Magnetostatics

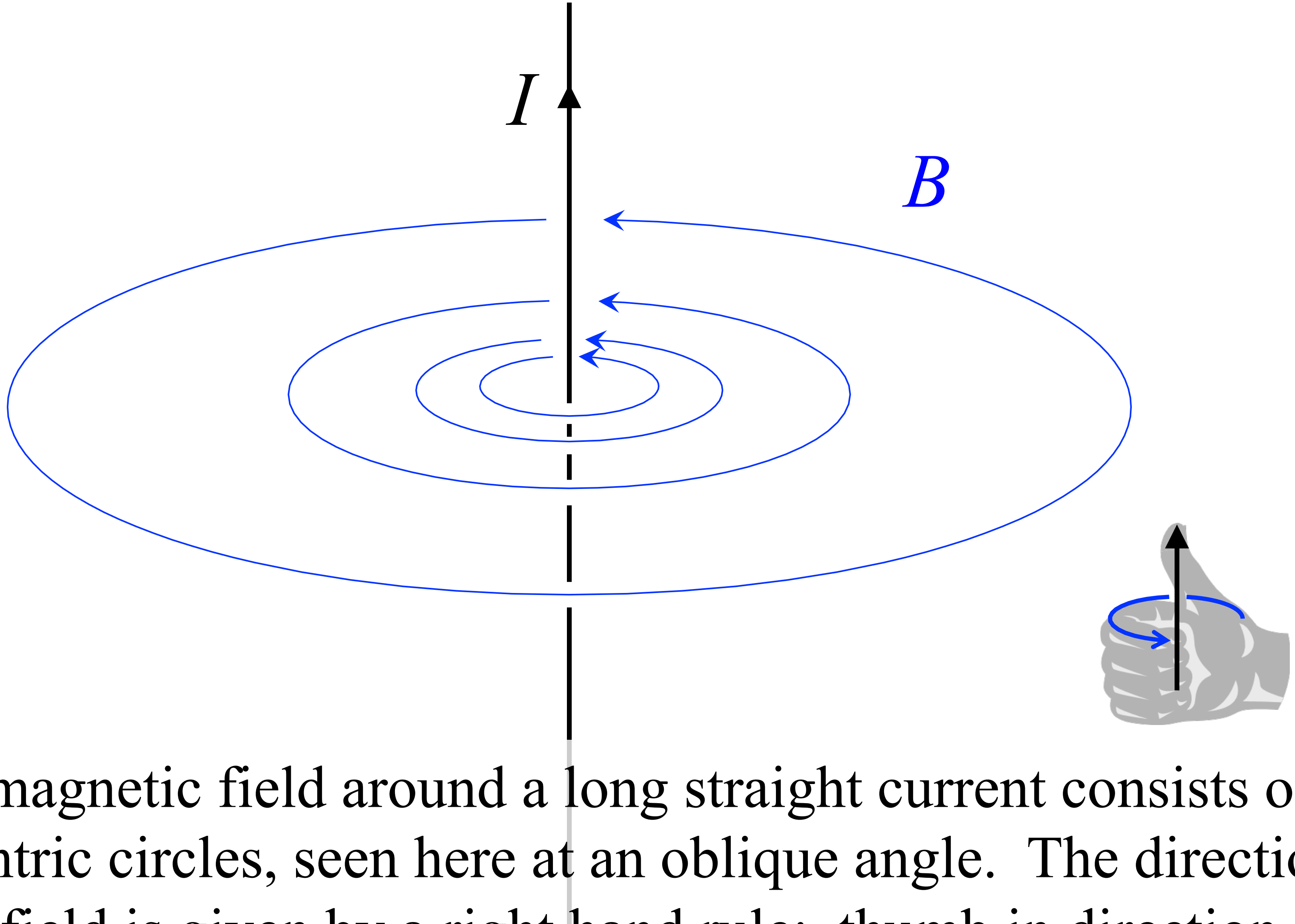
I. Field Basics – units, poles

II. Magnetic Force on Charge
   Mass Spectrometer
   Cyclotron

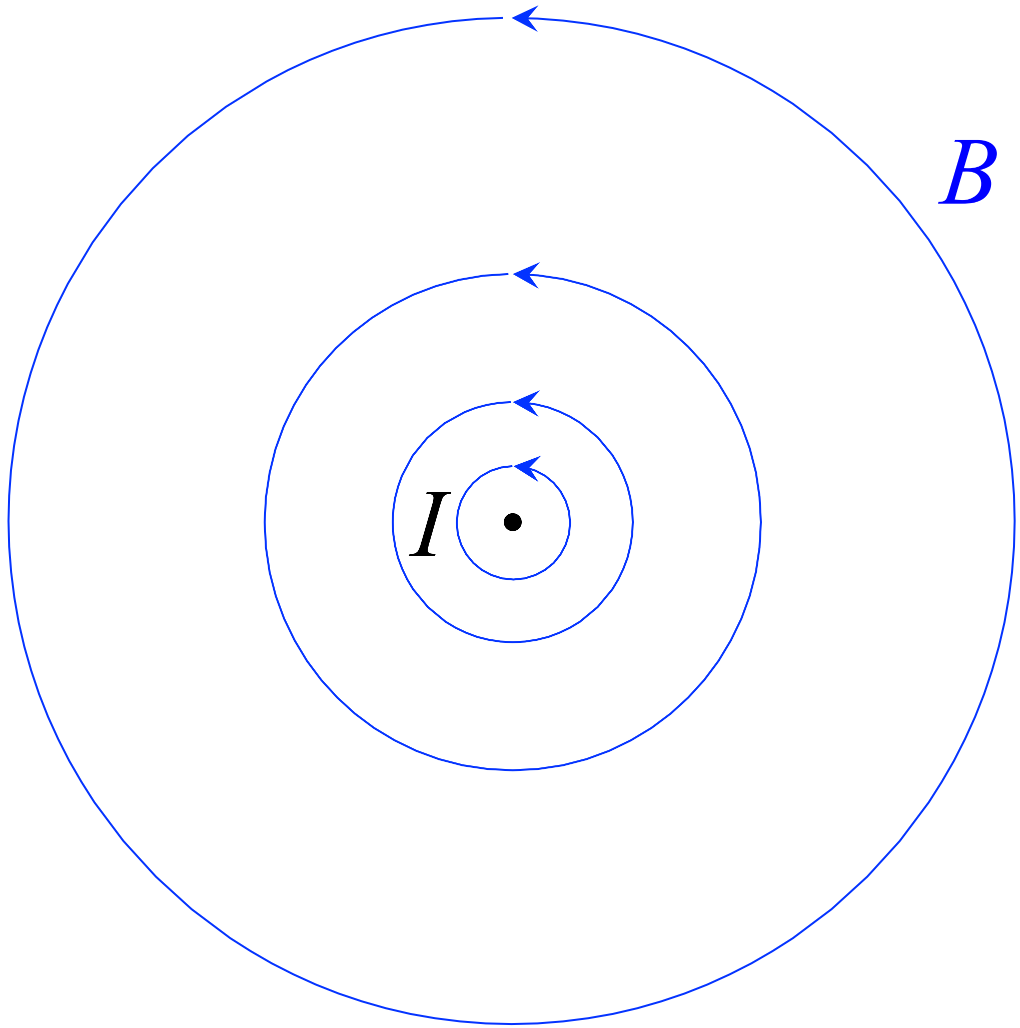
III. Magnetic Force on Current
   Motors and Meters

IV. Sources of Magnetic Fields
   Biot-Savart Law
   Ampere's Law
   Solenoids

	The student will be able to:	HW:
1	Define and illustrate the basic properties of magnetic fields and permanent magnets: field lines, north and south poles, magnetic compasses, Earth's magnetic field.	1 – 2 ✓
2	Solve problems relating magnetic force to the motion of a charged particle through a magnetic field, such as that found in a mass spectrometer.	3 – 10 ✓
3	Solve problems involving forces on a current carrying wire in a magnetic field and torque on a current carrying loop of wire in a magnetic field, such as that found in a motor.	11 – 18 ✓
4	State and apply the Biot-Savart Law and solve such problems that relate a magnetic field to the current that produced it.	19 – 24
5	State and apply Ampere's Law and Gauss's Law for magnetic fields and solve related problems such as those involving parallel wires, solenoids, and toroids.	25 – 40

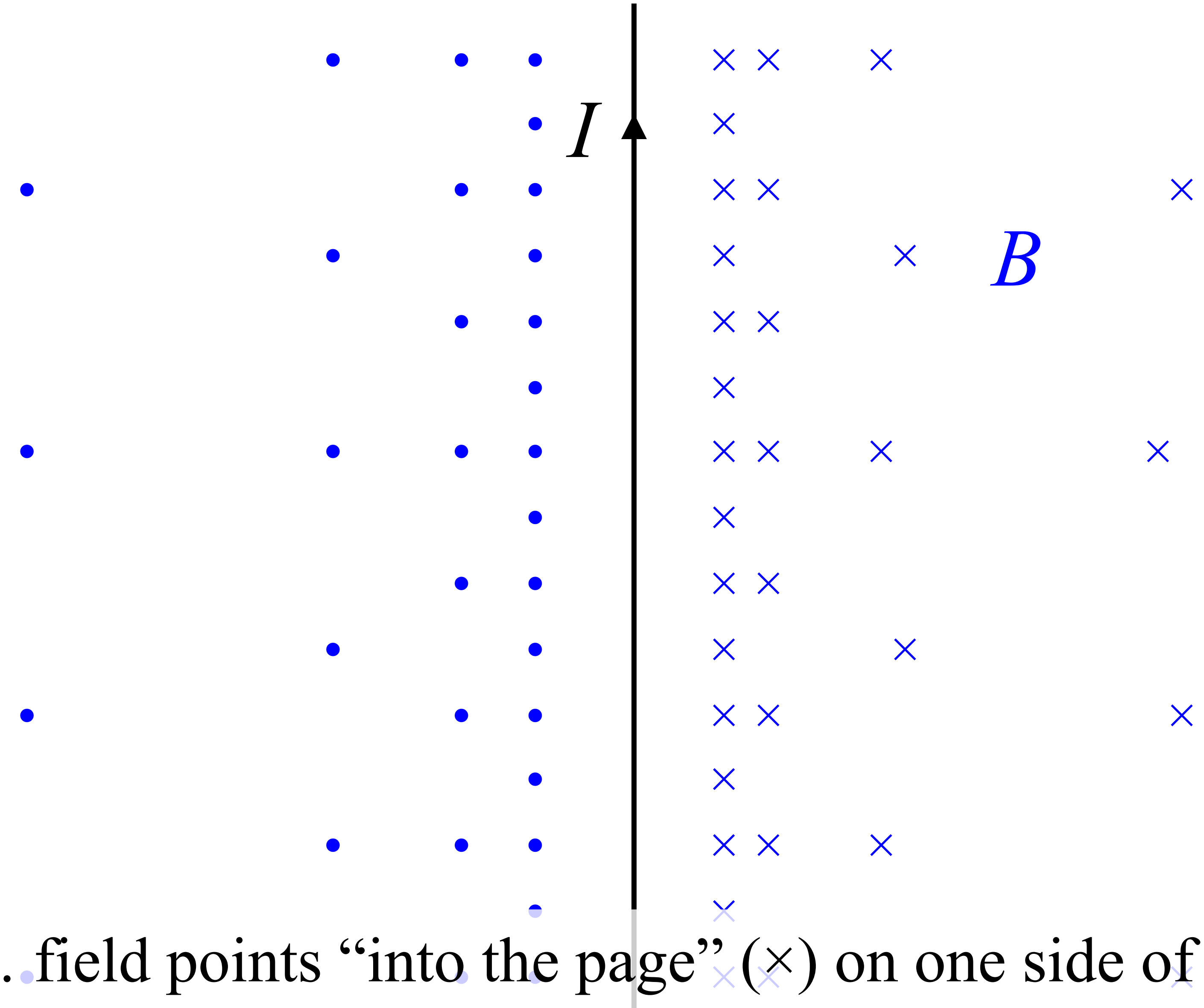


The magnetic field around a long straight current consists of concentric circles, seen here at an oblique angle. The direction of the field is given by a right hand rule: thumb in direction of current, fingers curl and point in direction of the field.



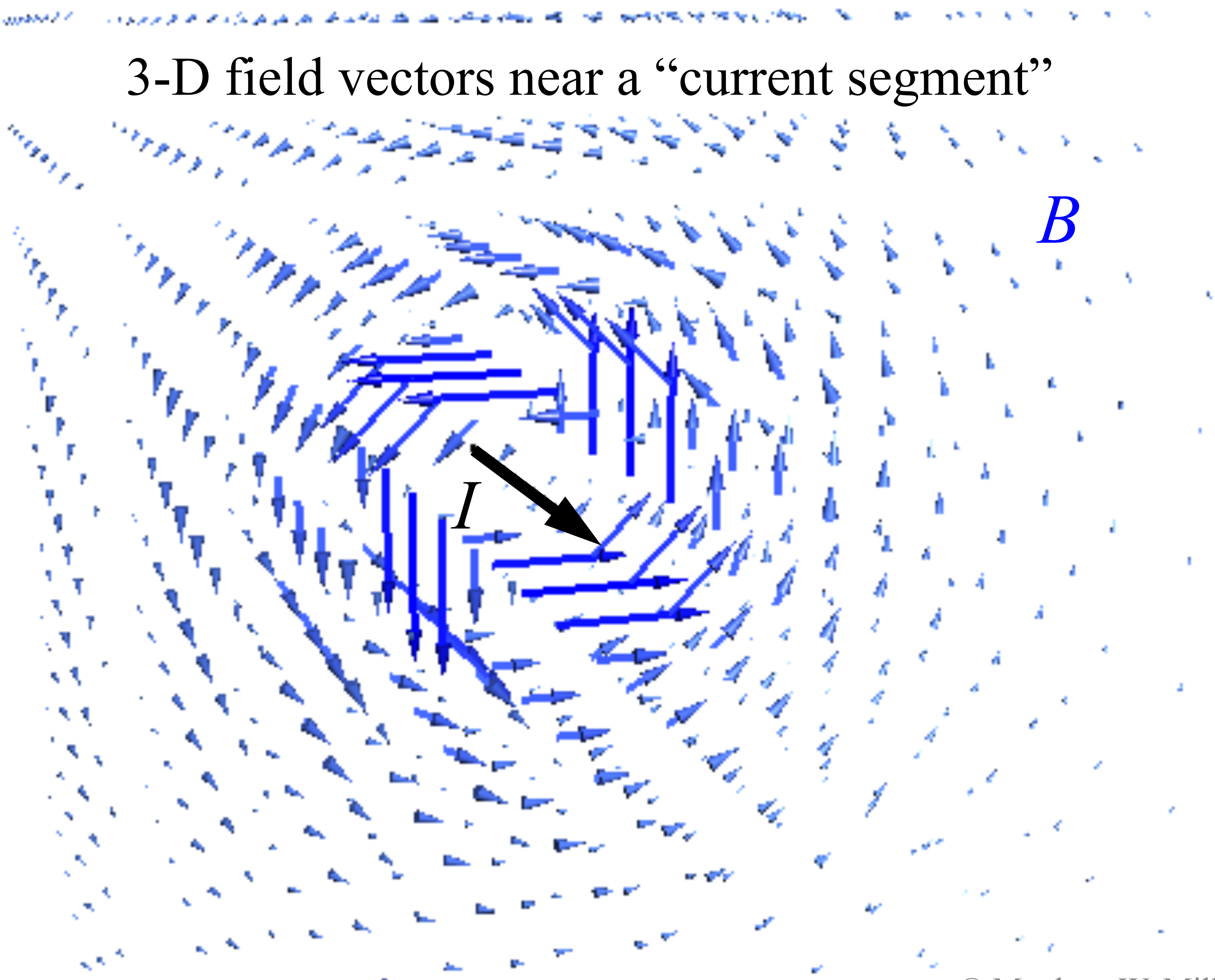
Same field seen “from above” such that the long straight current is “out of the page” – directly toward the eye.

A view of a plane that *contains* the long straight current…



… field points “into the page” (×) on one side of the current and “out of the page” (•) on the other side.

3-D field vectors near a "current segment"



Biot-Savart Law

The magnetic field produced by a current carrying wire:

$$\vec{B} = \int \frac{\mu_0 I d\vec{\ell} \times \hat{r}}{4\pi r^2}$$

I = current

$d\ell$ = incremental length of wire

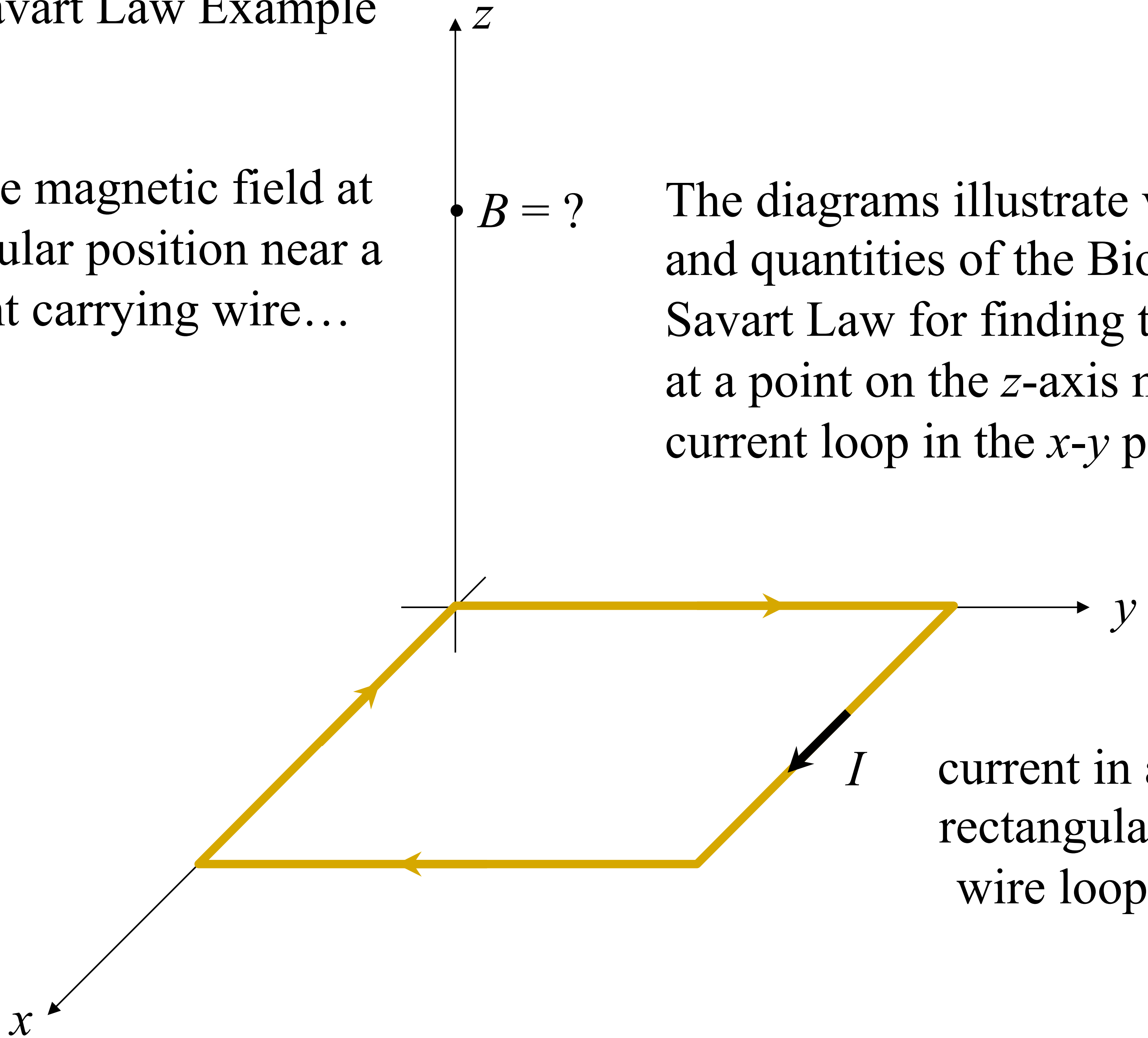
r = position relative to wire

μ_0 = permeability of free space ($4\pi \times 10^{-7}$ N/A^2)

Biot-Savart Law Example

Find the magnetic field at a particular position near a current carrying wire…

The diagrams illustrate vectors and quantities of the Biot-Savart Law for finding the field at a point on the *z*-axis near a current loop in the *x*-*y* plane…

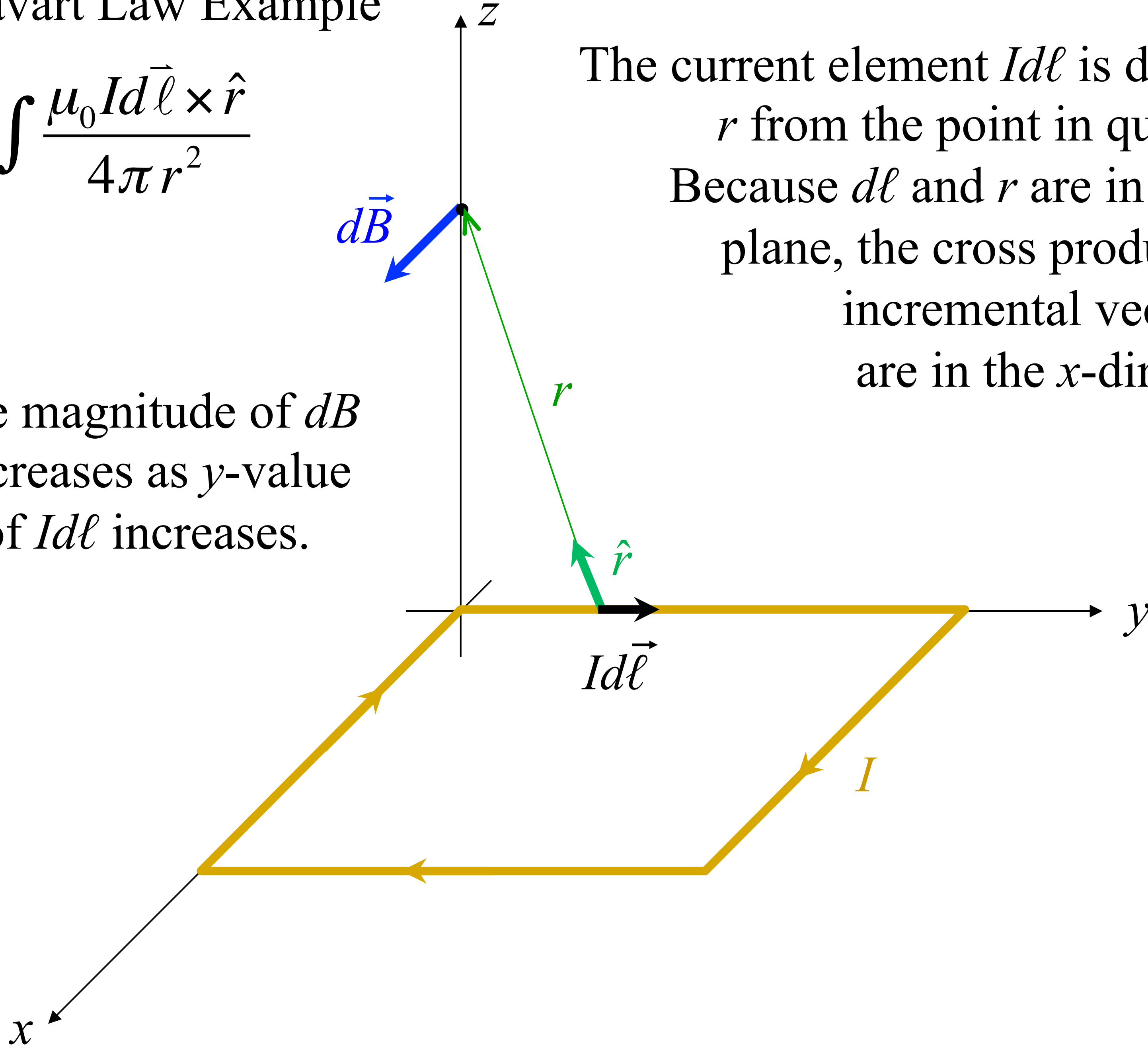


Biot-Savart Law Example

$$\vec{B} = \int \frac{\mu_0 I d\vec{\ell} \times \hat{r}}{4\pi r^2}$$

The current element $Id\ell$ is distance r from the point in question. Because $d\ell$ and r are in the y-z plane, the cross product and incremental vector dB are in the x-direction.

The magnitude of dB decreases as y-value of $Id\ell$ increases.



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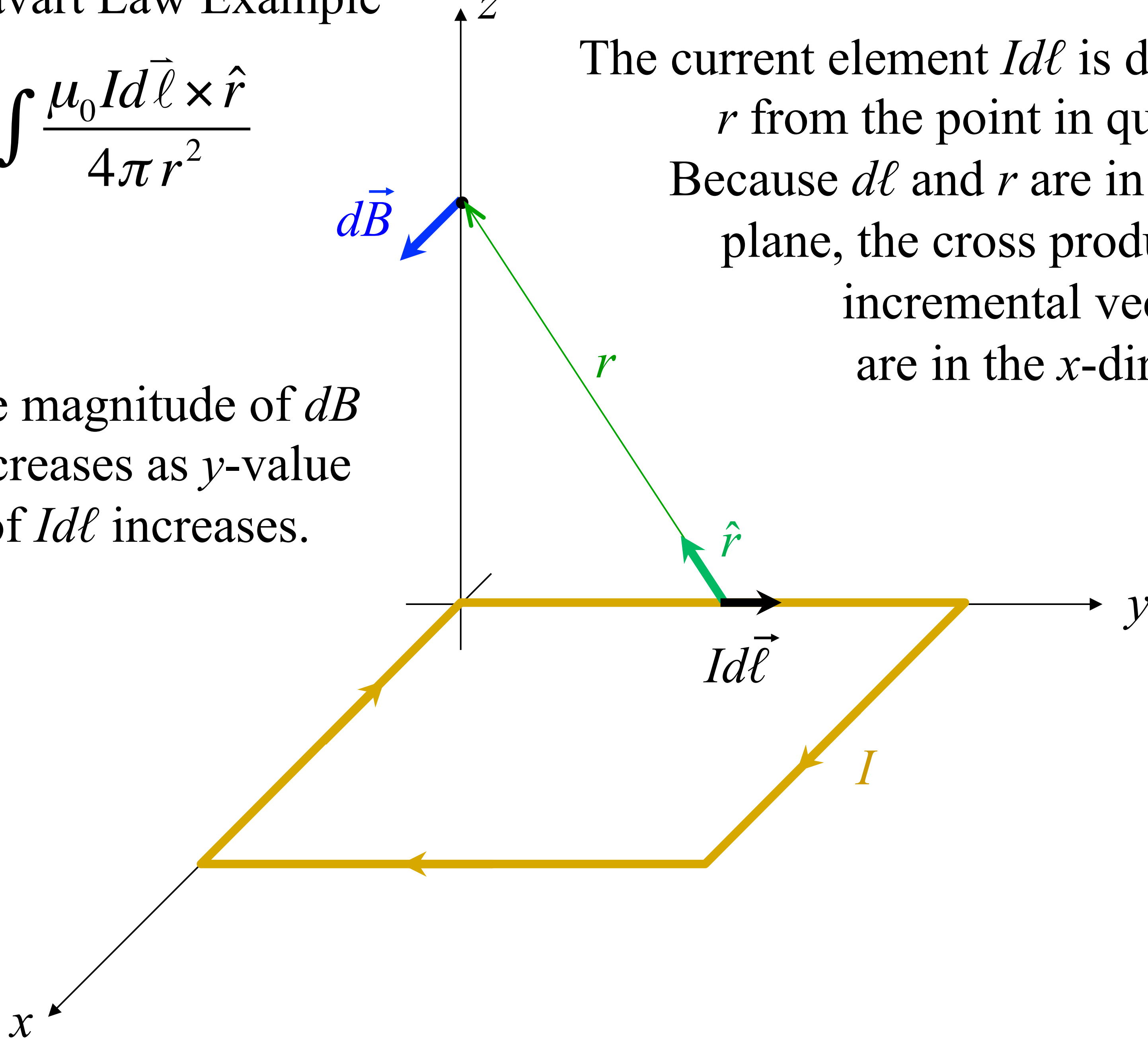
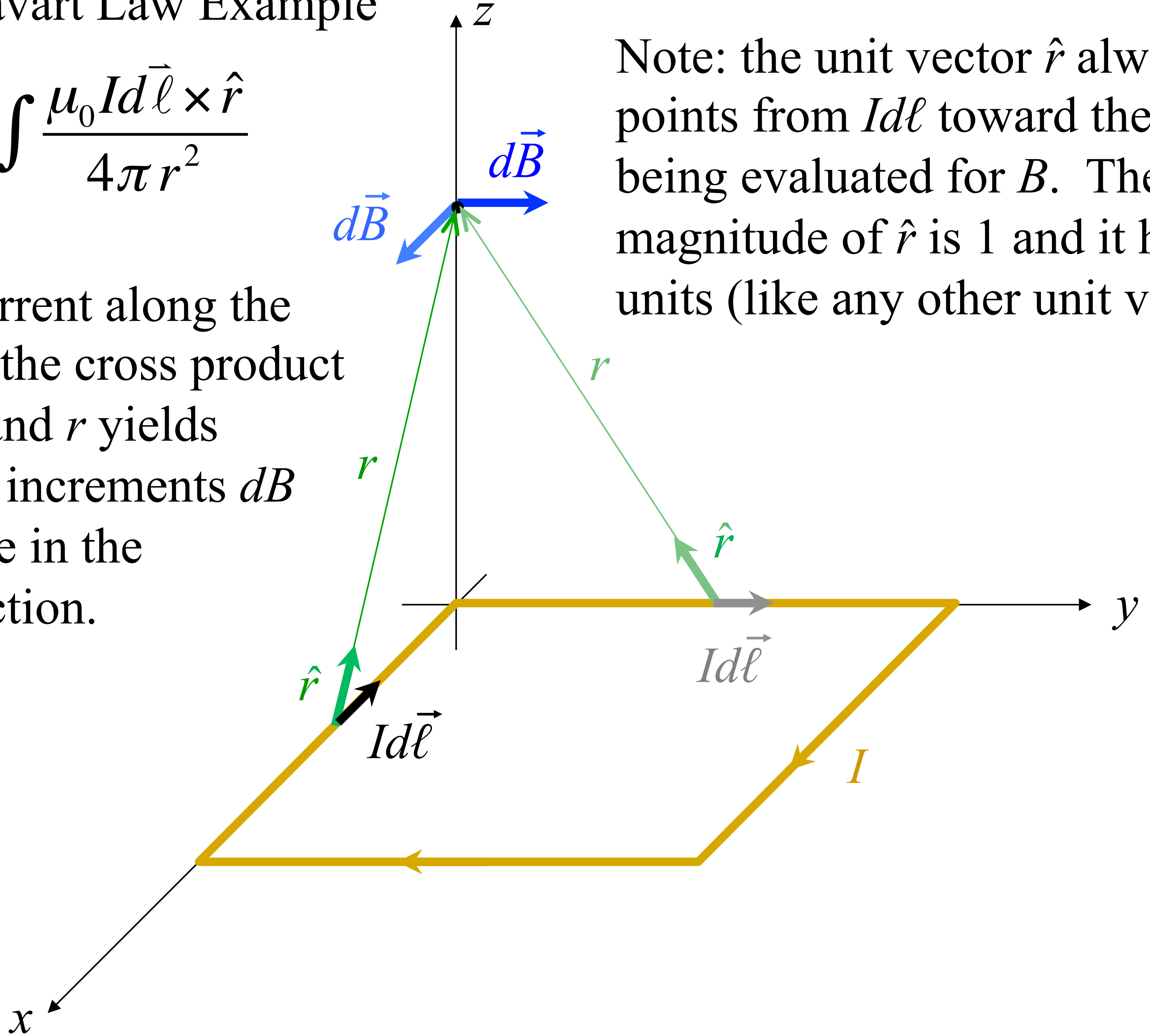


Biot-Savart Law Example

$$\vec{B} = \int \frac{\mu_0 I d\vec{\ell} \times \hat{r}}{4\pi r^2}$$

For current along the x-axis the cross product of $d\ell$ and r yields vector increments dB that are in the y-direction.

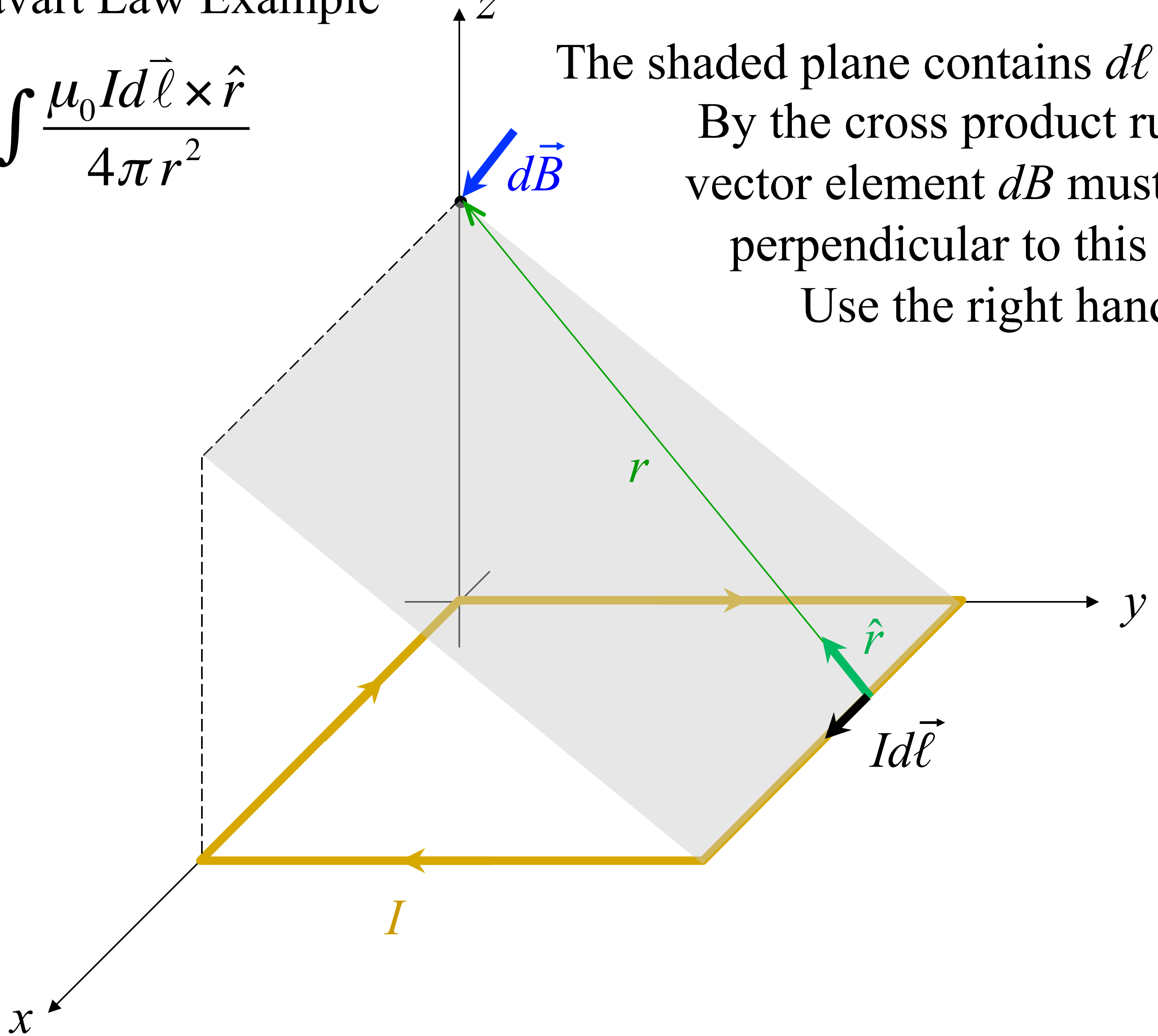


Note: the unit vector $\hat{r}$ always points from $Id\ell$ toward the point being evaluated for B. The magnitude of $\hat{r}$ is 1 and it has no units (like any other unit vector).

Biot-Savart Law Example

$$\vec{B} = \int \frac{\mu_0 I d\vec{\ell} \times \hat{r}}{4\pi r^2}$$

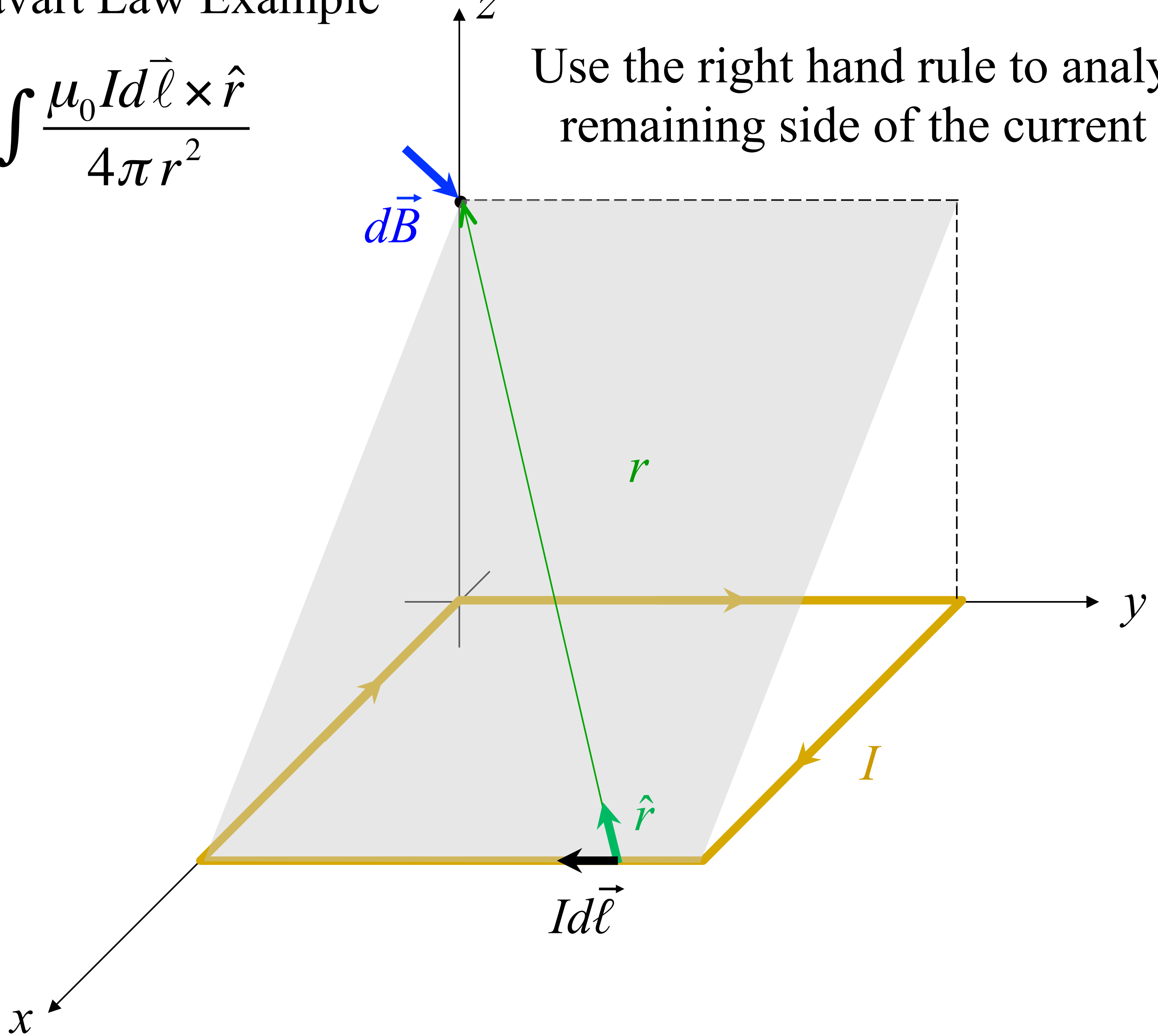
The shaded plane contains $d\ell$ and r.
By the cross product rule the
vector element dB must point
perpendicular to this plane.
Use the right hand rule!



Biot-Savart Law Example

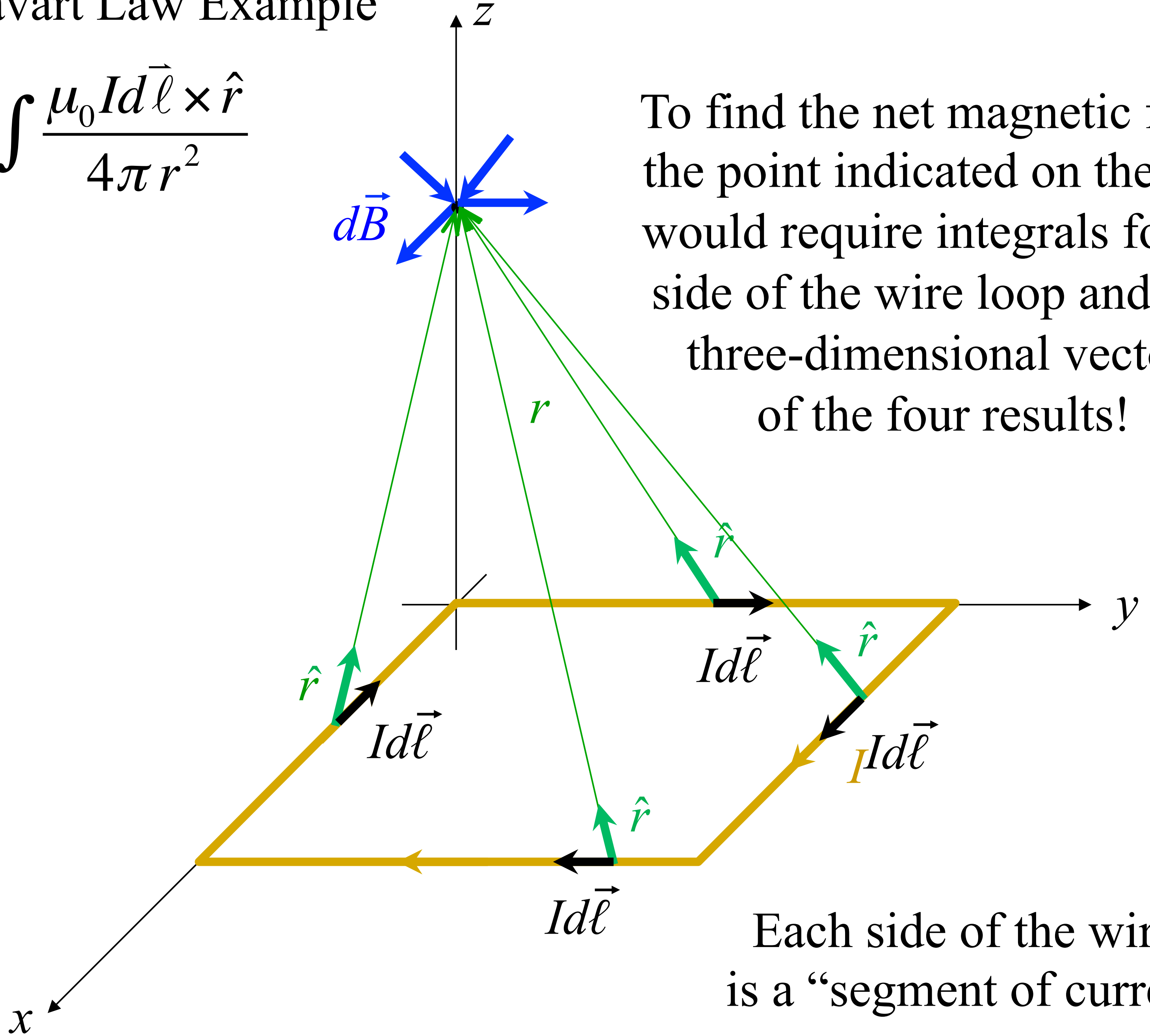
$$\vec{B} = \int \frac{\mu_0 I d\vec{\ell} \times \hat{r}}{4\pi r^2}$$

Use the right hand rule to analyze the remaining side of the current loop.



Biot-Savart Law Example

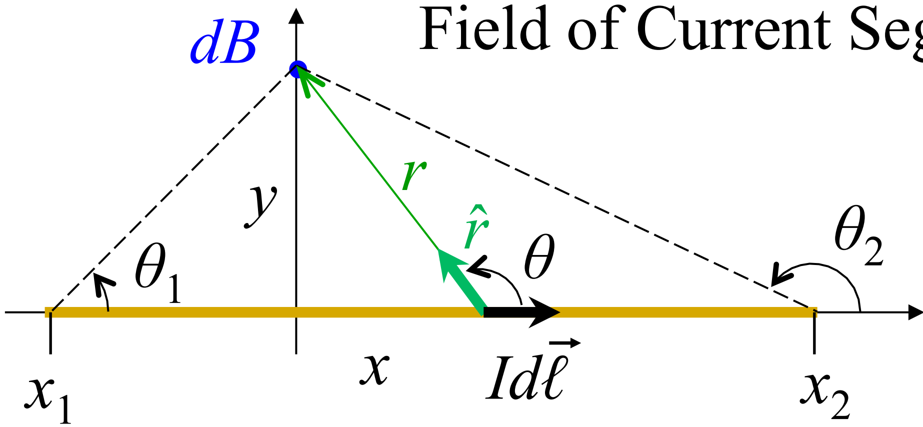
$$\vec{B} = \int \frac{\mu_0 I d\vec{\ell} \times \hat{r}}{4\pi r^2}$$



To find the net magnetic field at the point indicated on the *z*-axis would require integrals for each side of the wire loop and then a three-dimensional vector sum of the four results! Yikes!

Each side of the wire loop is a "segment of current"…

Field of Current Segment



$$\vec{B} = \int \frac{\mu_0 I d\vec{\ell} \times \hat{r}}{4\pi r^2}$$

A key part of Biot-Savart is the **cross product**!

Recall how a cross product works: $\vec{R} \times \vec{S} = RS \sin\theta$

The magnitude of $d\ell$ is dx and the magnitude of $\hat{r}$ is 1: $d\vec{\ell} \times \hat{r} = (dx)(1)\sin\theta$

So, the cross product has magnitude: $d\vec{\ell} \times \hat{r} = \sin\theta \, dx$

Then, it is necessary to choose one variable!

Put sin θ in terms of x or put dx in terms of $d\theta$.

$$d\vec{\ell} \times \hat{r} = \sin\theta \, dx$$

$$d\vec{\ell} \times \hat{r} = \frac{y}{\sqrt{x^2 + y^2}} dx$$

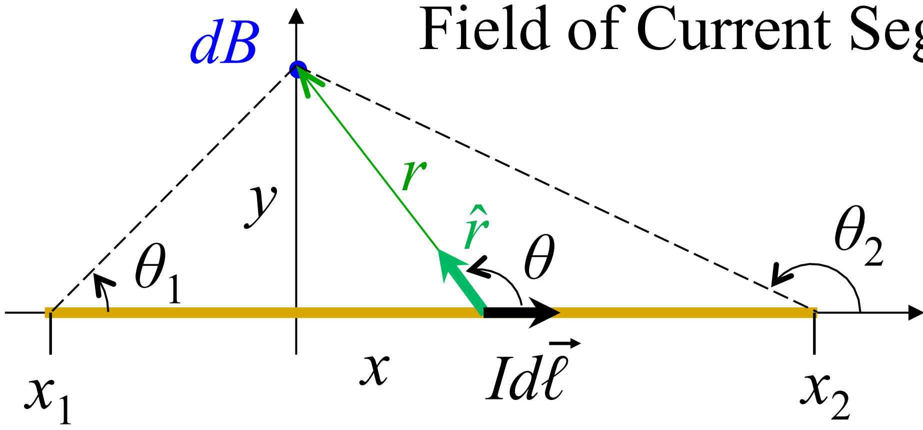
$$d\vec{\ell} \times \hat{r} = \frac{y dx}{\sqrt{x^2 + y^2}}$$

$$d\vec{\ell} \times \hat{r} = \sin\theta \, dx$$

$$d\vec{\ell} \times \hat{r} = \sin\theta \frac{y d\theta}{\sin^2\theta}$$

$$d\vec{\ell} \times \hat{r} = \frac{y d\theta}{\sin\theta}$$

Field of Current Segment



$$r^2 = x^2 + y^2$$

$$\sin\theta = \frac{y}{r} = \frac{y}{\sqrt{x^2+y^2}}$$

$$\cos\theta = -\frac{x}{r} = -\frac{x}{\sqrt{x^2+y^2}}$$

$$d\vec{\ell}\times\hat{r} = \sin\theta\, dx$$

$$d\vec{\ell}\times\hat{r} = \frac{y\,dx}{\sqrt{x^2+y^2}}$$

$$\vec{B} = \int \frac{\mu_0 I d\vec{\ell}\times\hat{r}}{4\pi r^2}$$

Integral in terms of x:

$$B = \int \frac{\mu_0 I}{4\pi\left(x^2+y^2\right)}\frac{y\,dx}{\sqrt{x^2+y^2}}$$

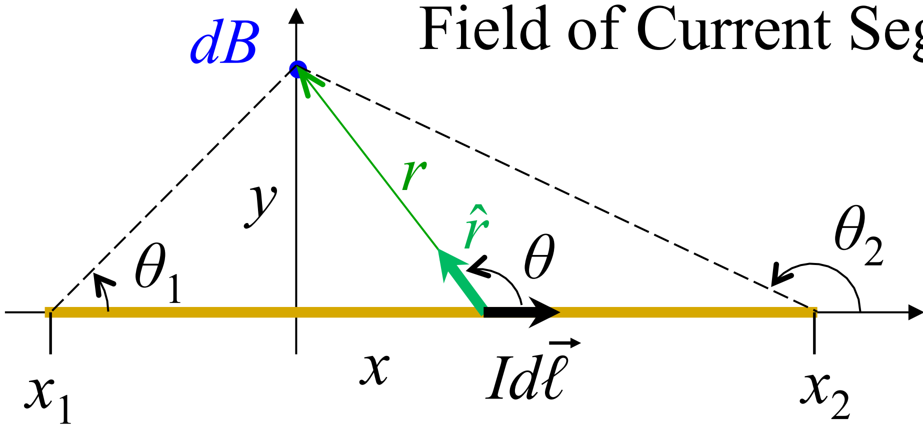
$$B = \int_{x_1}^{x_2} \frac{\mu_0 I y\,dx}{4\pi\left(x^2+y^2\right)^{\frac{3}{2}}}$$

$$B = \frac{\mu_0 I}{4\pi y}\left(\frac{x_2}{\sqrt{x_2^2+y^2}} - \frac{x_1}{\sqrt{x_1^2+y^2}}\right)$$

$$B = \frac{\mu_0 I}{4\pi y}\left(-\cos\theta_2 - \left(-\cos\theta_1\right)\right)$$

$$B = \frac{\mu_0 I}{4\pi y}\left(\cos\theta_1 - \cos\theta_2\right)$$

Field of Current Segment



$$\vec{B} = \int \frac{\mu_0 I d\vec{\ell} \times \hat{r}}{4\pi r^2}$$

Integral in terms of θ:

$$\sin\theta = \frac{y}{r}$$

$$r = \frac{y}{\sin\theta}$$

$$\tan\theta = -\frac{y}{x}$$

$$x = -\frac{y}{\tan\theta}$$

$$\frac{dx}{d\theta} = \frac{y}{\sin^2\theta}$$

$$dx = \frac{y d\theta}{\sin^2\theta}$$

$$d\vec{\ell} \times \hat{r} = \sin\theta \, dx$$

$$d\vec{\ell} \times \hat{r} = \sin\theta \frac{y d\theta}{\sin^2\theta}$$

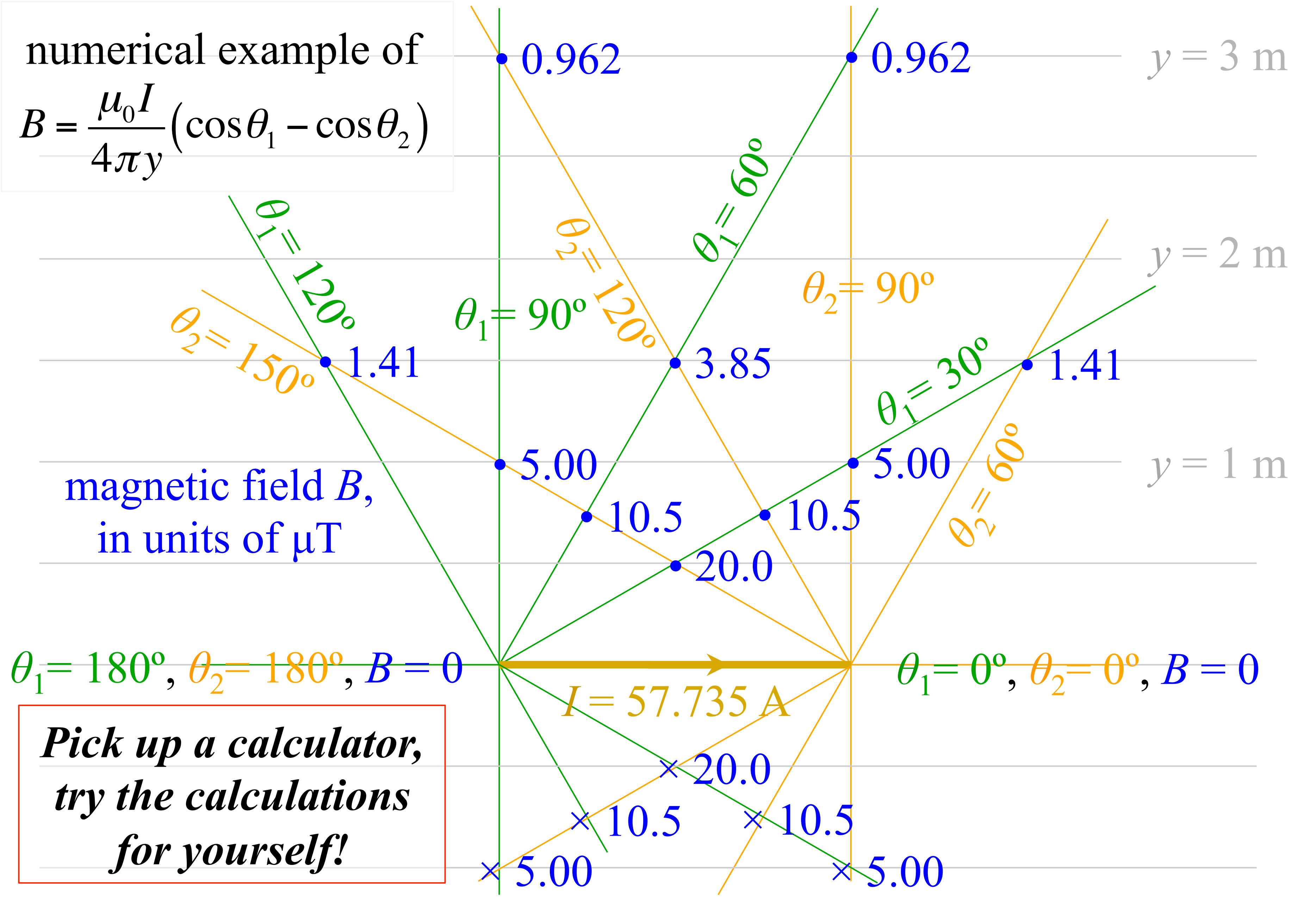
$$d\vec{\ell} \times \hat{r} = \frac{y d\theta}{\sin\theta}$$

$$B = \int \frac{\mu_0 I}{4\pi\left(\frac{y}{\sin\theta}\right)^2} \frac{y d\theta}{\sin\theta}$$

$$B = \int_{\theta_1}^{\theta_2} \frac{\mu_0 I \sin\theta d\theta}{4\pi y}$$

$$B = \frac{\mu_0 I}{4\pi y}\left(\cos\theta_1 - \cos\theta_2\right)$$

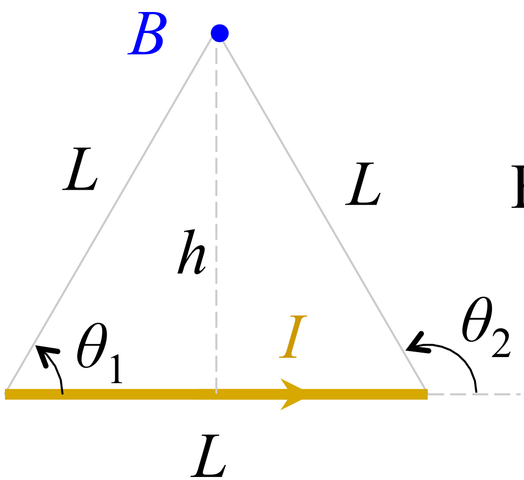
Field of Current Segment L = 1.732 m, I = 57.735 A



Consider an equilateral triangle, sides length L, with current I on one side:

$$B = \frac{\mu_0 I}{4\pi y}\left(\cos\theta_1 - \cos\theta_2\right) \quad \theta_1 = 60^\circ,\ \theta_2 = 120^\circ$$

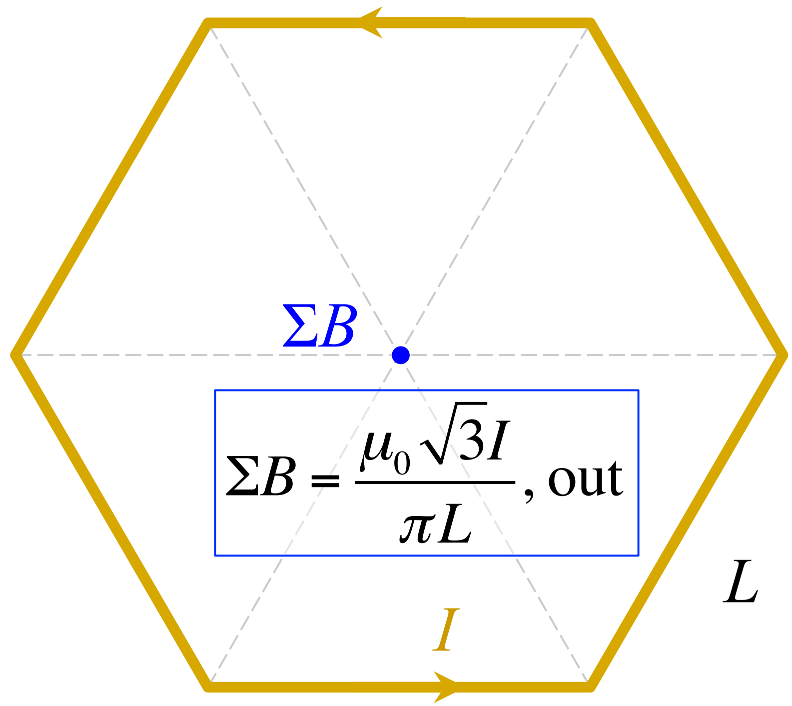
$$B = \frac{\mu_0 I}{4\pi h}\left(\frac{1}{2} - \left(-\frac{1}{2}\right)\right) = \frac{\mu_0 I}{4\pi h} = \frac{\mu_0 \sqrt{3} I}{6\pi L}$$

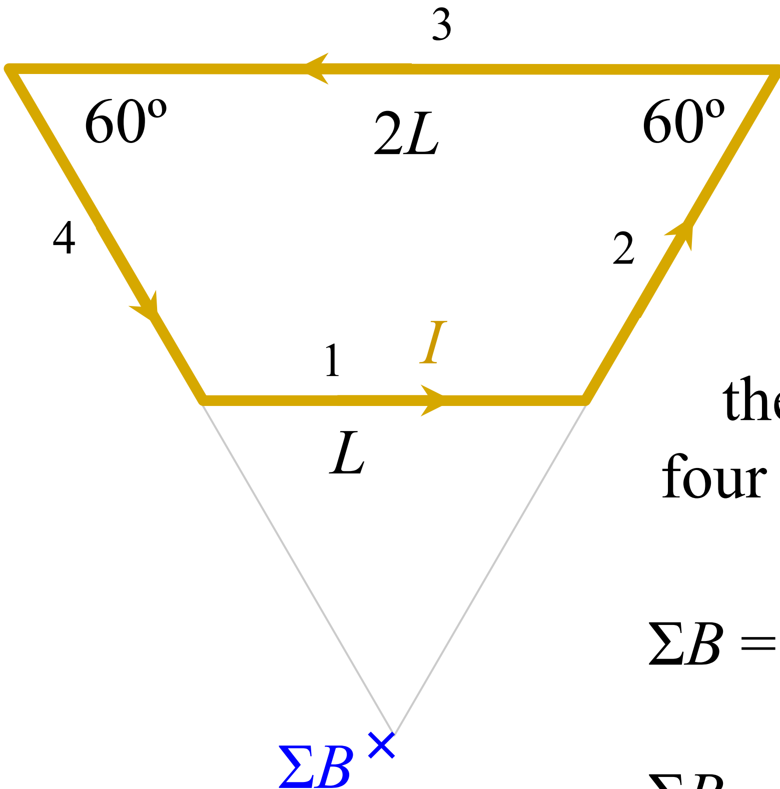


Field at the vertex: $B = \frac{\mu_0 \sqrt{3} I}{6\pi L}$, out of page

Superposition of the above gives the magnetic field at the center of a hexagonal loop of current:

$$\Sigma B = 6\left(\frac{\mu_0 \sqrt{3} I}{6\pi L}\right) = \frac{\mu_0 \sqrt{3} I}{\pi L}$$





$$\Sigma B = \frac{\mu_0 \sqrt{3} I}{12\pi L}, \text{ in}$$

Another example using: $B = \dfrac{\mu_0 \sqrt{3} I}{6\pi L}$
(derived previous page)

The magnetic field at the vertex of the larger equilateral triangle is the superposition of the fields from the four sides of the trapezoidal current loop.

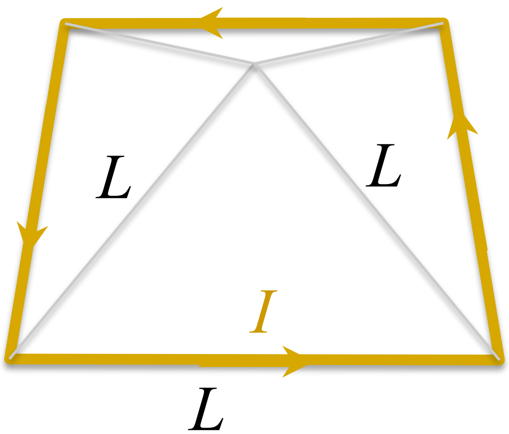
$$\Sigma B = B_1 + B_2 + B_3 + B_4$$

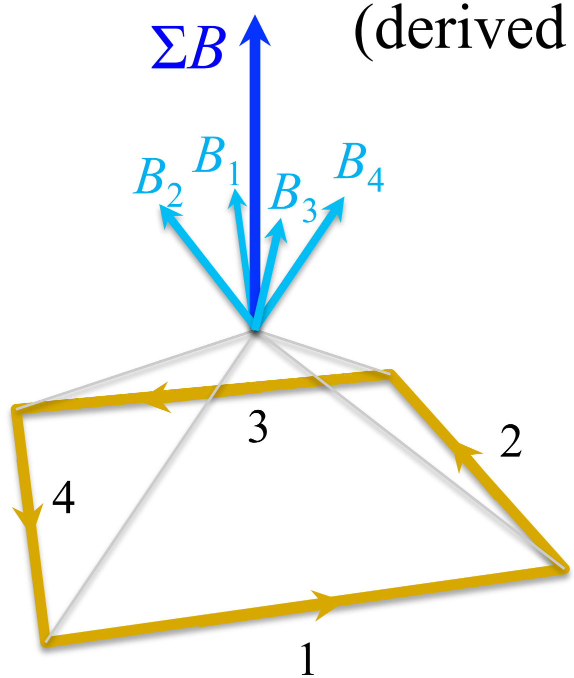
$$\Sigma B = B_1 + 0 + B_3 + 0$$

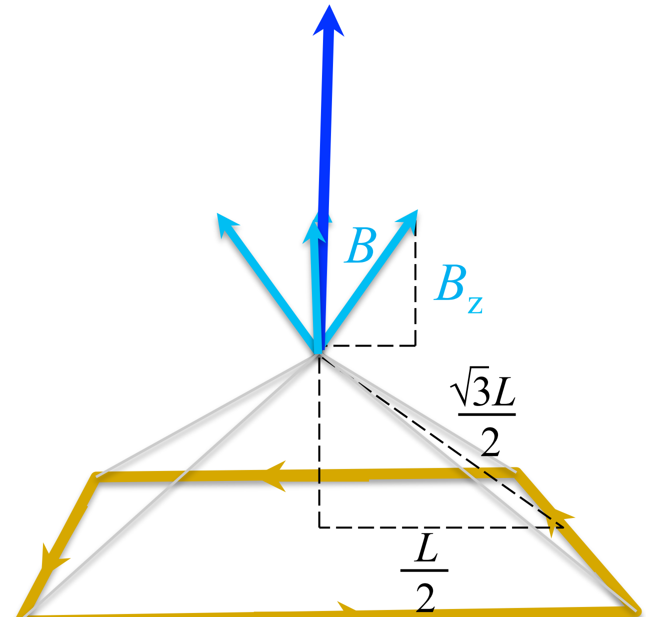
$$\Sigma B = \left(\frac{\mu_0 \sqrt{3} I}{6\pi L}, \text{ in} \right) + \left(\frac{\mu_0 \sqrt{3} I}{6\pi (2L)}, \text{ out} \right) = \frac{\mu_0 \sqrt{3} I}{12\pi L}, \text{ in}$$

Current square, field B at vertex of pyramid?

Another example using: $B = \frac{\mu_0 \sqrt{3} I}{6\pi L}$ (derived previously)







$$\frac{B_z}{B} = \frac{\frac{L}{2}}{\frac{\sqrt{3}L}{2}}$$

$$B_z = \frac{1}{\sqrt{3}} B$$

$$B_z = \frac{1}{\sqrt{3}} \frac{\mu_0 \sqrt{3} I}{6\pi L}$$

$$B_z = \frac{\mu_0 I}{6\pi L}$$

$$\Sigma B = 4 B_z$$

$$\Sigma B = 4\left(\frac{\mu_0 I}{6\pi L}\right)$$

$$\boxed{\Sigma B = \frac{2\mu_0 I}{3\pi L}}$$

Field of "Infinite" Linear Current

Apply the formula for the *finite* current segment to a situation where the length of wire increases to *infinity*:

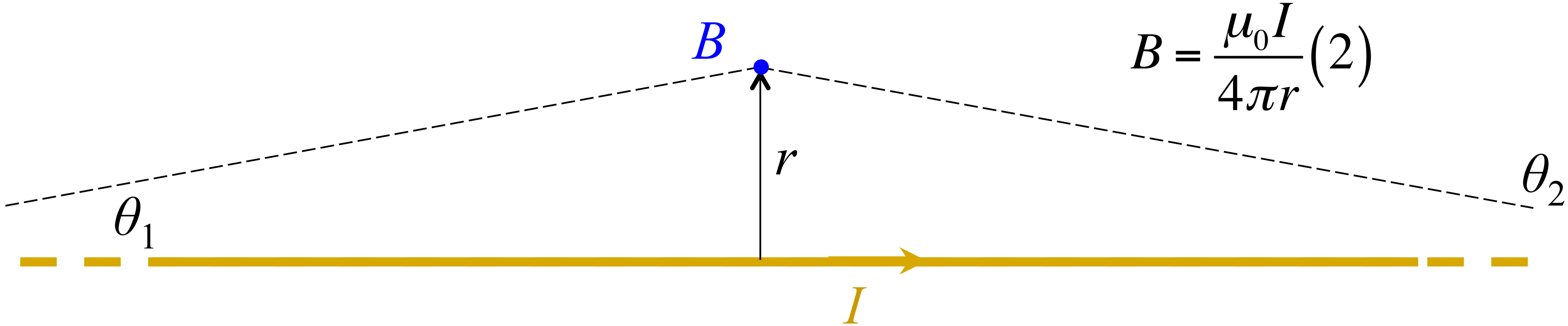
$$\theta_1 \to 0^\circ,\ \theta_2 \to 180^\circ$$

$$B = \frac{\mu_0 I}{4\pi y}\left(\cos\theta_1 - \cos\theta_2\right)$$

$$B = \frac{\mu_0 I}{4\pi r}\left(\cos 0^\circ - \cos 180^\circ\right)$$

$$B = \frac{\mu_0 I}{4\pi r}\left(1-(-1)\right)$$

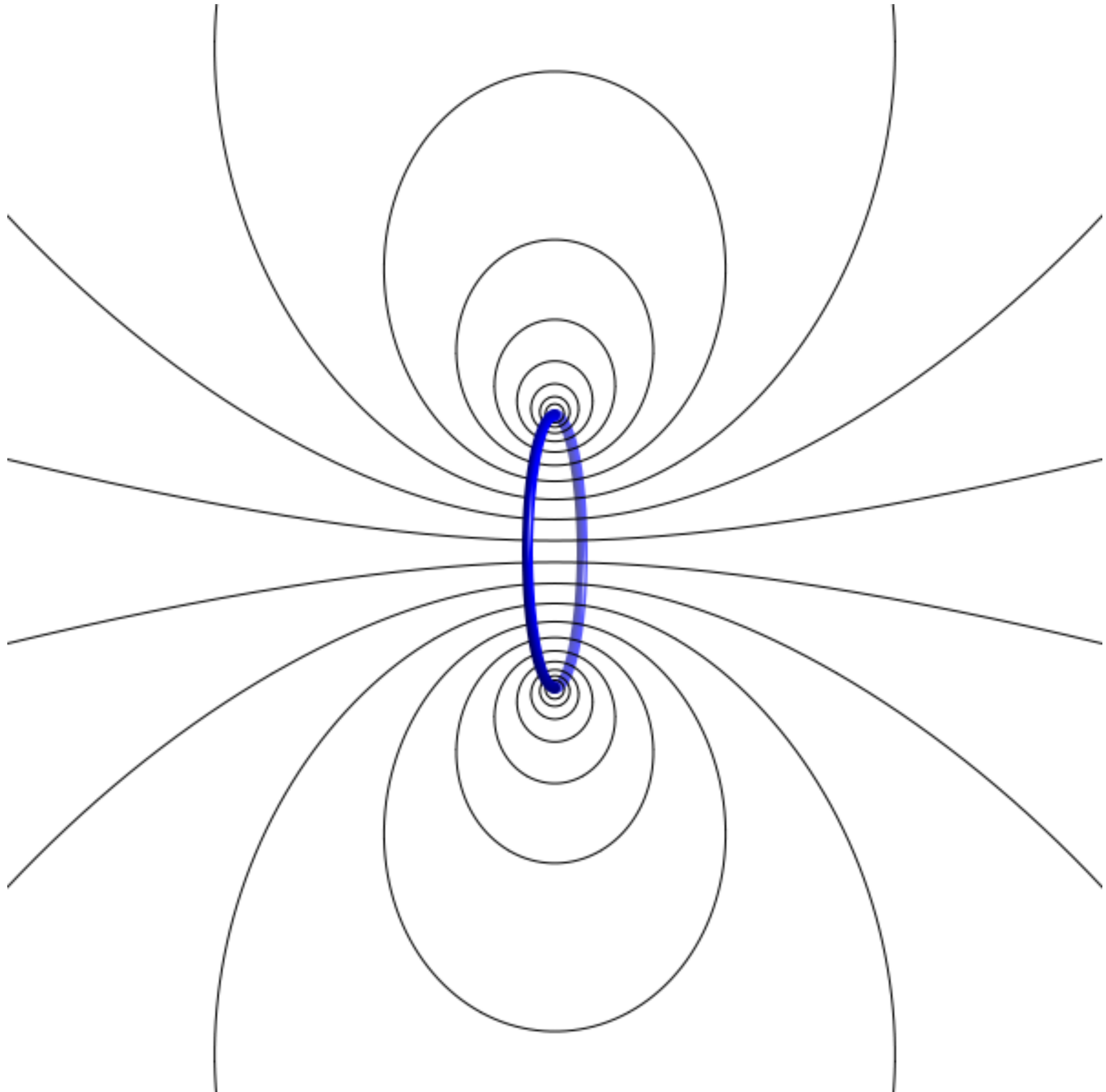
$$B = \frac{\mu_0 I}{4\pi r}(2)$$



The resulting formula is very useful and gives an accurate value for locations relatively near a relatively long current carrying wire ($r << L$).

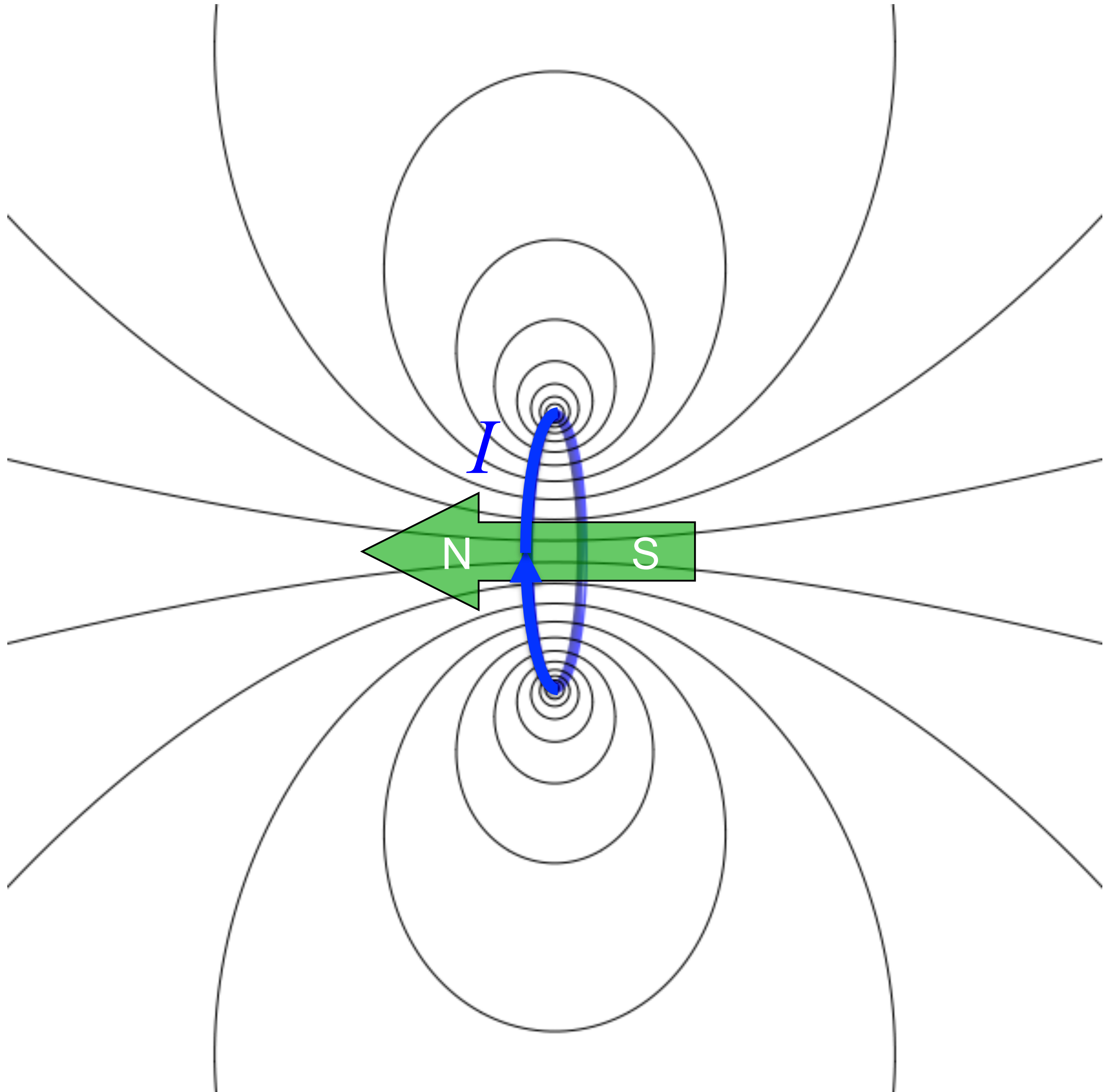
$$B = \frac{\mu_0 I}{2\pi r}$$

Magnetic Field – Ring of Current



Credit: Geek3, Wikipedia

Magnetic Field – Ring of Current



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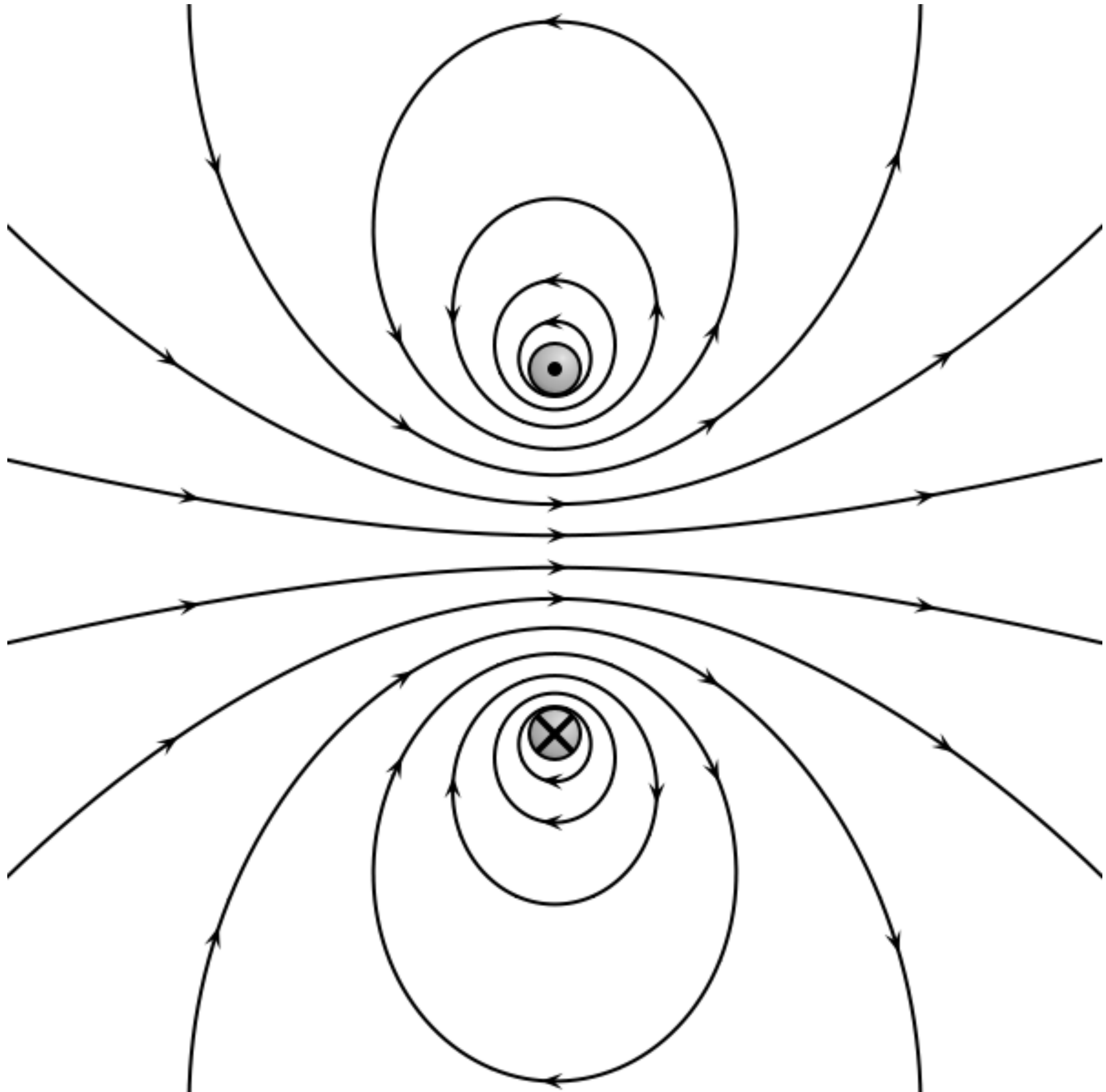

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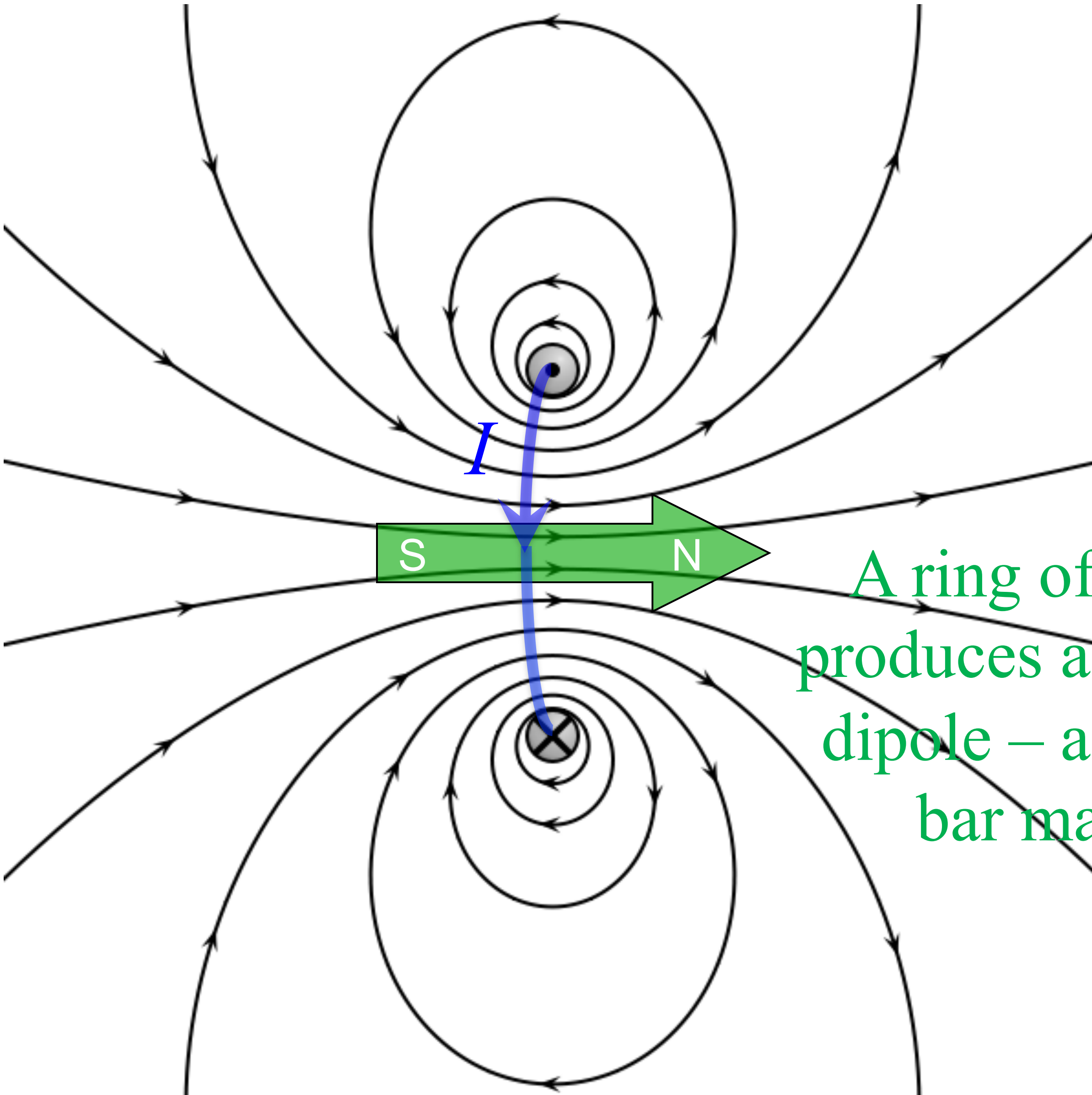
Magnetic Field – Ring of Current



A ring of current produces a magnetic dipole – acts like a bar magnet.

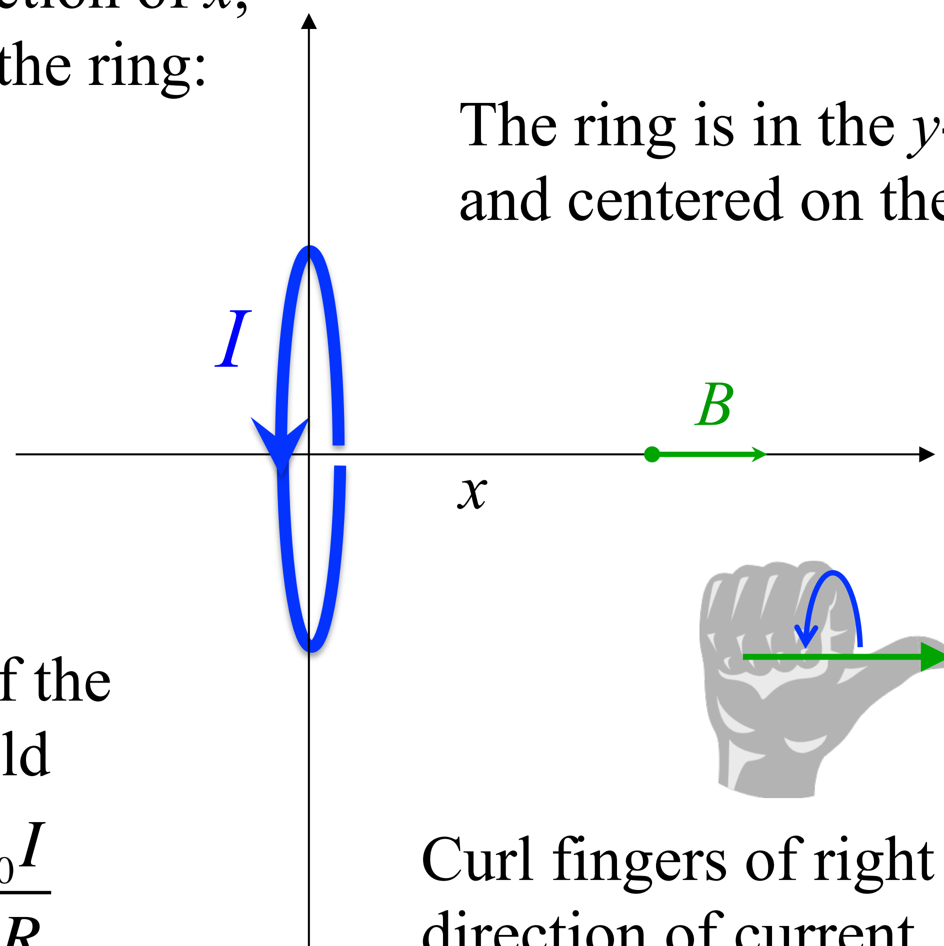
Credit: Geek3, Wikipedia

Magnetic Field – Ring of Current

The Biot-Savart Law gives the magnetic field as a function of x, position on the axis of the ring:

$$B = \frac{\mu_0 I R^2}{2\left(R^2 + x^2\right)^{\frac{3}{2}}}$$

The ring is in the y-z plane and centered on the origin.



At the exact center of the ring $x = 0$ and the field simplifies to:

$$B = \frac{\mu_0 I}{2R}$$

Curl fingers of right hand in direction of current, field points in direction of thumb!