Impulse

Relating Force and Momentum

	The student will be able to:	HW:
1	Determine the center of mass for a set of objects or particles and/or a continuous distribution of mass.	1 – 7
2	Apply Newton's 2nd Law to a system of particles and solve related problems either with the presence or absence of external forces.	8 – 12
3	State and apply the Law of Conservation of Momentum and solve related problems.	13 – 23
4	Define and apply elasticity and solve related problems.	24 – 30
5	Define and apply the concept of impulse and solve problems that relate momentum, force, and impulse.	31 – 38
6	Solve problems involving variable mass such as that of a rocket.	39 – 40

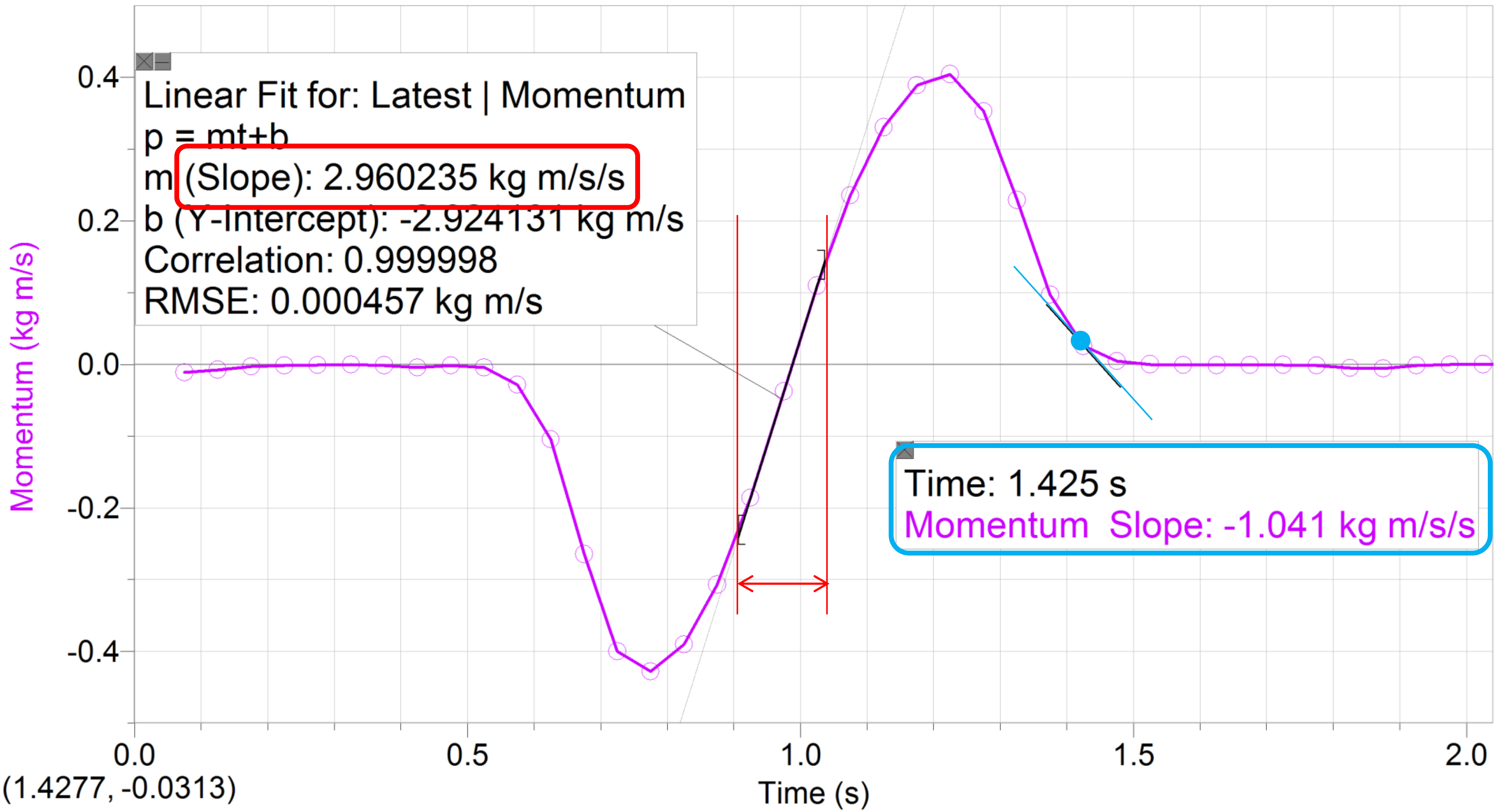
Relating Force to Momentum

- Forces cause acceleration and change in velocity – however this is also change in momentum.
- An alternate form of Newton's 2nd Law relates force to change in momentum:

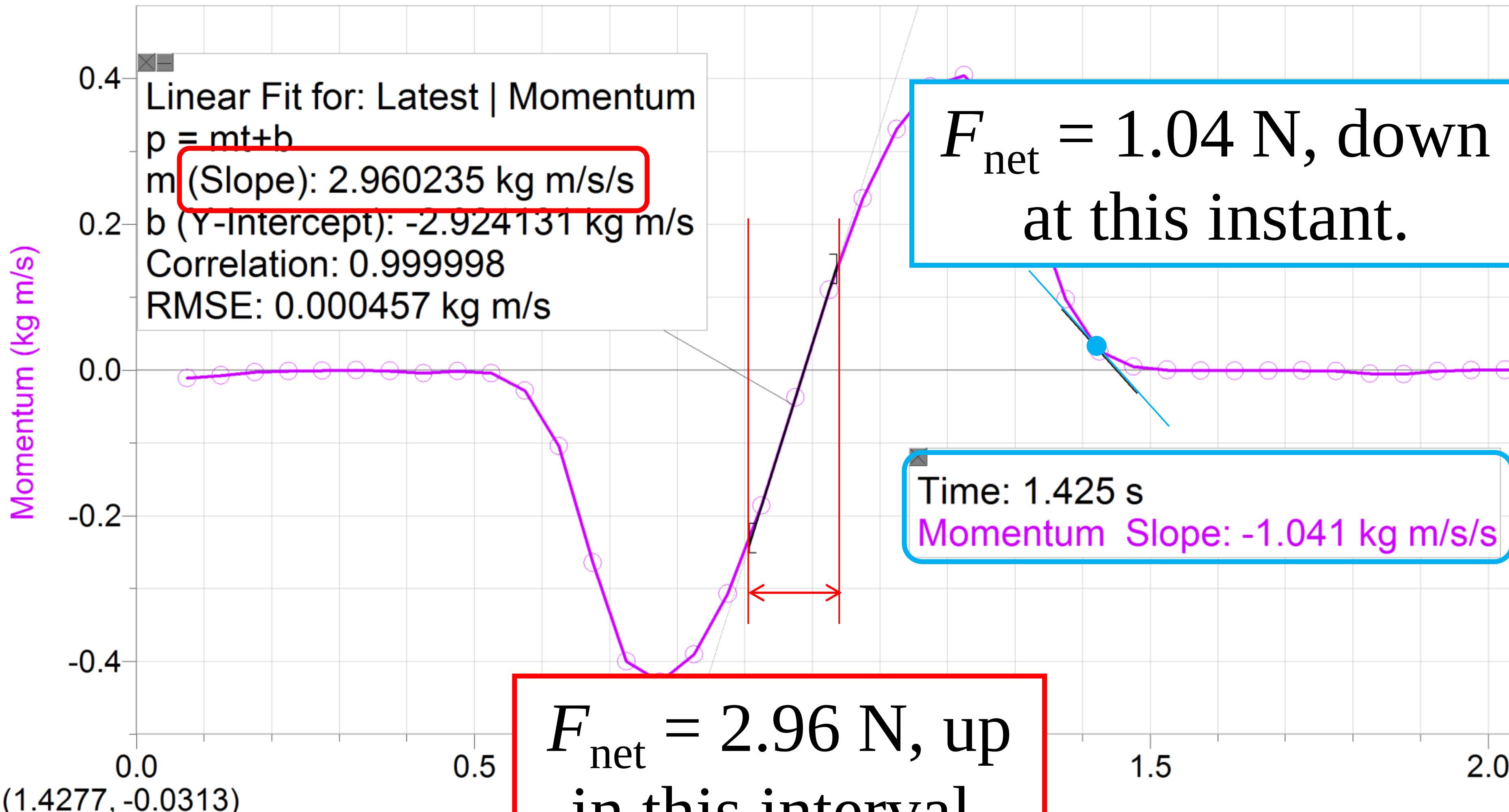
$$\Sigma\vec{F} = \frac{d\,\vec{p}}{dt}$$

Net force equals rate of change in momentum.

Momentum vs. Time
Object Moving Vertically



Momentum vs. Time
Object Moving Vertically



F_{net} = 1.04 N, down at this instant.

F_{net} = 2.96 N, up in this interval.

More morphing and shenanigans …

$$\Sigma \vec{F} = \frac{d\vec{p}}{dt}$$

$$\int \Sigma \vec{F}\, dt = \int d\vec{p}$$

$$\int \Sigma \vec{F}\, dt = \Delta \vec{p}$$

$$\Sigma \int \vec{F}\, dt = \Delta \vec{p}$$

This derives from Newton's 2nd Law.

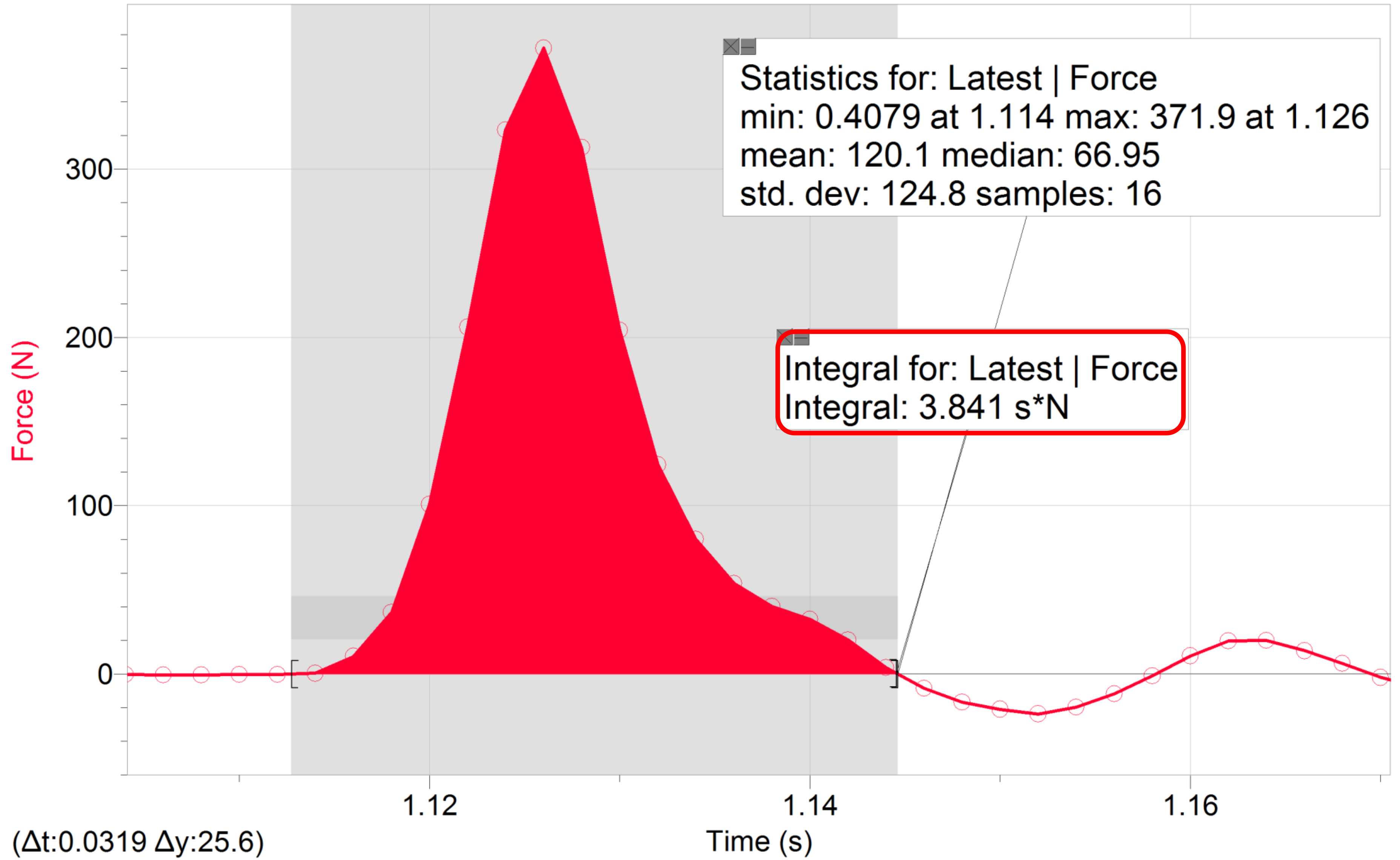
Define **impulse**, *J*, as the product of force and time…

$$\vec{J} = \int \vec{F}\, dt$$

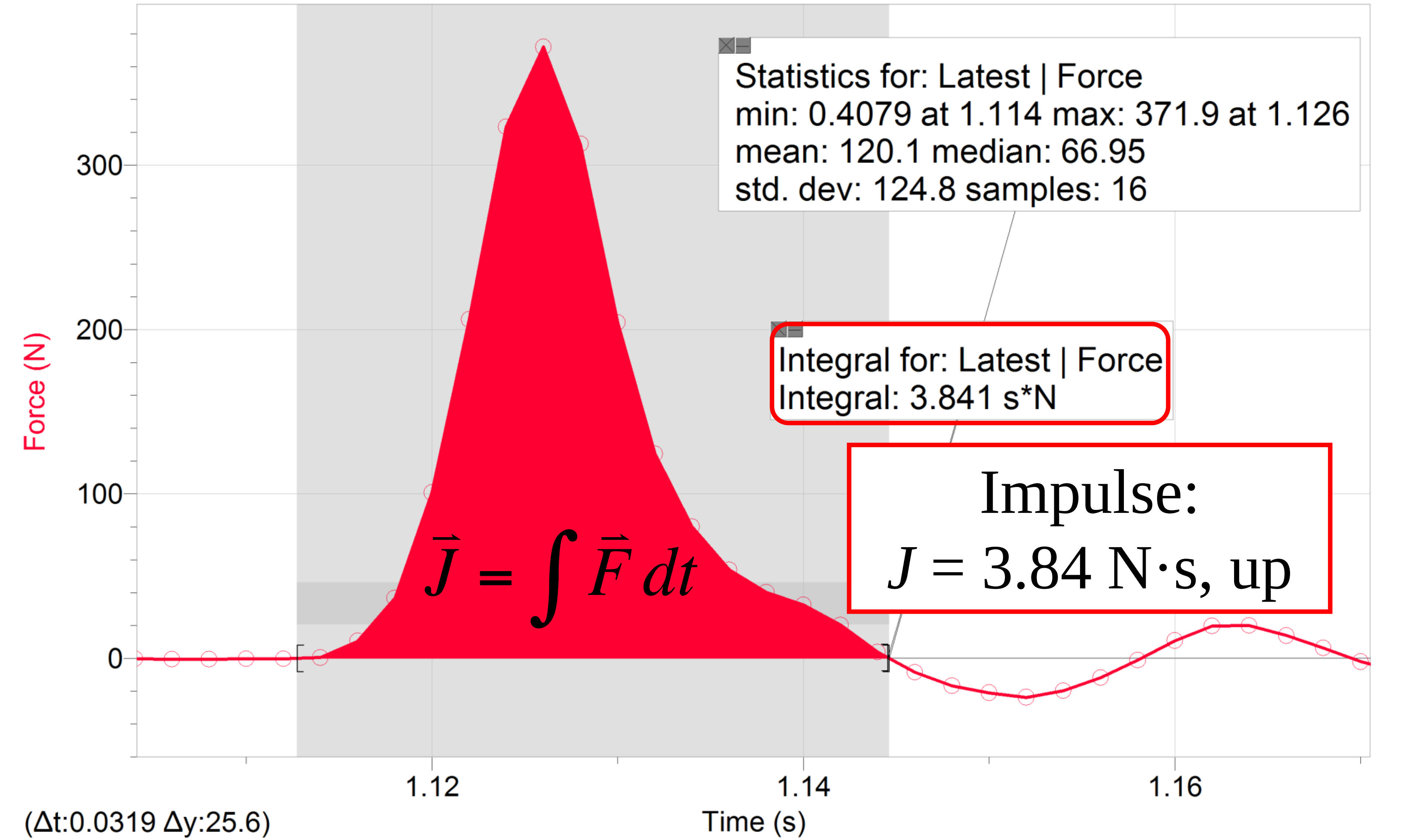
… then *net* impulse equals change in momentum!

$$\Sigma \vec{J} = \Delta \vec{p}$$

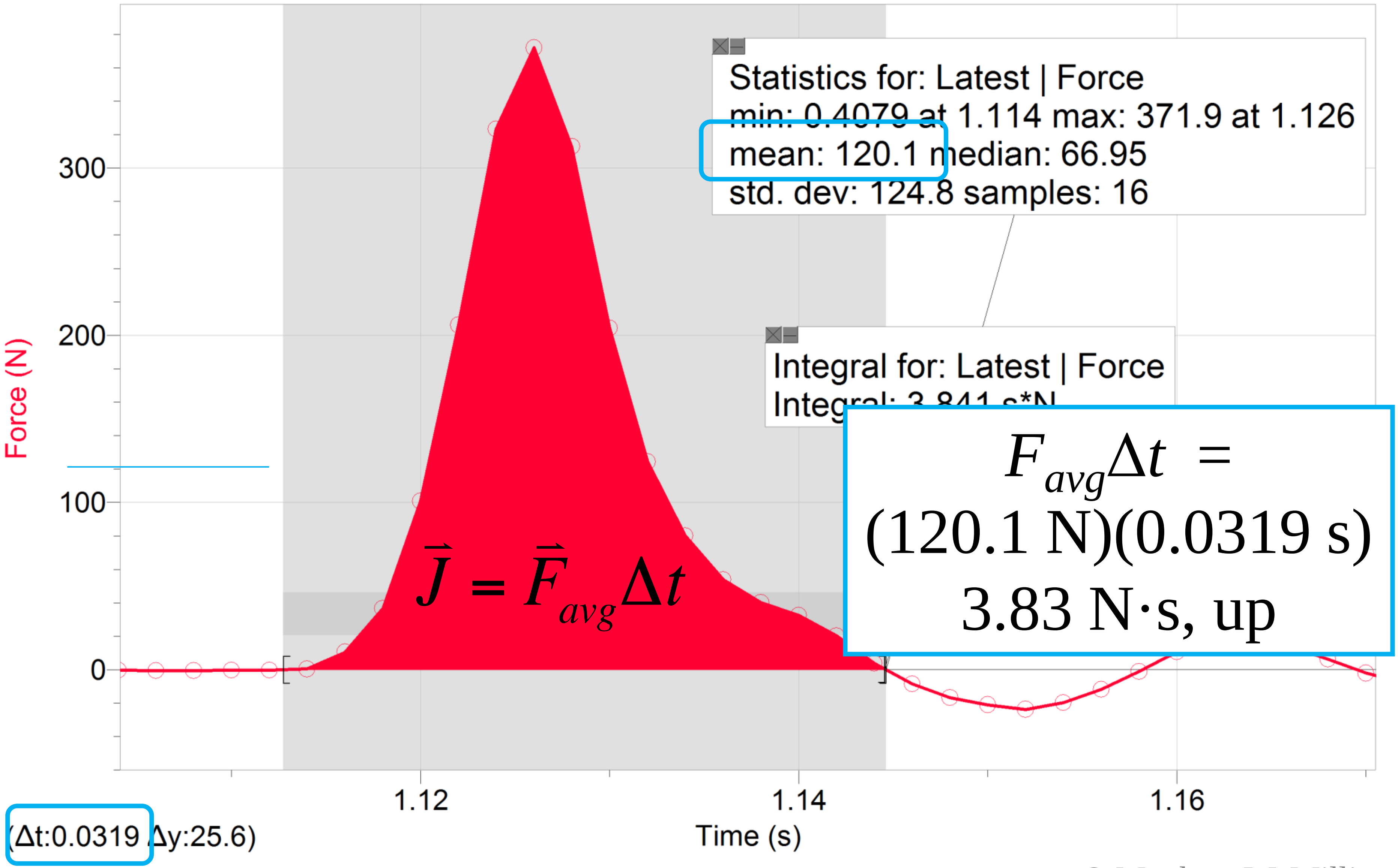
Normal Force vs. Time
Object Bouncing



Normal Force vs. Time
Object Bouncing



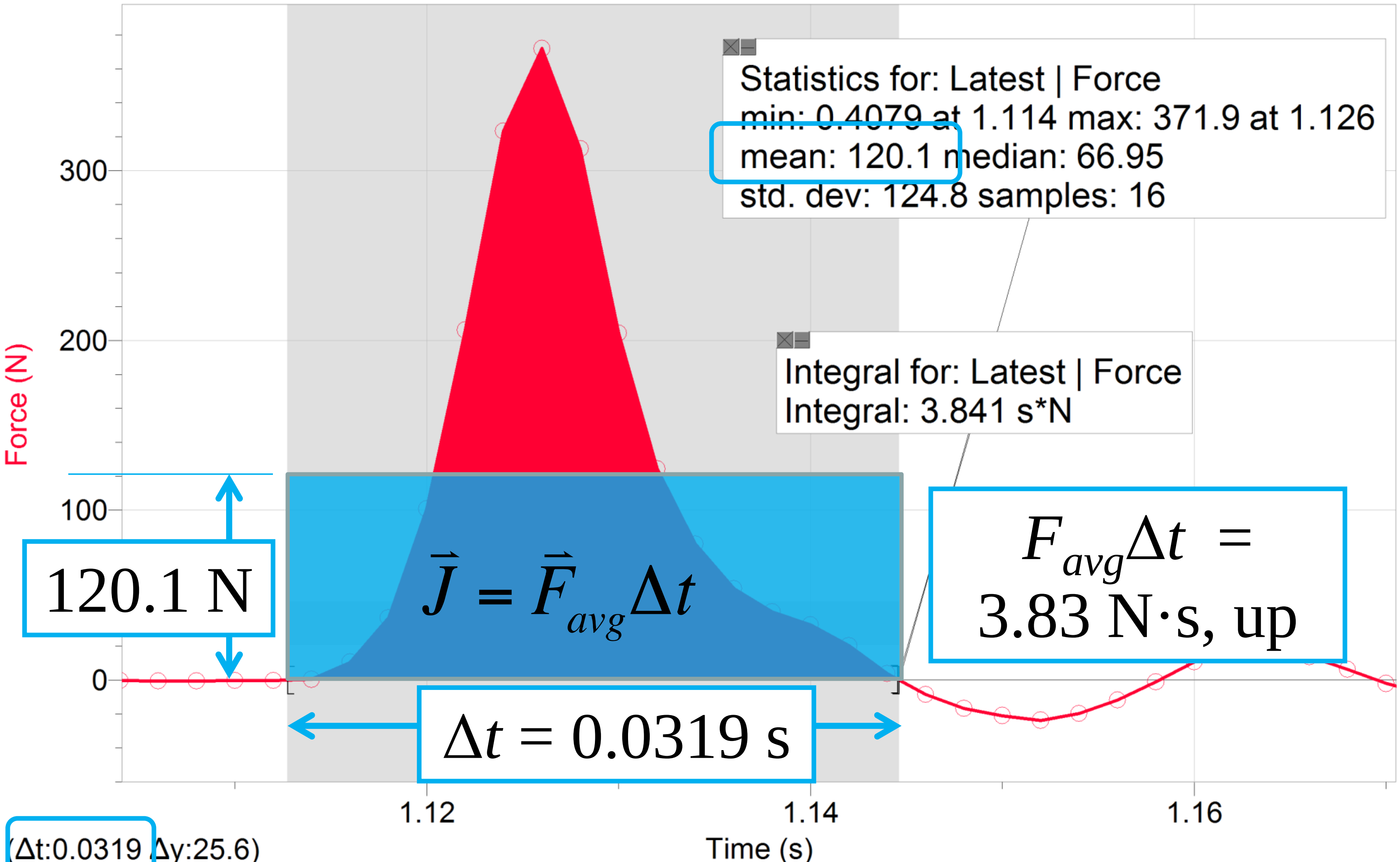
Normal Force vs. Time
Object Bouncing



$$\vec{J} = \vec{F}_{avg}\Delta t$$

$$F_{avg}\Delta t =$$
$$(120.1\text{ N})(0.0319\text{ s})$$
3.83 N·s, up

Normal Force vs. Time
Object Bouncing



Variable Mass Systems

Motion of a Rocket

	The student will be able to:	HW:
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	The student will be able to:	HW:
6	Solve problems involving variable mass such as that of a rocket.	39 – 40

Object with Changing Mass

If an object loses or gains mass then it is interacting with the mass that it loses or gains.

$$\Sigma \vec{F} = \frac{d\vec{p}}{dt} = \frac{d\left(m\vec{v}\right)}{dt}$$

$$\Sigma \vec{F} = m\frac{d\vec{v}}{dt} + \frac{dm}{dt}\vec{v}$$

Note: use of the chain rule is not a correct derivation of the result, but rather illustrates intuitively the effect of changing mass.

Net external force on system

Rate of change in total momentum of system

Rocket Motion

$$\Sigma\vec{F} = m\frac{d\vec{v}}{dt} + \frac{dm}{dt}\vec{v}_e$$

rate of change in momentum of rocket

rate of change in momentum of exhaust particles = thrust of the engine!

Rocket Motion

$$\Sigma\vec{F} = m\frac{d\vec{v}}{dt} + \frac{dm}{dt}\vec{v}_e$$

m = mass of rocket; it is <u>not</u> constant! (changes at rate *dm*/*dt*)

dv/dt = rate of change in velocity of rocket

dm/dt = “burn rate”

v_e = velocity of exhaust gases *relative to rocket* (typically a constant value)