Magnetostatics

I. Field Basics – units, poles

II. Magnetic Force on Charge
    Mass Spectrometer
    Cyclotron

III. Magnetic Force on Current
    Motors and Meters

IV. Sources of Magnetic Fields
    Biot-Savart Law
    Ampere's Law
    Solenoids

	The student will be able to:	HW:
1	Define and illustrate the basic properties of magnetic fields and permanent magnets: field lines, north and south poles, magnetic compasses, Earth's magnetic field.	1 – 2 ✓
2	Solve problems relating magnetic force to the motion of a charged particle through a magnetic field, such as that found in a mass spectrometer.	3 – 10 ✓
3	Solve problems involving forces on a current carrying wire in a magnetic field and torque on a current carrying loop of wire in a magnetic field, such as that found in a motor.	11 – 18 ✓
4	State and apply the Biot-Savart Law and solve such problems that relate a magnetic field to the current that produced it.	19 – 24 ✓
5	State and apply Ampere's Law and Gauss's Law for magnetic fields and solve related problems such as those involving parallel wires, solenoids, and toroids.	25 – 40

Gauss' s Law for Magnetic Fields

For any arbitrary closed surface:

$$\oint \vec{B} \cdot d\vec{A} = 0$$

B = magnetic field

A = area of the surface (normal vector)

Because there are no point sources for magnetic fields, the field lines always form continuous loops. Any magnetic field line entering a closed surface will at some point exit that surface so that the net flux is zero. The closed surface can contain magnetic elements are not, it doesn't matter!

Ampere's Law

For any arbitrary open surface:

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I$$

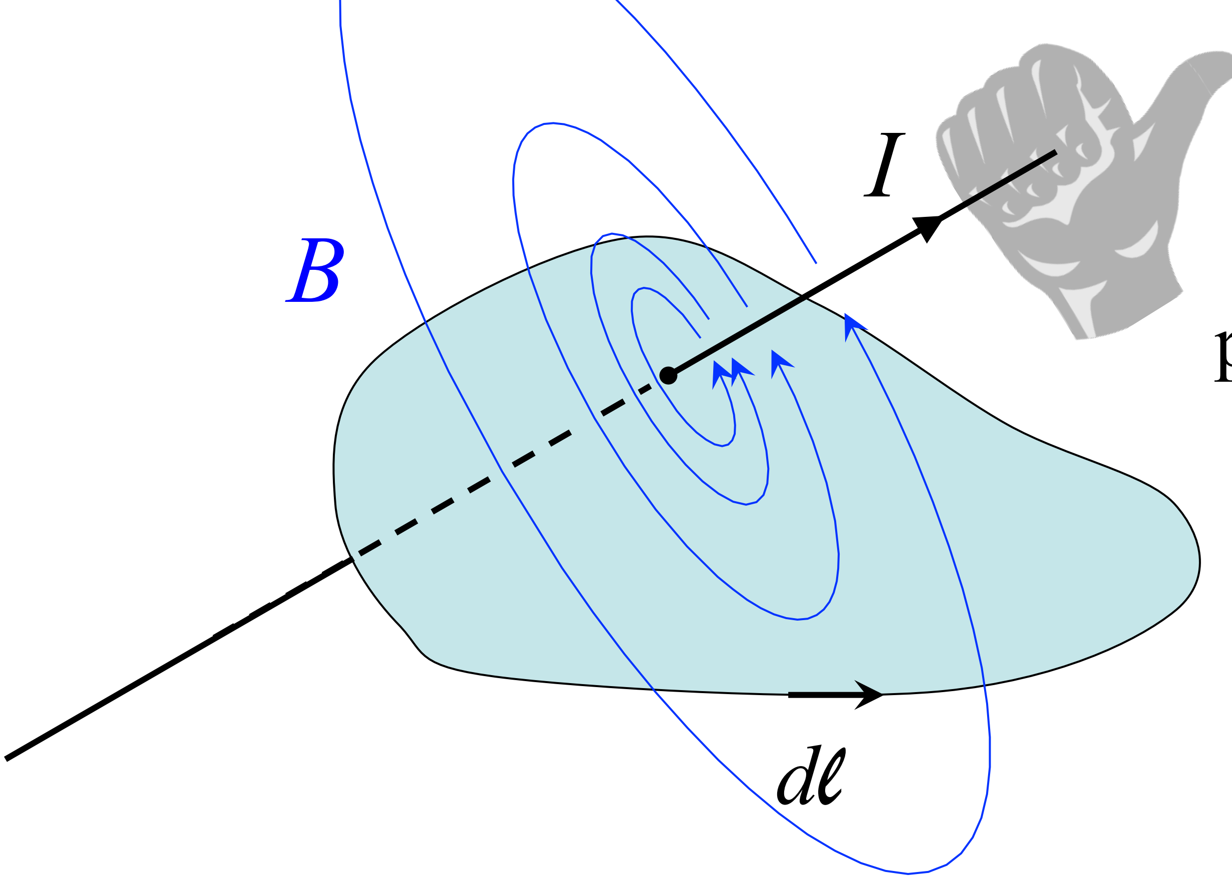
B = magnetic field

ℓ = length of perimeter of the surface

μ_0 = permeability of free space

I = net current *passing through* the surface

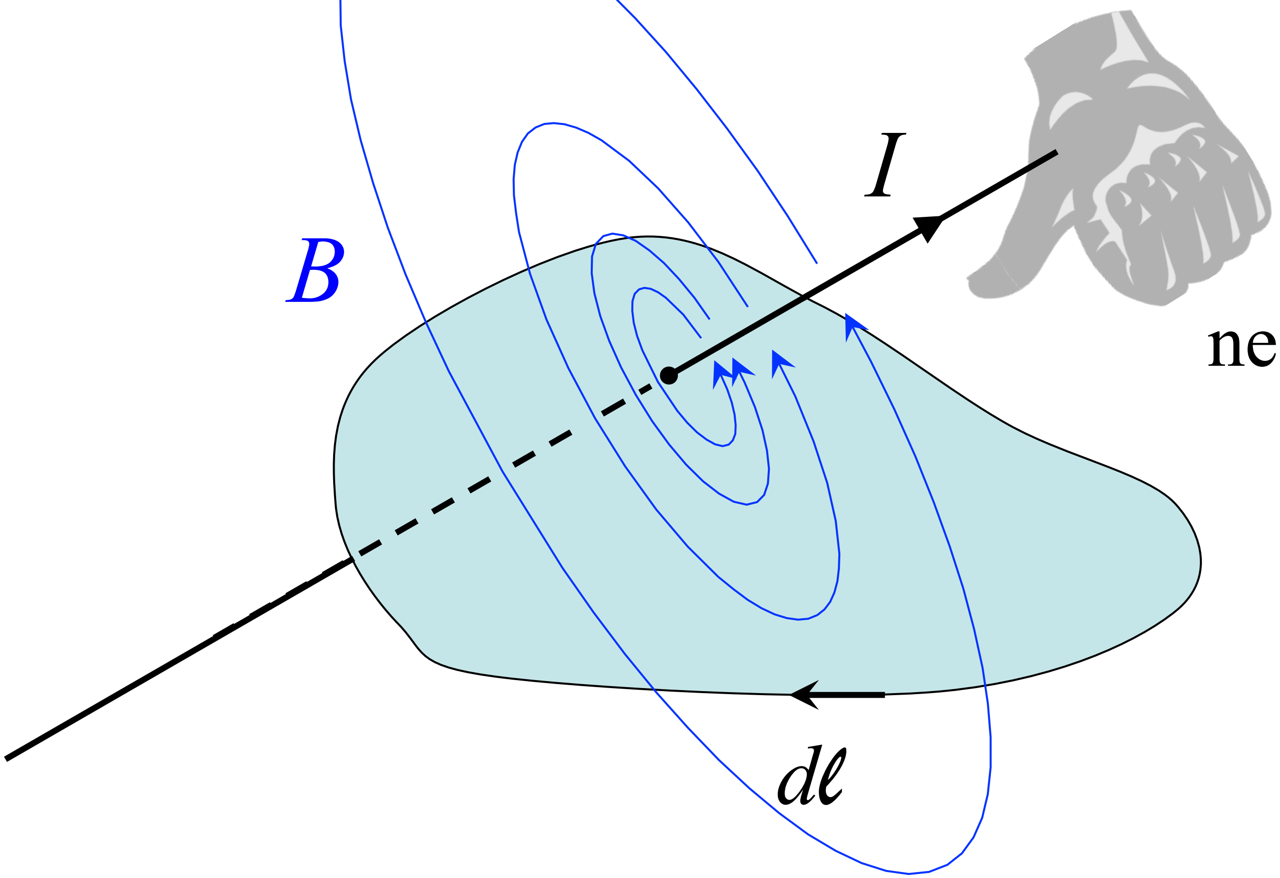
note: direction of ℓ and sign of I determined by right hand rule



I would be positive by RHR

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I$$

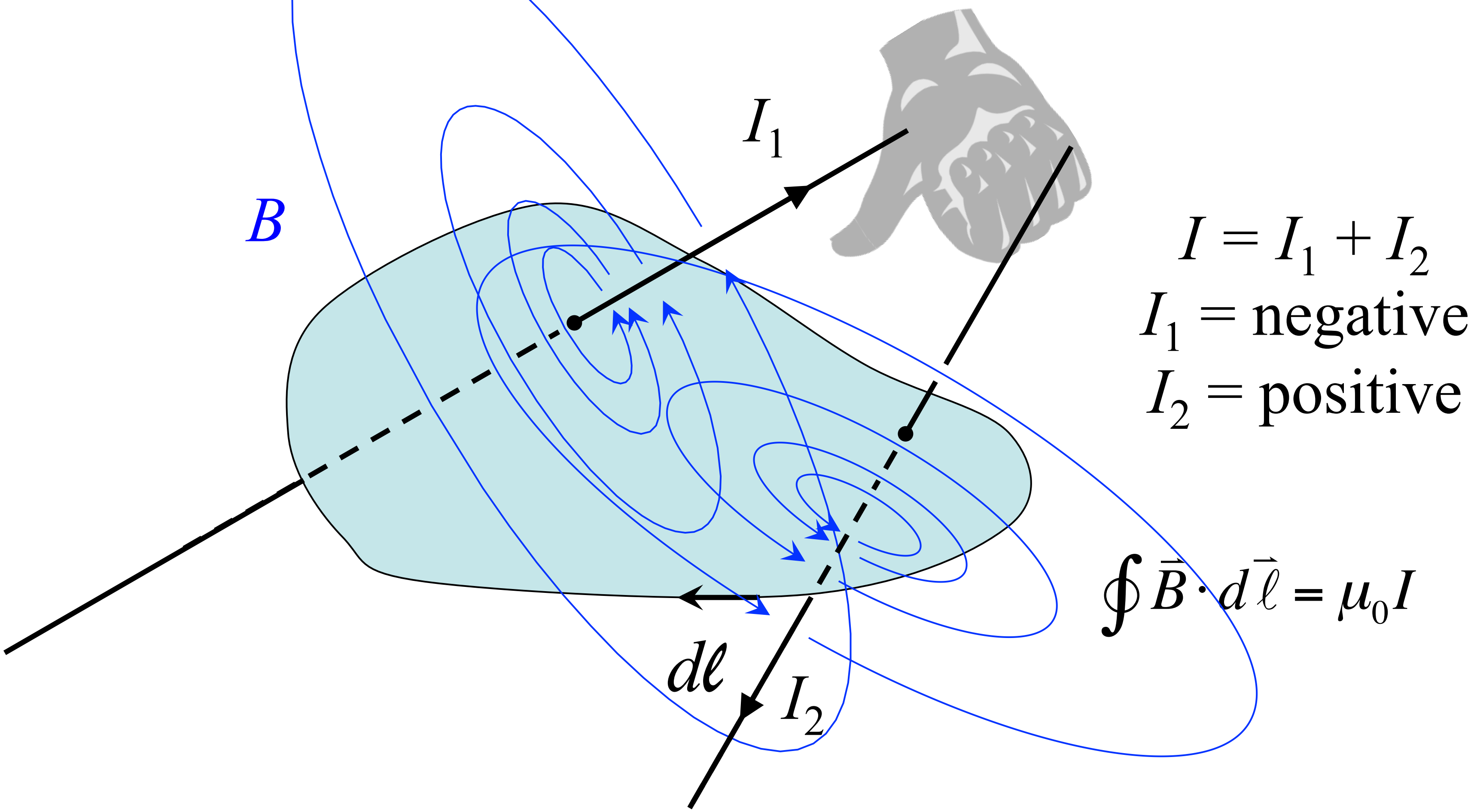
The light blue surface is "pierced" by the current *I*. In Ampere's Law, *dℓ* is an incremental length of a path along the perimeter of the surface. Choose a direction for *dℓ*, curl fingers of right hand in that direction, thumb points in direction of positive current "enclosed" by the path.



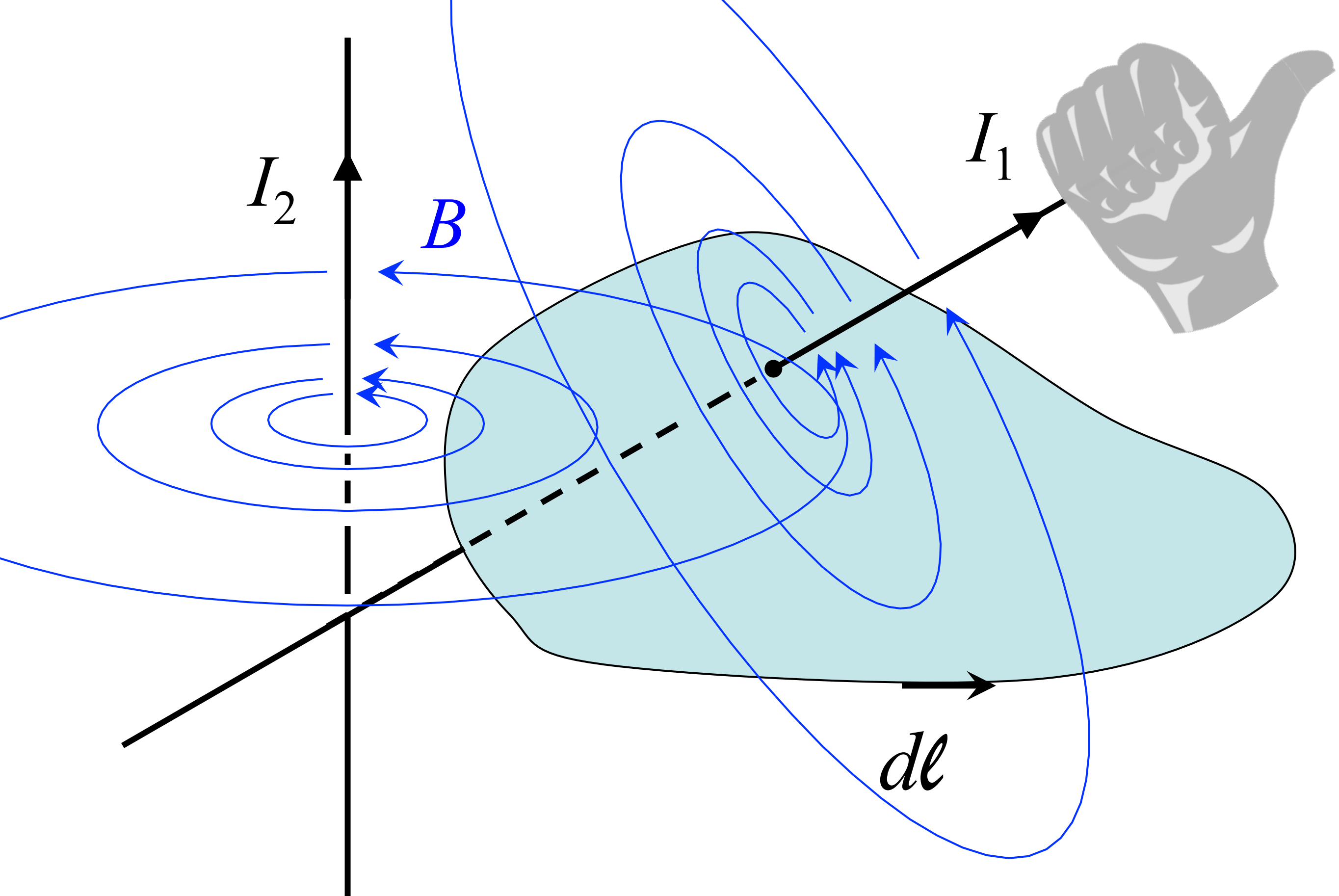
I would be negative by RHR

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I$$

Exact same situation as before but now take *dℓ* to point oppositely. This time the right hand rule shows the current to be negative for Ampere's Law. The surface can be real or imaginary. Only currents passing through this surface are included.



If there is more than one current passing through the "Amperian" surface then the "*I*" in Ampere's Law is the summation of all current, some of which might be negative as in this case.



$I = I_1$

I_1 = positive

I_2 = excluded

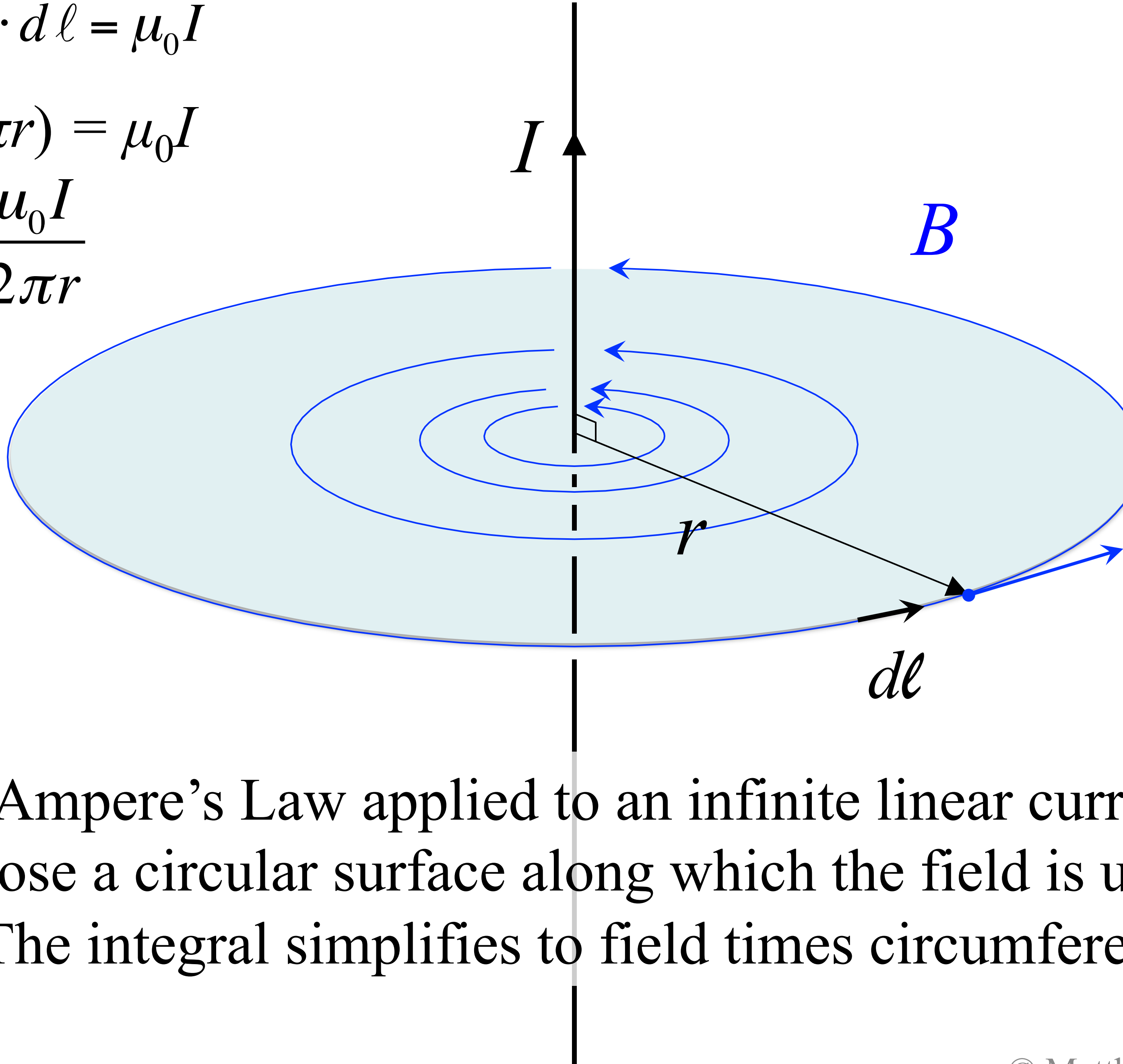
$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I$$

Here there is a current, I_2, which comes near but does not pass through the *open* surface. In that sense it is not "contained" and should not be included in Ampere's Law. This is similar to whether or not charge is located inside a *closed* surface when using Gauss's Law for electric fields.

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I$$

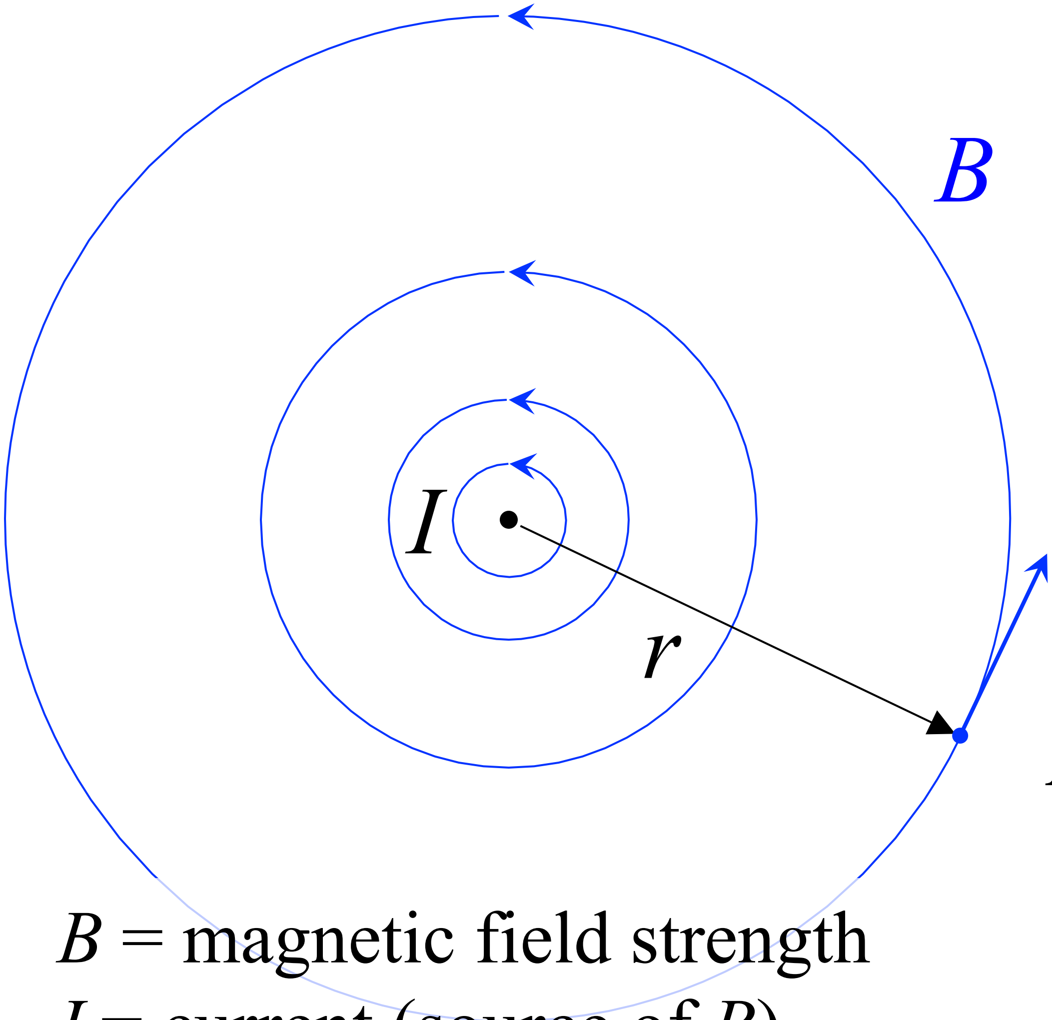
$$B(2\pi r) = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}$$



Ampere's Law applied to an infinite linear current:
Choose a circular surface along which the field is uniform.
The integral simplifies to field times circumference.

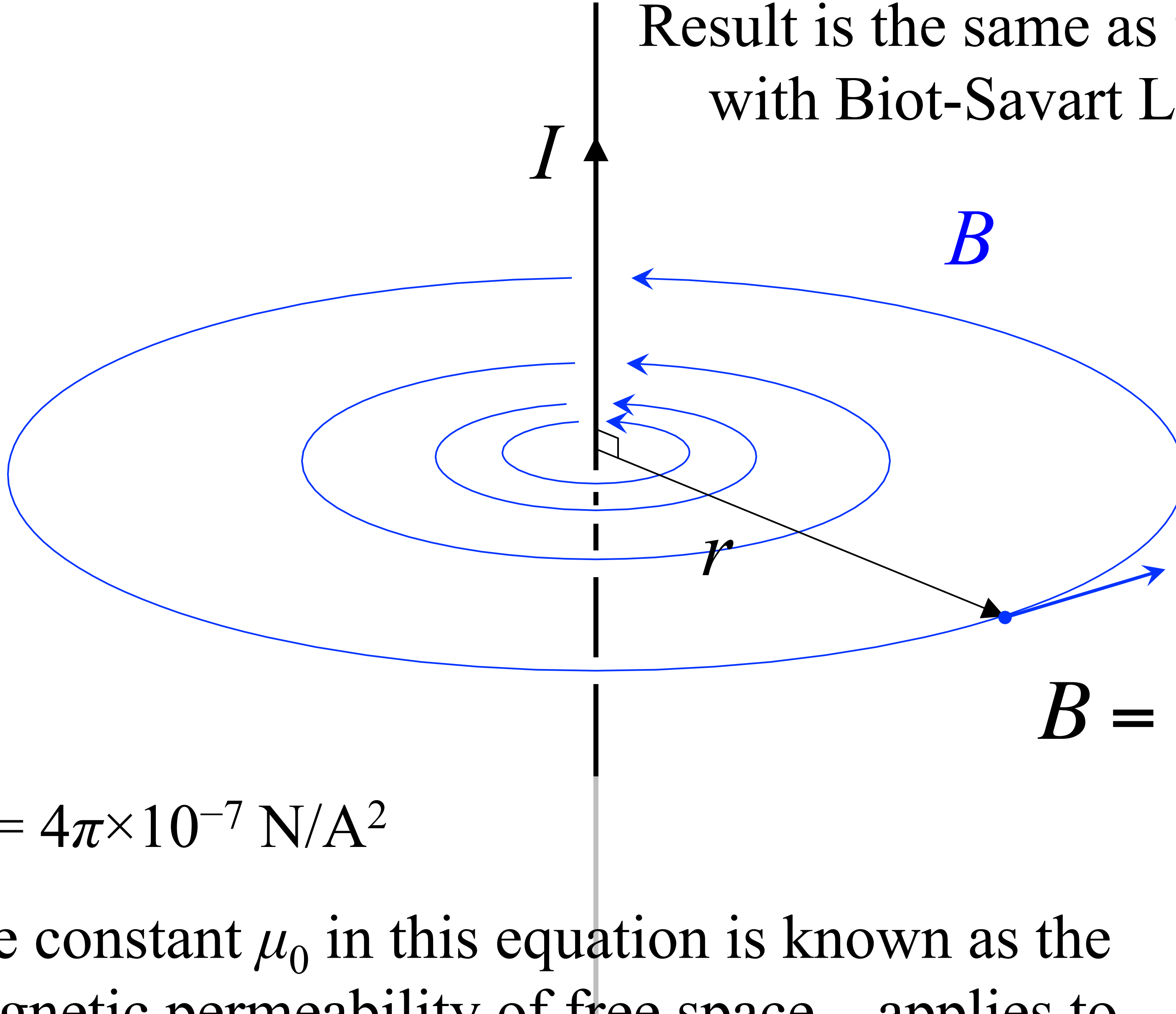
Magnetic Field of "infinite" Linear Current



$$B = \frac{\mu_0 I}{2\pi r}$$

B = magnetic field strength
I = current (source of B)
r = perpendicular distance from I
μ_0 = permeability constant

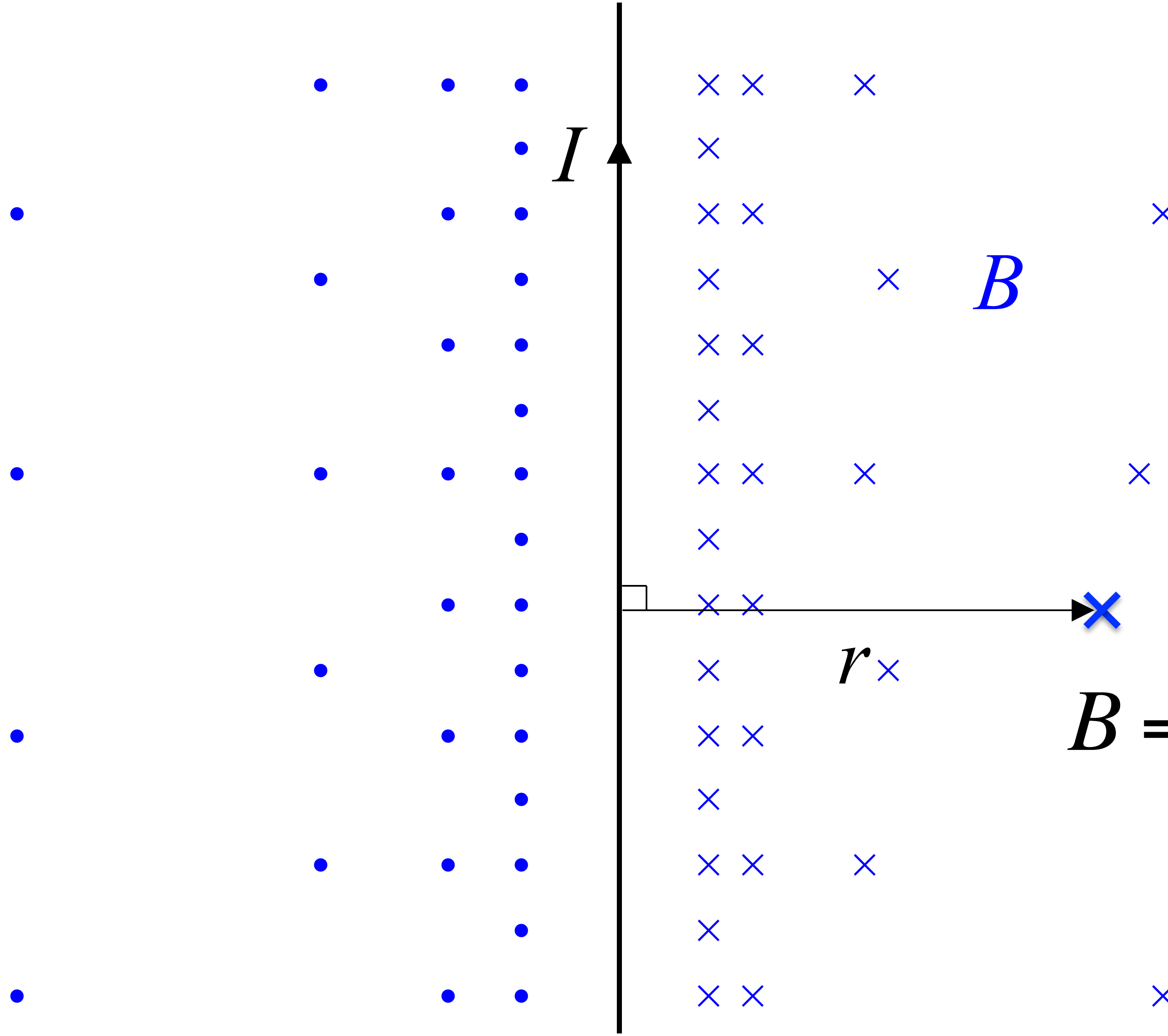
Result is the same as found with Biot-Savart Law



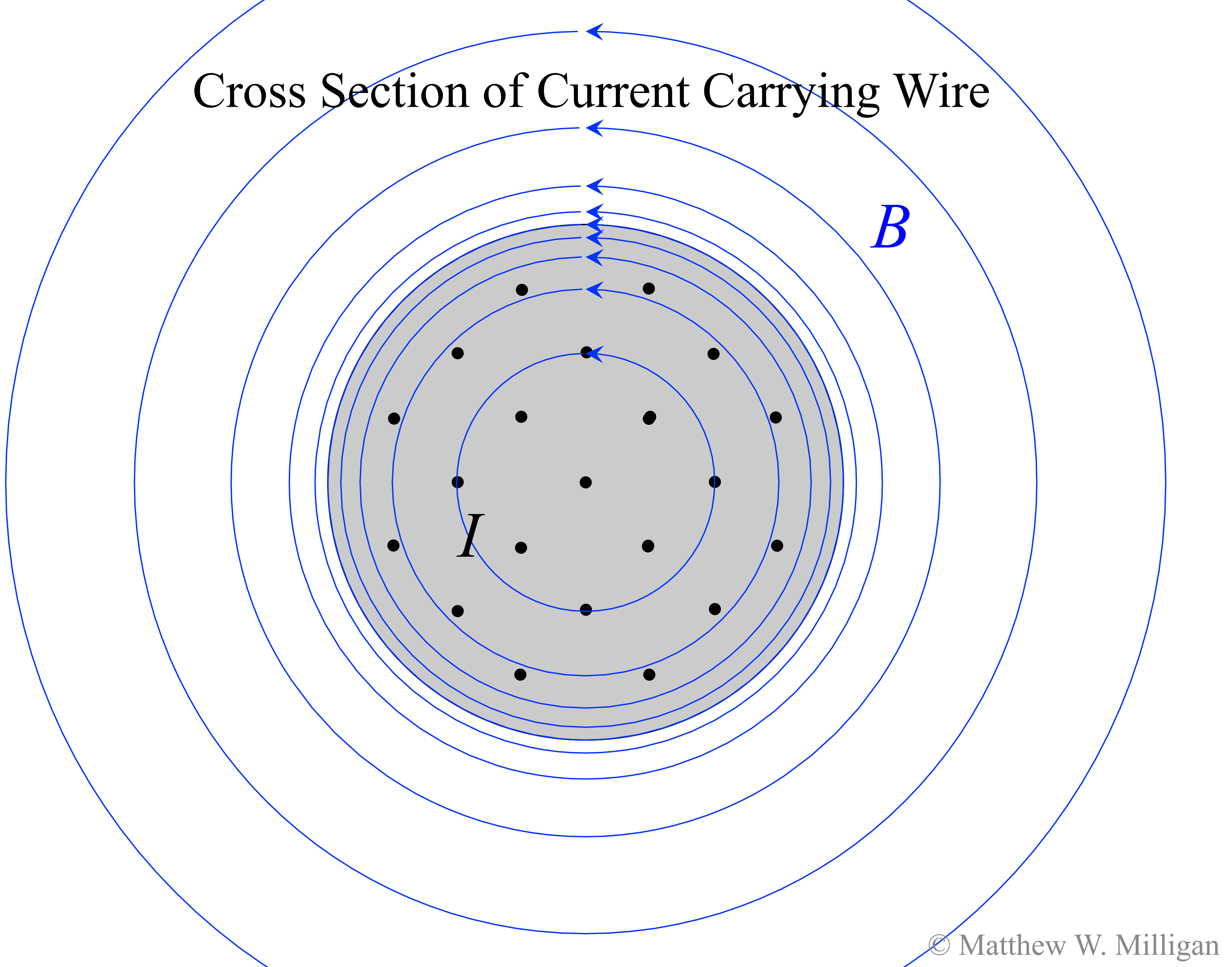
$$B = \frac{\mu_0 I}{2\pi r}$$

$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{N/A^2}$

The constant μ_0 in this equation is known as the magnetic permeability of free space – applies to a vacuum or essentially the same for air.



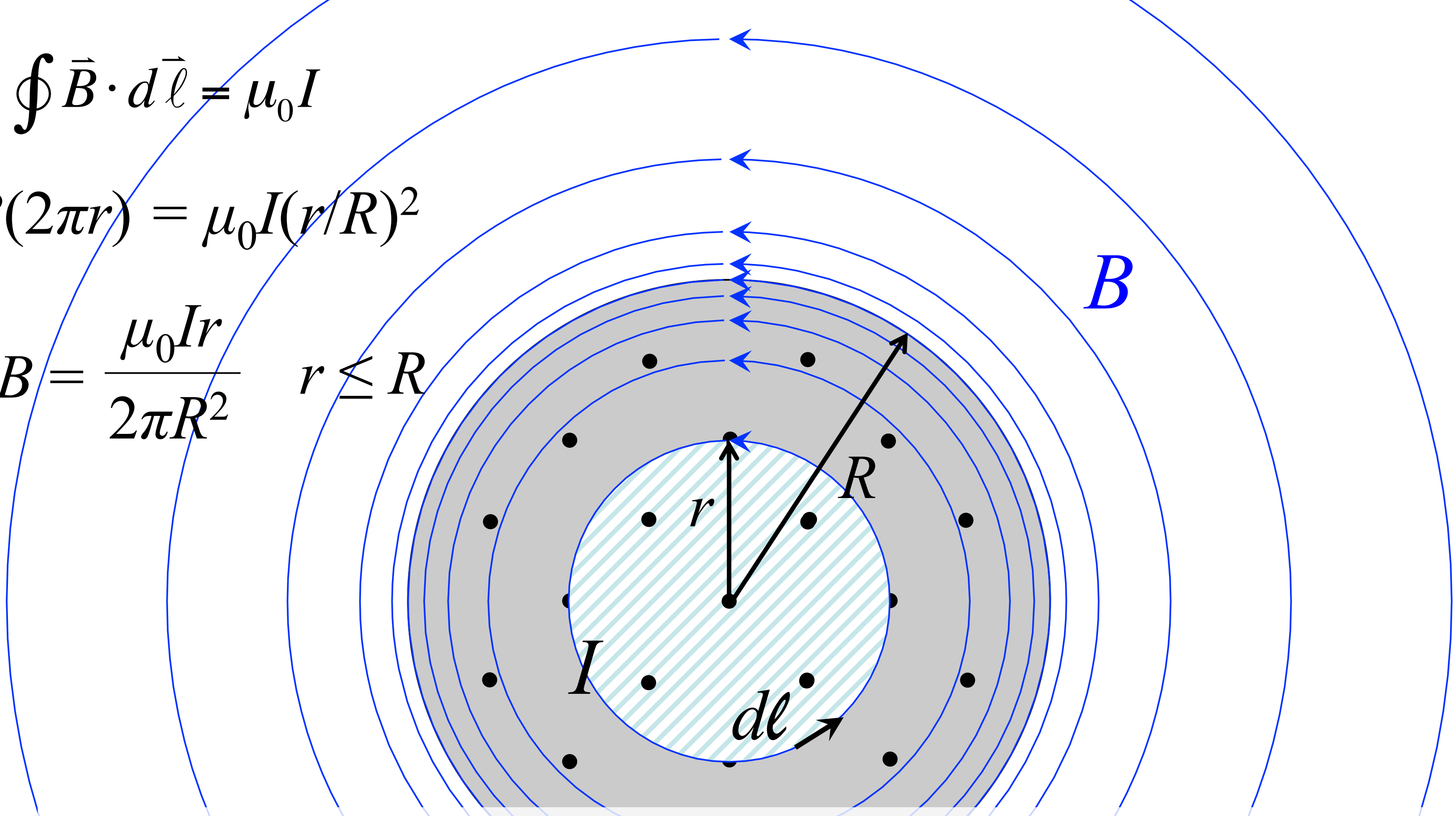
$$B = \frac{\mu_0 I}{2\pi r}$$


Cross Section of Current Carrying Wire
B
I
© Matthew W. Milligan

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I$$

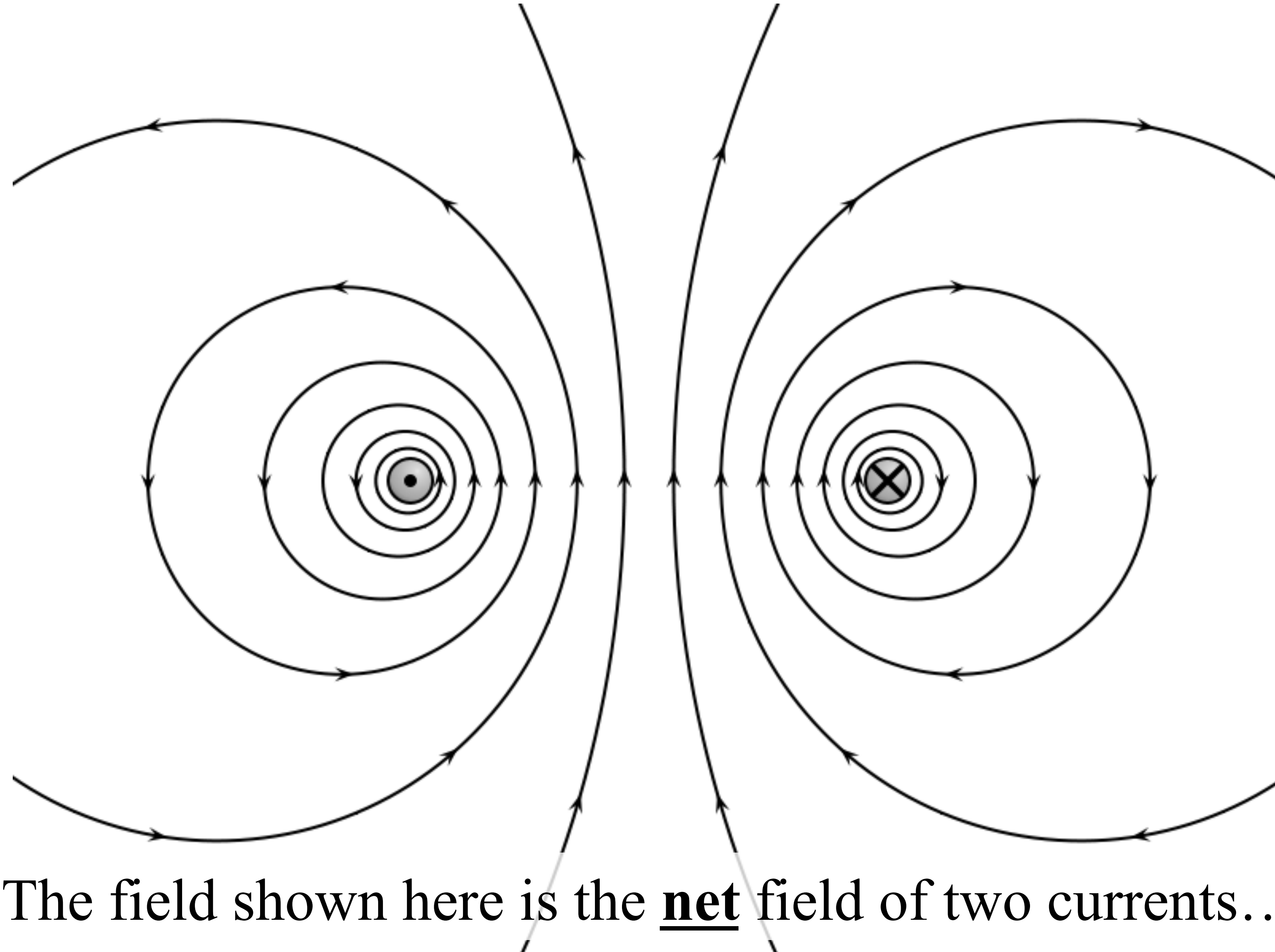
$$B(2\pi r) = \mu_0 I (r/R)^2$$

$$B = \frac{\mu_0 I r}{2\pi R^2} \quad r \leq R$$



Ampere's Law is here applied to points *inside* a current carrying wire. Notice that only part of the current is within the perimeter of the chosen surface. This solution assumes uniform current density.

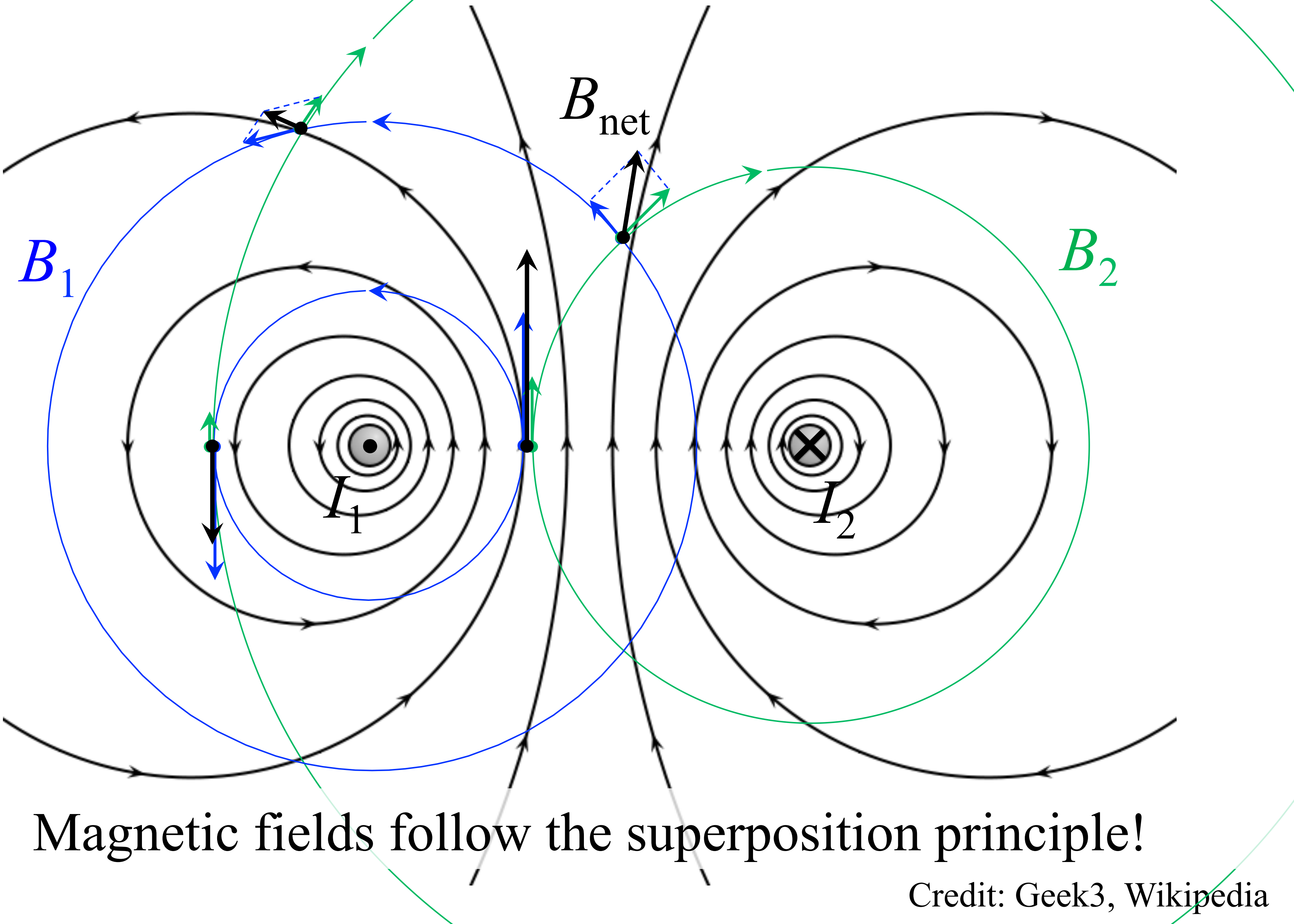
Magnetic Field – Antiparallel Currents



The field shown here is the **net** field of two currents…

Credit: Geek3, Wikipedia

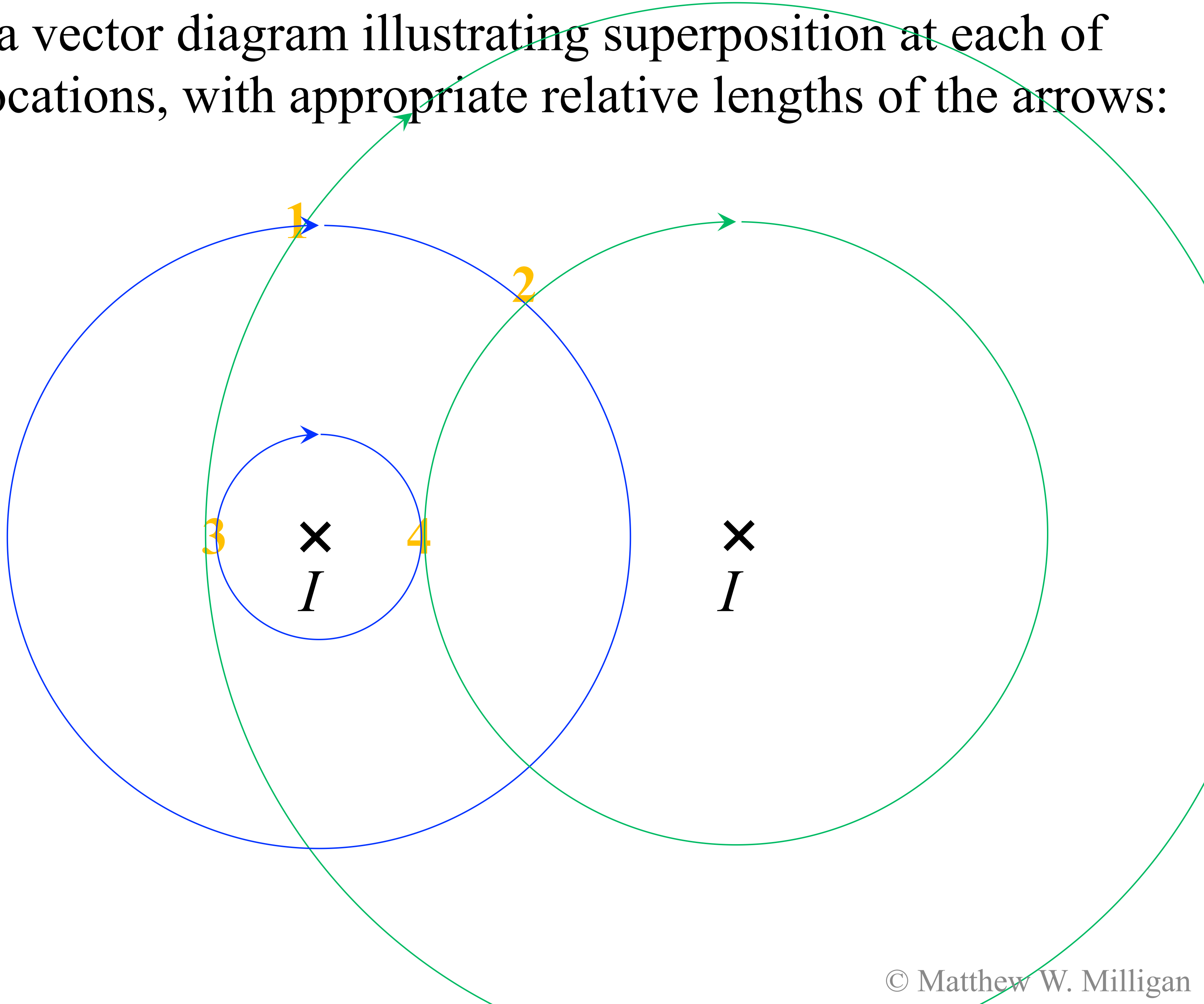
Magnetic Field – Antiparallel Currents

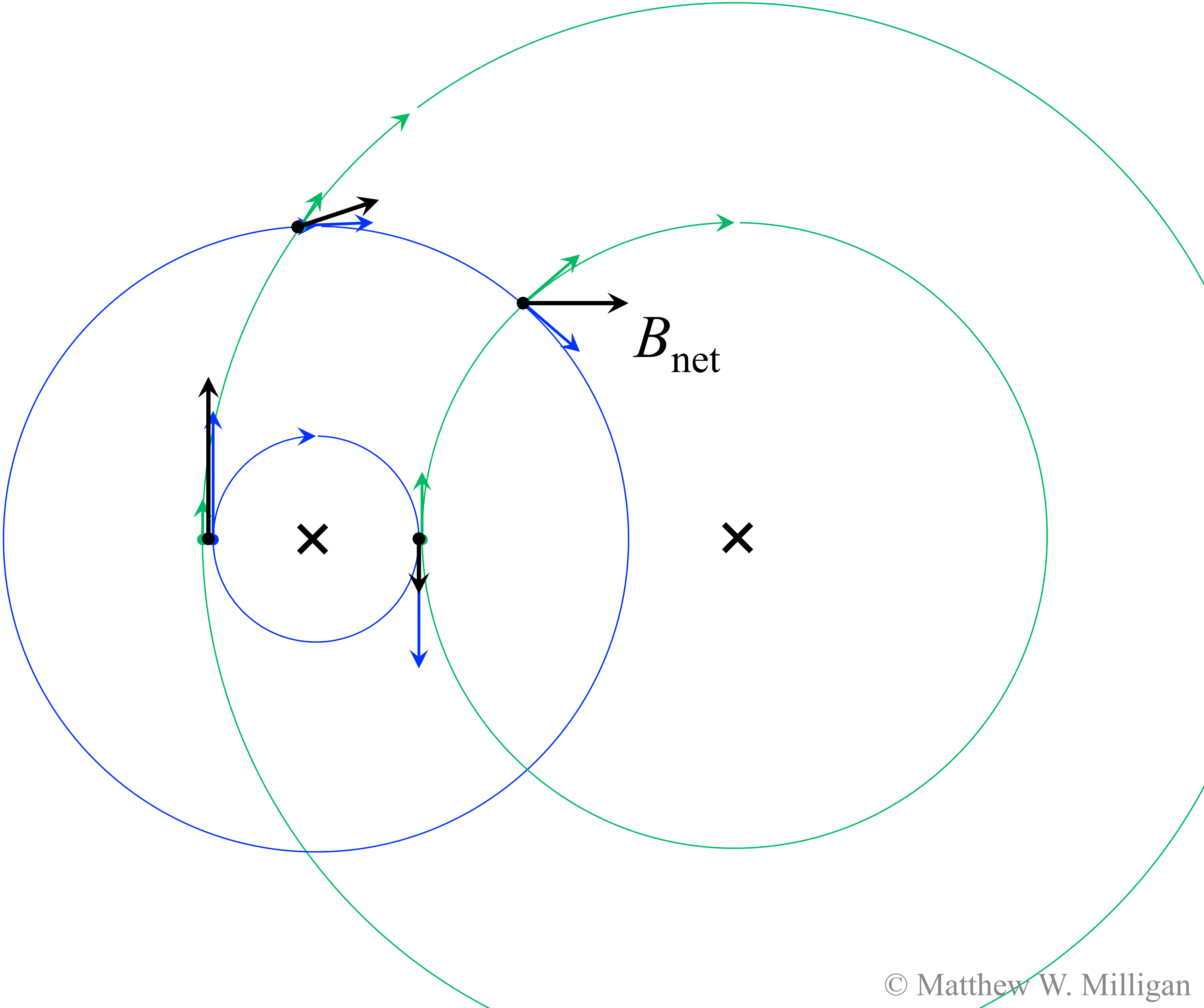


Magnetic fields follow the superposition principle!

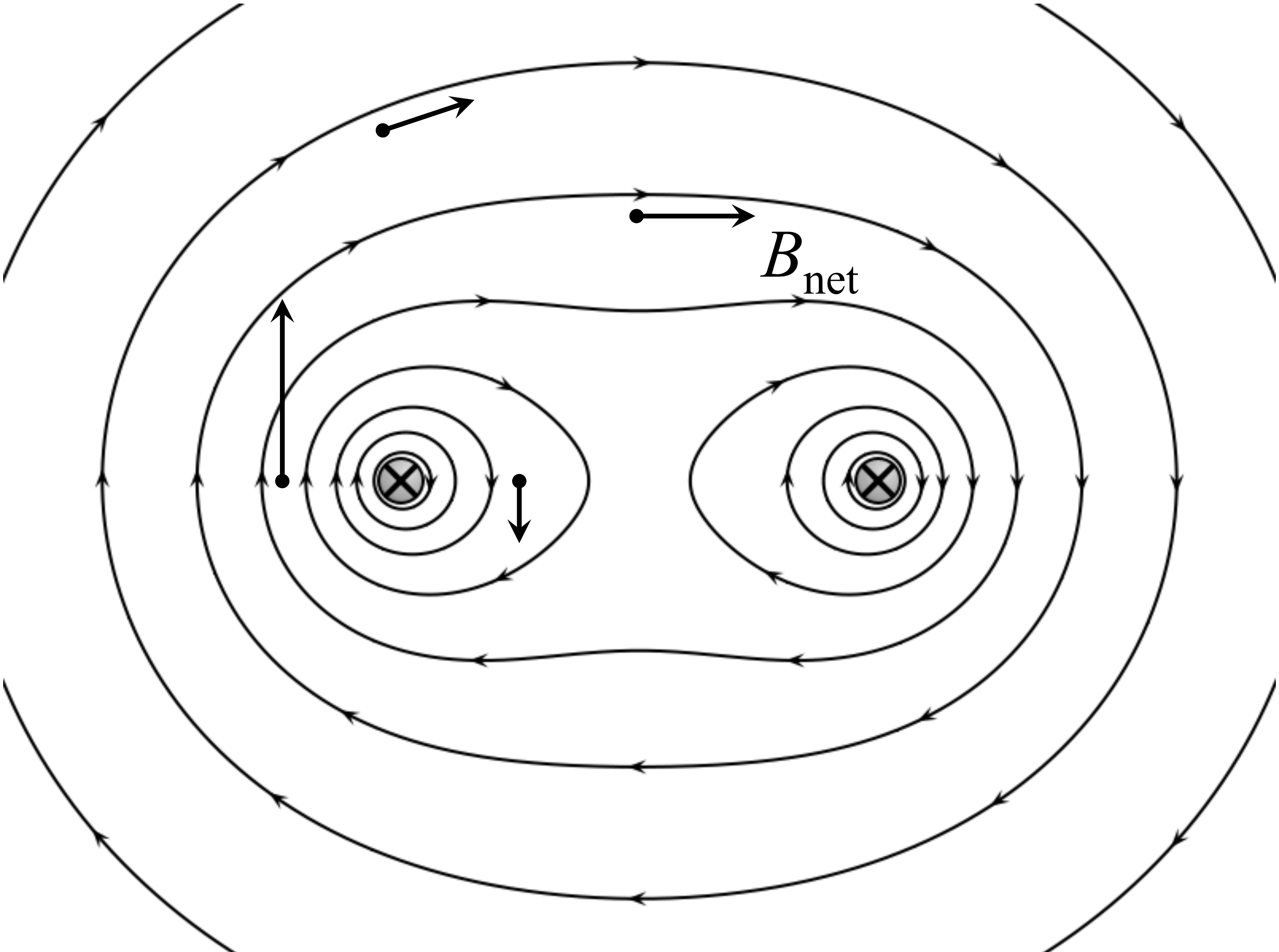
Credit: Geek3, Wikipedia

Draw a vector diagram illustrating superposition at each of four locations, with appropriate relative lengths of the arrows:



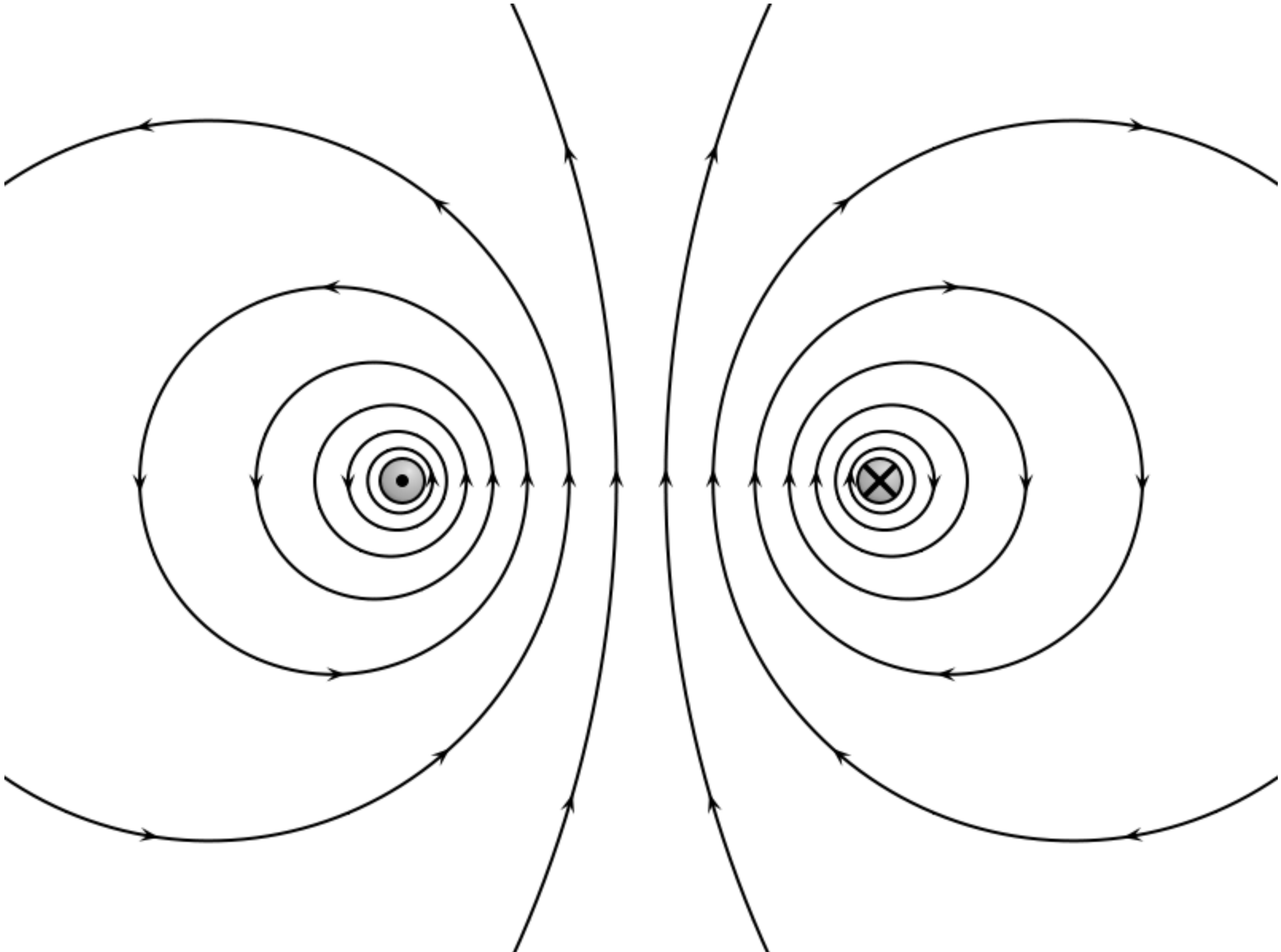

B_{net}

Magnetic Field – Parallel Currents



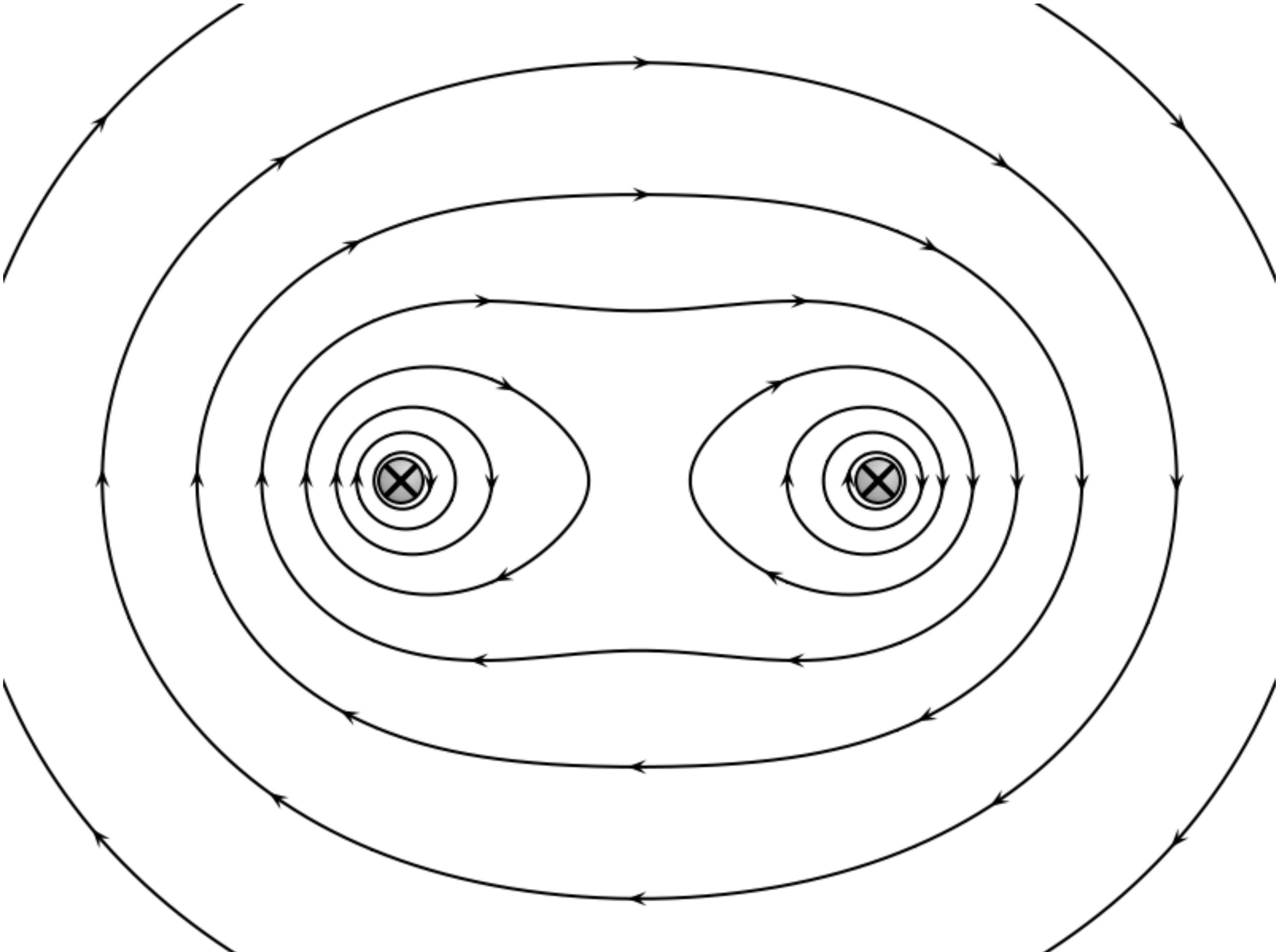
Credit: Geek3, Wikipedia

Magnetic Field – Antiparallel Currents



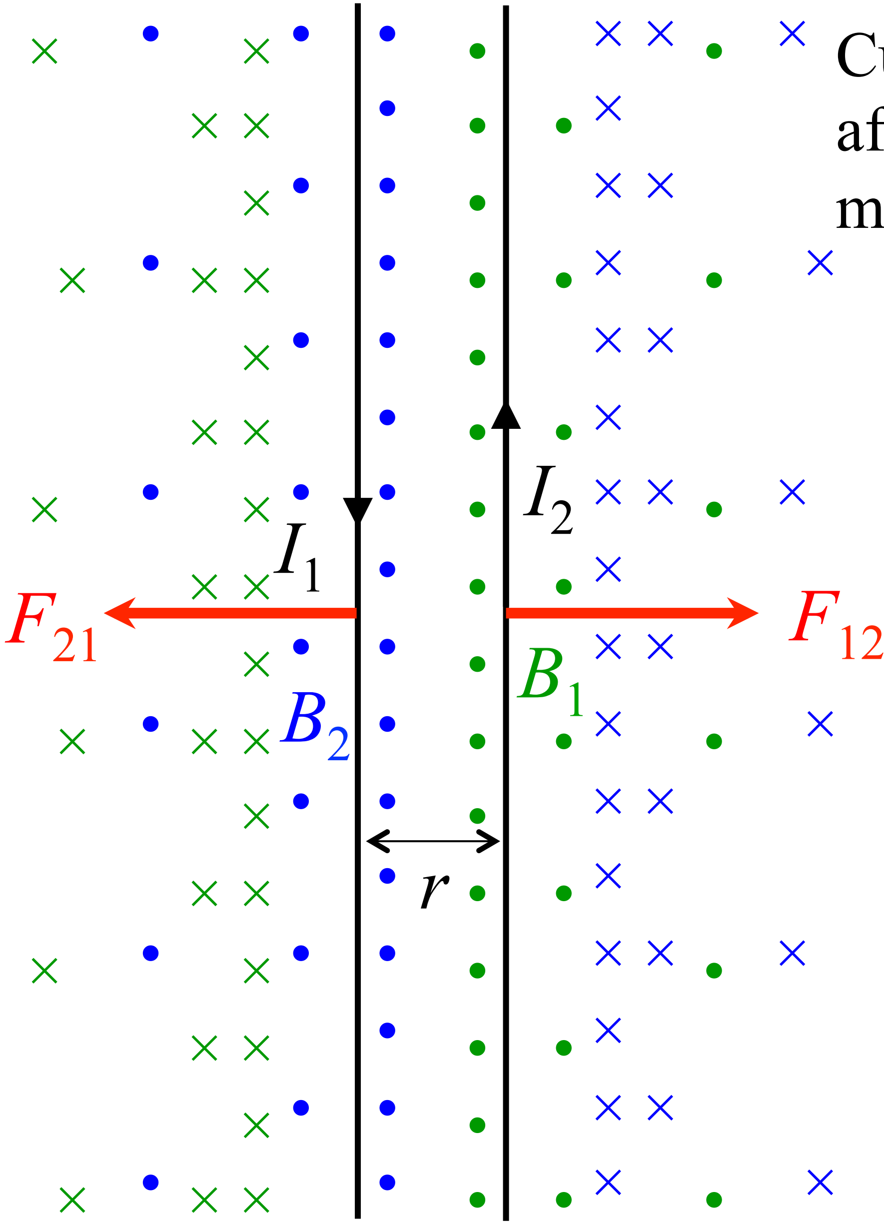
Credit: Geek3, Wikipedia

Magnetic Field – Parallel Currents



Credit: Geek3, Wikipedia

Parallel Current Carrying Wires



Current 1 creates a magnetic field that affects current 2. Current 2 creates a magnetic field that affects current 1:

$$B_1 = \frac{\mu_0 I_1}{2\pi r} \qquad B_2 = \frac{\mu_0 I_2}{2\pi r}$$

$$F_{12} = I_2 L B_1 \qquad F_{21} = I_1 L B_2$$

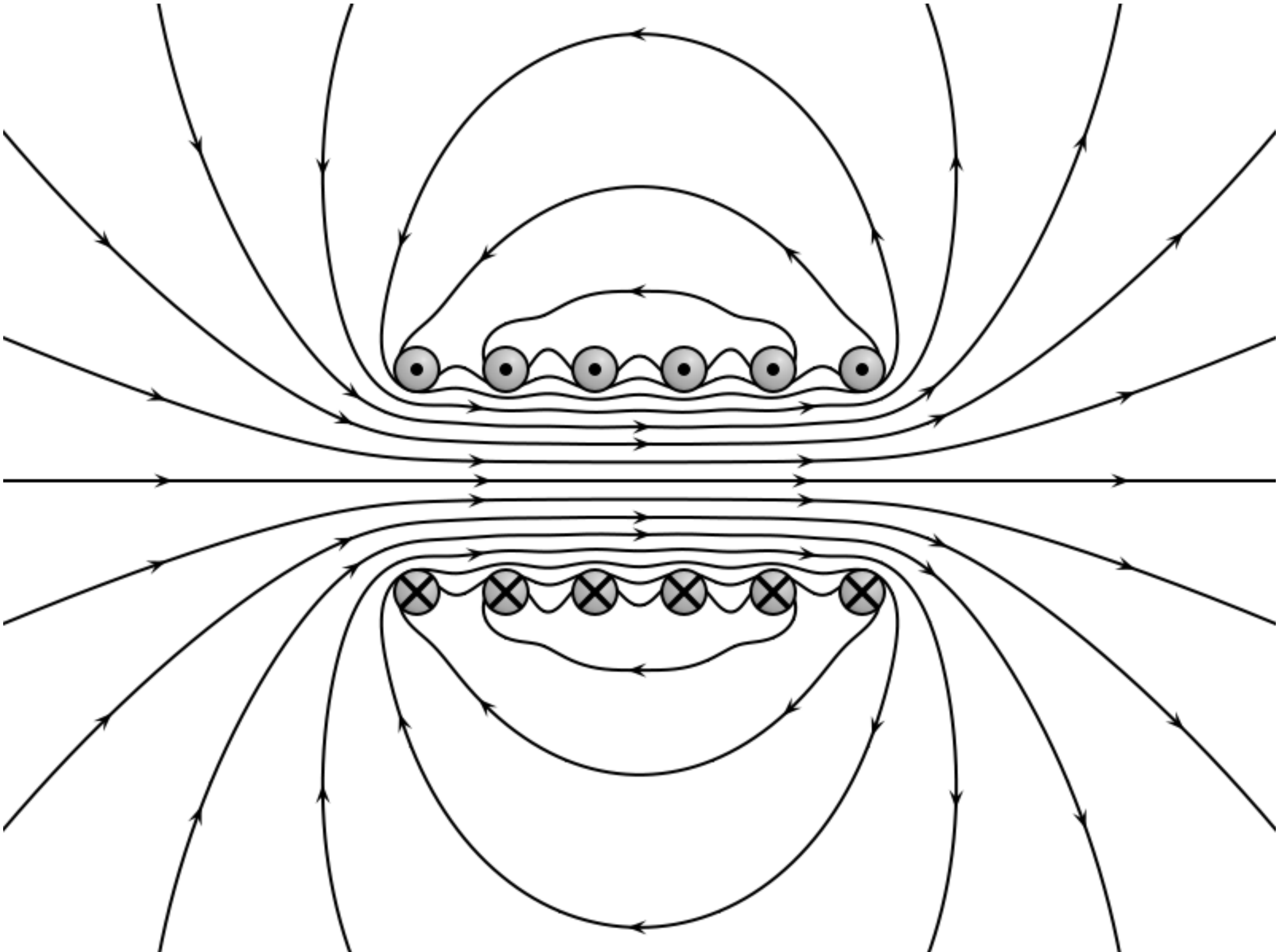
$$F_{12} = I_2 L \frac{\mu_0 I_1}{2\pi r} \qquad F_{21} = I_1 L \frac{\mu_0 I_2}{2\pi r}$$

$$\frac{F_{12}}{L} = \frac{\mu_0 I_1 I_2}{2\pi r} \qquad \frac{F_{21}}{L} = \frac{\mu_0 I_1 I_2}{2\pi r}$$

$$\frac{F}{L} = \frac{\mu_0 I_1 I_2}{2\pi r}$$

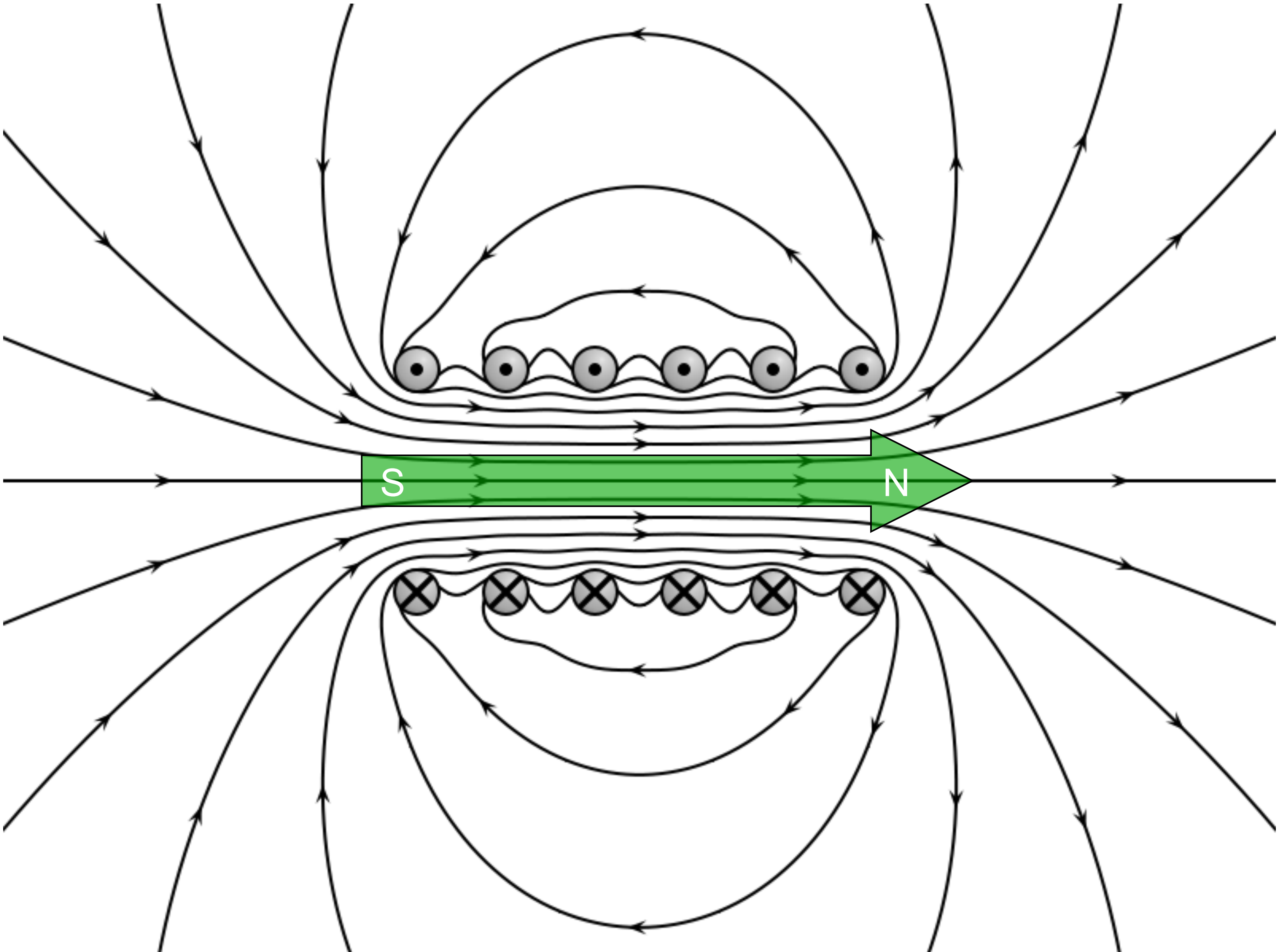
Force per length, parallel wires
Opposite currents repel,
like currents attract!

Magnetic Field – Solenoid



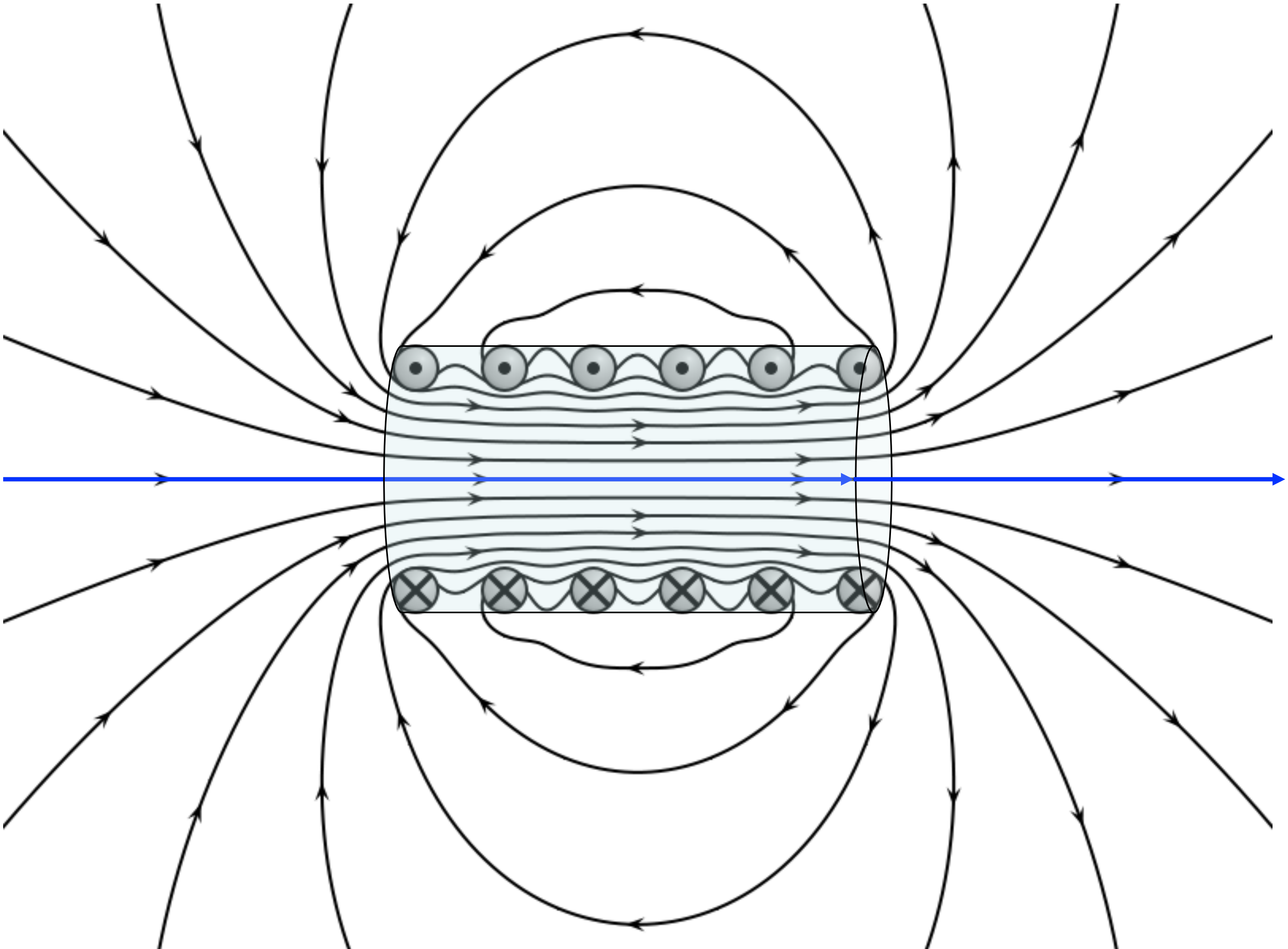
Credit: Geek3, Wikipedia

Magnetic Field – Solenoid



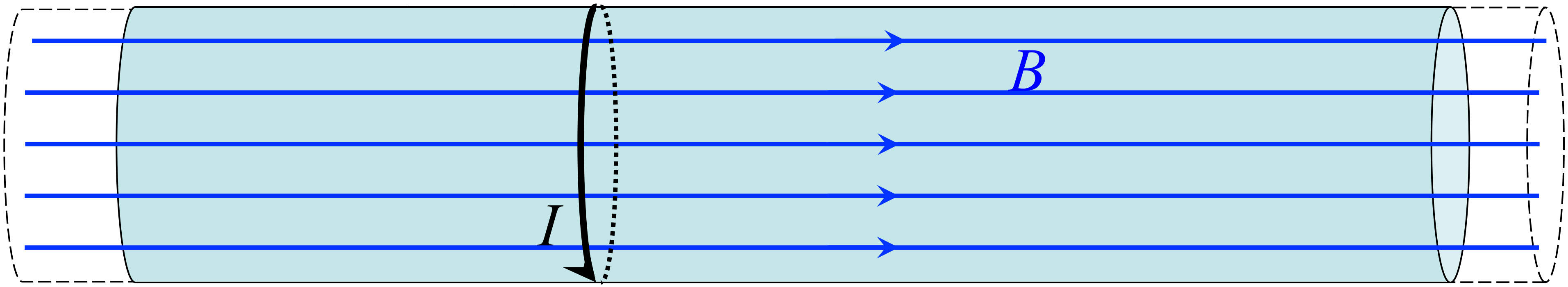
Credit: Geek3, Wikipedia

Magnetic Field – Solenoid



Credit: Geek3, Wikipedia

Magnetic Field – Ideal "Infinite" Solenoid



$$B = \mu_0 n I$$

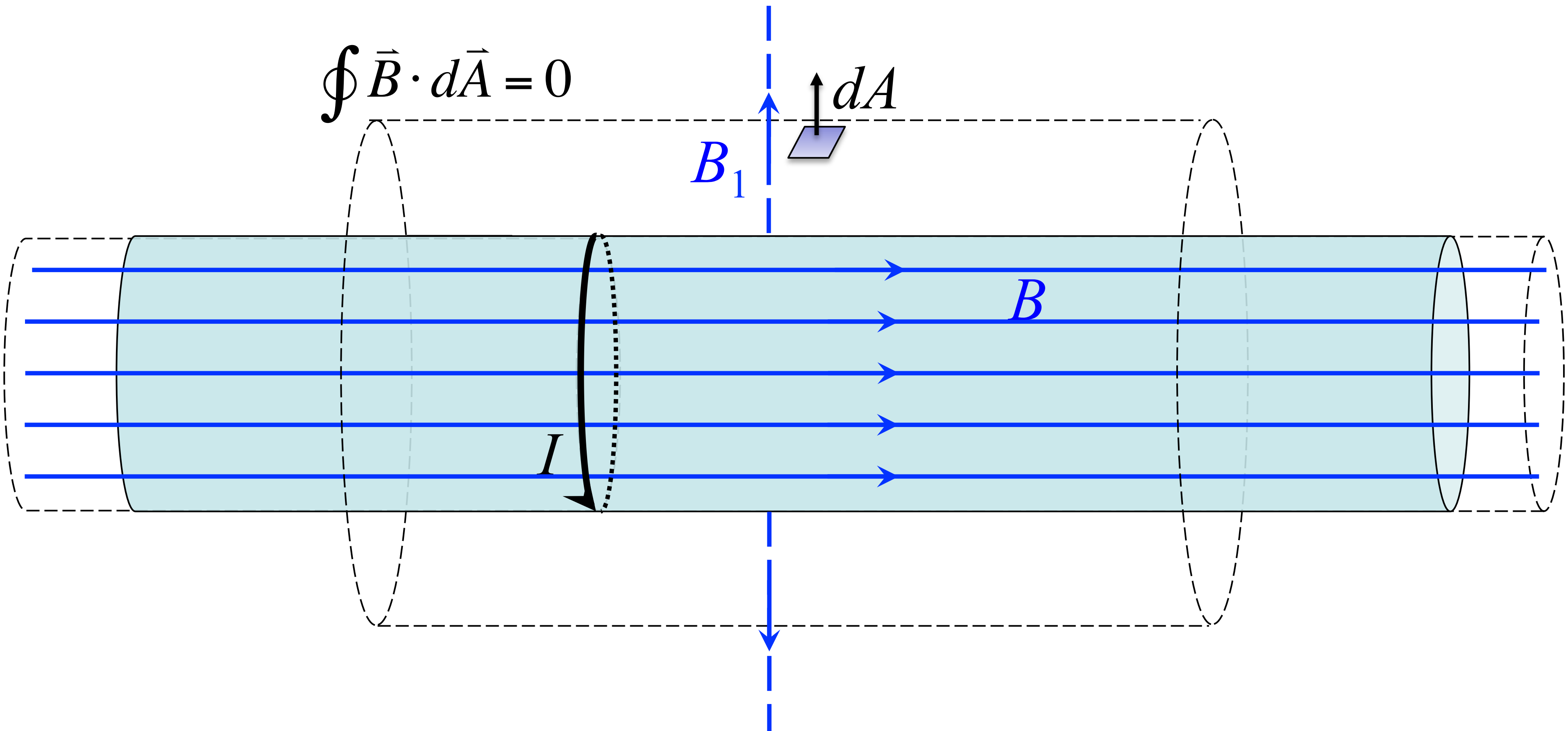
or

$$B = \frac{\mu_0 N I}{L}$$

B = field *anywhere* inside (uniform)
n = number of turns per length
N = number of turns (or coils) of wire
L = length of cylinder
I = current

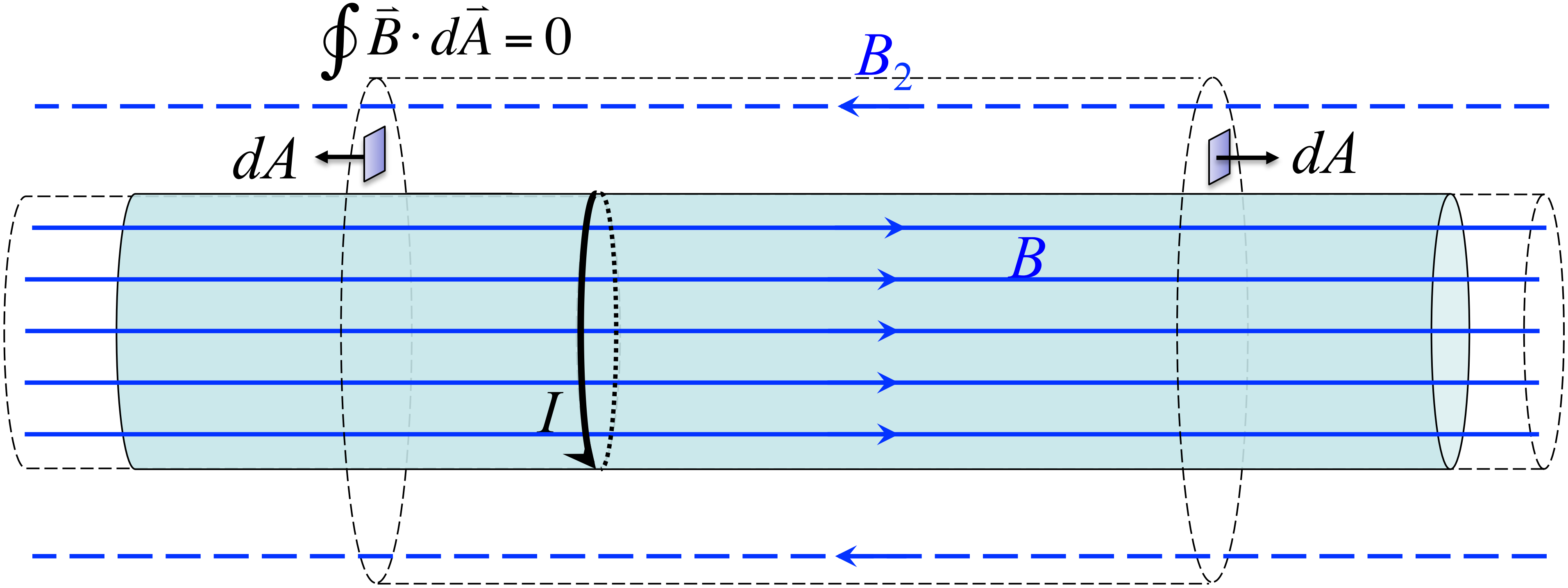
Ideal "Infinite" Solenoid – Derivation of *B*

Suppose there were a field component B_1 pointing radially away from the solenoid. The surrounding cylindrical cylinder would have a magnetic flux if that were the case – a violation of Gauss's Law for magnetic fields. Therefore B_1 cannot exist and must equal zero.



Ideal "Infinite" Solenoid – Derivation of B

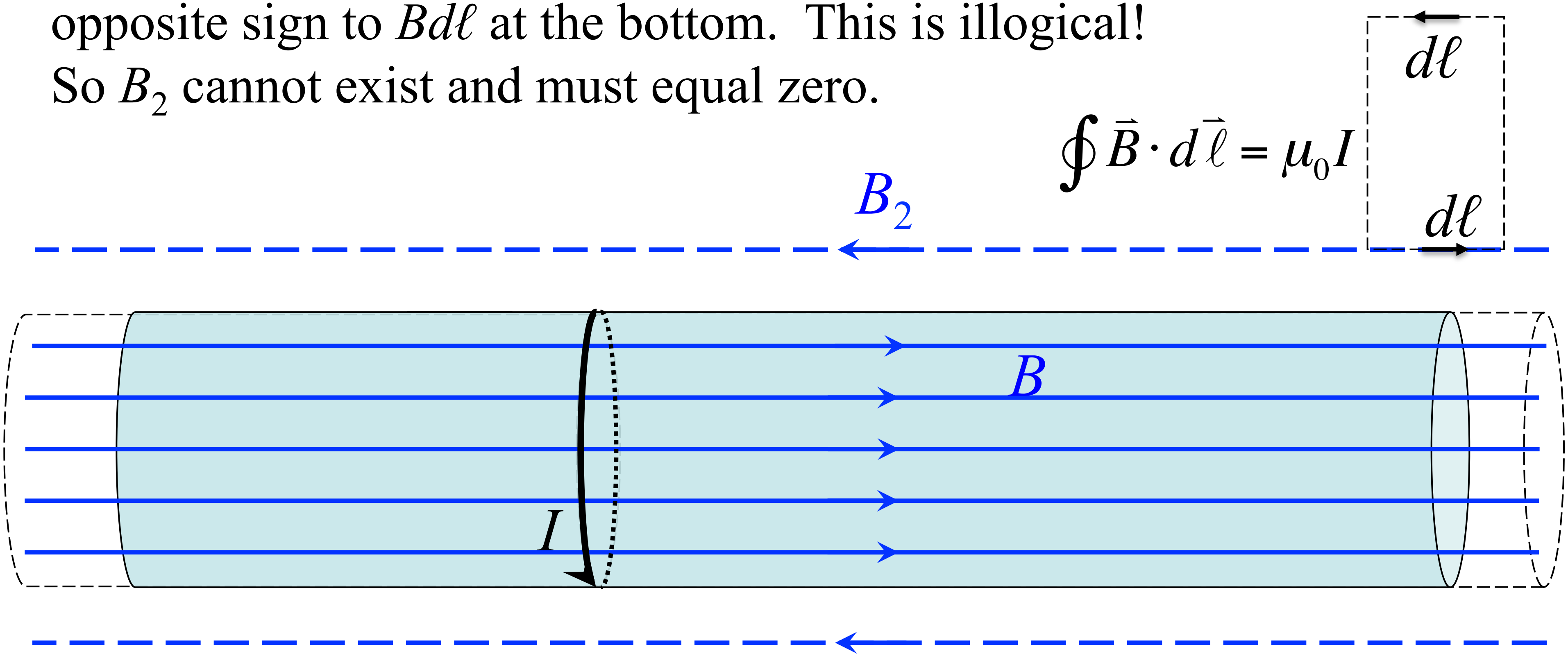
Suppose there were a field component B_2 outside the solenoid, parallel to its axis. This would *not* be a violation of Gauss's Law for magnetic fields because the net magnetic flux would be zero. Notice the same can be said for the field inside the solenoid.



Ideal "Infinite" Solenoid – Derivation of B

However, the field B_2 is inconsistent with Ampere's Law. Applied to the rectangle shown, the current I is zero. Only if B_2 is uniform "to infinity" would $Bd\ell$ at the top of any such rectangle be equal and opposite sign to $Bd\ell$ at the bottom. This is illogical!
So B_2 cannot exist and must equal zero.

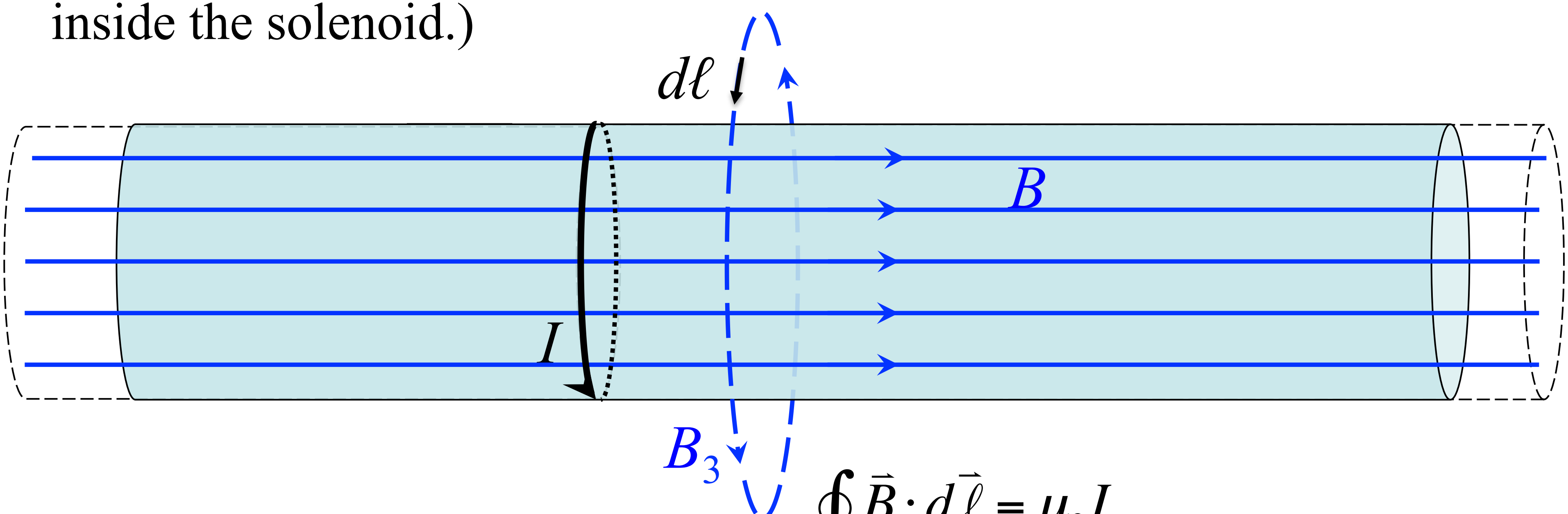
$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I$$



(The field *inside* a *real* solenoid must loop around on the *outside* and B_2 in fact exists, though it is negligible compared to the field inside.)

Ideal "Infinite" Solenoid – Derivation of B

Suppose there were a field component B_3 in a concentric circle surrounding the solenoid. If the current is idealized as a "sheet" there is zero current passing *through* the open surface that is bounded by B_3. In that case B_3 must be zero to satisfy Ampere's Law. (The coil of wire in a real solenoid will pass through this surface and B_3 may in fact be nonzero – but still negligible, especially compared to the field inside the solenoid.)

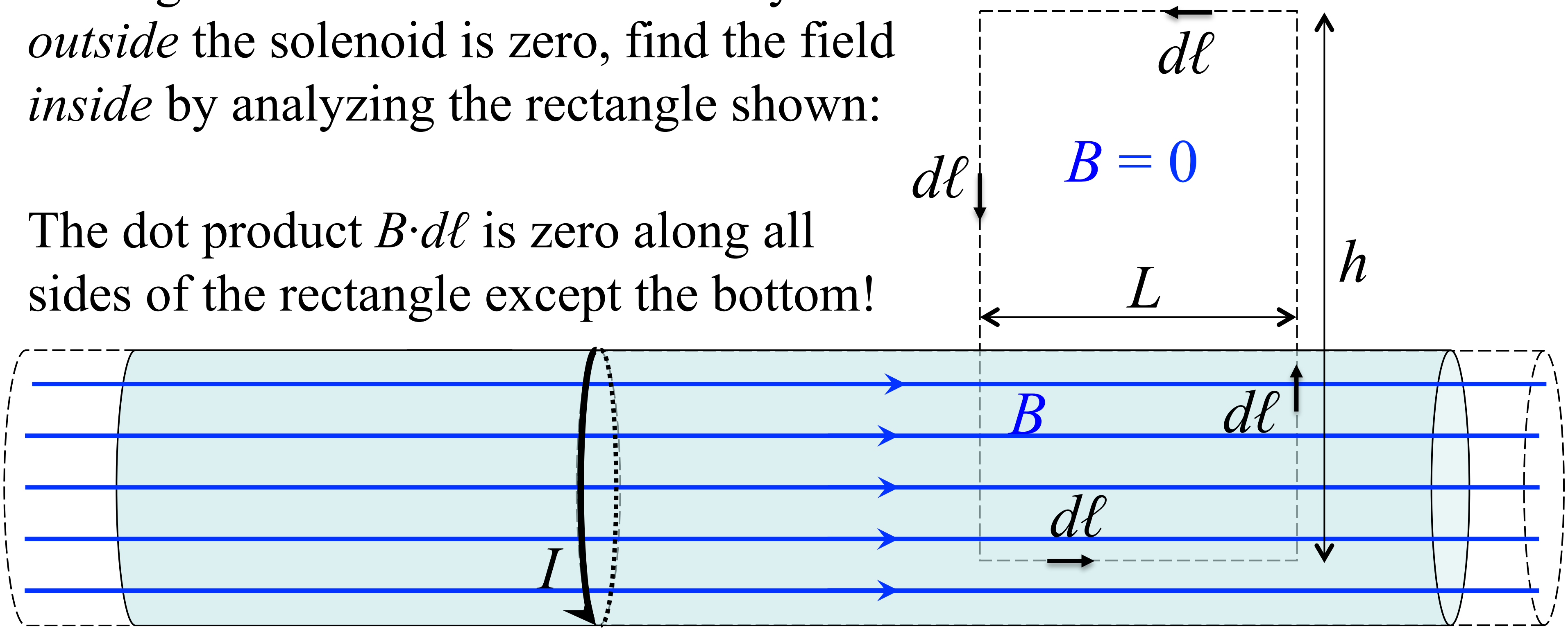


$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I$$

Ideal "Infinite" Solenoid – Derivation of B

Having established that the field anywhere *outside* the solenoid is zero, find the field *inside* by analyzing the rectangle shown:

The dot product $B \cdot d\ell$ is zero along all sides of the rectangle except the bottom!



Note that the bottom of the rectangle could be *anywhere* inside the solenoid and the analysis would be the same. Therefore the magnetic field is uniform throughout its interior.

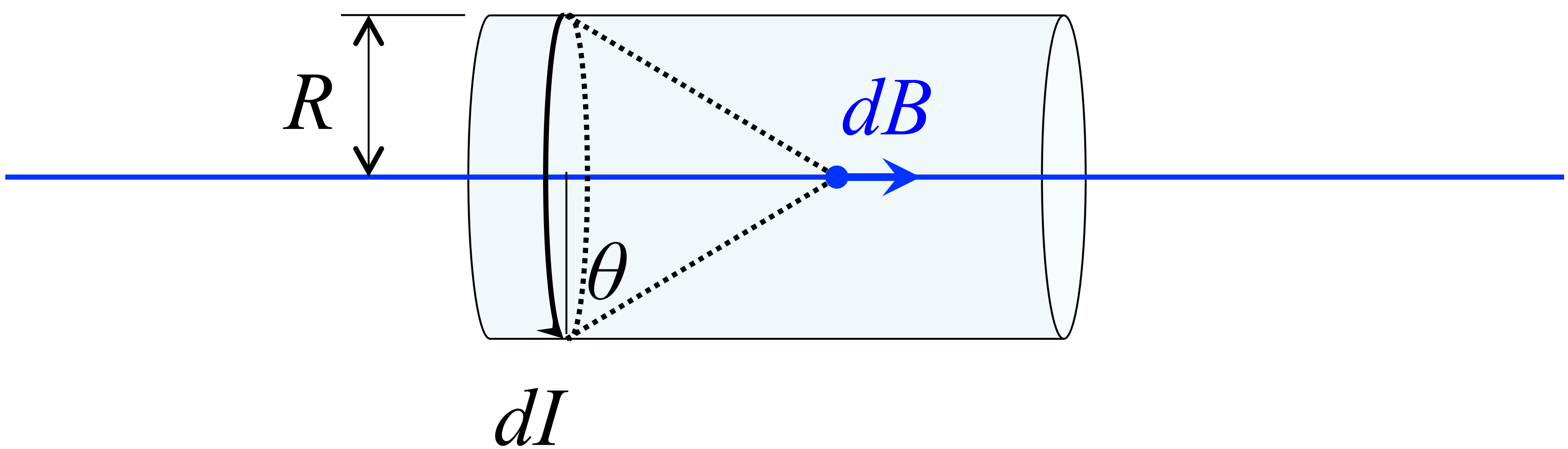
$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I$$

$$BL = \mu_0 (NI)$$

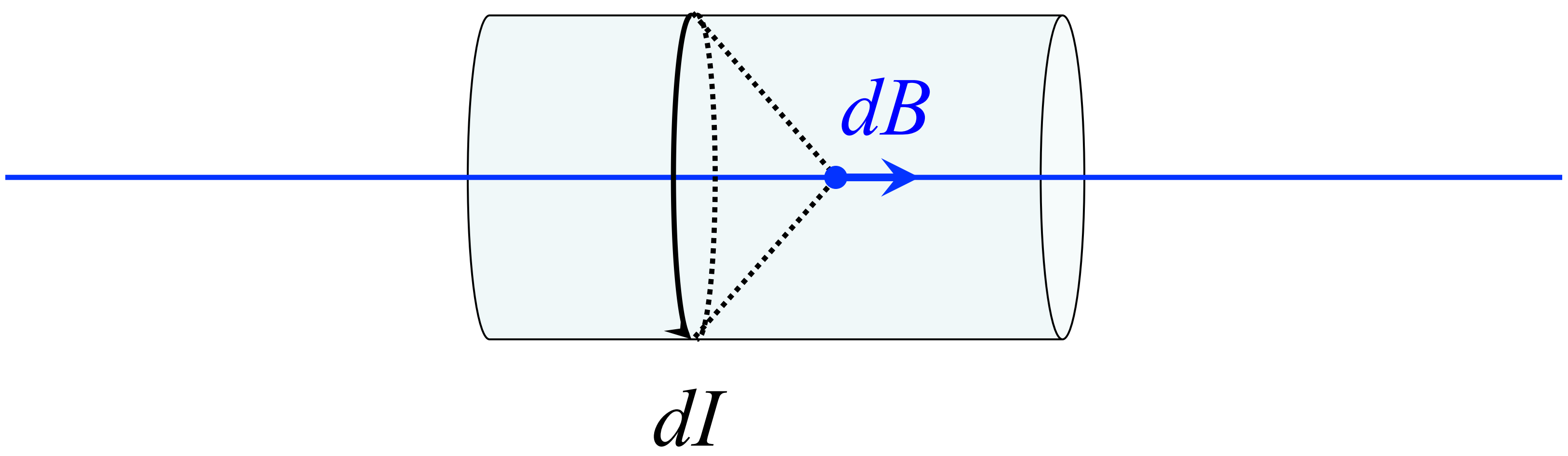
$$B = \frac{\mu_0 NI}{L} = \mu_0 nI$$

Magnetic Field – Finite Solenoid

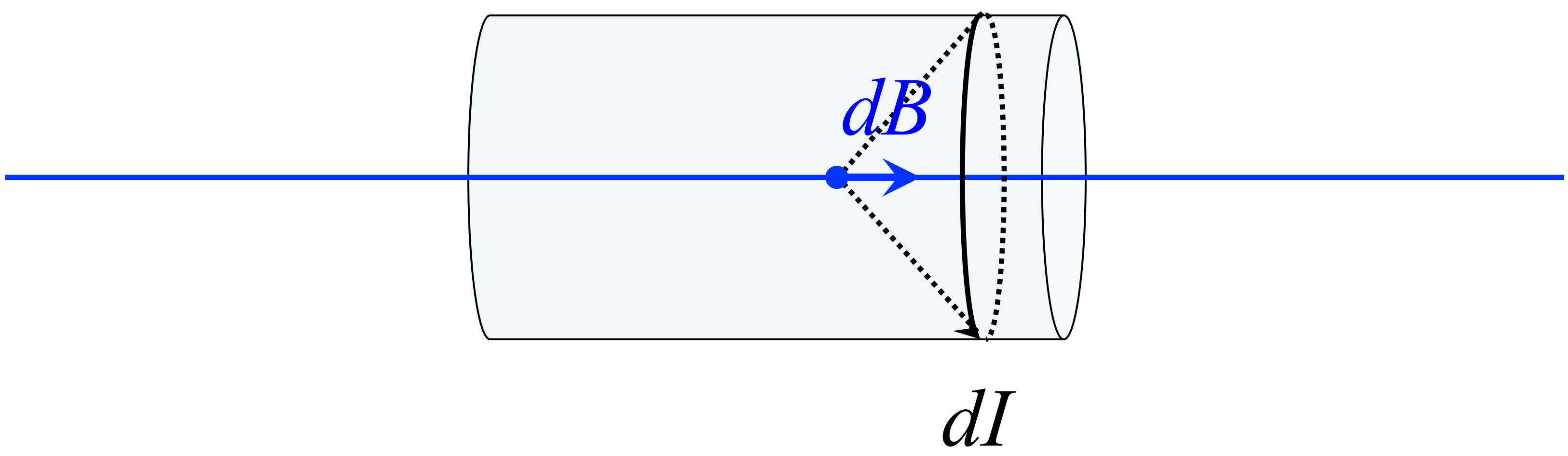
$$dB = \frac{\mu_0 dI \cos^3\theta}{2R} \quad \text{(from HW)}$$



Magnetic Field – Finite Solenoid

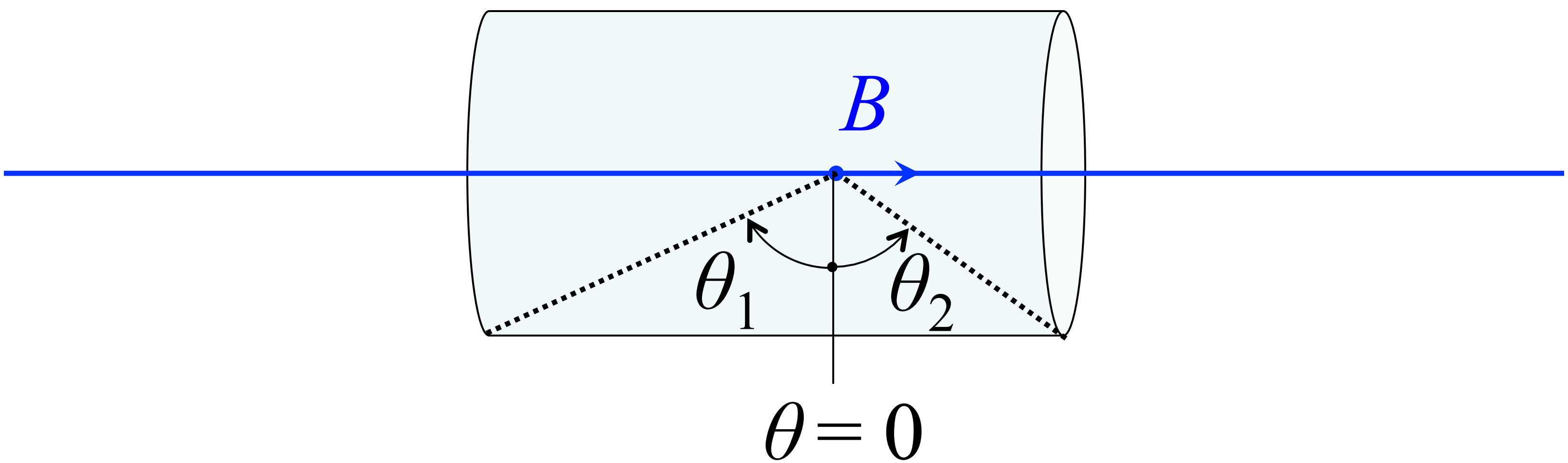


Magnetic Field – Finite Solenoid



Magnetic Field – Finite Solenoid

$$B = \frac{\mu_0 n I}{2}\left(\sin\theta_2 - \sin\theta_1\right)$$

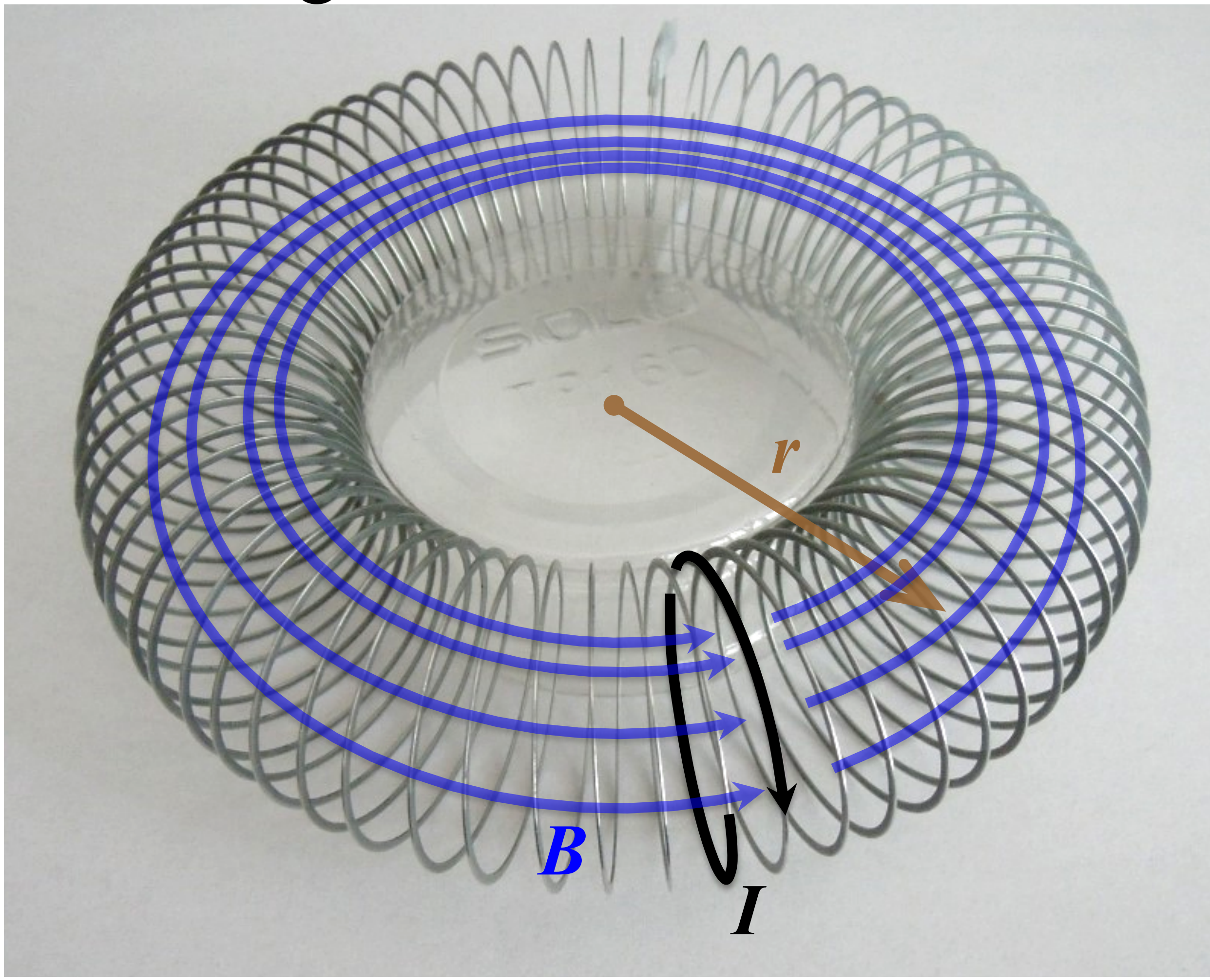


B = field at a point along the axis <u>only</u>

n = number of turns per length

θ = angle shown, may be positive or negative

Magnetic Field – Toroid



A toroid is similar to a solenoid but with a torus shape. However, Ampere's Law reveals the field is not uniform but rather inversely proportional to radius shown above!