Magnetostatics

I. Field Basics – units, poles

II. Magnetic Force on Charge
- Mass Spectrometer
- Cyclotron

III. Magnetic Force on Current
- Motors and Meters

IV. Sources of Magnetic Fields
- Biot-Savart Law
- Ampere's Law
- Solenoids

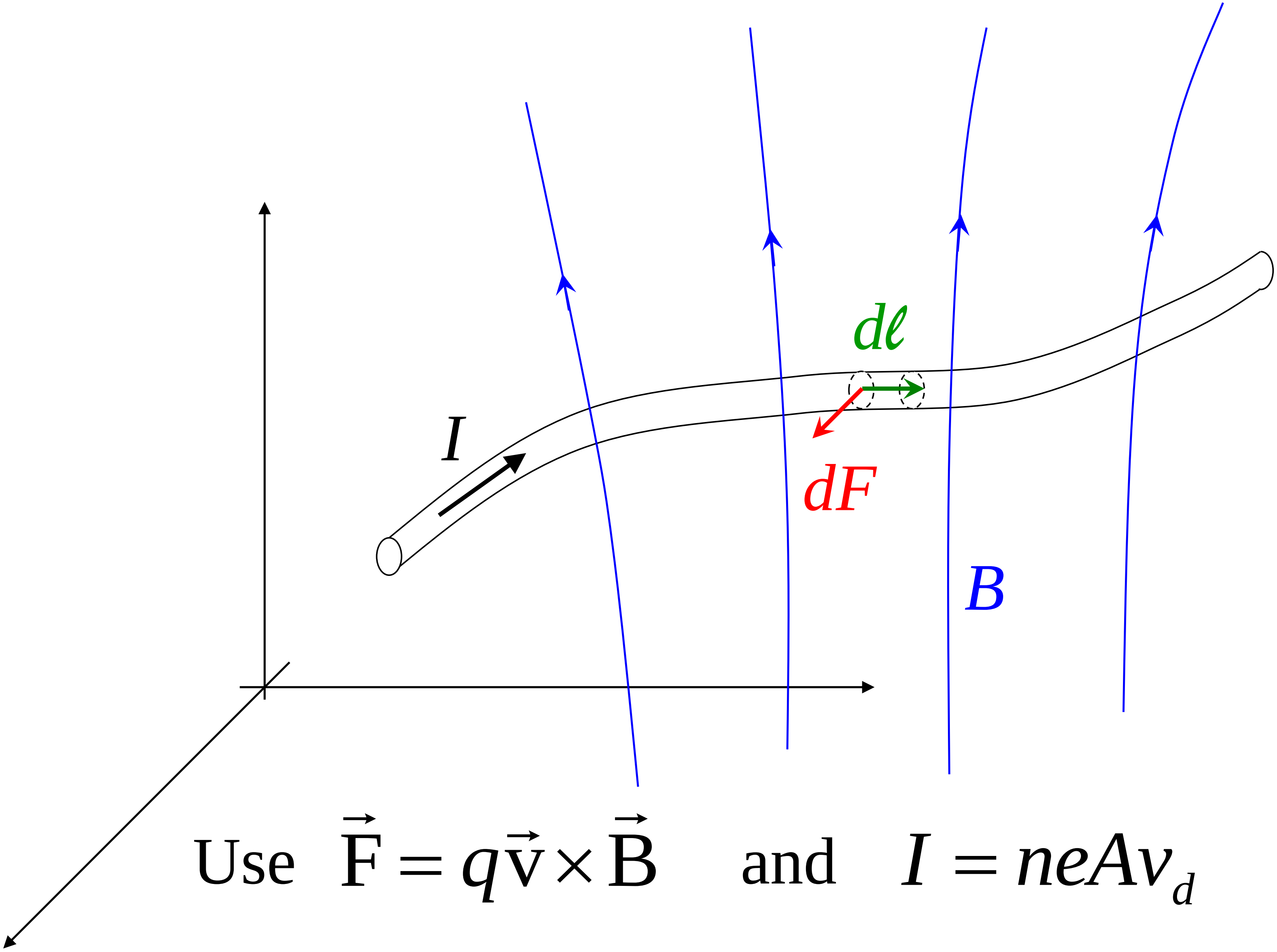
	The student will be able to:	HW:
1	Define and illustrate the basic properties of magnetic fields and permanent magnets: field lines, north and south poles, magnetic compasses, Earth's magnetic field.	✓ 1 – 2
2	Solve problems relating magnetic force to the motion of a charged particle through a magnetic field, such as that found in a mass spectrometer.	✓ 3 – 10
3	Solve problems involving forces on a current carrying wire in a magnetic field and torque on a current carrying loop of wire in a magnetic field, such as that found in a motor.	11 – 18
4	State and apply the Biot-Savart Law and solve such problems that relate a magnetic field to the current that produced it.	19 – 24
5	State and apply Ampere's Law and Gauss's Law for magnetic fields and solve related problems such as those involving parallel wires, solenoids, and toroids.	25 – 40

Magnetic Force on Current Carrying Wire

A current carrying wire is affected by a magnetic field as shown in the equation:

$$\vec{F}_m = \int I \, d\vec{\ell} \times \vec{B}$$

F = force on the wire
I = current in the wire
B = magnetic field
$d\ell$ = infinitesimal length of wire in direction of current



Use $\vec{F} = q\vec{v} \times \vec{B}$ and $I = neAv_d$

to show: $\vec{F} = \int I\, d\vec{\ell} \times \vec{B}$

Special Case

A ***straight*** current carrying wire affected by a ***uniform*** magnetic field results in:

$$\vec{F}_m = I\vec{\ell} \times \vec{B}$$

F = force on the wire
I = current in the wire
B = magnetic field
ℓ = length of wire

Special Case

A ***straight*** current carrying wire affected by a ***uniform*** magnetic field results in:

$$F_M = I\ell B\sin\theta$$

$$F_M = I\ell_{\perp}B = I\ell B_{\perp}$$

F = force on the wire

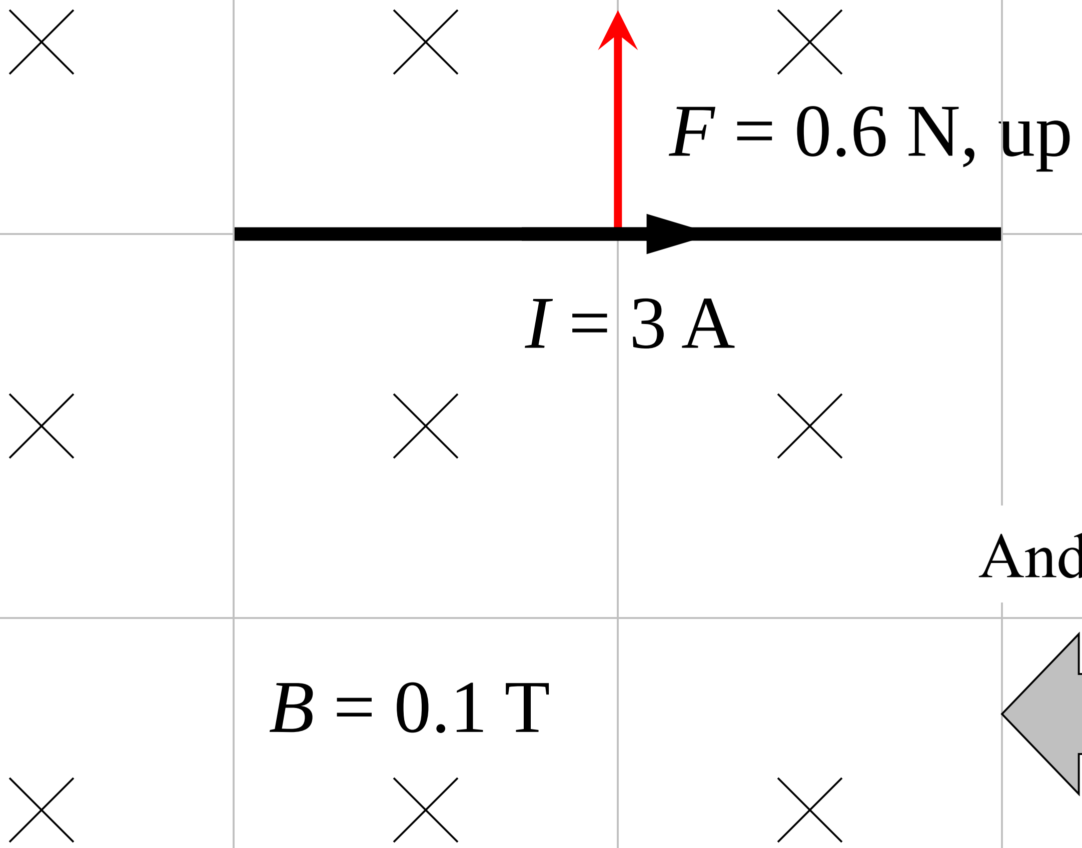
I = current in the wire

B = magnetic field

ℓ = length of wire

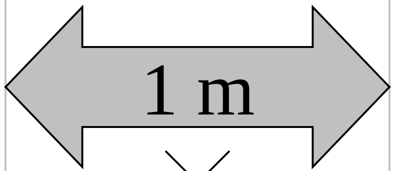
Determine the magnetic force:

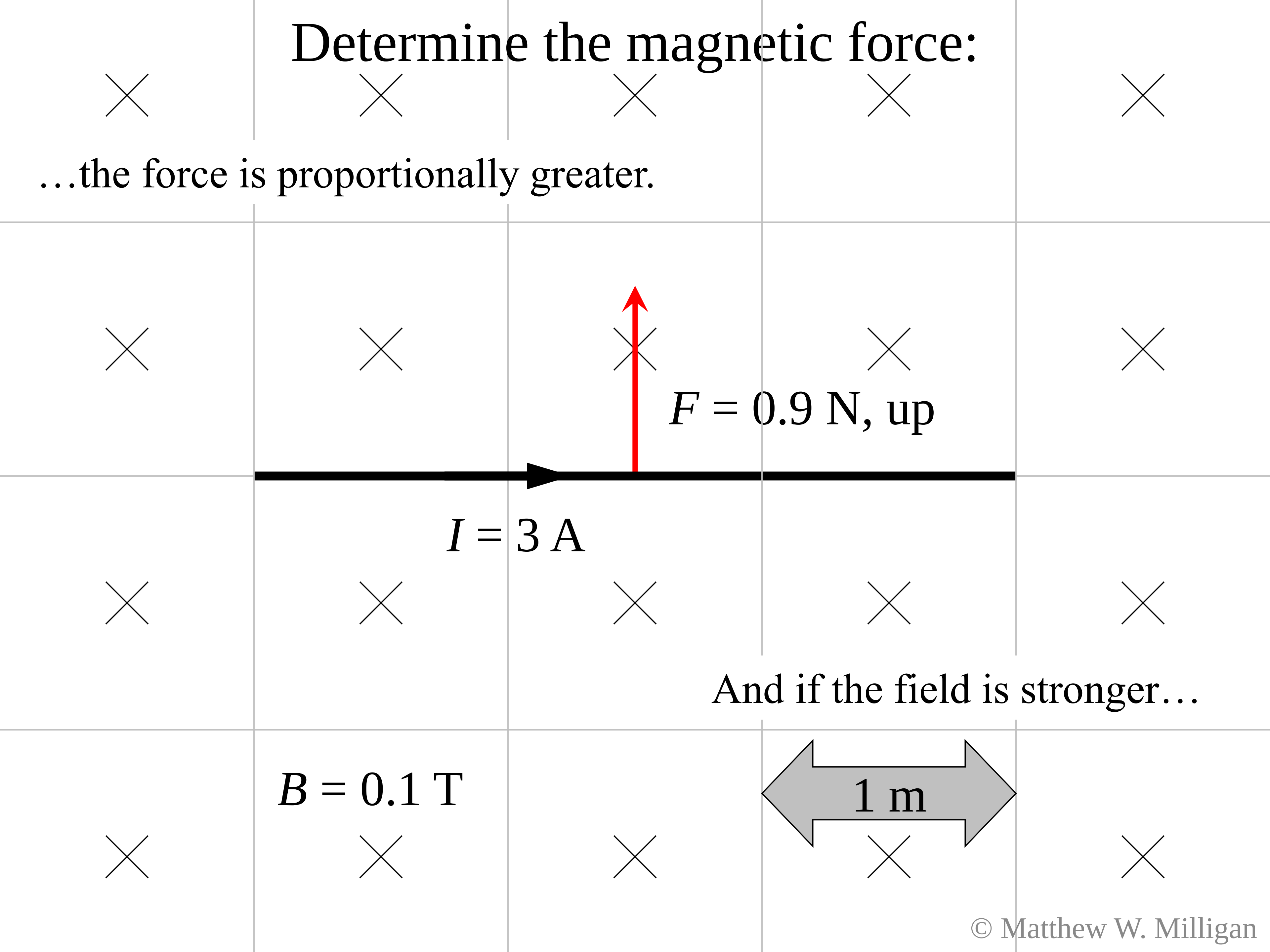
For a straight wire with current perpendicular to the magnetic field, the force is simply the product $F = IlB$, in this case $F = (3\text{ A})(2\text{ m})(0.1\text{ N/mA})$.



direction given by the right hand rule!

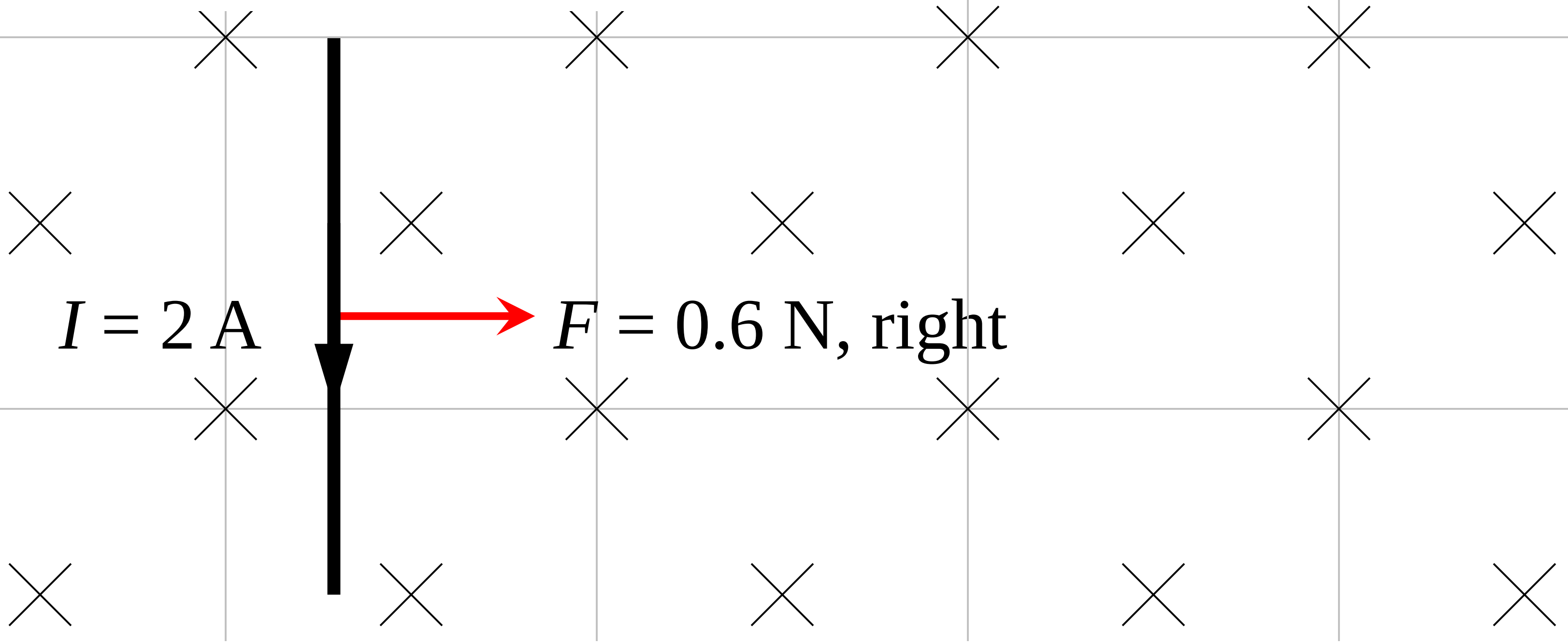
And if the wire is longer…



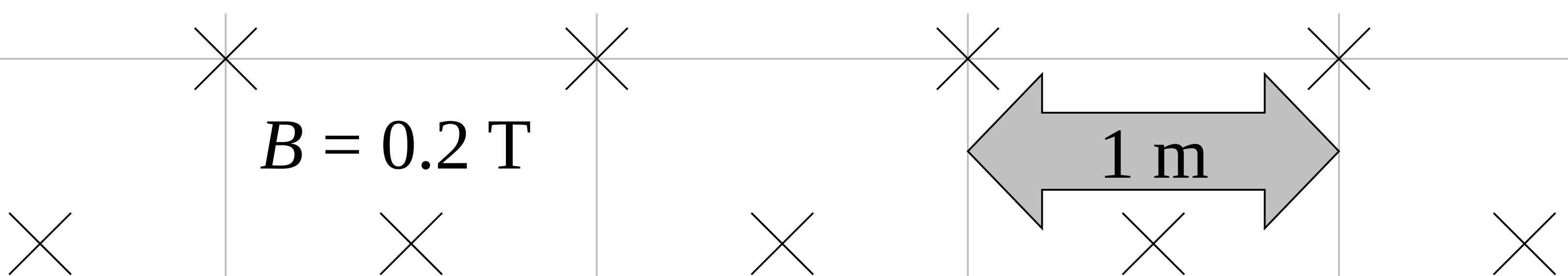

Determine the magnetic force:
…the force is proportionally greater.
F = 0.9 N, up
I = 3 A
And if the field is stronger…
B = 0.1 T
1 m
© Matthew W. Milligan

Determine the magnetic force:

…the force is proportionally greater.

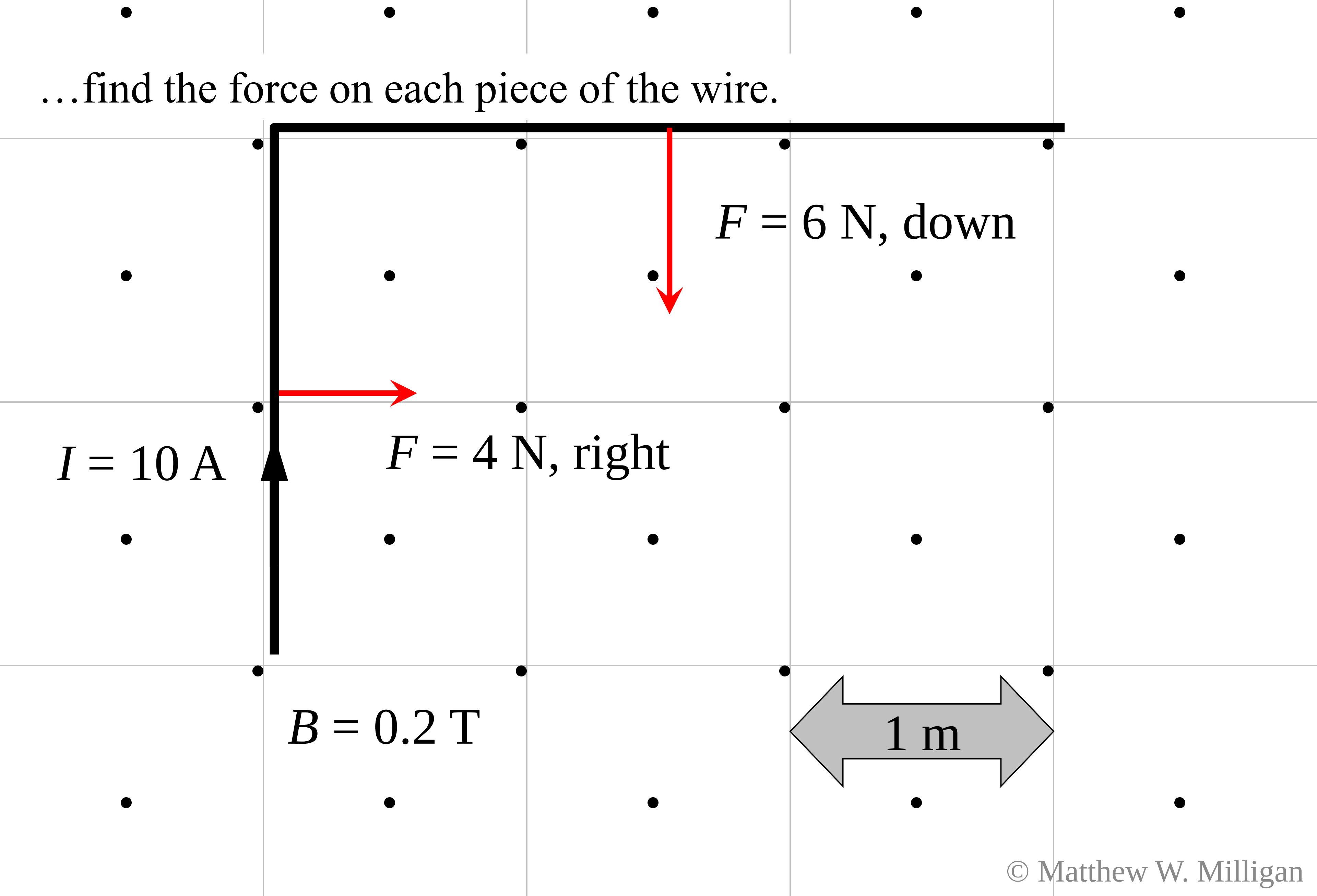


And if field and/or current is reversed and wire is bent…

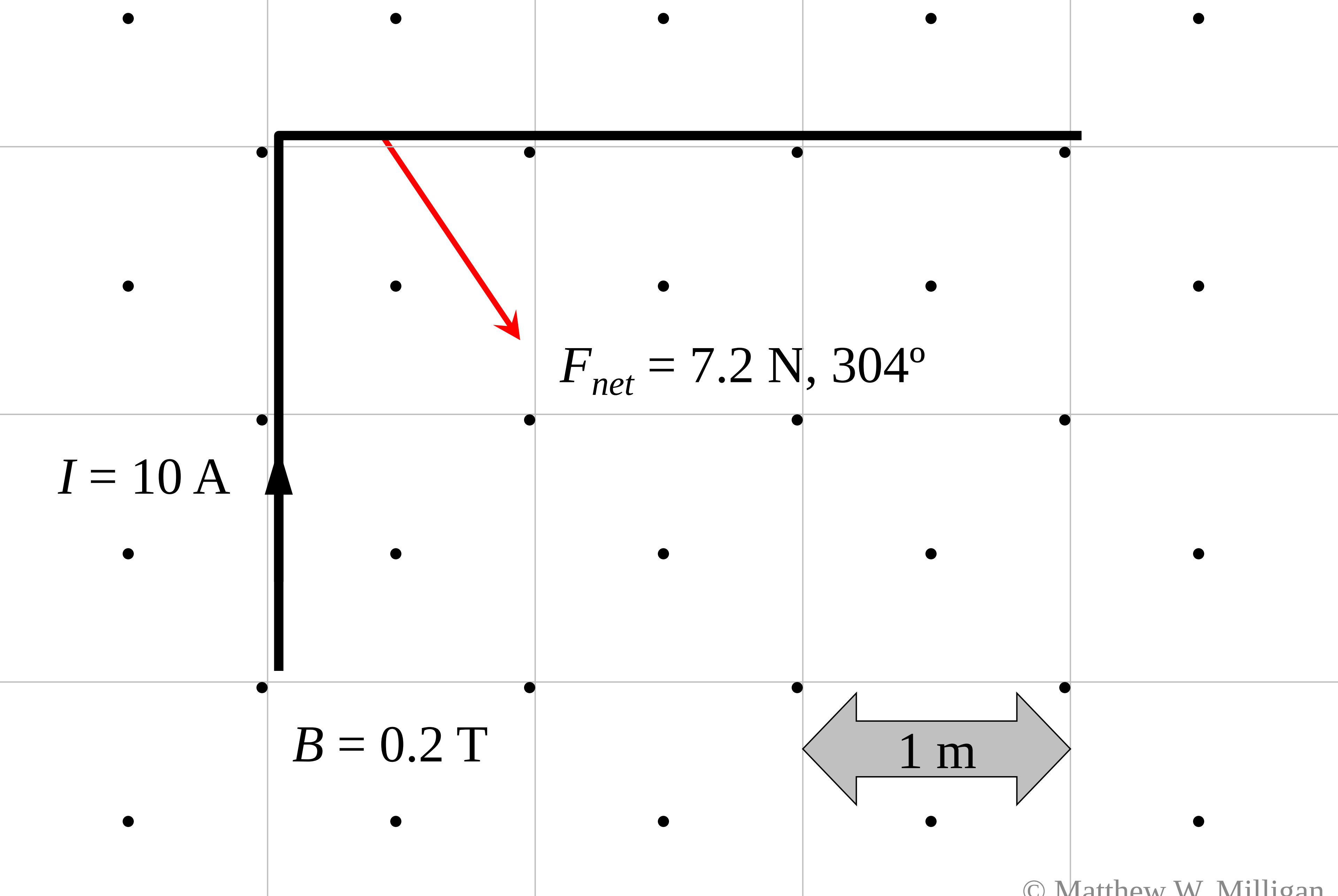


Determine the magnetic force:

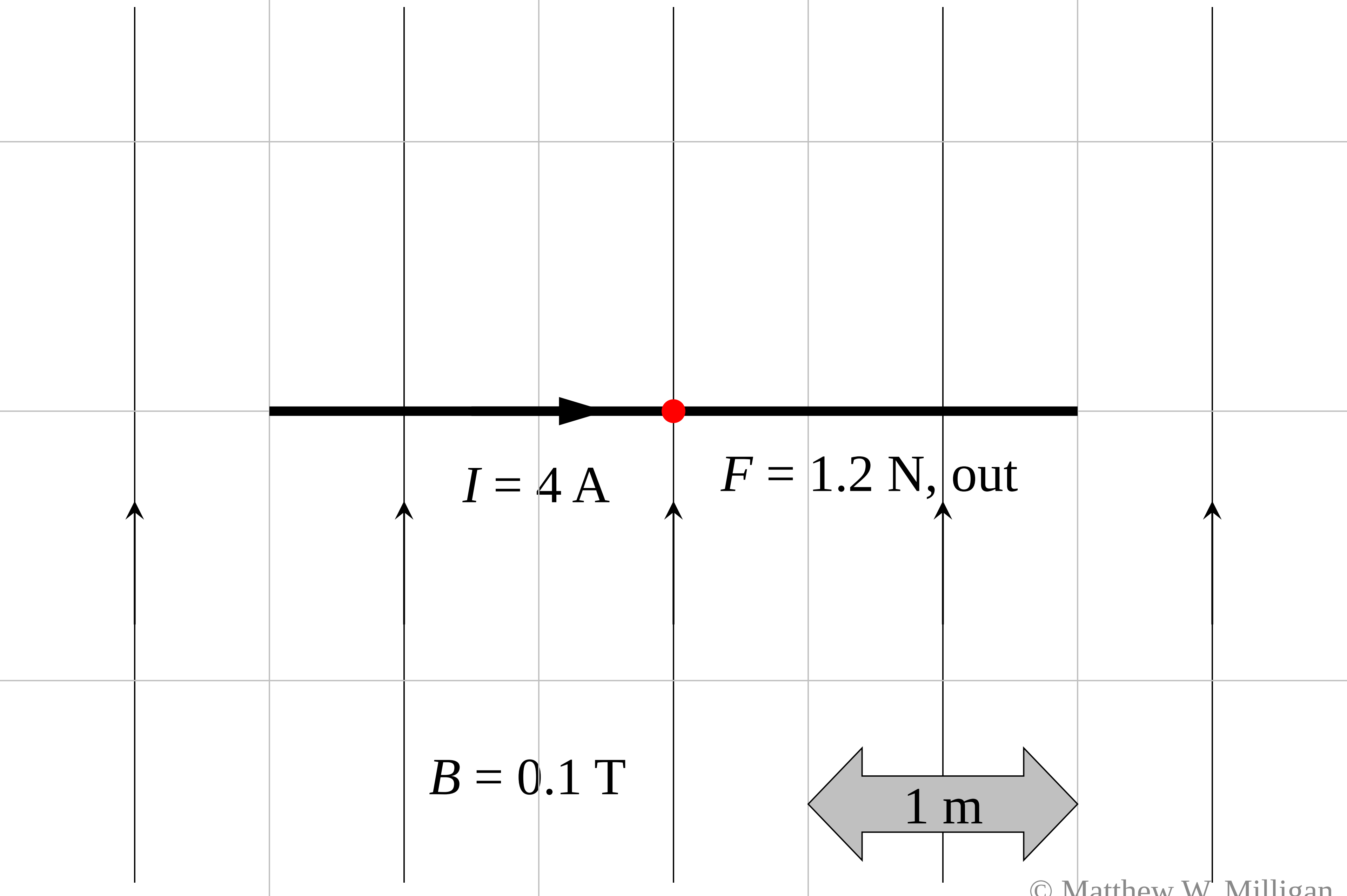
…find the force on each piece of the wire.



Determine the magnetic force:

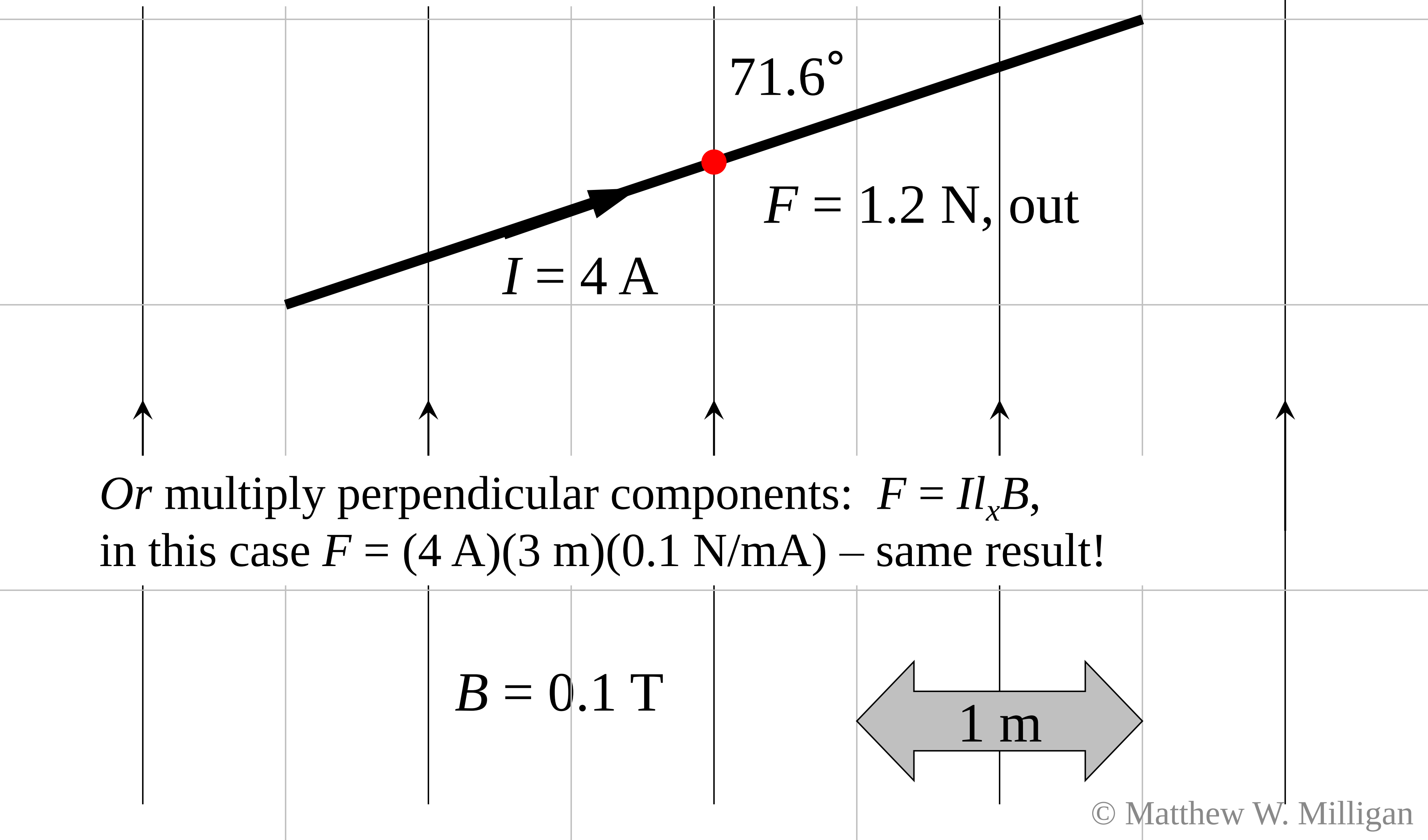


Determine the magnetic force:



Determine the magnetic force:

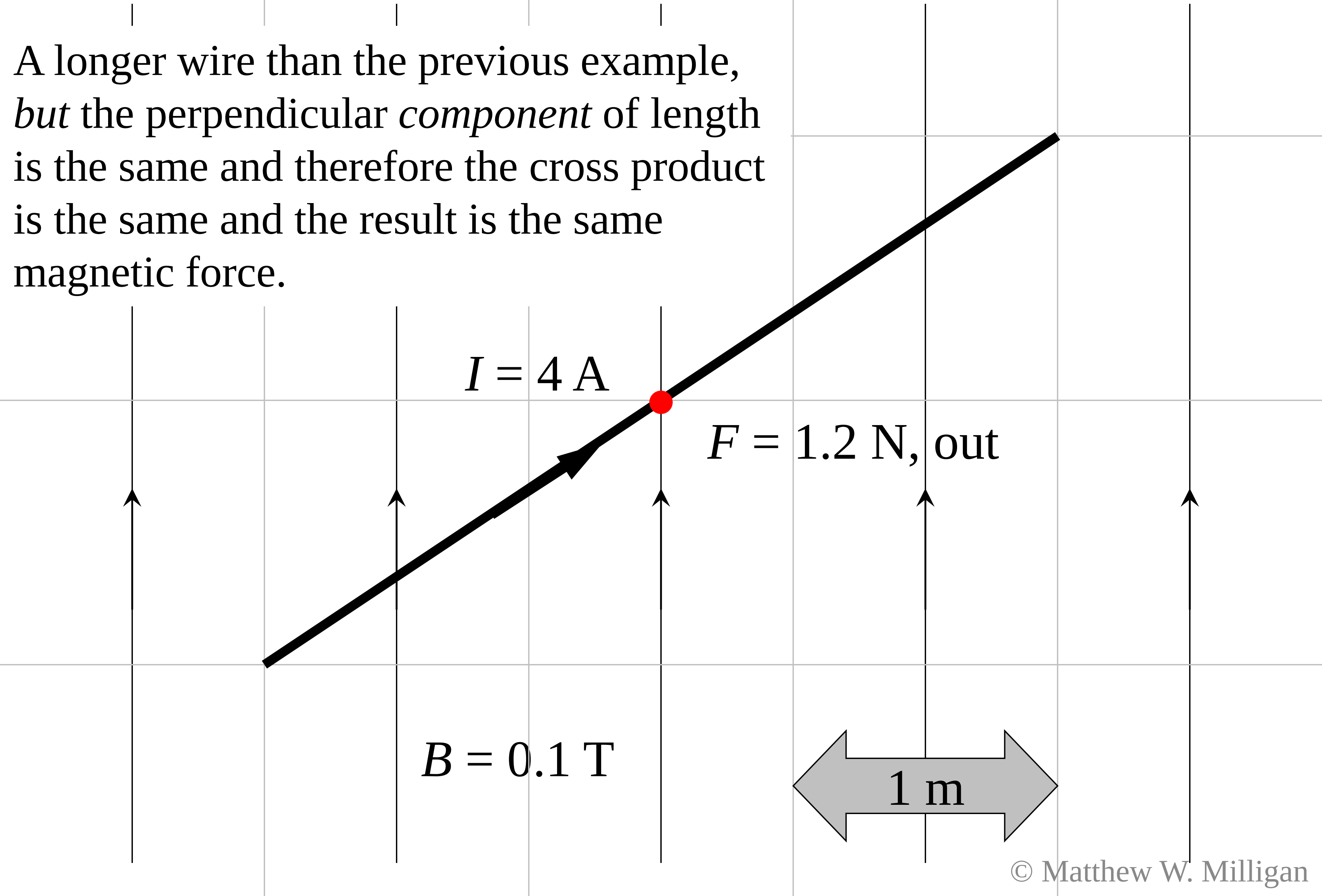
If current is *not* perpendicular use: $F = IlB \sin\theta$,
in this case F = (4 A)(3.16 m)(0.1 N/mA)(sin 71.6º).



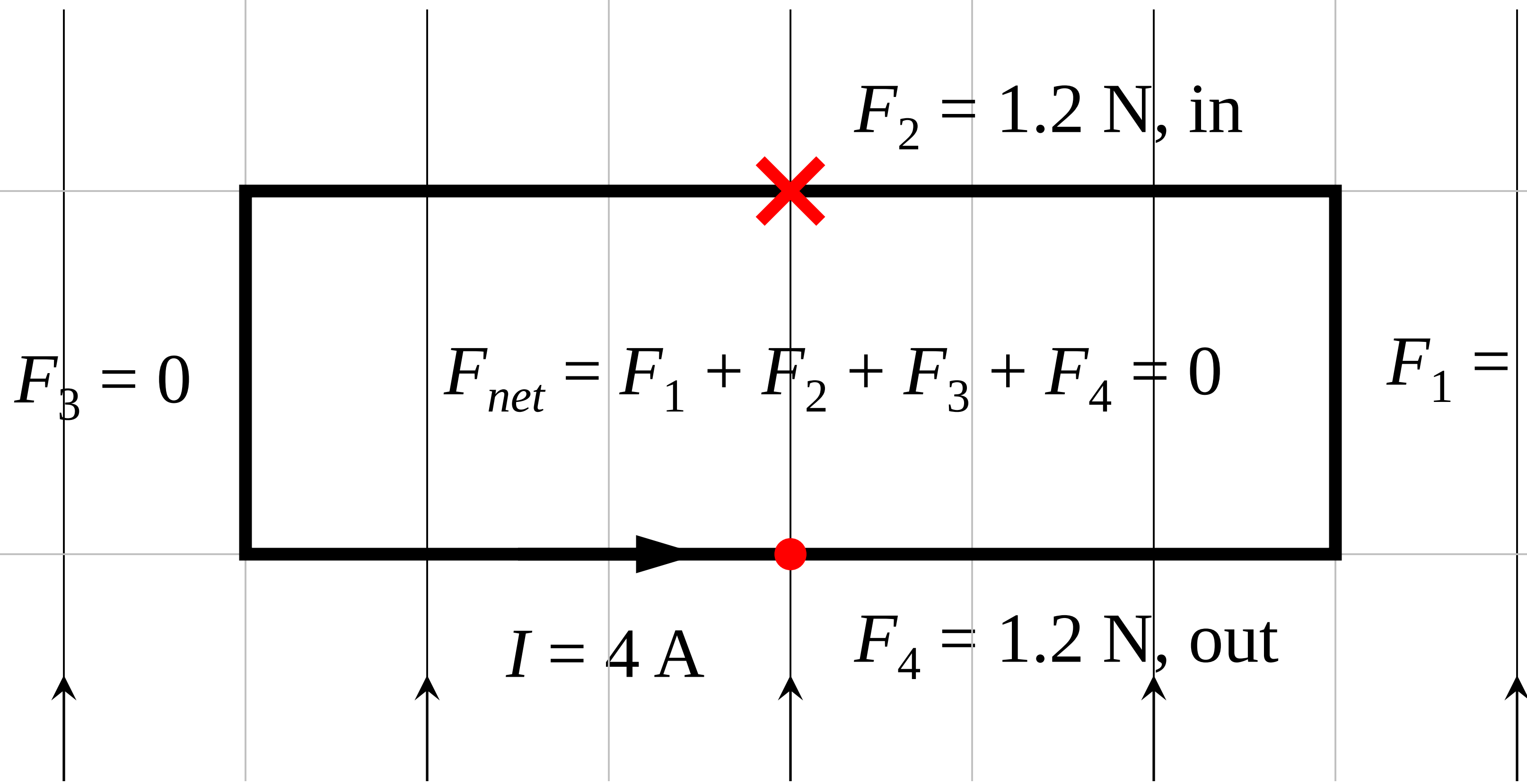
Or multiply perpendicular components: $F = Il_xB$,
in this case F = (4 A)(3 m)(0.1 N/mA) – same result!

Determine the magnetic force:

A longer wire than the previous example, *but* the perpendicular *component* of length is the same and therefore the cross product is the same and the result is the same magnetic force.



Determine the magnetic force:

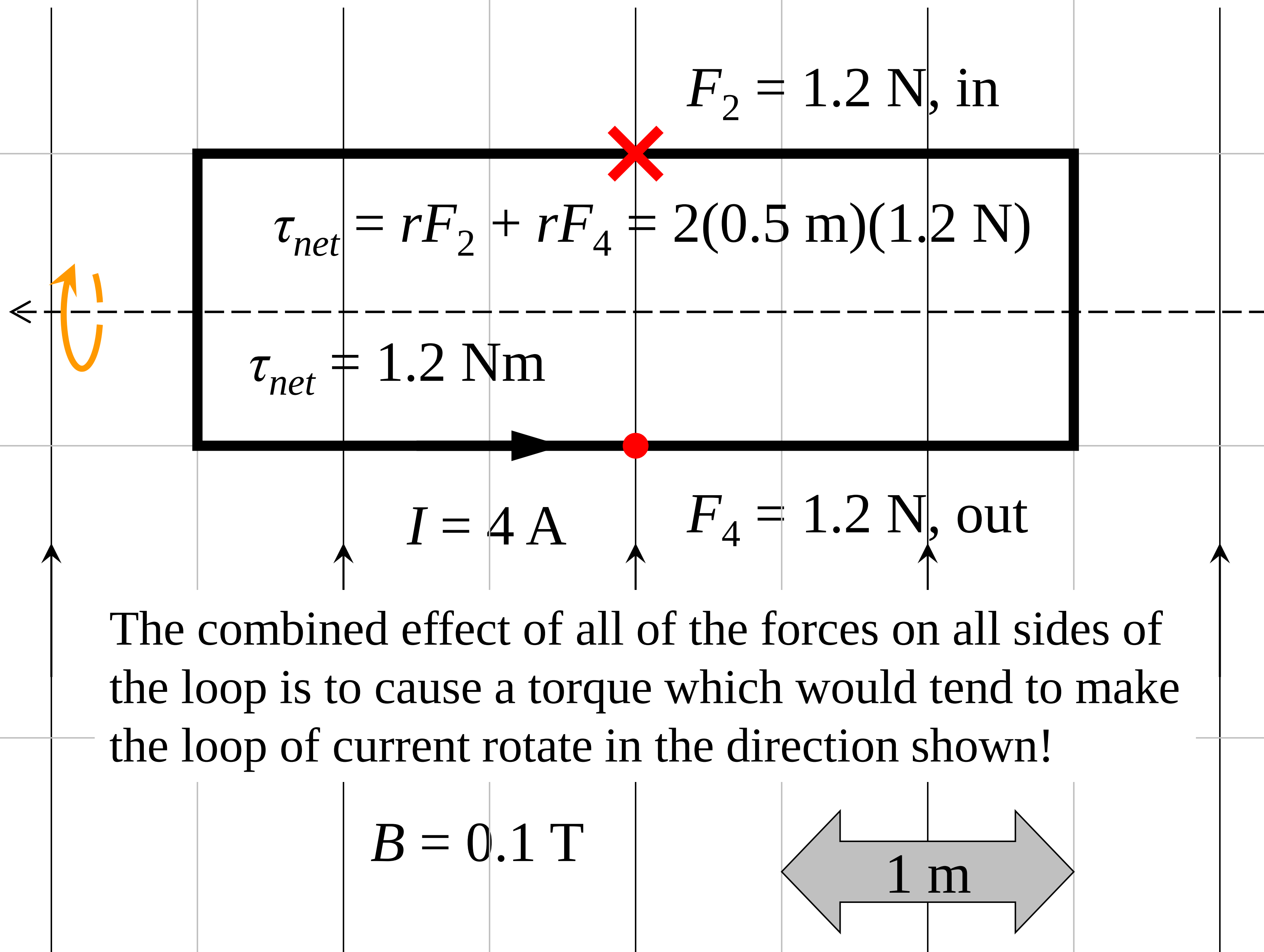


Net *force* on the loop of current is *zero*, but what about *torque*?

B = 0.1 T

1 m

Determine the magnetic torque:

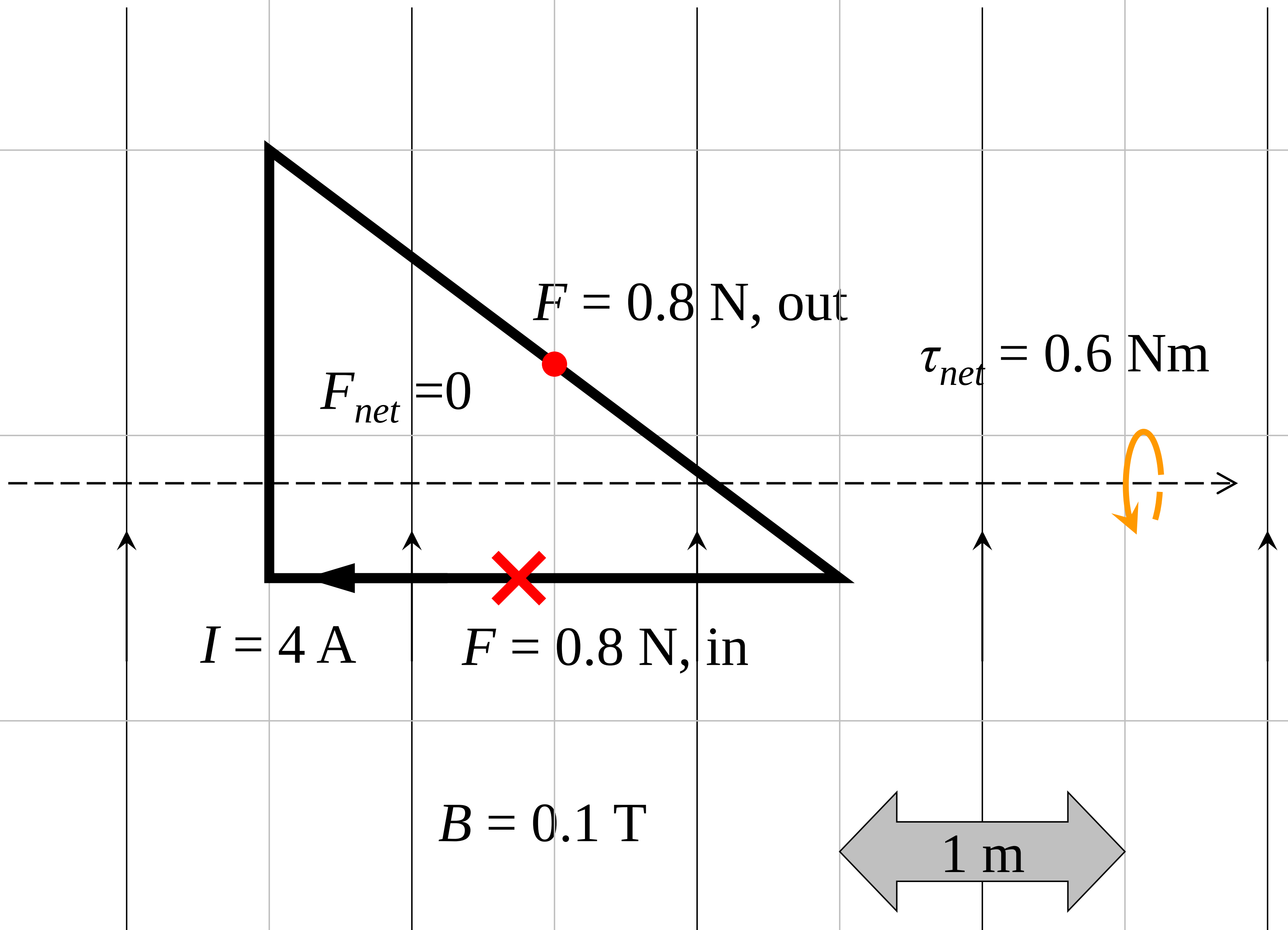


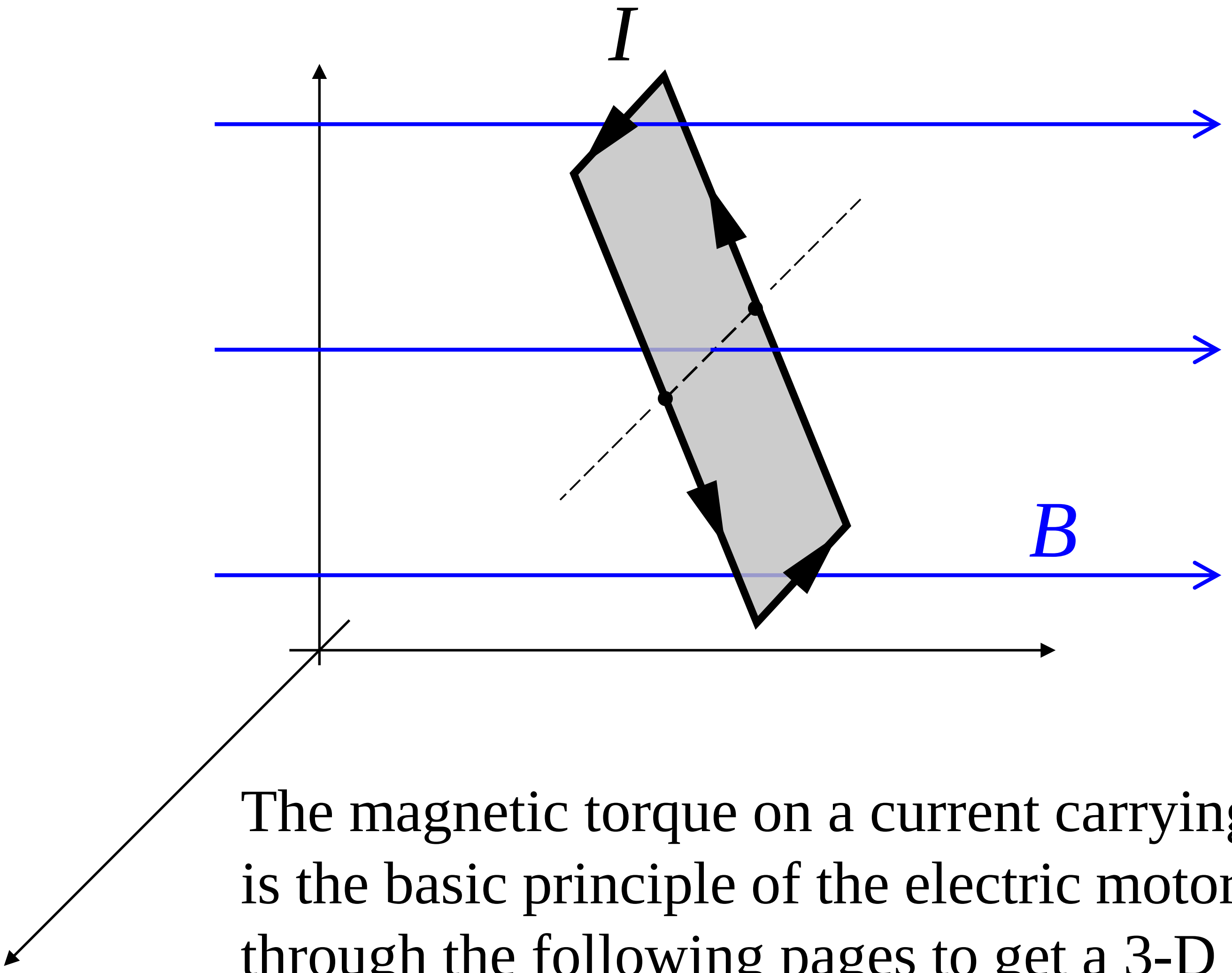
$$\tau_{net} = rF_2 + rF_4 = 2(0.5\text{ m})(1.2\text{ N})$$

$$\tau_{net} = 1.2\text{ Nm}$$

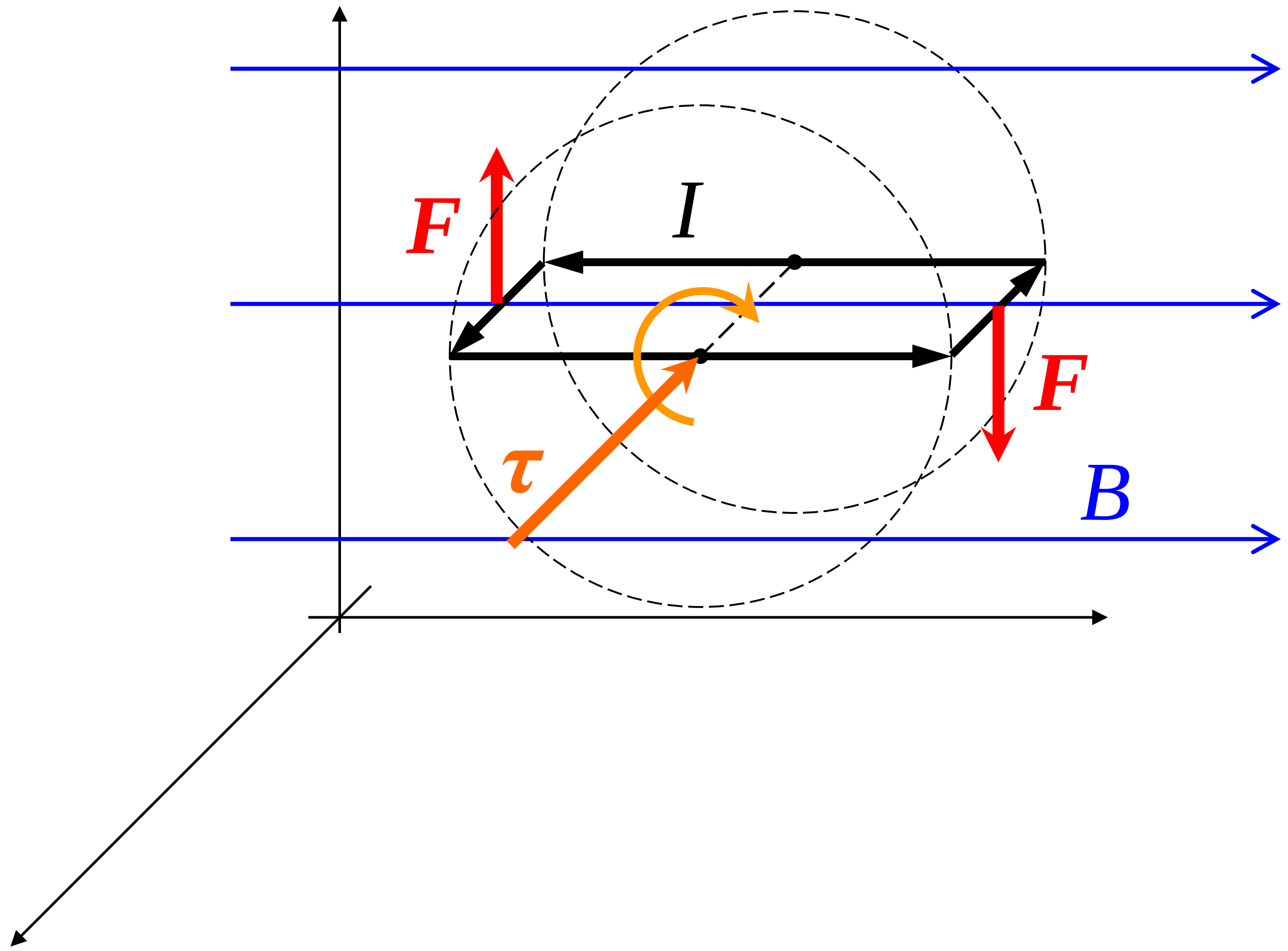
The combined effect of all of the forces on all sides of the loop is to cause a torque which would tend to make the loop of current rotate in the direction shown!

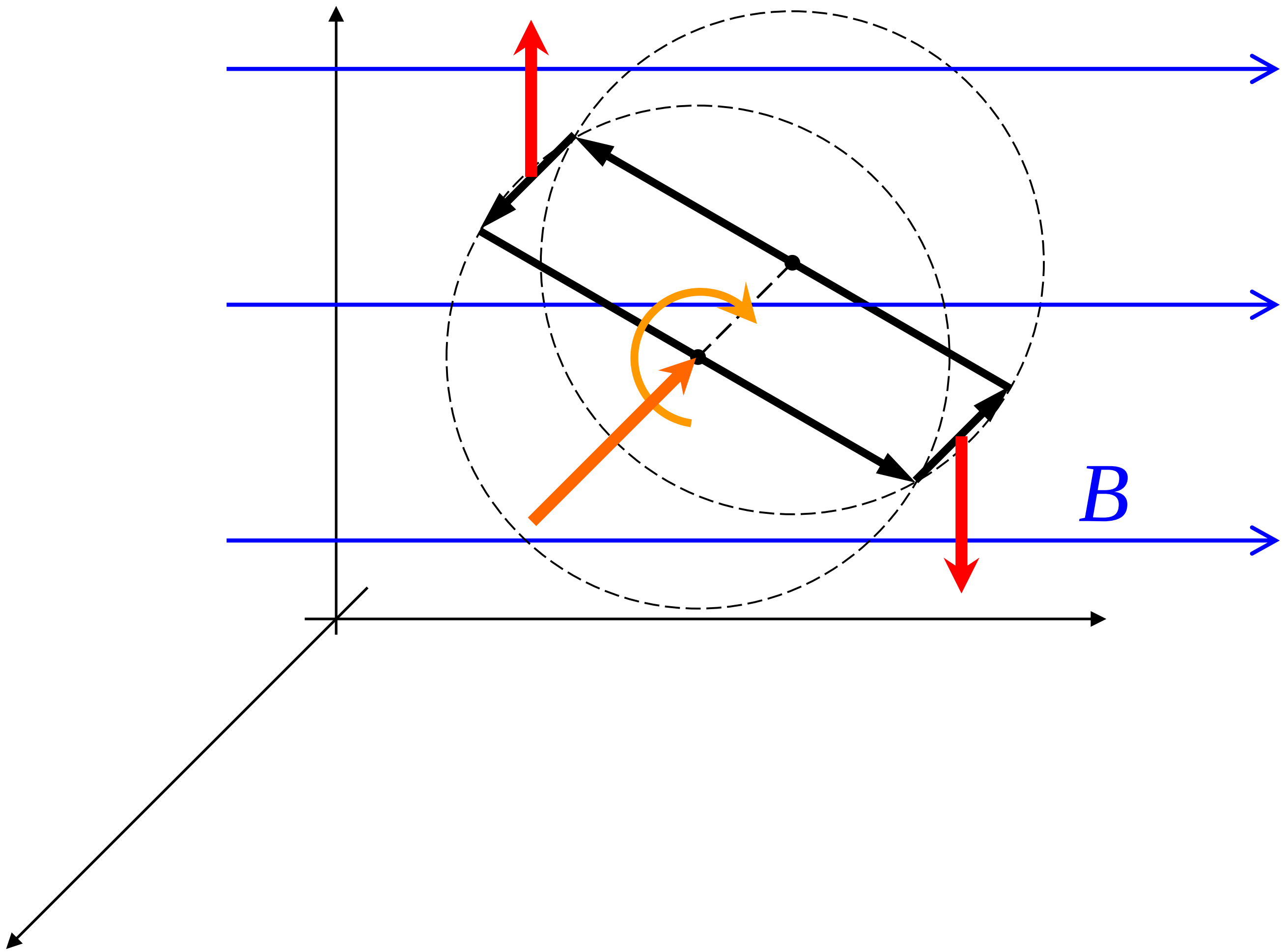
Determine the magnetic force and torque:

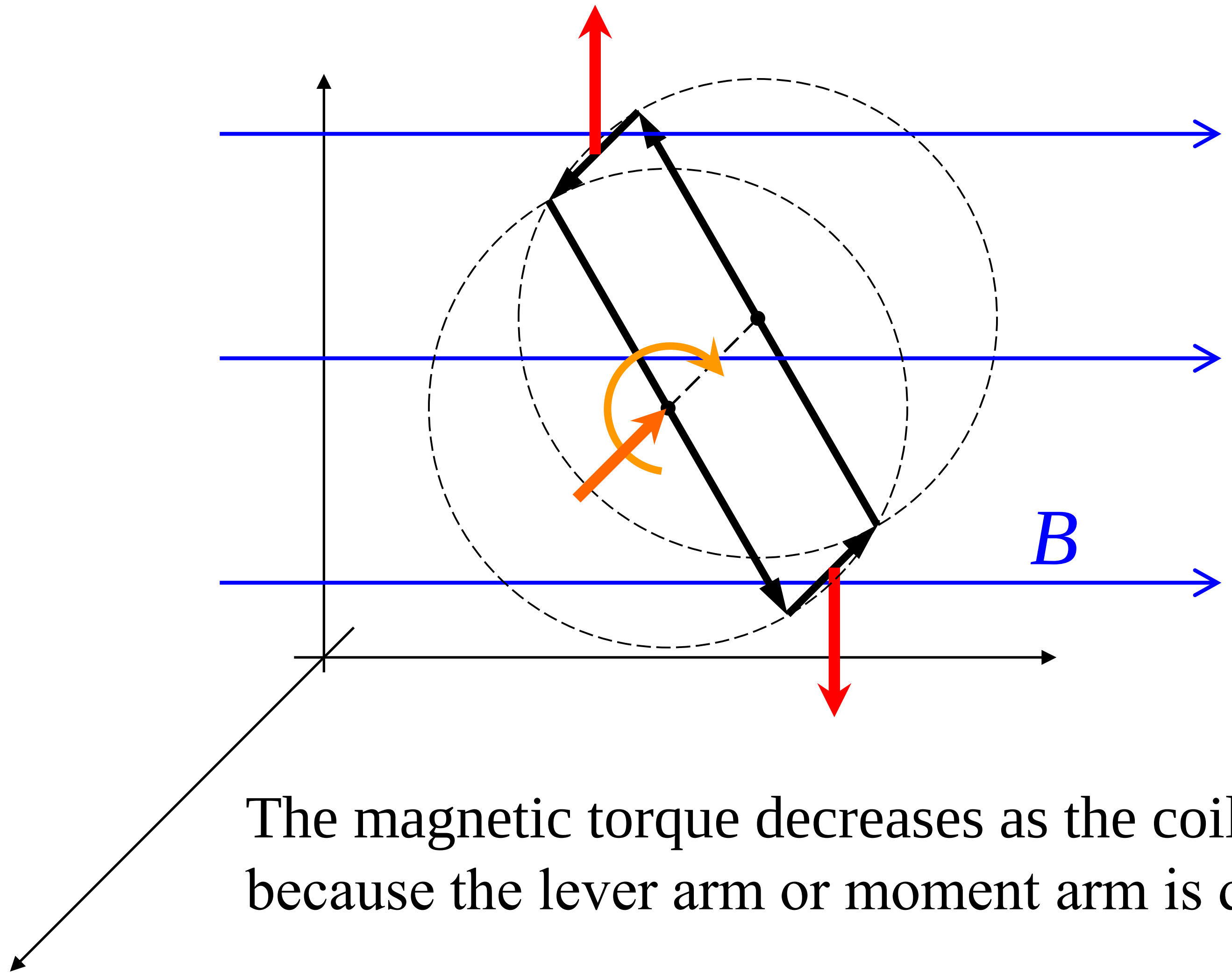




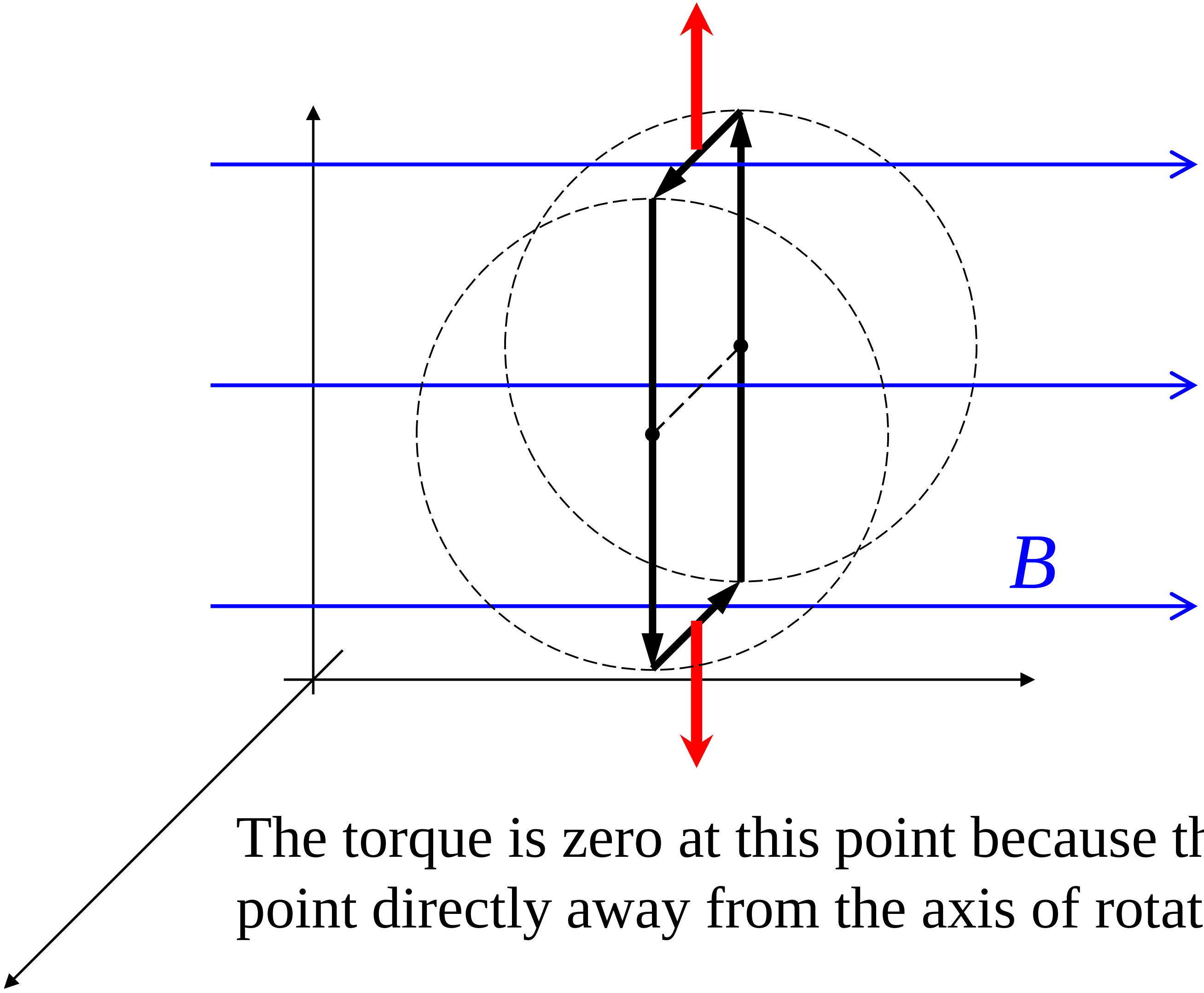
The magnetic torque on a current carrying loop of wire is the basic principle of the electric motor. Flip through the following pages to get a 3-D perspective of the changing torque as the loop rotates in a uniform magnetic field…


F
I
F
B
τ

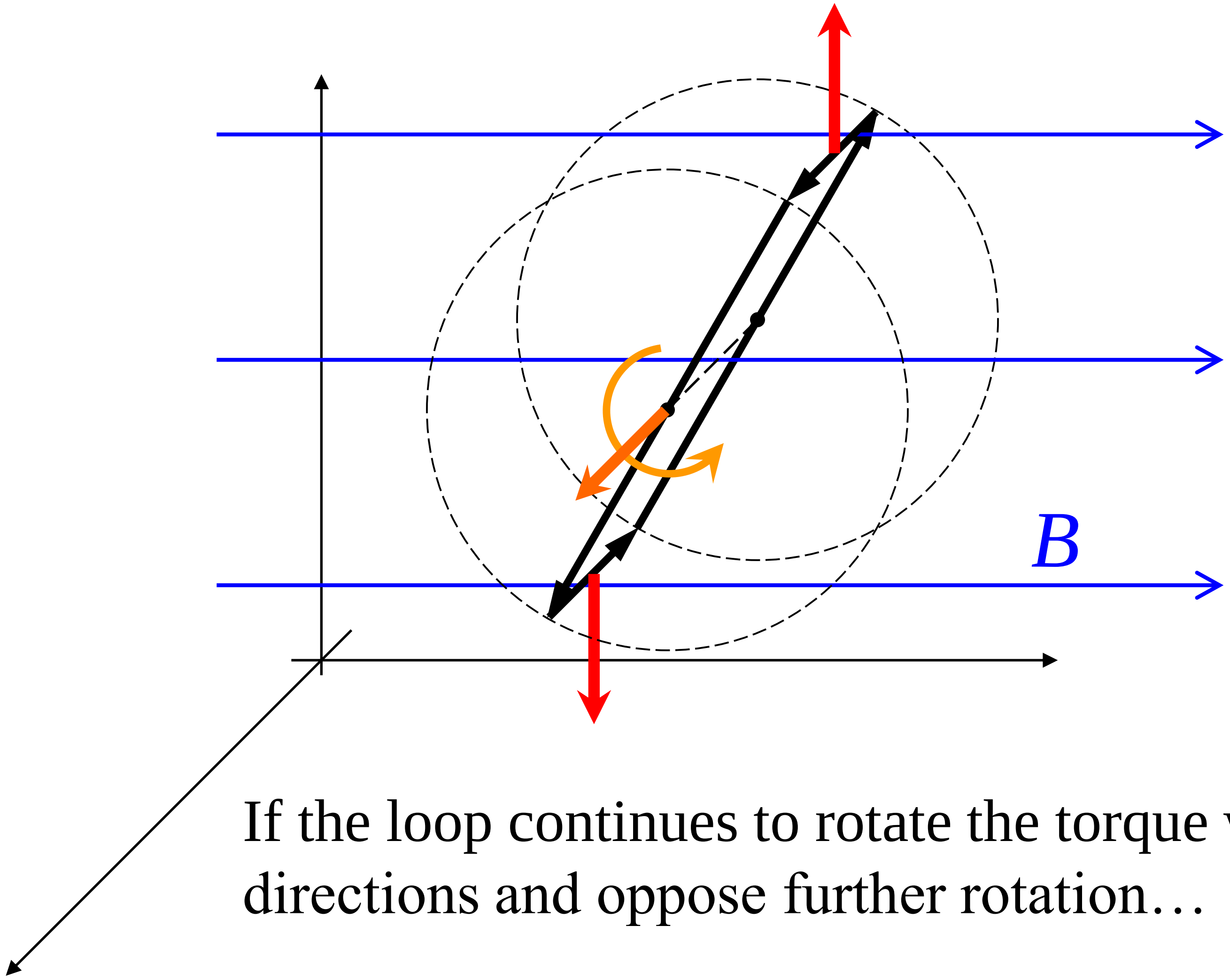

B



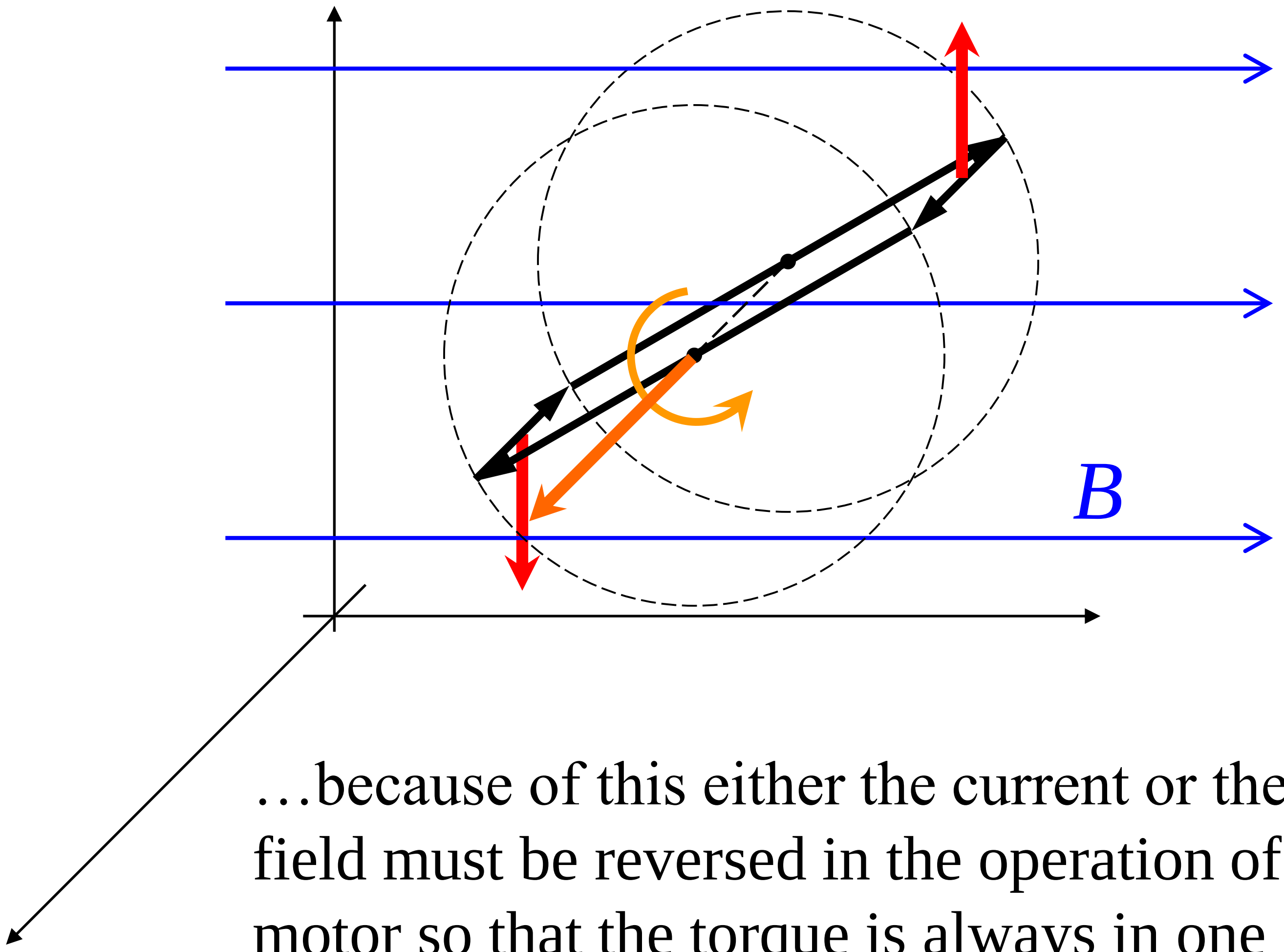
The magnetic torque decreases as the coil rotates because the lever arm or moment arm is decreasing…



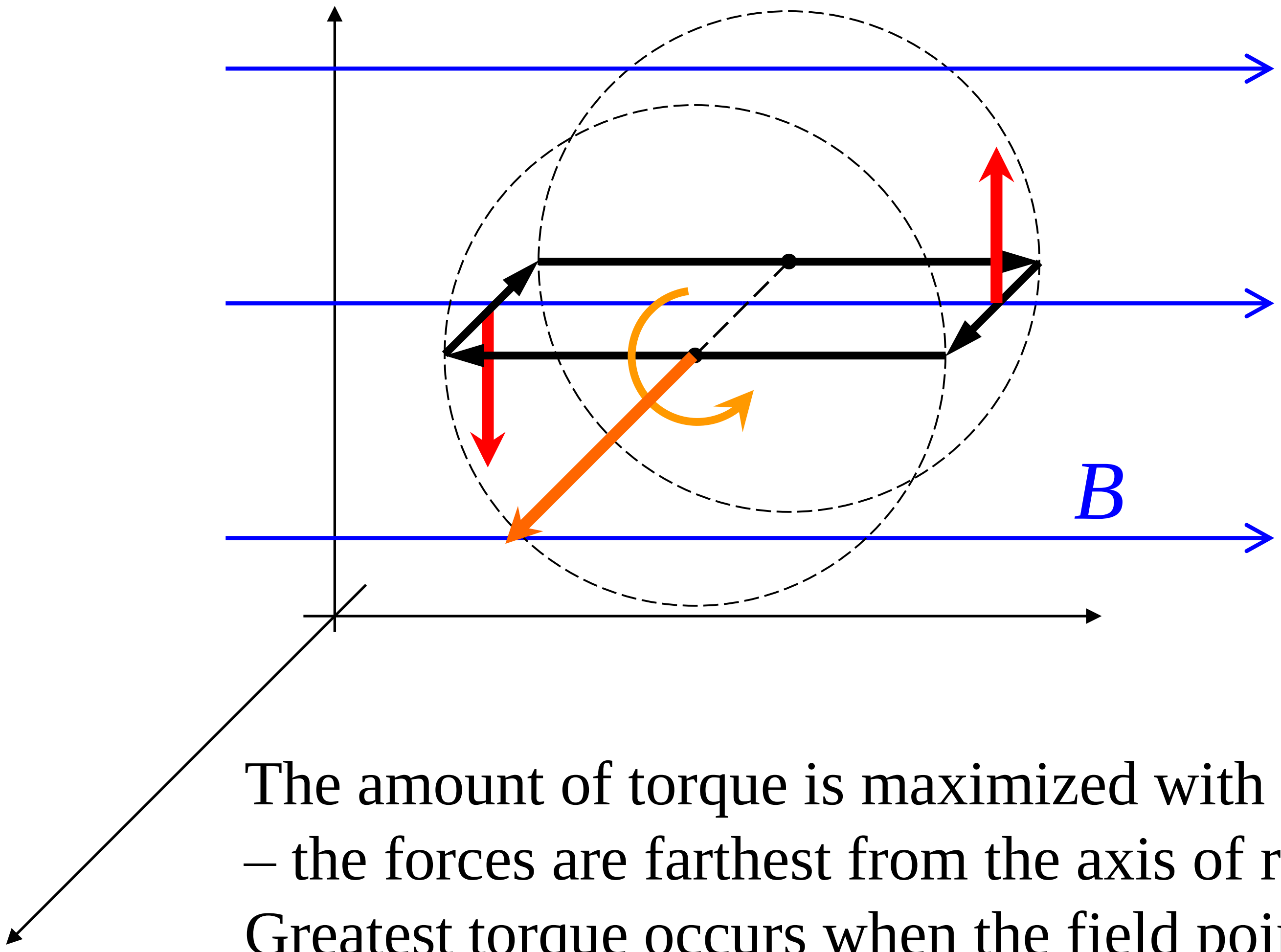
The torque is zero at this point because the forces point directly away from the axis of rotation.



If the loop continues to rotate the torque will reverse directions and oppose further rotation…

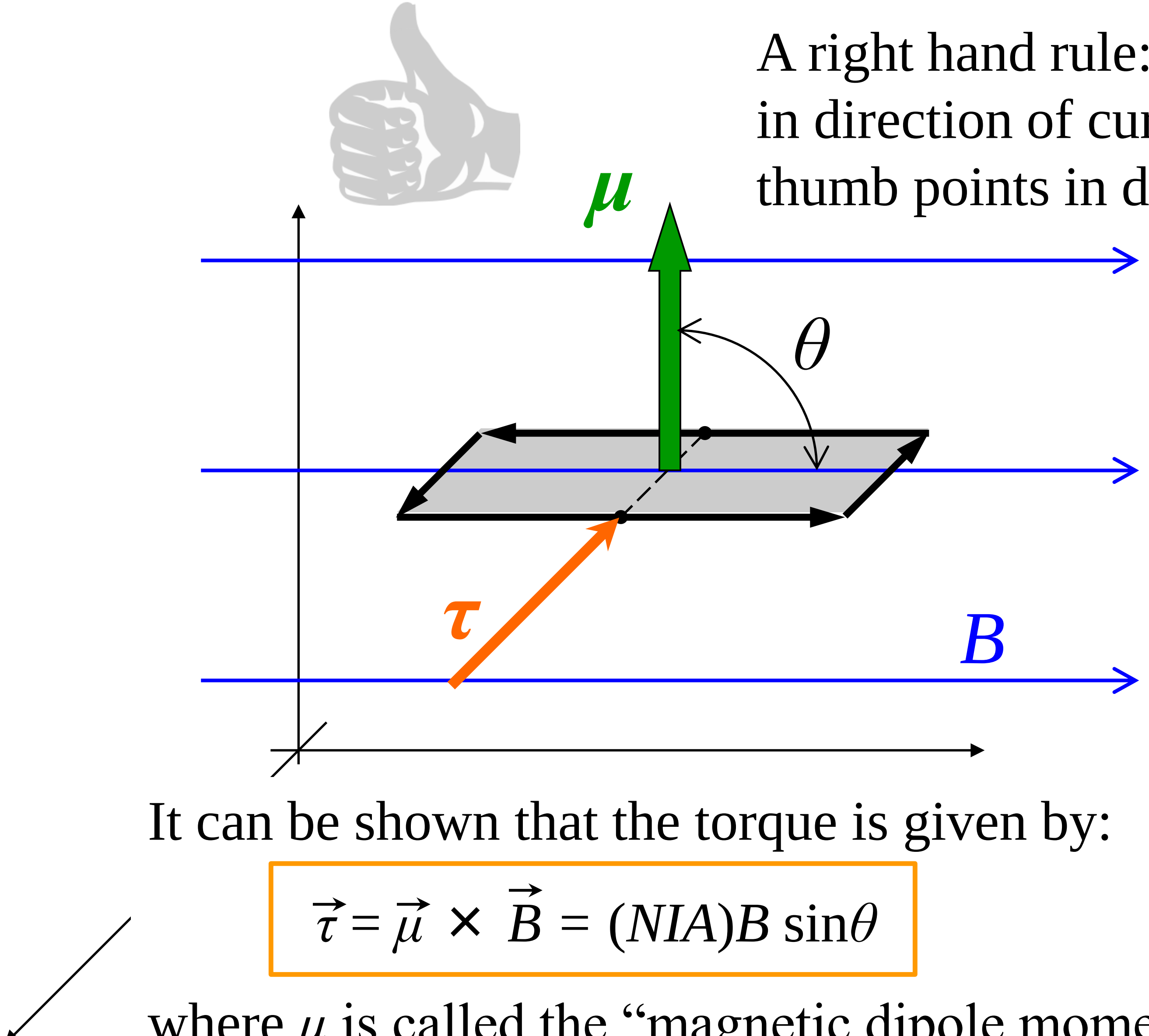


…because of this either the current or the magnetic field must be reversed in the operation of an electric motor so that the torque is always in one direction.



The amount of torque is maximized with this orientation – the forces are farthest from the axis of rotation. Greatest torque occurs when the field points across the area bound by the loop of current carrying wire.

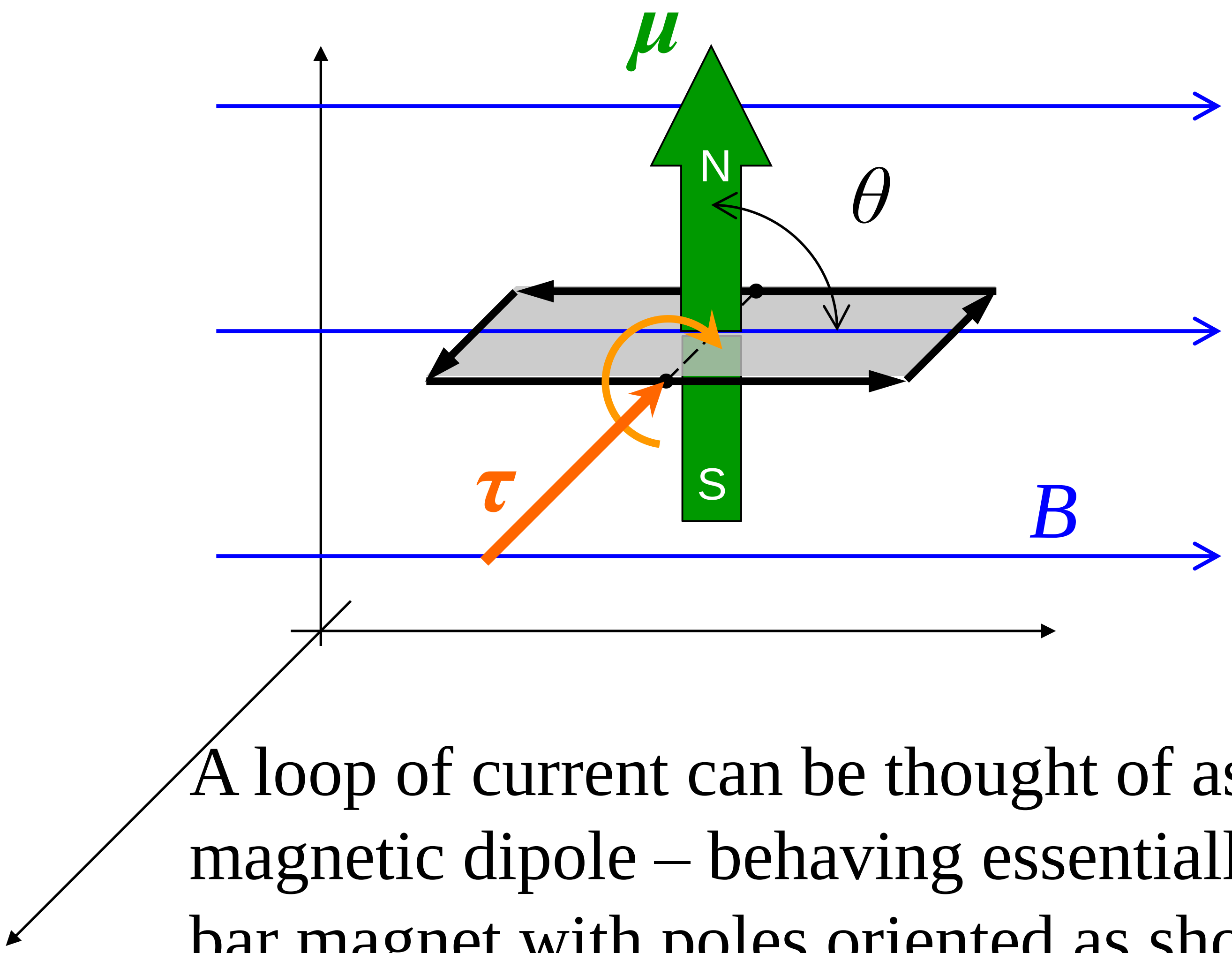
A right hand rule: curl fingers in direction of current I and thumb points in direction of μ.



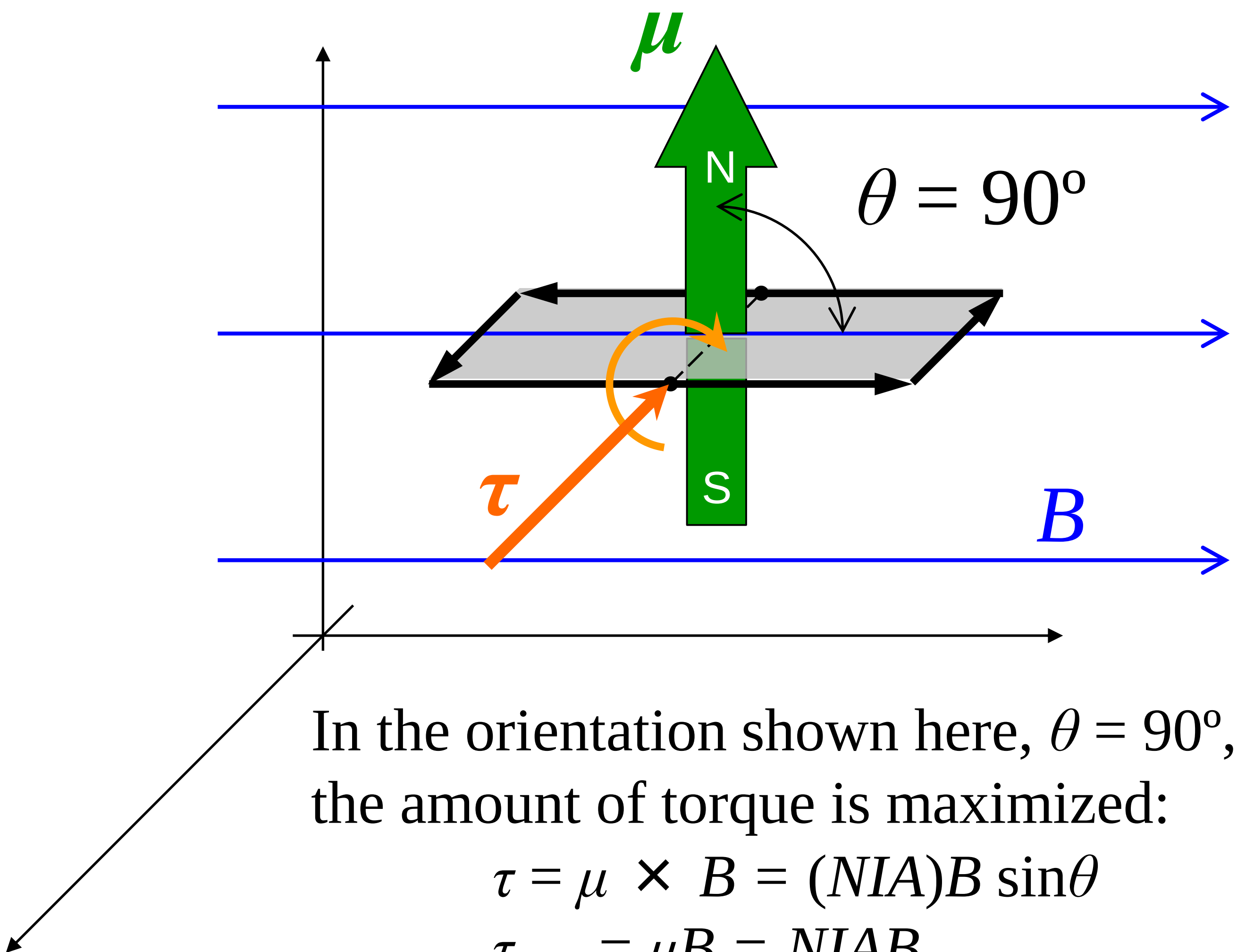
It can be shown that the torque is given by:

$$\vec{\tau} = \vec{\mu} \times \vec{B} = (NIA)B \sin\theta$$

where μ is called the "magnetic dipole moment" and equals the product of number of turns N, current I, and the area A bound by the current.



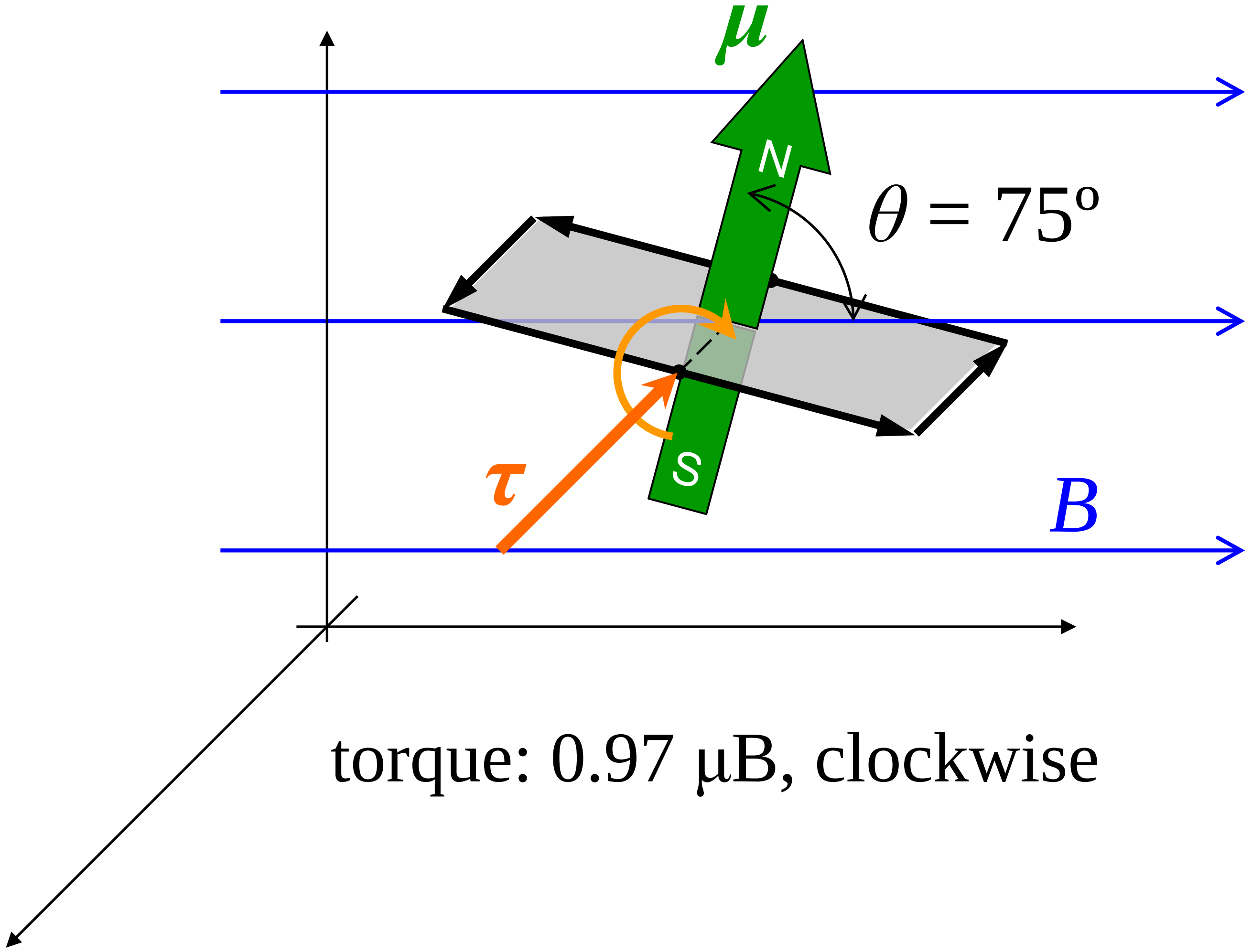
A loop of current can be thought of as a magnetic dipole – behaving essentially as a bar magnet with poles oriented as shown here.



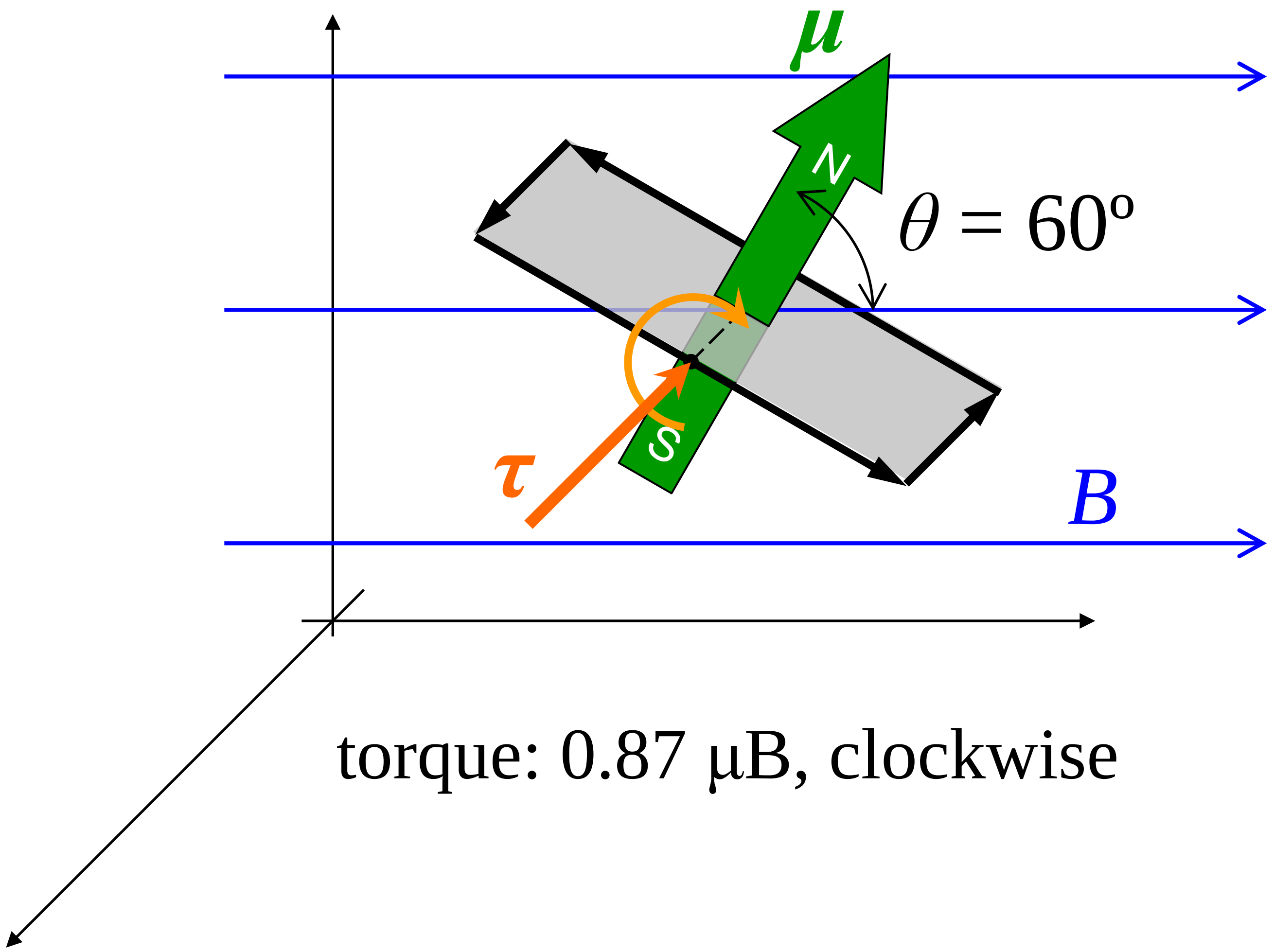
In the orientation shown here, $\theta = 90^o$, the amount of torque is maximized:

$$\tau = \mu \times B = (NIA)B \sin\theta$$

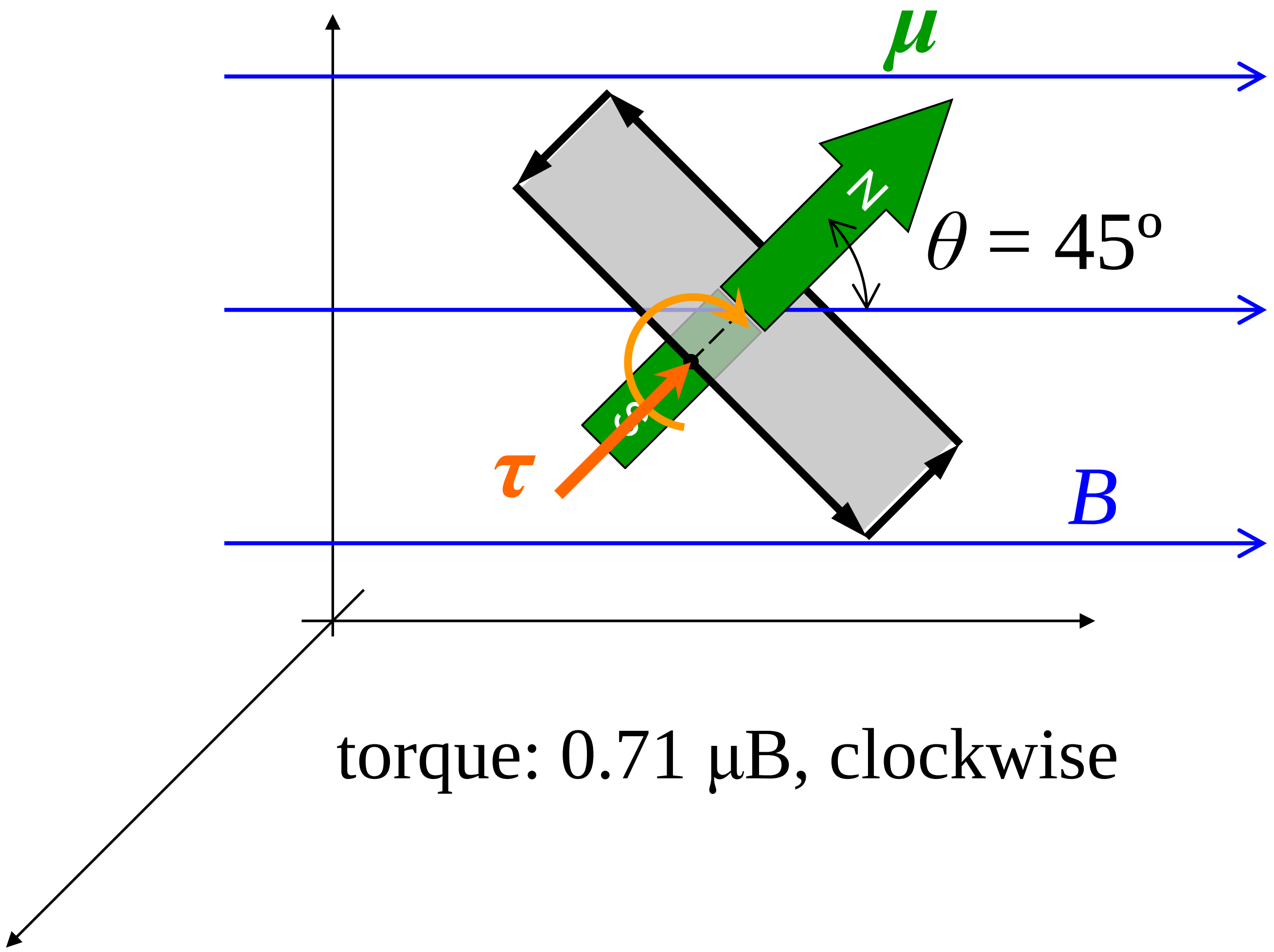
$$\tau_{max} = \mu B = NIAB$$



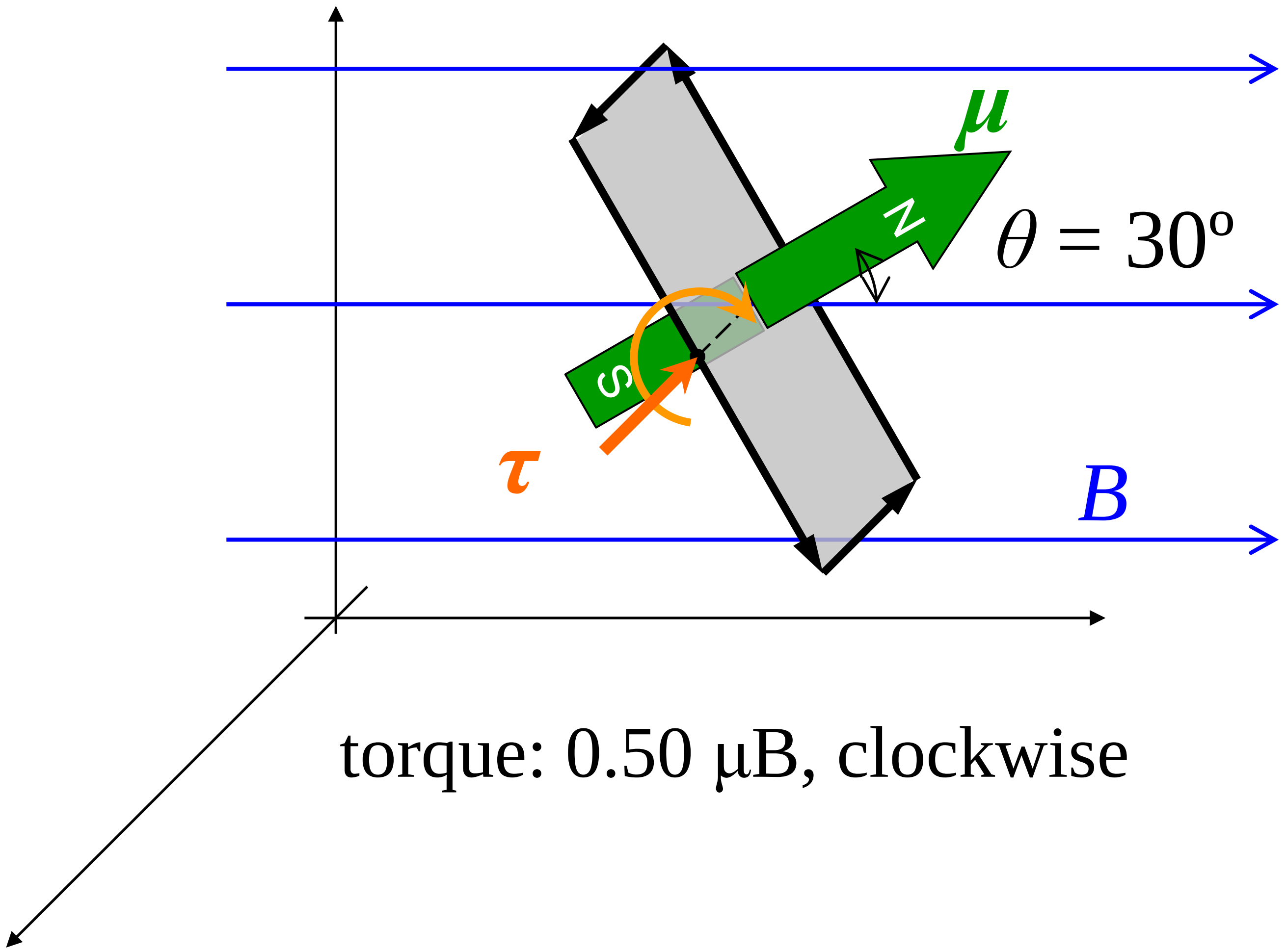
torque: 0.97 μB, clockwise



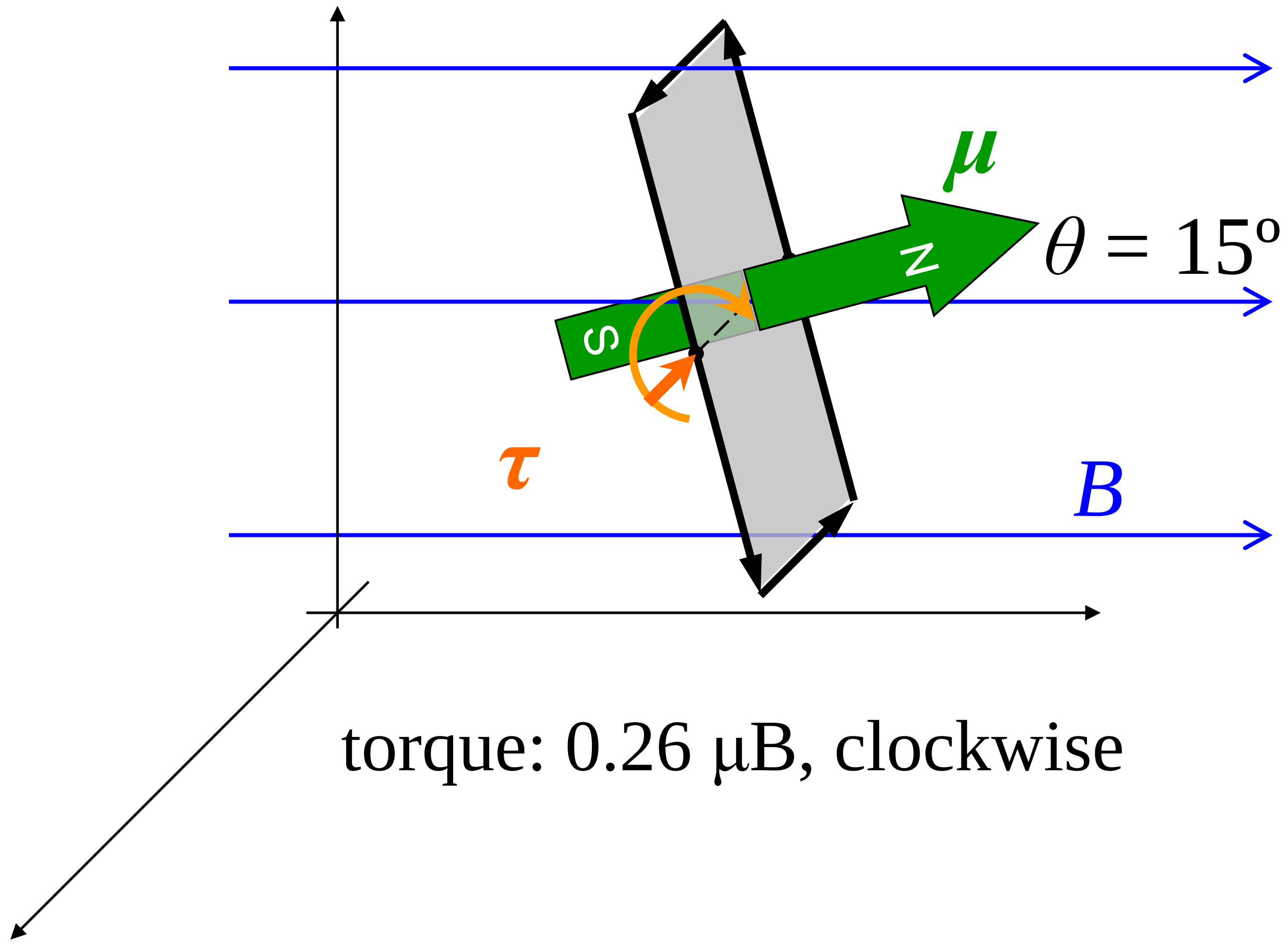
torque: 0.87 μB, clockwise



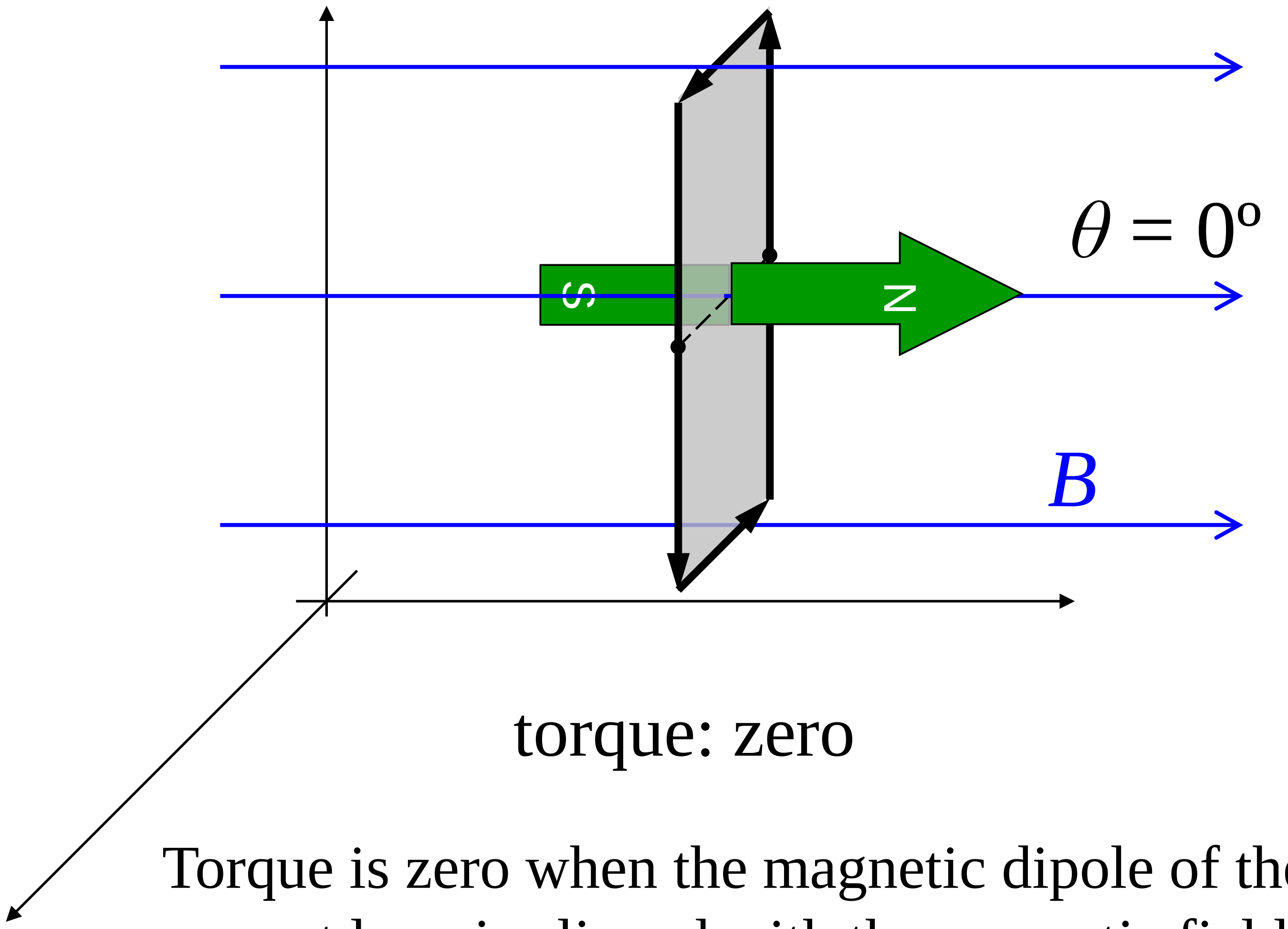
torque: 0.71 μB, clockwise



torque: 0.50 μB, clockwise

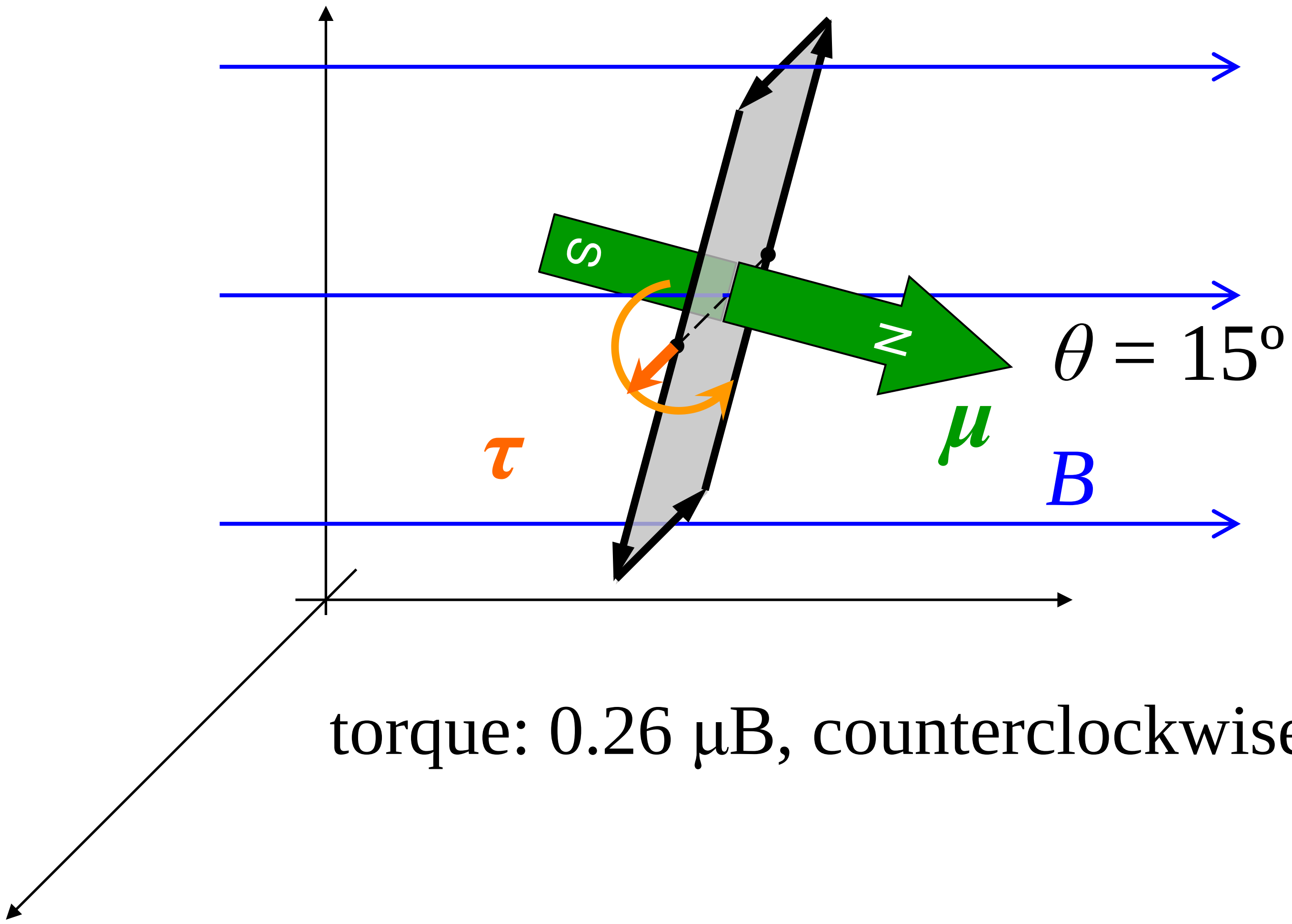


torque: 0.26 μB, clockwise

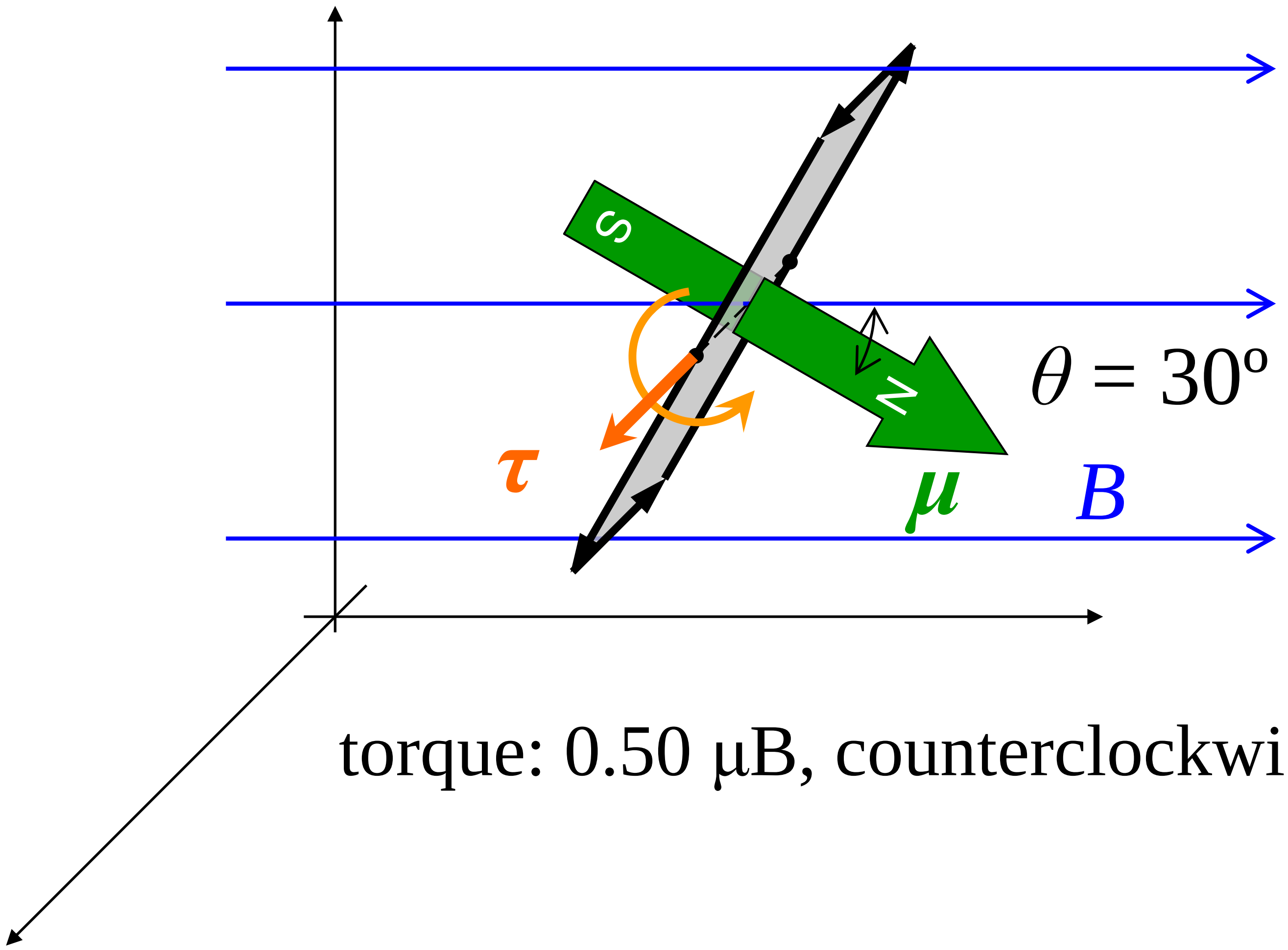


torque: zero

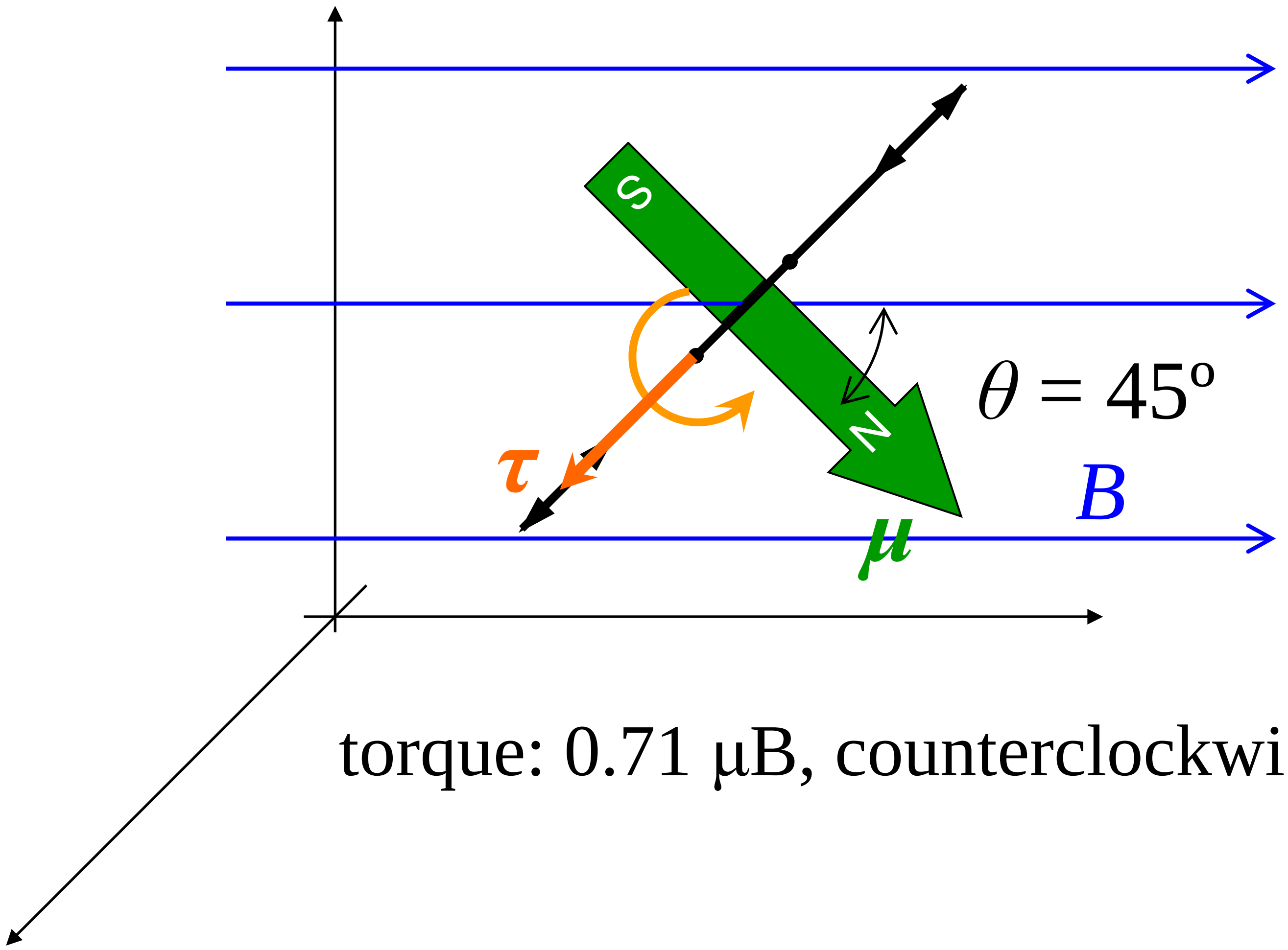
Torque is zero when the magnetic dipole of the current loop is aligned with the magnetic field – an angular position of equilibrium.



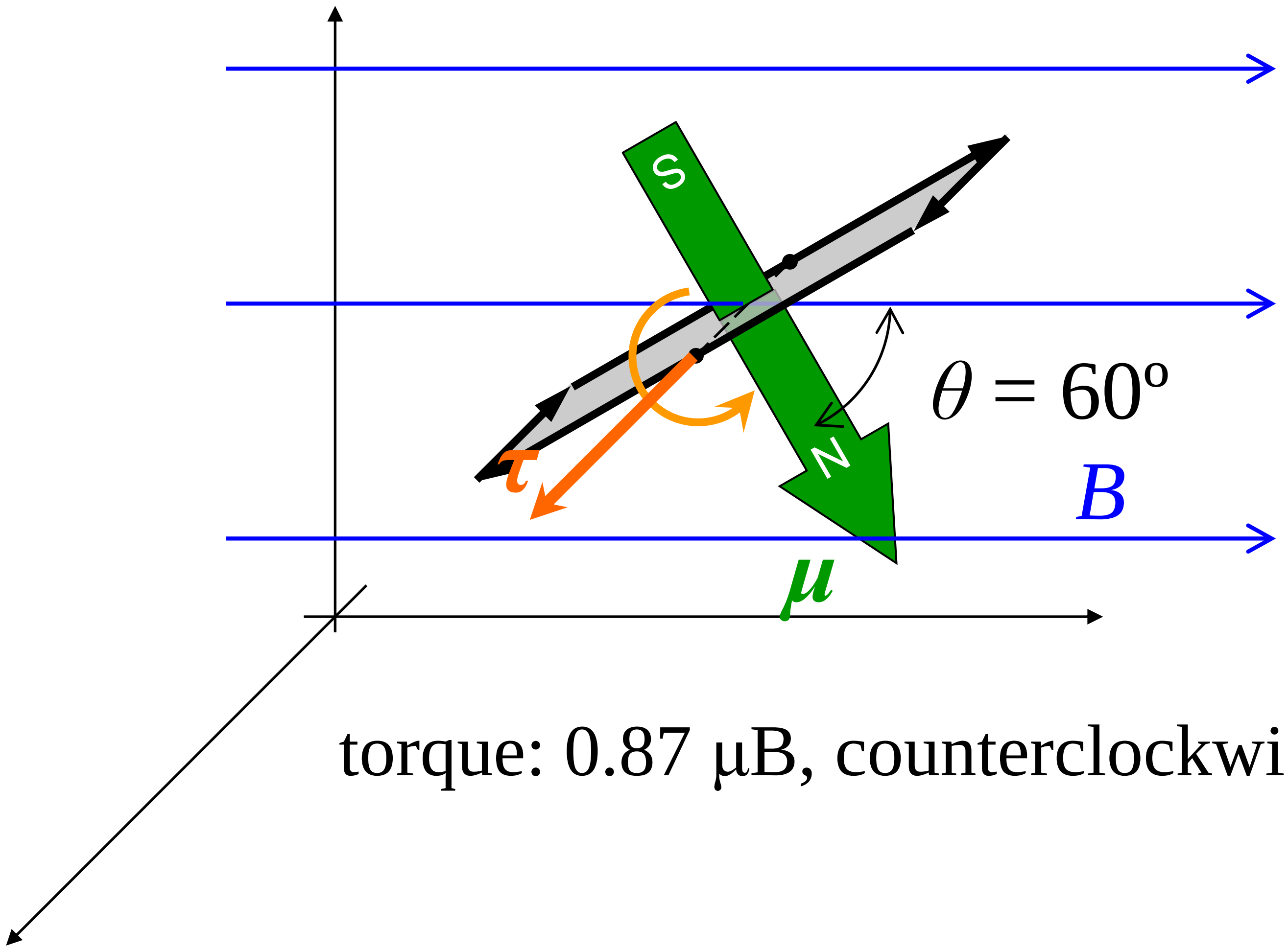
torque: 0.26 μB, counterclockwise



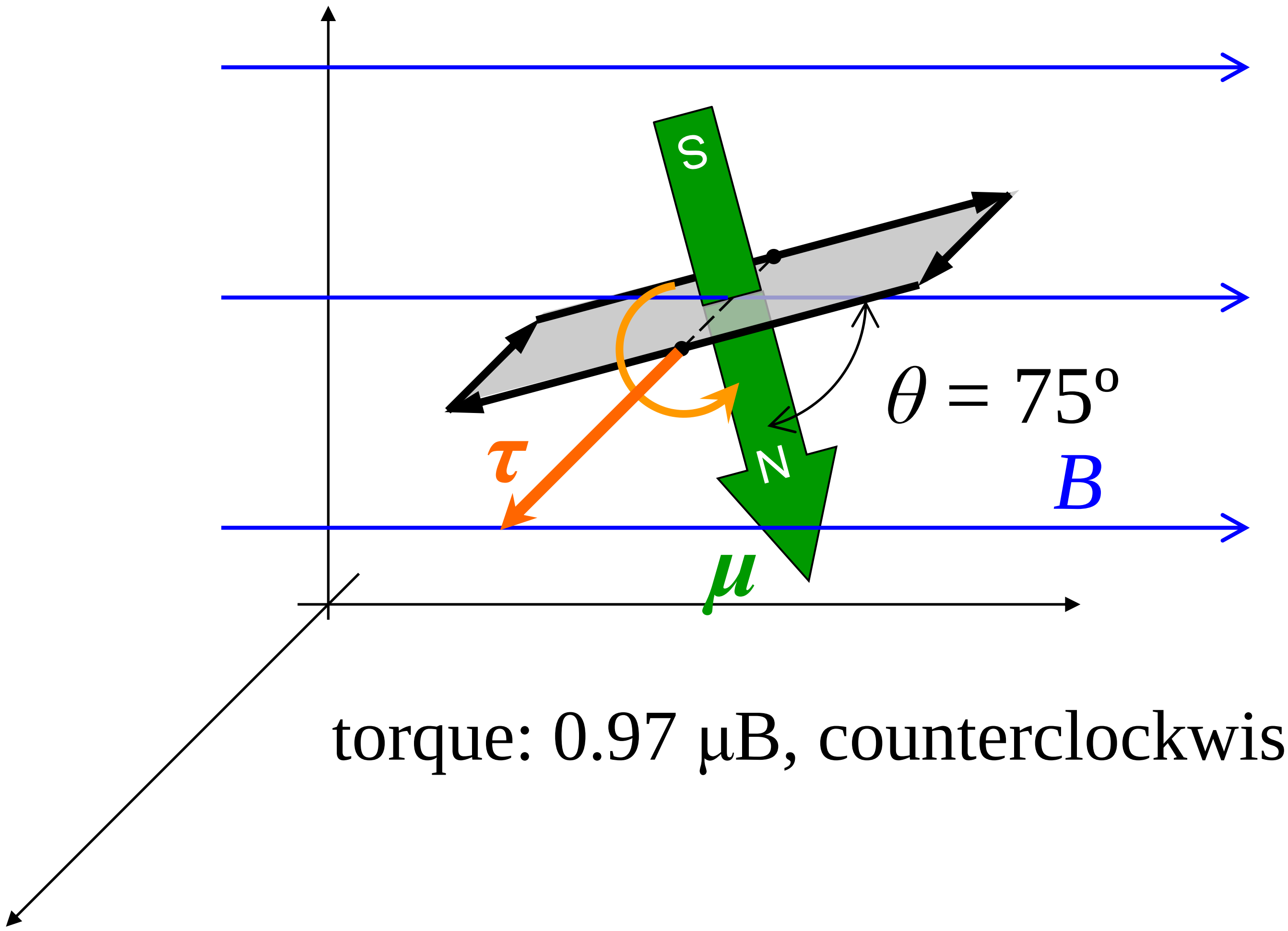
torque: 0.50 μB, counterclockwise



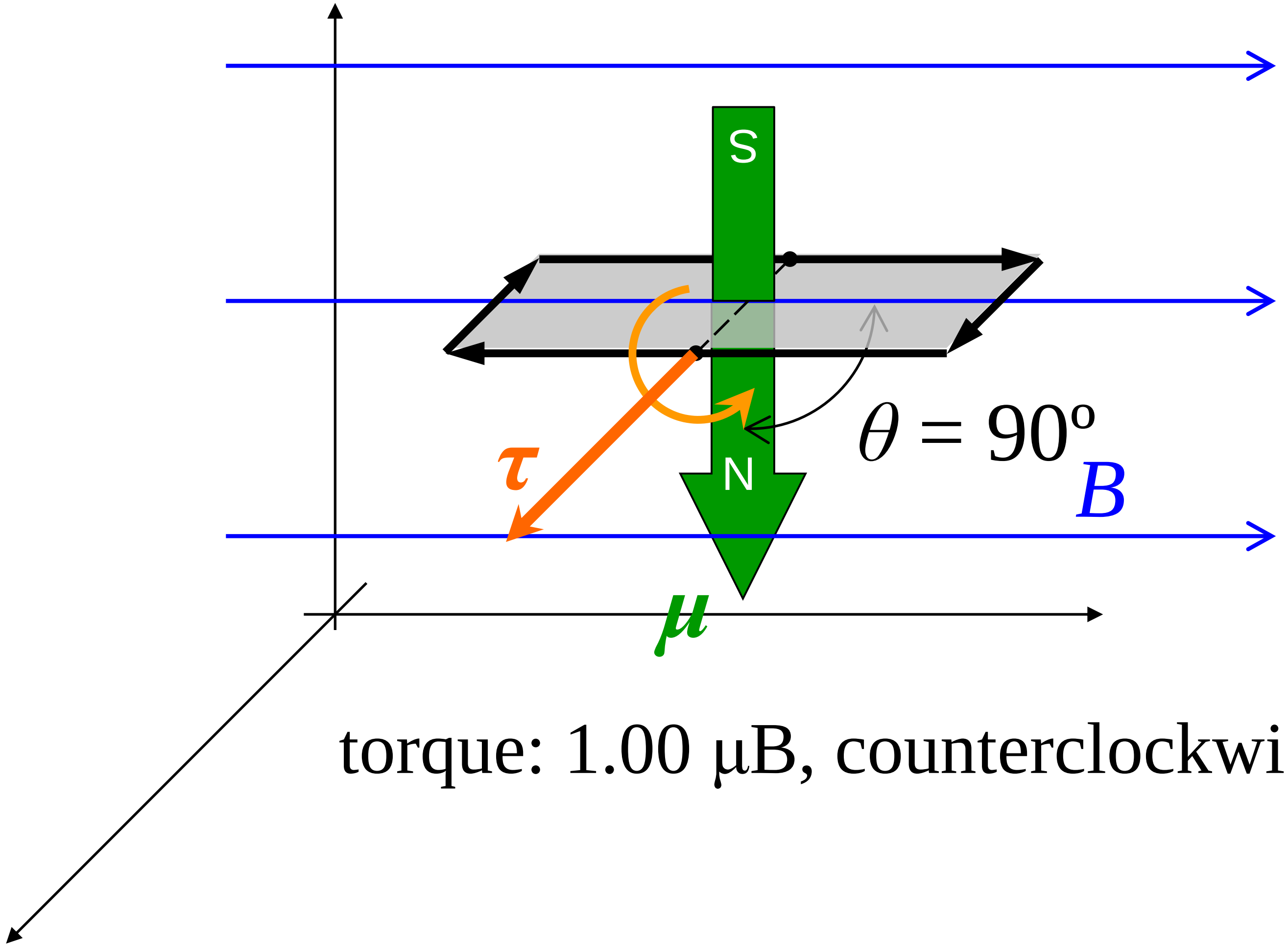
torque: 0.71 μB, counterclockwise



torque: 0.87 μB, counterclockwise



torque: 0.97 μB, counterclockwise



torque: 1.00 μB, counterclockwise