Advanced Kinematics

I. Vector addition/subtraction
II. Components
III. Relative Velocity
IV. Projectile Motion
V. Use of Calculus (nonuniform acceleration)
VI. Parametric Equations

	The student will be able to:	HW:
1	Calculate the components of a vector given its magnitude and direction. ✓	1 – 2
2	Calculate the magnitude and direction of a vector given its components. ✓	3 – 4
3	Use vector components as a means of analyzing/solving 2-D motion problems. ✓	5 – 6
4	Add or subtract vectors analytically (using trigonometric calculations). ✓	7 – 9
5	Use vector addition or subtraction as a means of solving relative motion problems. ✓	10 – 15
6	State the horizontal and vertical relations for projectile motion and use the same to solve projectile problems. ✓	16 – 24
7	Use derivatives to determine speed, velocity, or acceleration and solve for extrema and/or zeros.	25 – 27
8	Use integrals to determine distance, displacement, change in speed or velocity and solve for functions thereof given initial conditions.	28 – 31
9	Solve problems involving parametric equations that describe motion components	32 – 34

Use of Calculus in Kinematics

- An instantaneous rate of change can be defined mathematically as a limit.
- In calculus it is shown that this type of limit is equivalent to a "derivative".
- By definition position, velocity, and acceleration are functions of time that are always related by the rules of derivatives.

Instantaneous Velocity and Acceleration

$$\vec{v} = \lim_{\Delta t \to 0} \left(\frac{\Delta \vec{r}}{\Delta t} \right)$$

$$\vec{a} = \lim_{\Delta t \to 0} \left(\frac{\Delta \vec{v}}{\Delta t} \right)$$

Awkward!

Instantaneous Velocity and Acceleration

$$\vec{v} = \frac{d\vec{r}}{dt}$$

$$\vec{a} = \frac{d\vec{v}}{dt}$$

Not awkward – and equivalent to the previous notation. Derivatives allow precise and concise definitions for instantaneous rates.

Instantaneous Velocity and Acceleration

$$\vec{v} = \frac{d\vec{r}}{dt}$$

The derivative of an object's position with respect to time is the object's velocity.

$$\vec{a} = \frac{d\vec{v}}{dt}$$

The derivative of an object's velocity with respect to time is the object's acceleration.

Instantaneous Speed

$$v = \frac{dl}{dt}$$

The derivative of an object's distance with respect to time is the object's speed.

Note: l = distance

Special Case – One-dimensional Motion

- The derivatives just given apply to any and all types of motions.
- However it is easiest to understand and apply to an object moving in one-dimension.
- In this special case position is simply a coordinate on a number line – typically "x".
- Directions of vectors are represented by +/–.
- Speed is the absolute value of velocity.

Instantaneous Velocity and Acceleration

For linear motion with position x along the x-axis, instantaneous velocity v and acceleration a:

$$v = \frac{dx}{dt}$$

$$a = \frac{dv}{dt}$$

For convenience the vector symbols are often omitted. But the quantities are still vectors and the sign (+/–) of the quantity equates with direction.

Velocity and Acceleration in 3-D Space

let position, $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2x}{dt^2}\hat{i} + \frac{d^2y}{dt^2}\hat{j} + \frac{d^2z}{dt^2}\hat{k}$$

Components of vector rates are derivatives of components!