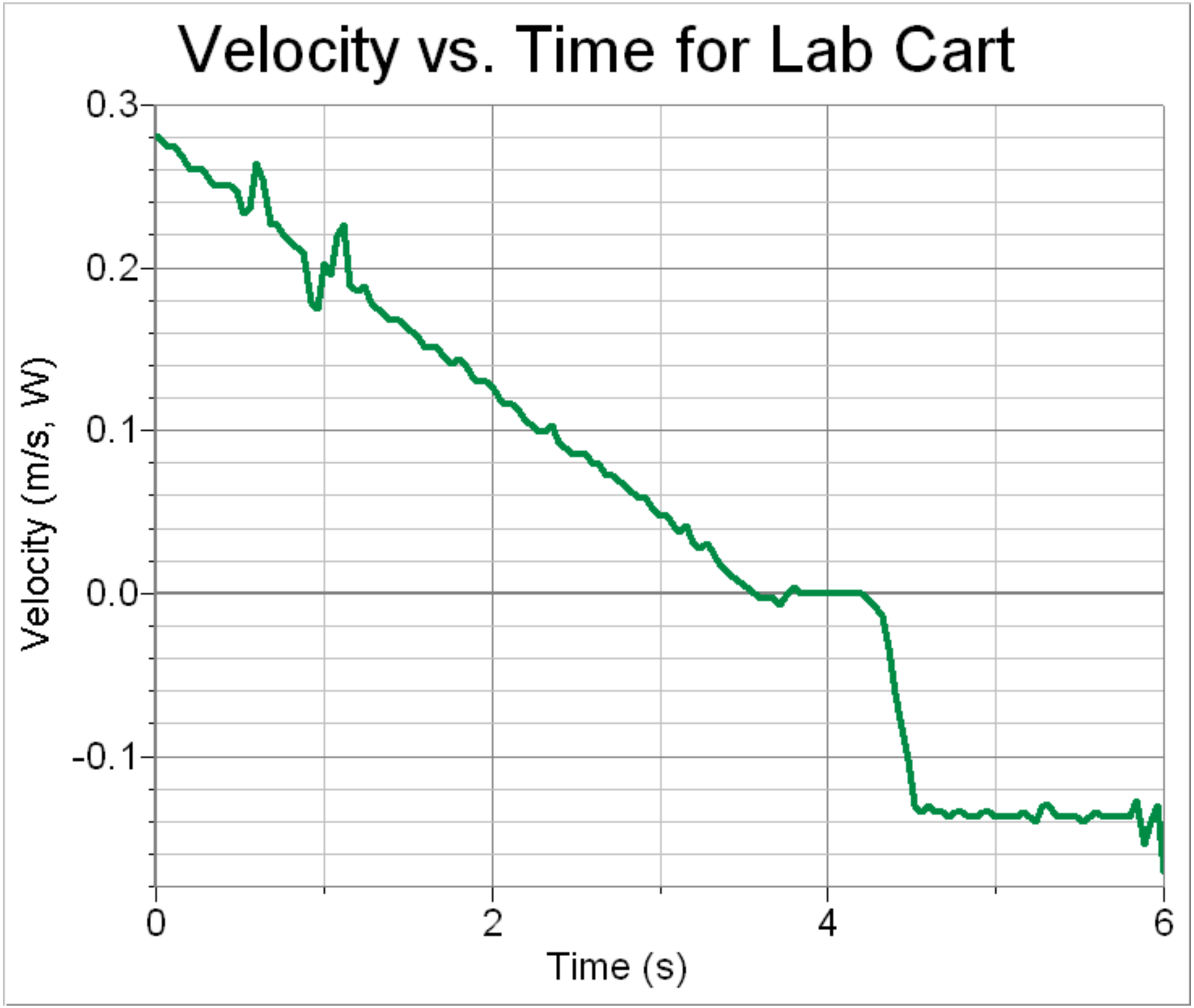
Speed vs. Time for Lab Cart
Speed (m/s)
0.4
0.2
0.0
0
1
2
3
4
Time (s)

1. Draw lines of best fit on each linear segment and use these to find the acceleration of each interval.
2. Determine the area under the curve and compare to the distance shown on the previous graph.


Speed vs. Time for Lab Cart
Speed (m/s)
0.0
0.2
0.4
0
1
2
3
4
Time (s)
(1.05, 0.37)
(2.30, 0.37)
(0.79, 0)
(3.20, 0.00)

1. $a_1 = 0$, $a_2 = 1.4$ m/s^2, $a_3 = 0$, $a_4 = -0.40$ m/s^2, $a_5 = 0$

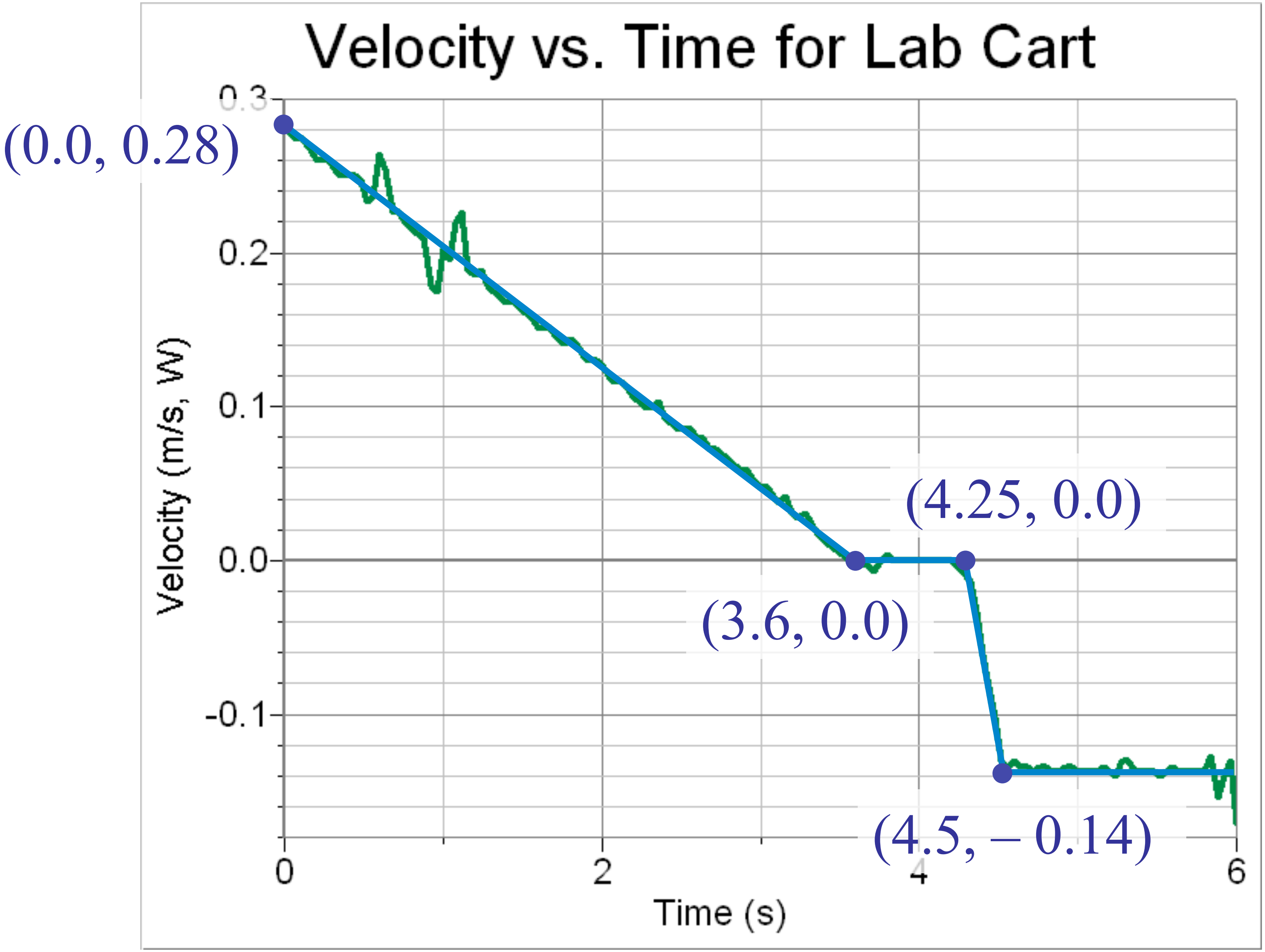
2. distance = area = 0.68 m


Velocity vs. Time for Lab Cart
Velocity (m/s, W)
0.3
0.2
0.1
0.0
-0.1
0
2
4
6
Time (s)

1. Draw lines of best fit on each linear segment and use these to find the acceleration of each interval.

2. Use the area method to determine the displacement of the cart during the first 4 seconds and during the last 2 seconds. Compare to the position vs. time graph.

1. $\mathbf{a}_1 = 0.078\ \text{m/s}^2$, E; $\mathbf{a}_2 = 0$,
   $\mathbf{a}_3 = 0.56\ \text{m/s}^2$, E; $\mathbf{a}_4 = 0$
2. $\mathbf{d}_1 = 0.50$ m, W; $\mathbf{d}_2 = 0.23$ m, E



Velocity vs. Time for Lab Cart
Velocity (m/s, W)
Time (s)
0.3
0.2
0.1
0.0
-0.1
0
2
4
6
(0.0, 0.28)
(4.25, 0.0)
(3.6, 0.0)
(4.5, – 0.14)