Unit Outline

I. Center of Mass
discrete, continuous

II. Motion of a System of Particles

III. Conservation of Momentum
frame of reference

IV. Impulse

V. Variable Mass

	The student will be able to:	HW:
1	Determine the center of mass for a set of objects or particles and/or a continuous distribution of mass.	1 – 7
2	Apply Newton's 2^{nd} Law to a system of particles and solve related problems either with the presence or absence of external forces.	8 – 12
3	State and apply the Law of Conservation of Momentum and solve related problems.	13 – 23
4	Define and apply elasticity and solve related problems.	24 – 30
5	Define and apply the concept of impulse and solve problems that relate momentum, force, and impulse.	31 – 38
6	Solve problems involving variable mass such as that of a rocket.	39 – 40

Conservation of Momentum

Recall Newton's 2nd Law for a system of particles:

$$\Sigma \vec{F}_{ext} = \left(\Sigma m\right)\vec{a}_{cm}$$

If the net external force on a system is zero then the acceleration of the center of mass will be zero.

$$\vec{a}_{cm} = 0$$

It then would also be true that the velocity of the center of mass of such a system would be constant:

$$\vec{v}_{cm} = \text{constant}$$

Conservation of Momentum

If net external force is zero then:

$$\vec{v}_{cm} = \frac{\sum m_i \vec{v}_i}{\sum m_i}$$

$$\left(\sum m_i\right)\vec{v}_{cm} = \sum m_i \vec{v}_i = \text{constant}$$

By defining linear momentum as the product of mass and velocity it then becomes:

$$\vec{p}_{system} = \left(\sum m_i\right)\vec{v}_{cm} = \sum \vec{p}_i = \text{constant}$$

Conservation of Momentum

The total momentum of an isolated system of objects will remain constant over time.

For two objects that interact with one another:

$$\vec{p}_1 + \vec{p}_2 = \vec{p}_1' + \vec{p}_2'$$

Conservation of Momentum

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For two objects that interact with one another:

$$\vec{p}_1 + \vec{p}_2 = \vec{p}_1' + \vec{p}_2'$$

$$\underbrace{m_1\vec{v}_1 + m_2\vec{v}_2}_{\text{total momentum } \textit{before} \text{ an interaction}} = \underbrace{m_1\vec{v}_1' + m_2\vec{v}_2'}_{\text{total momentum } \textit{after} \text{ the interaction}}$$

total momentum *before* an interaction

total momentum *after* the interaction

Conservation of Momentum

The total momentum of an isolated system of objects will remain constant over time.

The same reasoning may be extended to interactions of three or more objects…

$$\vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \ldots = \vec{p}_1' + \vec{p}_2' + \vec{p}_3' + \ldots$$

$$\sum \vec{p}_i = \sum \vec{p}_i'$$

Elasticity

Characterizing Collisions

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Variety in Collisions

- All collisions will illustrate conservation of momentum and conservation of energy.
- However, depending on the nature of the objects involved, only a certain amount of kinetic energy will remain after the collision.
- In certain situations there may be conservation of *kinetic* energy.
- This occurs if and only if internal forces are conservative.

Elasticity

In a perfectly **elastic** collision the total kinetic energy of the system remains *constant*.

The total kinetic energy of the system will be *reduced* in an **inelastic** collision.

In a "perfectly inelastic" collision the objects stick together and the reduction in kinetic energy is maximized.

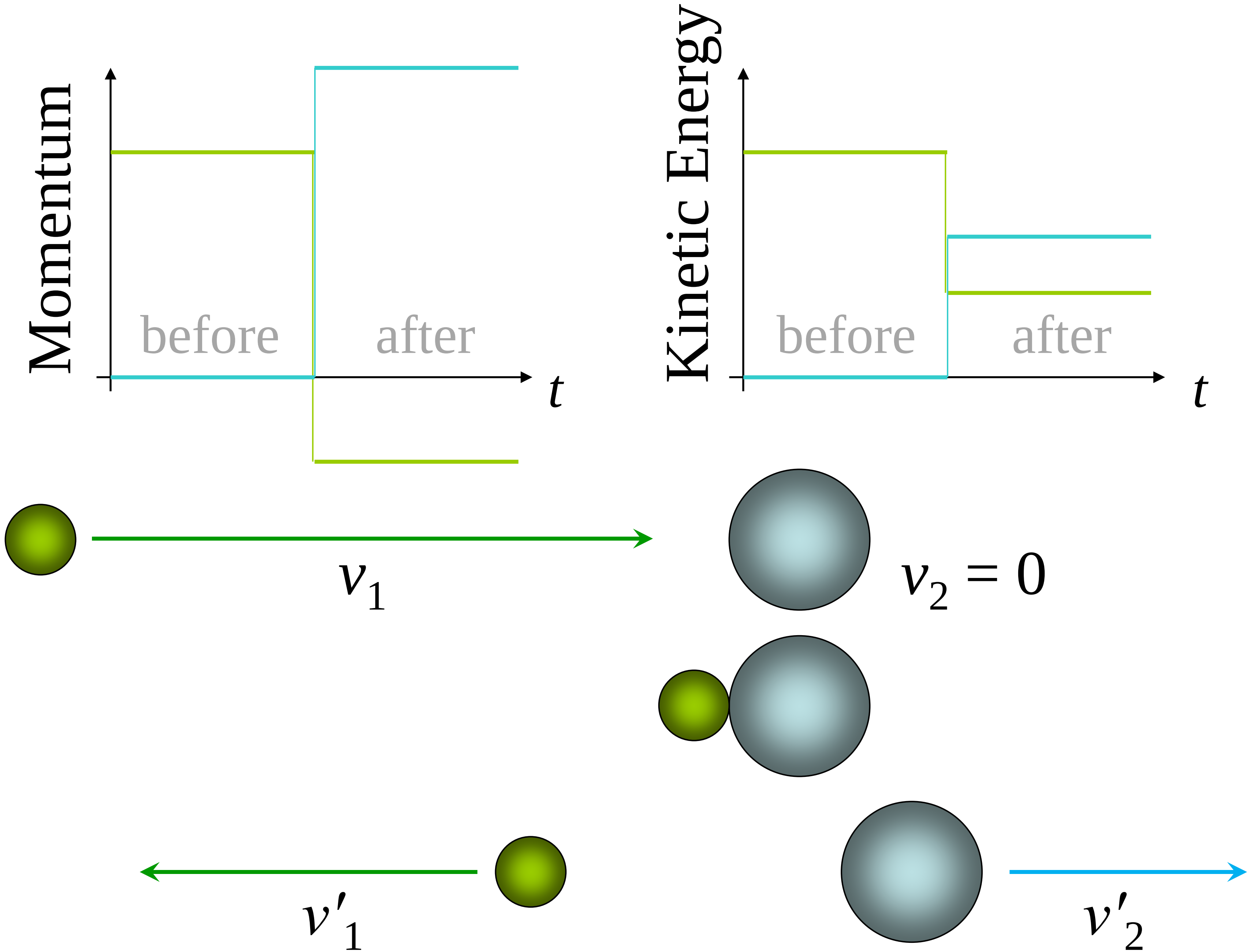
Quantifying Elasticity

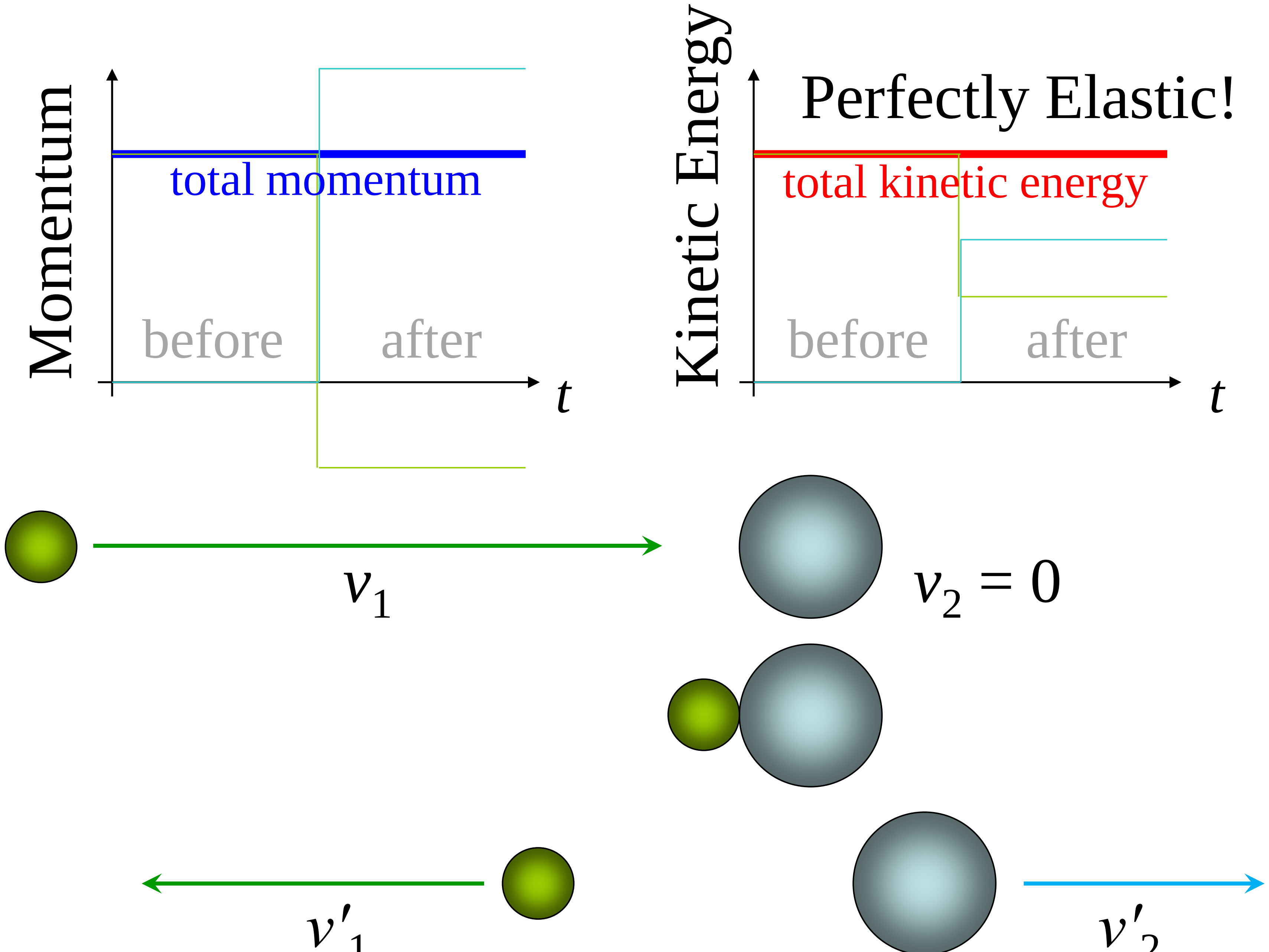
For two objects in a perfectly **elastic** collision:

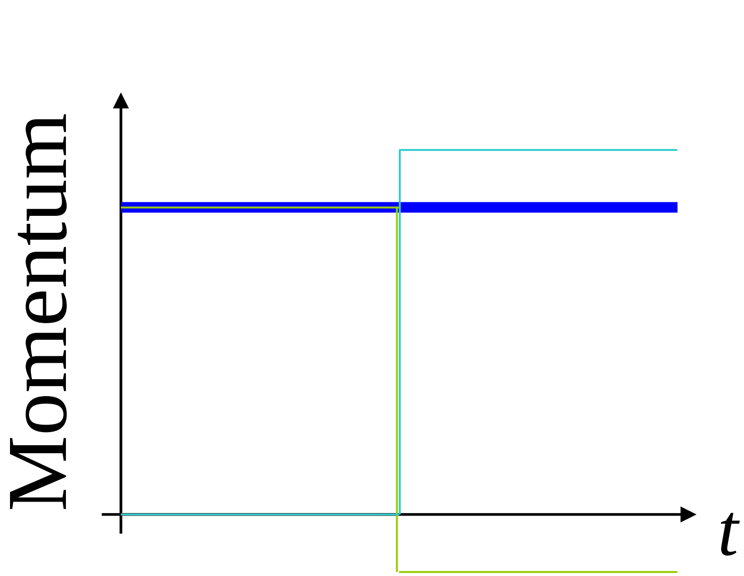
$$K_1 + K_2 = K_1' + K_2'$$

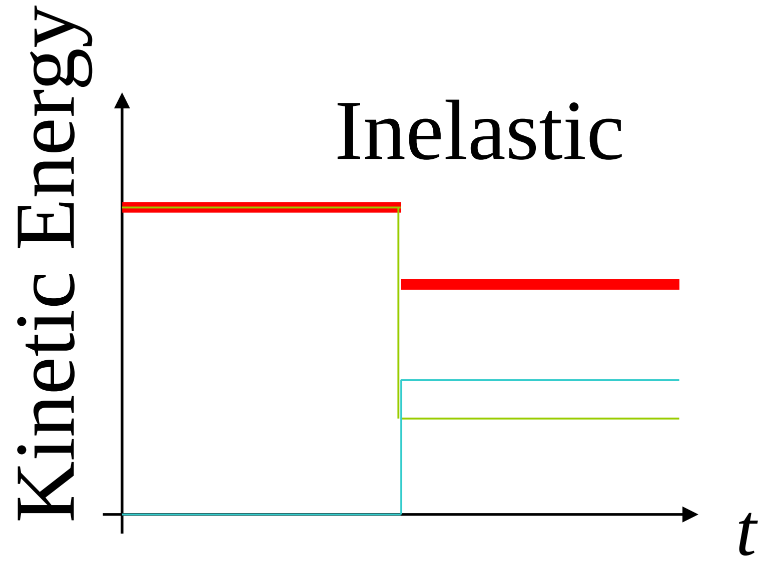
For two objects in an **inelastic** collision:

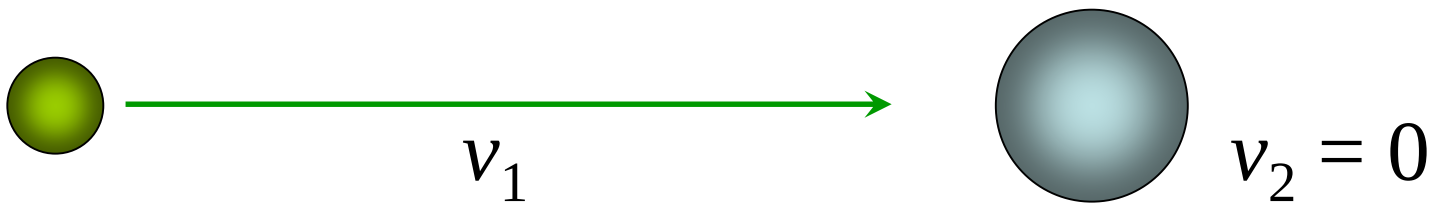
$$K_1 + K_2 > K_1' + K_2'$$


Momentum
before
after
t
Kinetic Energy
before
after
t
v_1
$v_2 = 0$
v'_1
v'_2

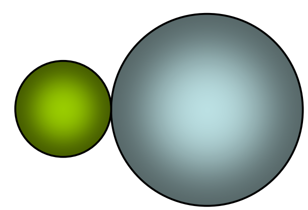

Momentum
total momentum
before
after
t
Kinetic Energy
Perfectly Elastic!
total kinetic energy
before
after
t
v1
v2 = 0
v′1
v′2

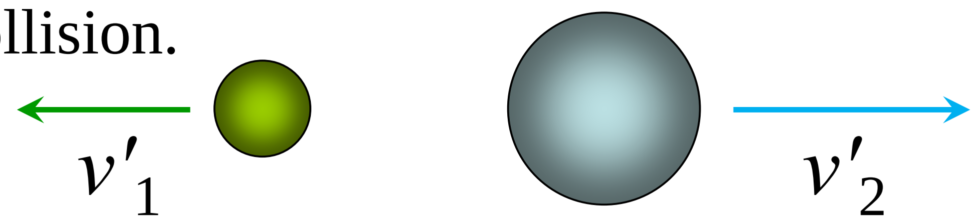




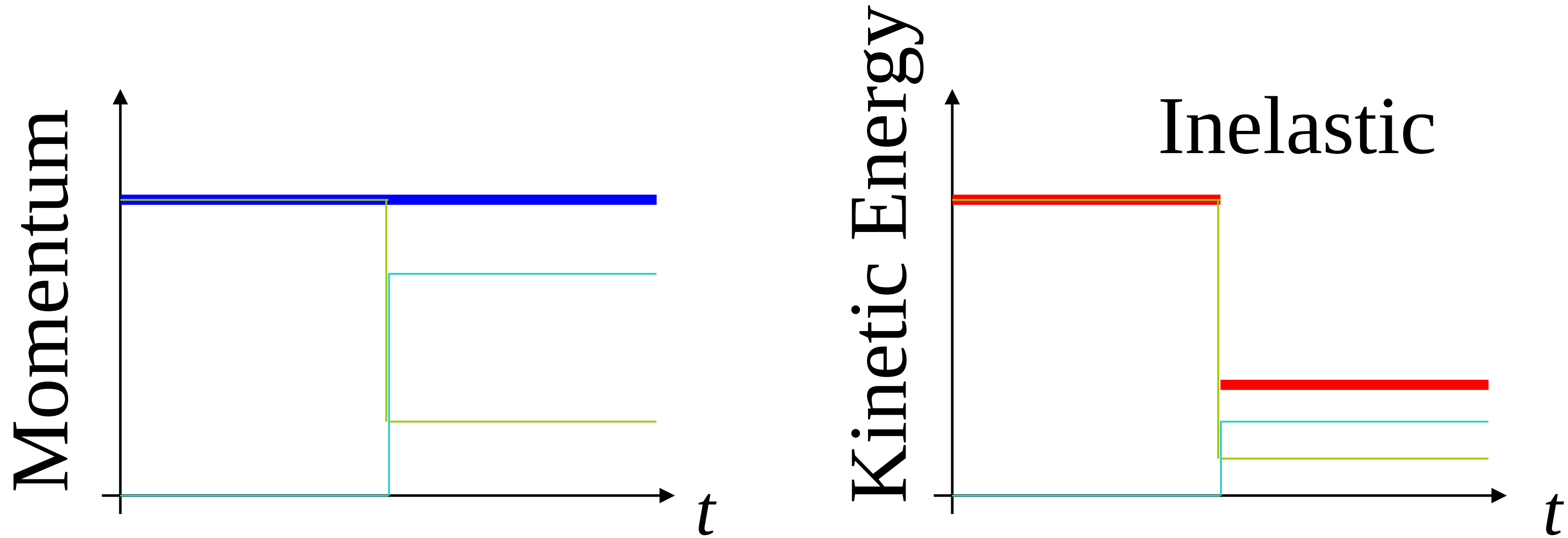


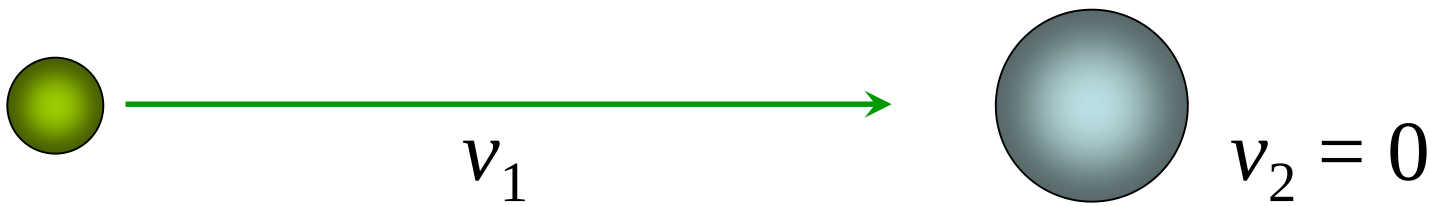
There is a range of possibilities when objects collide. Momentum is always conserved, but kinetic energy is reduced in an inelastic collision.



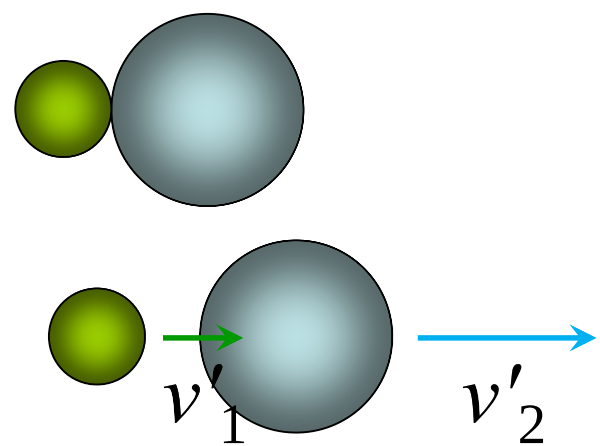


Inelastic

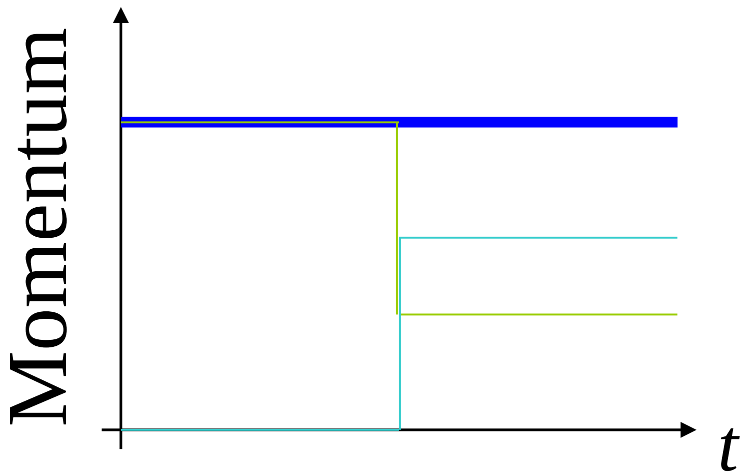


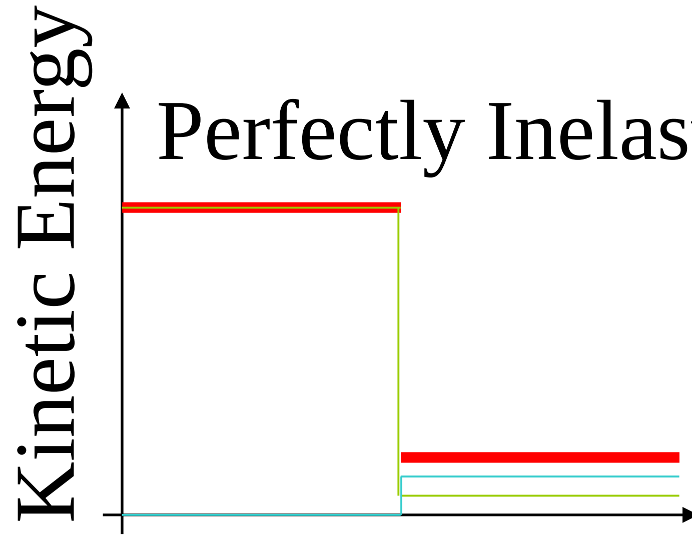


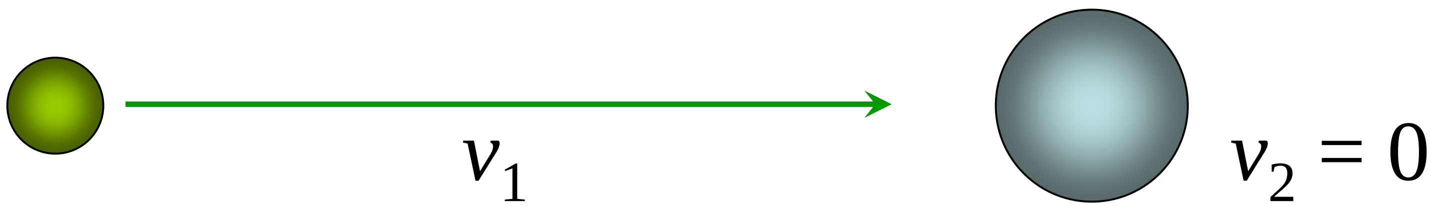
This collision is less elastic than the previous page. It is convenient to think of elasticity as a measure of "bounciness". The objects shown here are not very "bouncy"!



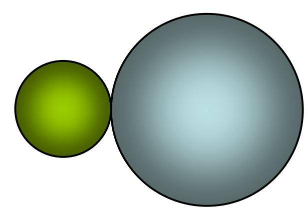
Perfectly Inelastic!

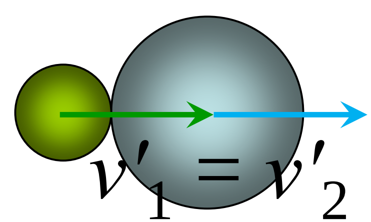






If objects stick together in a collision the kinetic energy remaining afterward is minimized. This is considered "perfectly inelastic" – (the opposite being elastic). Note that total momentum is unaffected!

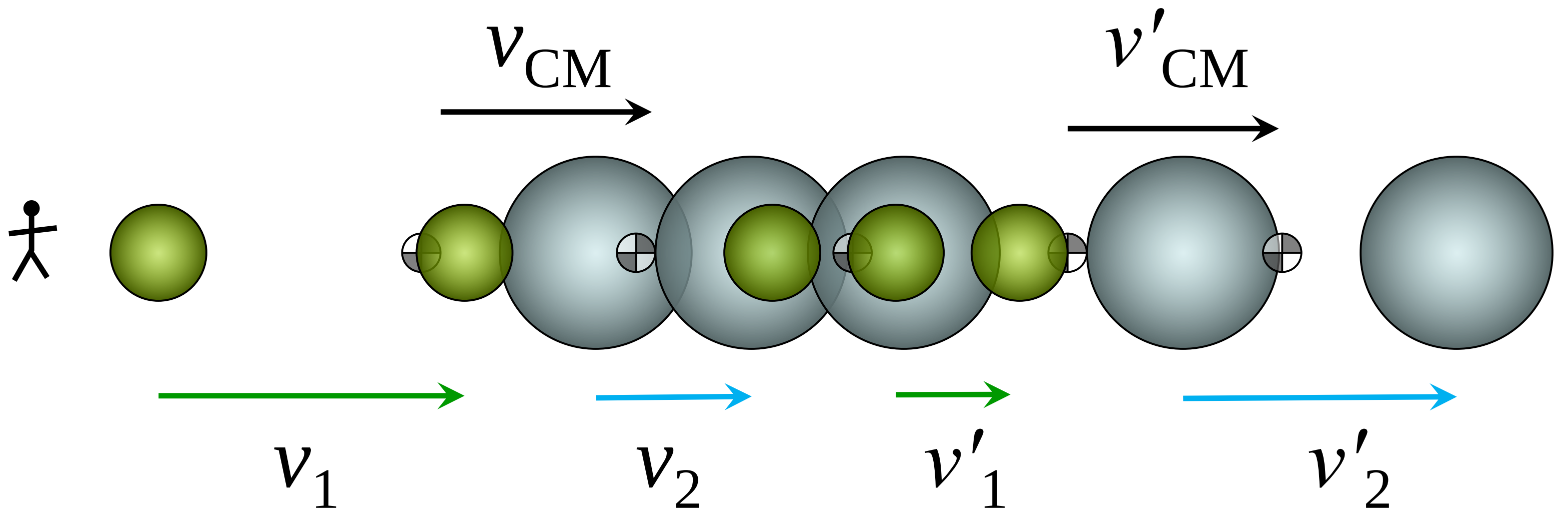




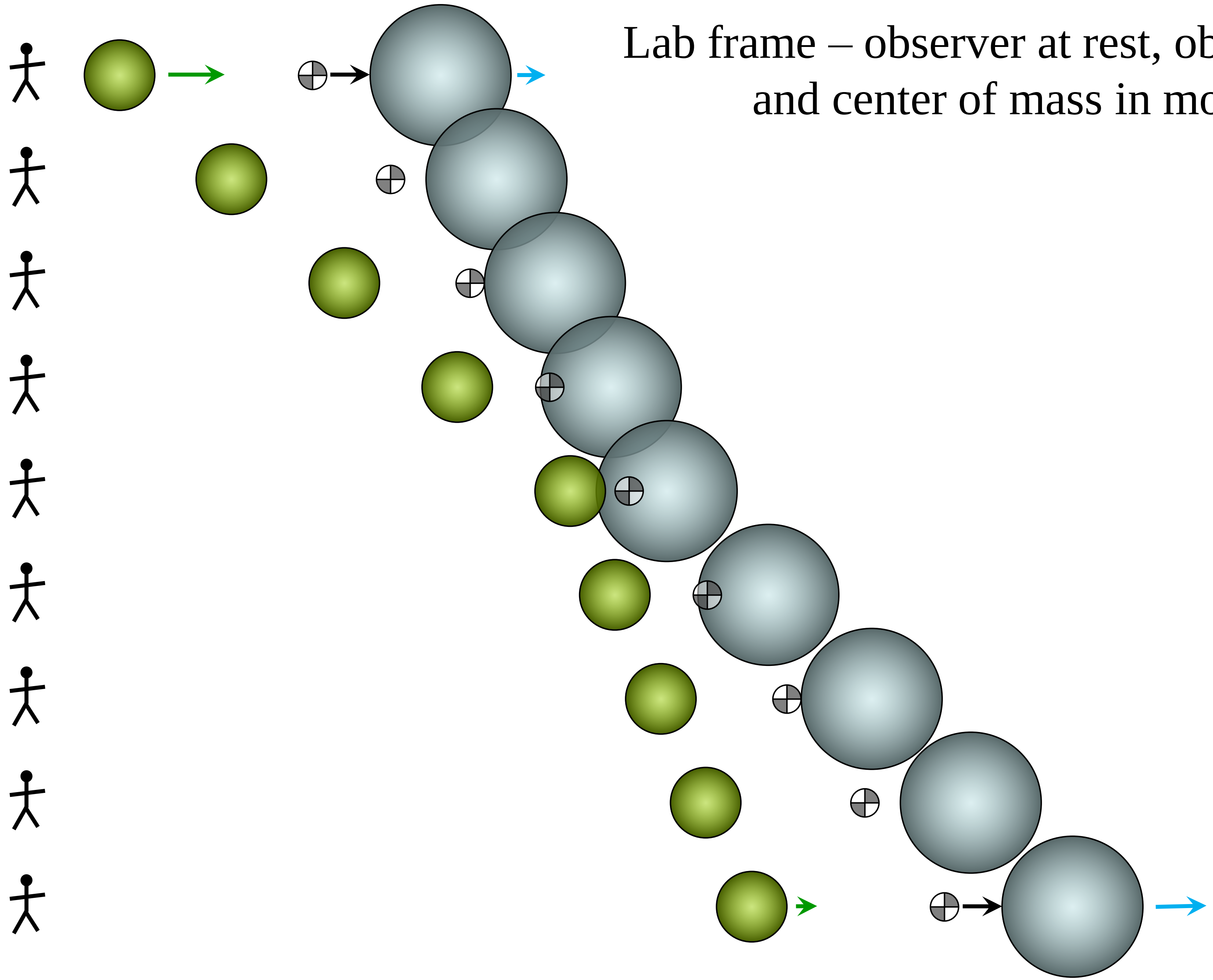
Collisions in Different Frames

- Any inertial frame of reference may be used to analyze a collision.
- In the center of mass frame of reference the total momentum is always zero.
- For elastic collisions the rebound speed is equal to the impact speed relative to the center of mass.
- For elastic collisions in one dimension the velocity of one object relative to the other is simply reversed afterward.

Here is a "stop action" sequence showing before and after a collision as seen by an observer at rest. The black and white symbol is the center of mass of the two objects. The velocity of the center of mass is unaffected by the collision.

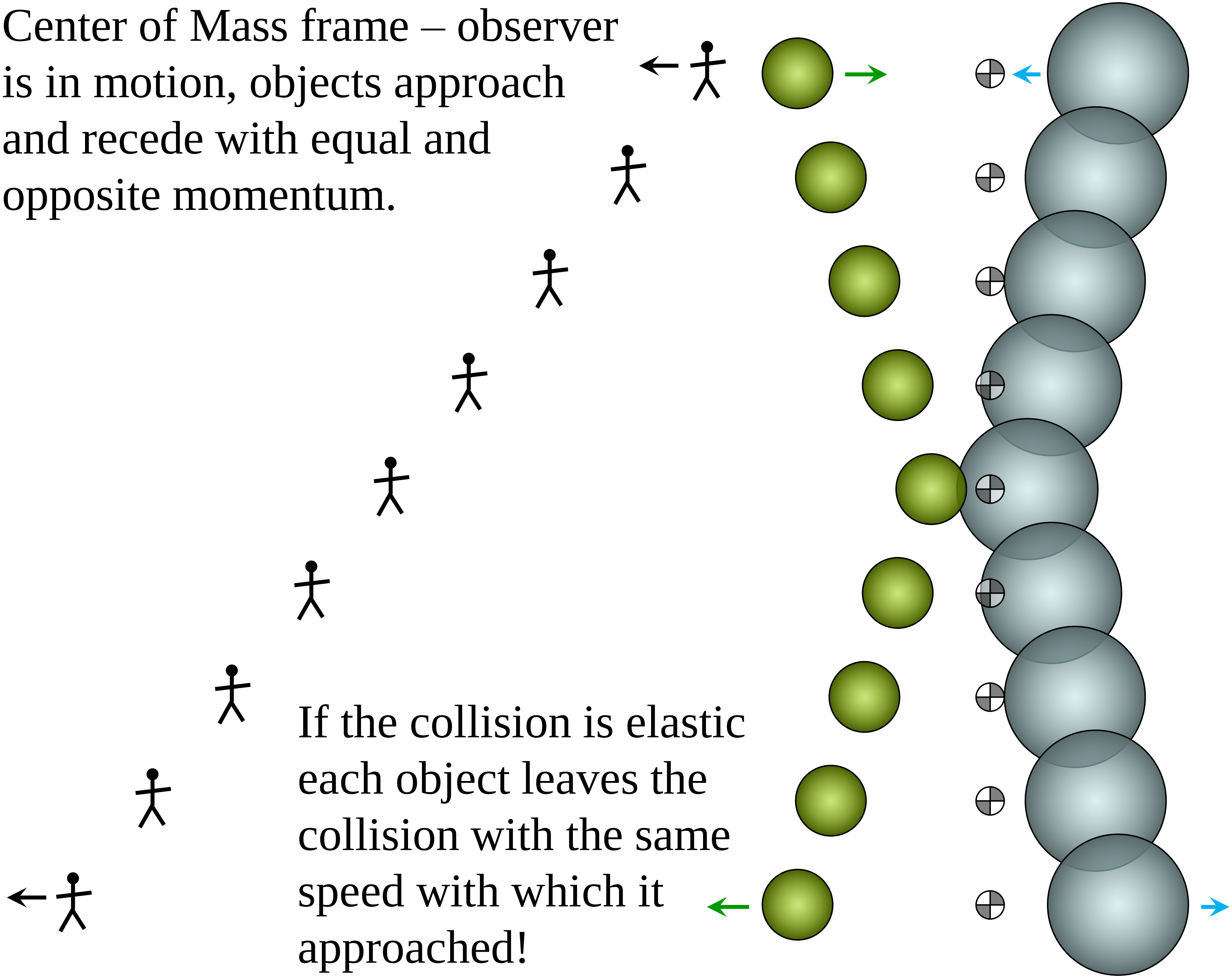


In this frame of reference the velocities and speeds of the two objects are different before and after the collision. But, what if the same collision were viewed in a frame of reference moving at the same velocity as the center of mass…



Lab frame – observer at rest, objects and center of mass in motion.

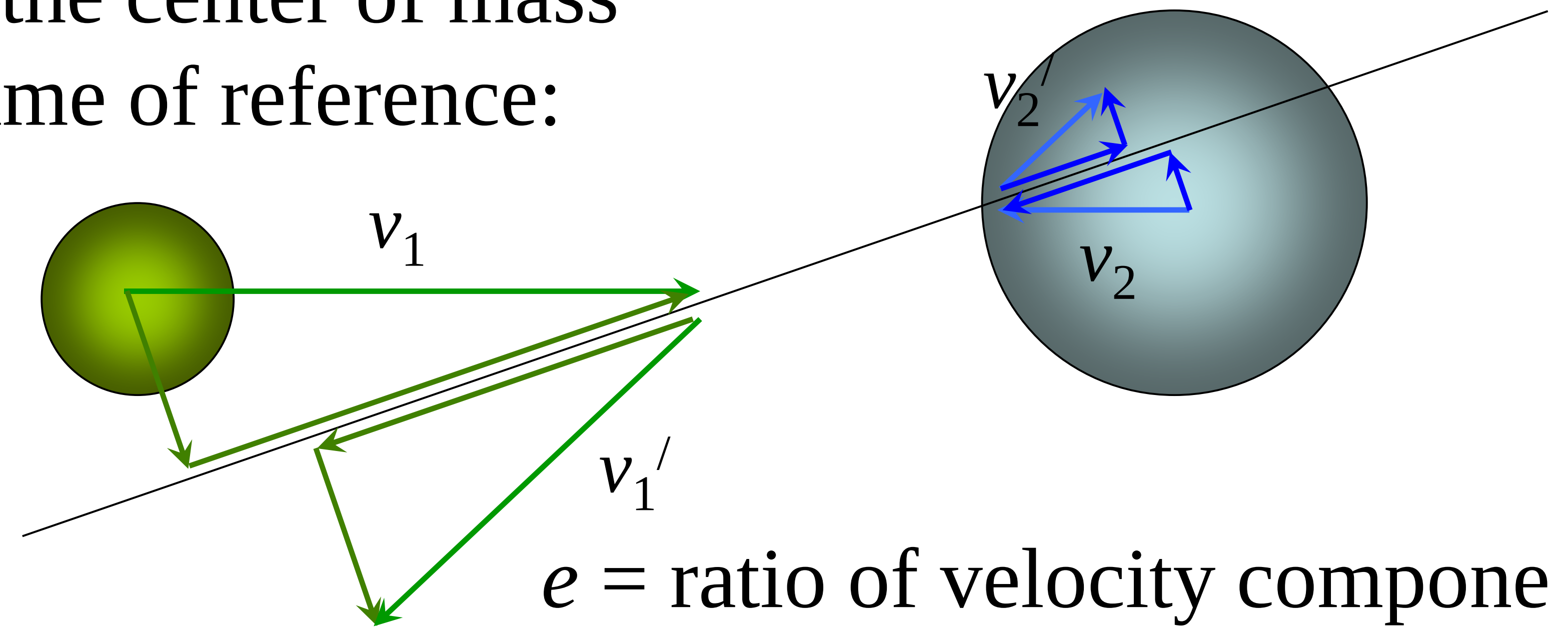
Center of Mass frame – observer is in motion, objects approach and recede with equal and opposite momentum.



If the collision is elastic each object leaves the collision with the same speed with which it approached!

Coefficient of Restitution

In the center of mass frame of reference:



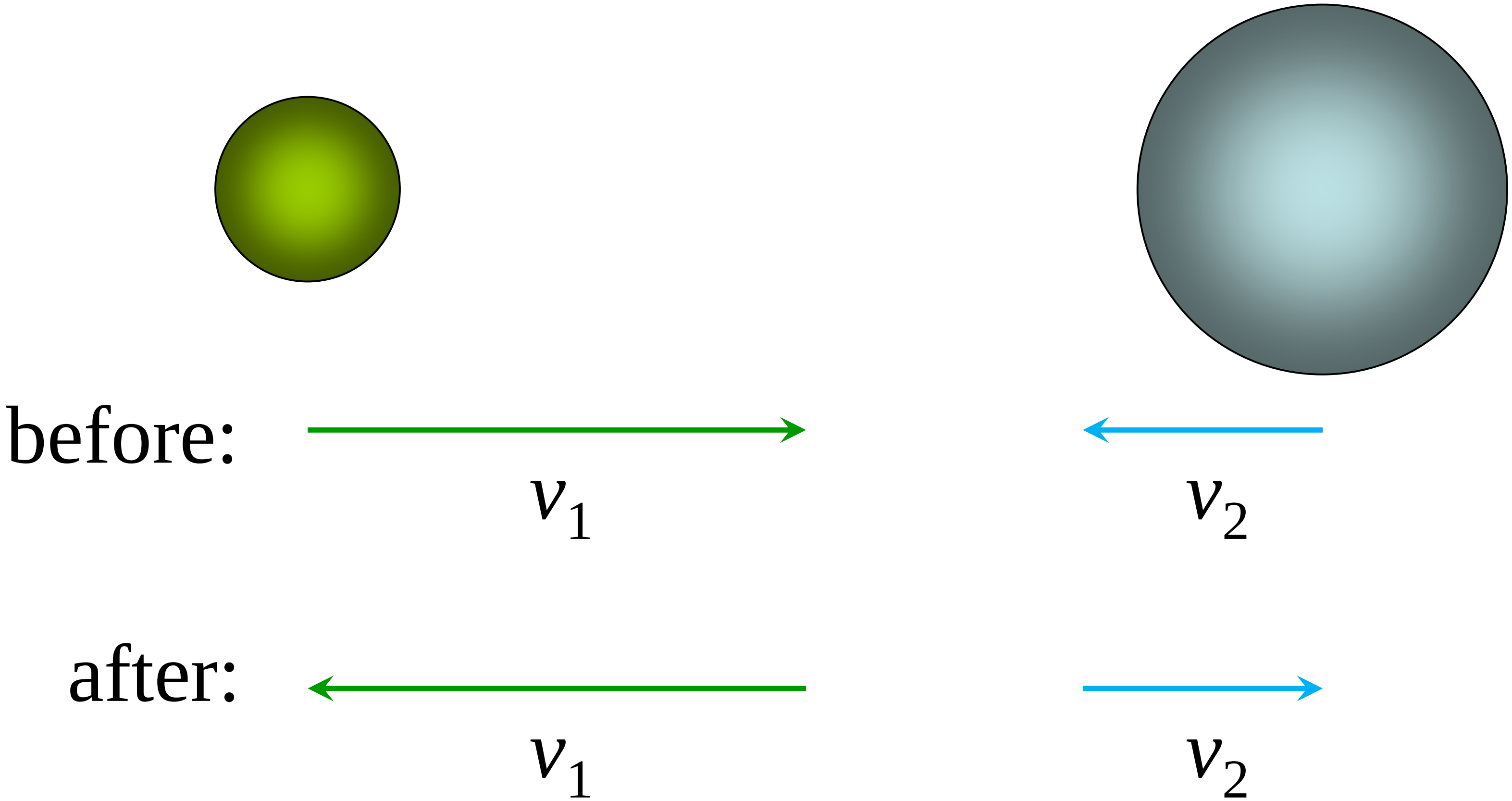
e = ratio of velocity components along the "line of collision"

$$0 \leq e \leq 1$$

note: components perpendicular to this line are unchanged by the collision

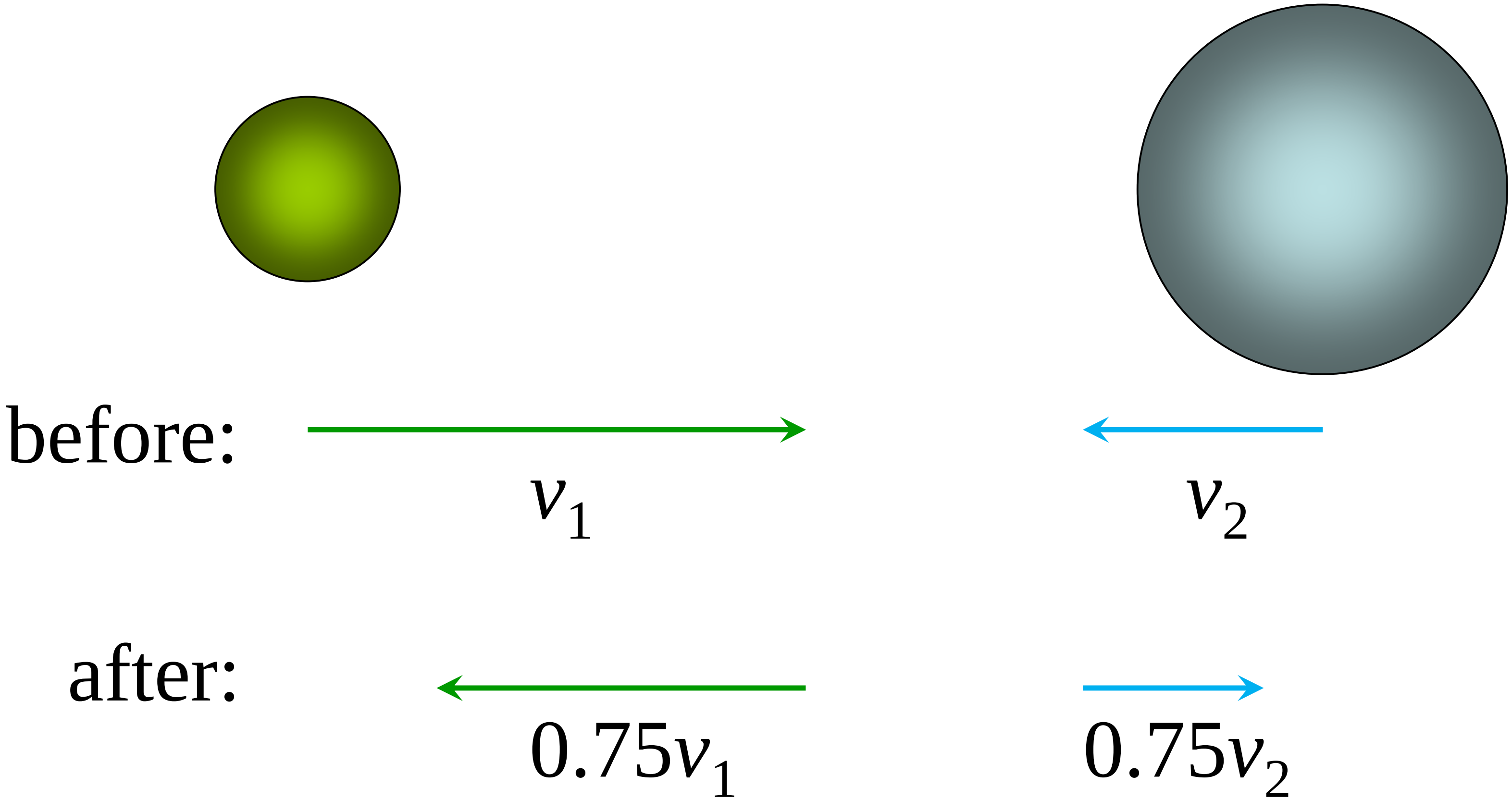
1-D collision in CM frame

$$e = 1$$



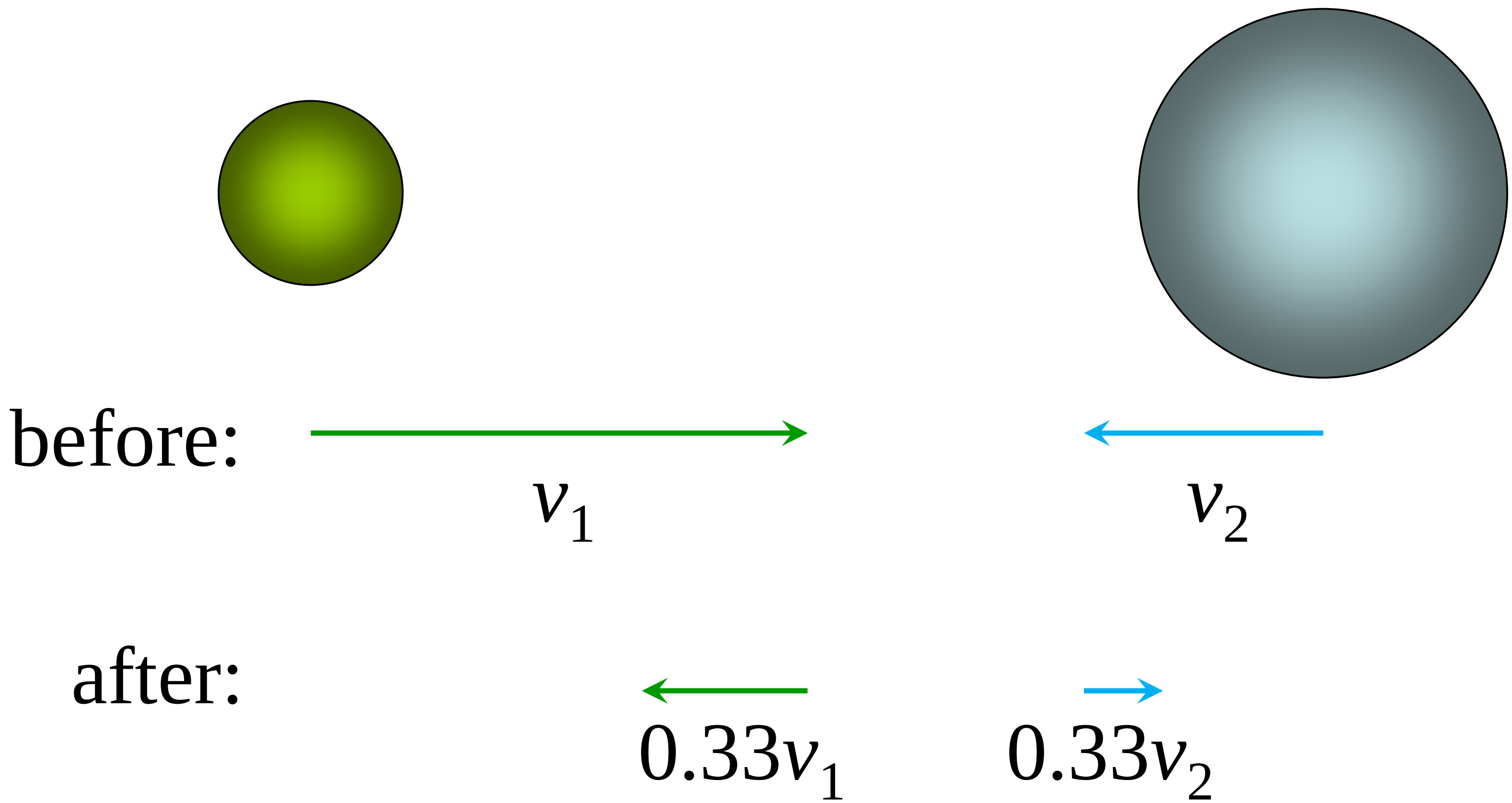
1-D collision in CM frame

$e = 0.75$



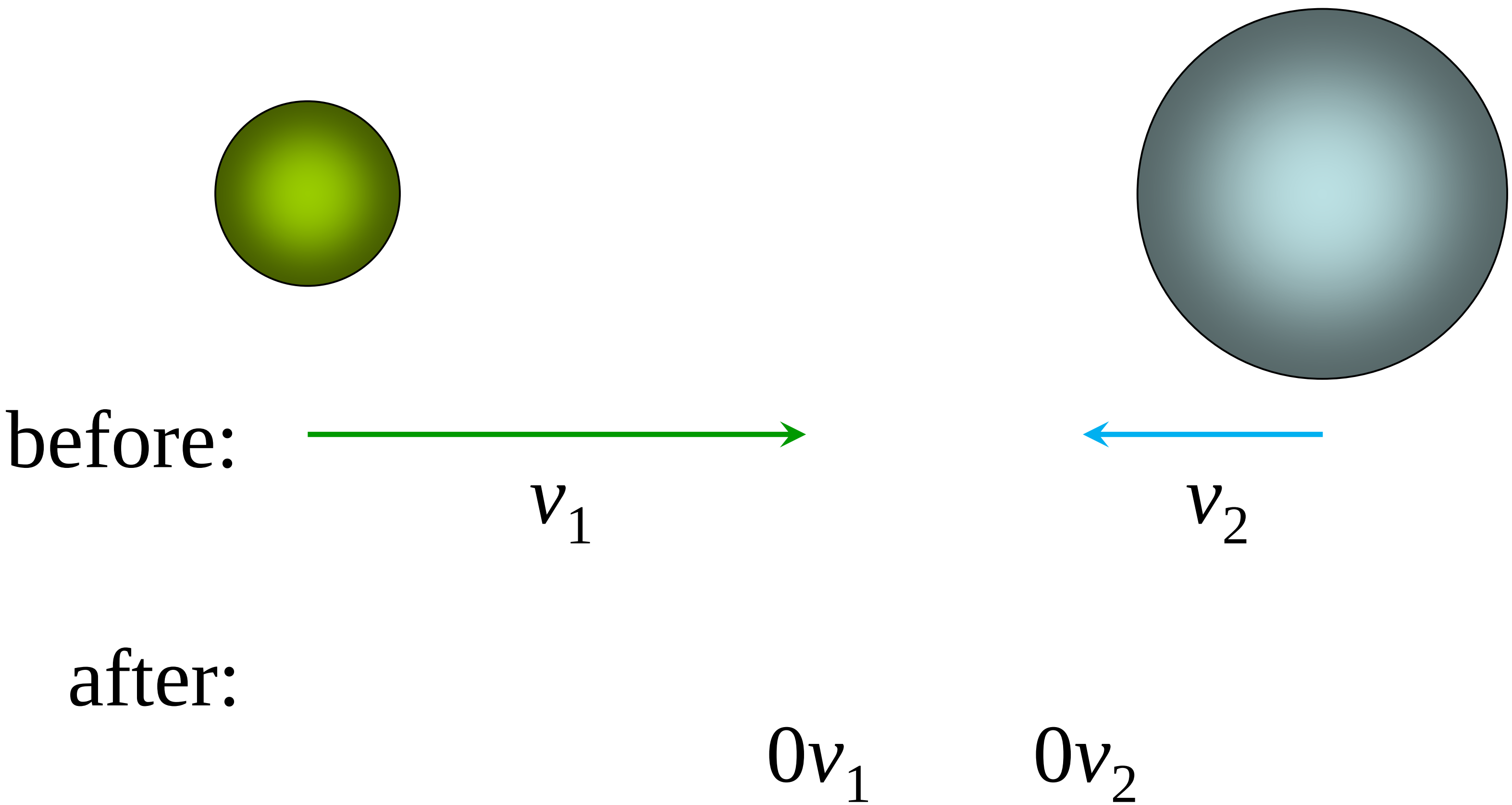
1-D collision in CM frame

$e = 0.33$

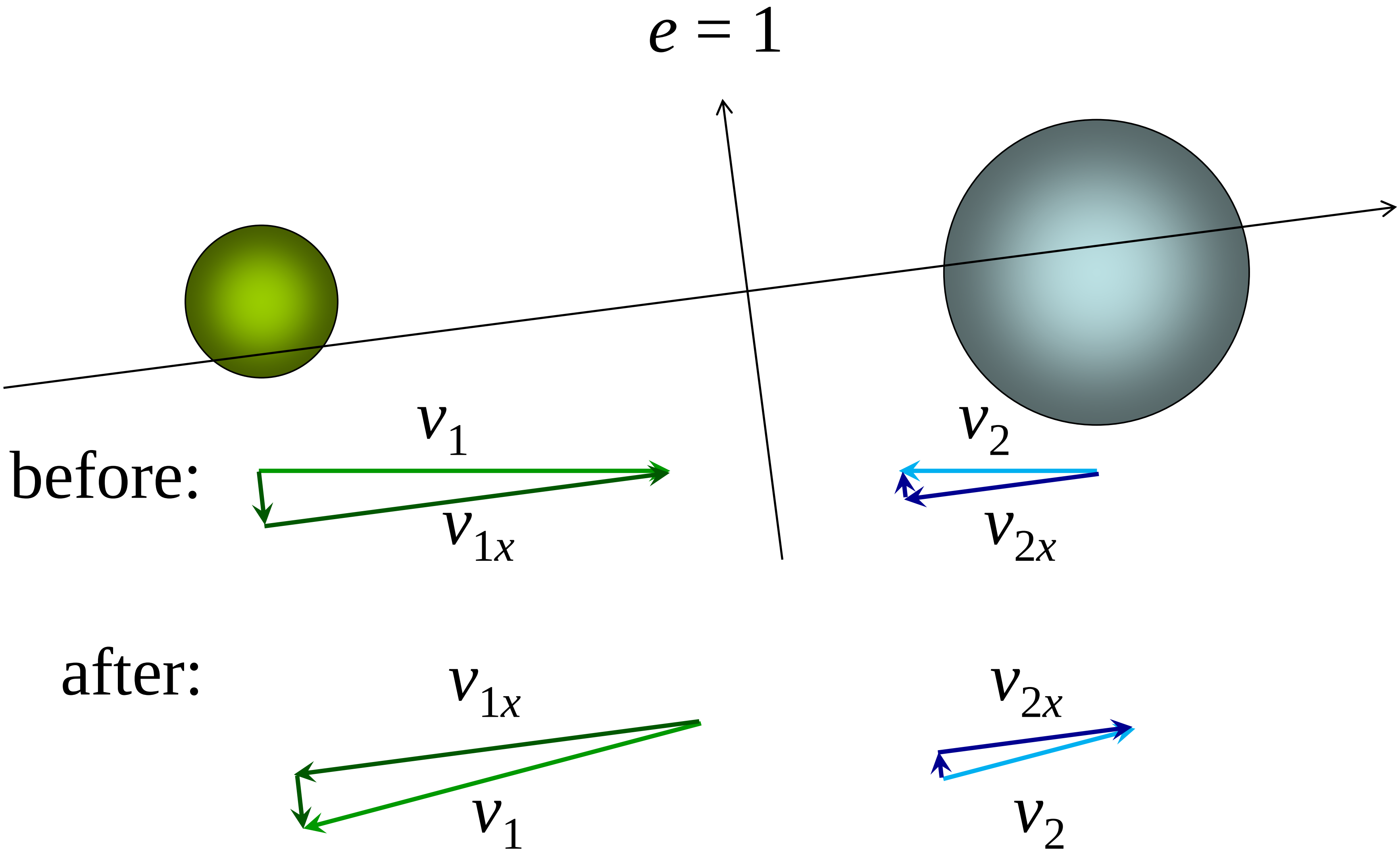


1-D collision in CM frame

$e = 0$

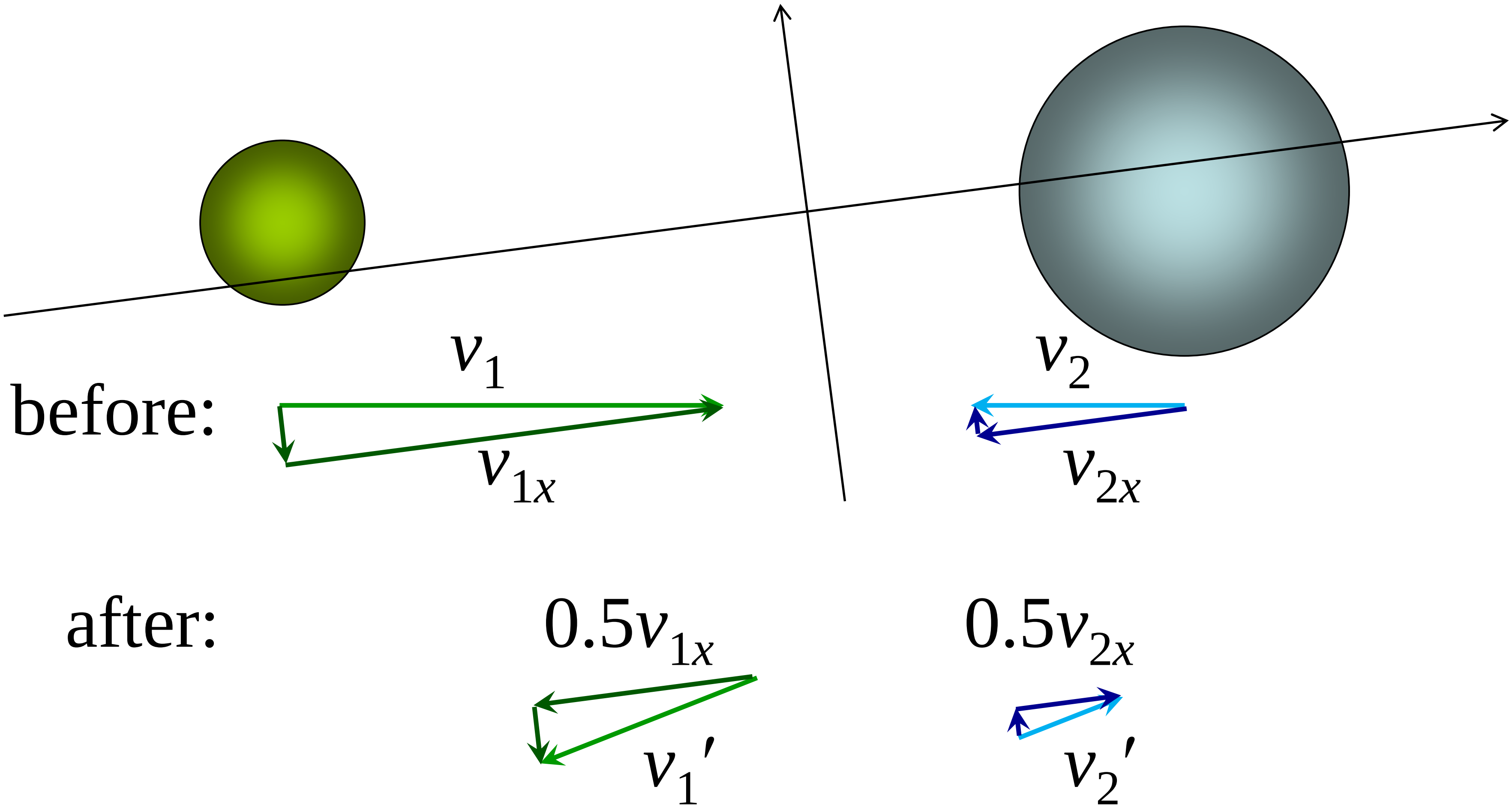


2-D collision in CM frame

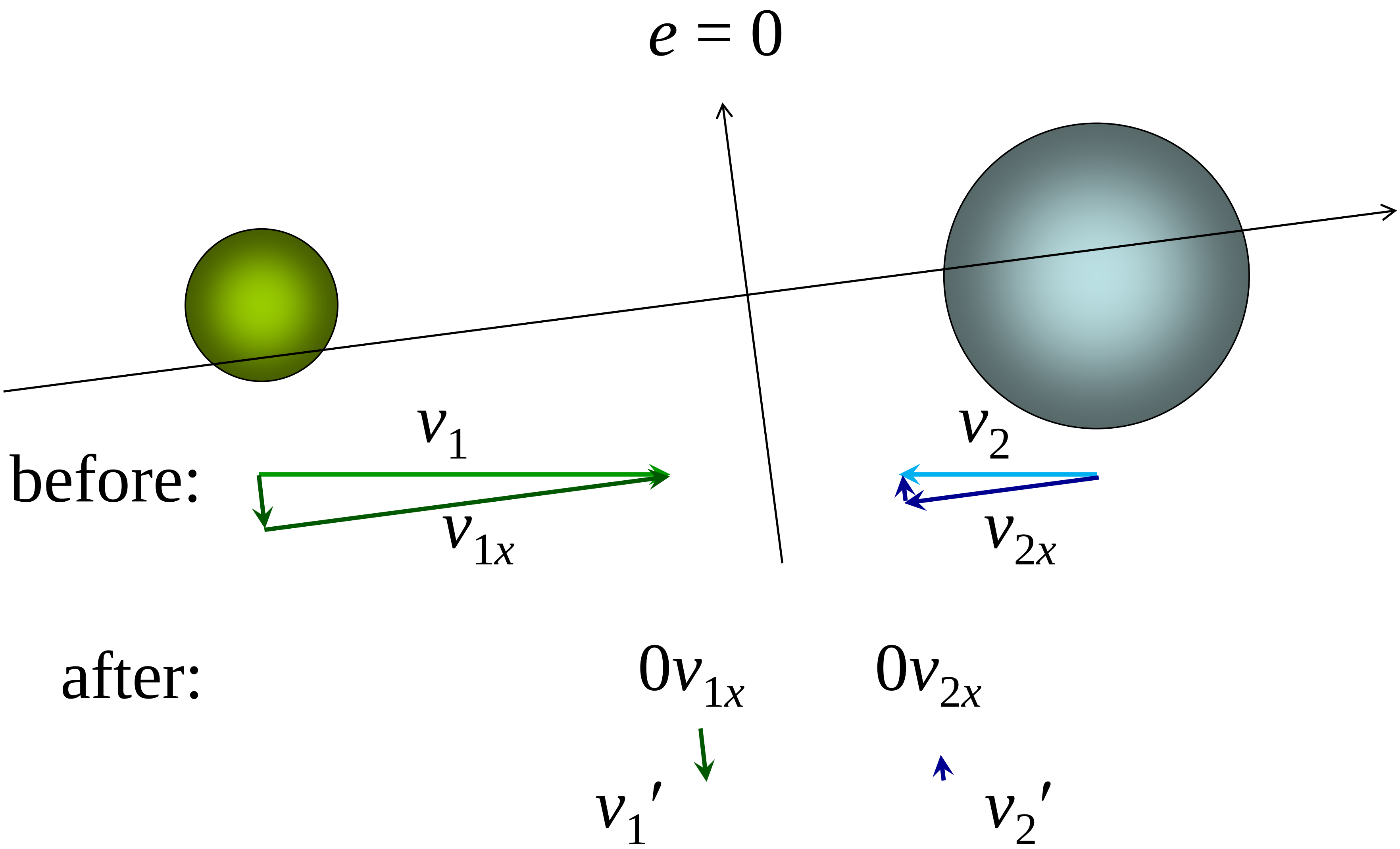


2-D collision in CM frame

$$e = 0.5$$



2-D collision in CM frame



Use PhET Collision Lab to set this up and adjust “elasticity” – observe the effect on the components of the velocities. Compare also speed and kinetic energy and note that the percentage is not based on these quantities.