Unit Outline

I. Center of Mass
   discrete, continuous

II. Motion of a System of Particles

III. Conservation of Momentum
   frame of reference

IV. Impulse

V. Variable Mass

	The student will be able to:	HW:
1	Determine the center of mass for a set of objects or particles and/or a continuous distribution of mass.	1 – 7
2	Apply Newton's 2nd Law to a system of particles and solve related problems either with the presence or absence of external forces.	8 – 12
3	State and apply the Law of Conservation of Momentum and solve related problems.	13 – 23
4	Define and apply elasticity and solve related problems.	24 – 30
5	Define and apply the concept of impulse and solve problems that relate momentum, force, and impulse.	31 – 38
6	Solve problems involving variable mass such as that of a rocket.	39 – 40

Position of center of mass: $\vec{r}_{cm} = \dfrac{\sum m_i \vec{r}_i}{\sum m_i}$

Differentiate to get velocity…

Position of center of mass: $\vec{r}_{cm} = \dfrac{\sum m_i \vec{r}_i}{\sum m_i}$

Velocity of center of mass: $\vec{v}_{cm} = \dfrac{\sum m_i \vec{v}_i}{\sum m_i}$

Differentiate to get acceleration…

Position of center of mass: $$\vec{r}_{cm} = \frac{\sum m_i \vec{r}_i}{\sum m_i}$$

Velocity of center of mass: $$\vec{v}_{cm} = \frac{\sum m_i \vec{v}_i}{\sum m_i}$$

Acceleration of center of mass: $$\vec{a}_{cm} = \frac{\sum m_i \vec{a}_i}{\sum m_i}$$

The motion of a system of masses has properties based on these equations that follow from the definition of the center of mass. Taking the system to be an “entity”, these three values describe the entity’s position, velocity, and acceleration.

$$\vec{r}_{cm} = \frac{\sum m_i \vec{r}_i}{\sum m_i}$$

$$\vec{v}_{cm} = \frac{\sum m_i \vec{v}_i}{\sum m_i}$$

$$\vec{a}_{cm} = \frac{\sum m_i \vec{a}_i}{\sum m_i}$$

$$\vec{a}_{cm} = \frac{\sum m_i \vec{a}_i}{\sum m_i}$$

$$\left(\sum m_i\right)\vec{a}_{cm} = \sum m_i \vec{a}_i$$

$$\Sigma \vec{F}_{ext} = \left(\Sigma m\right)\vec{a}_{cm}$$

Systems of Particles

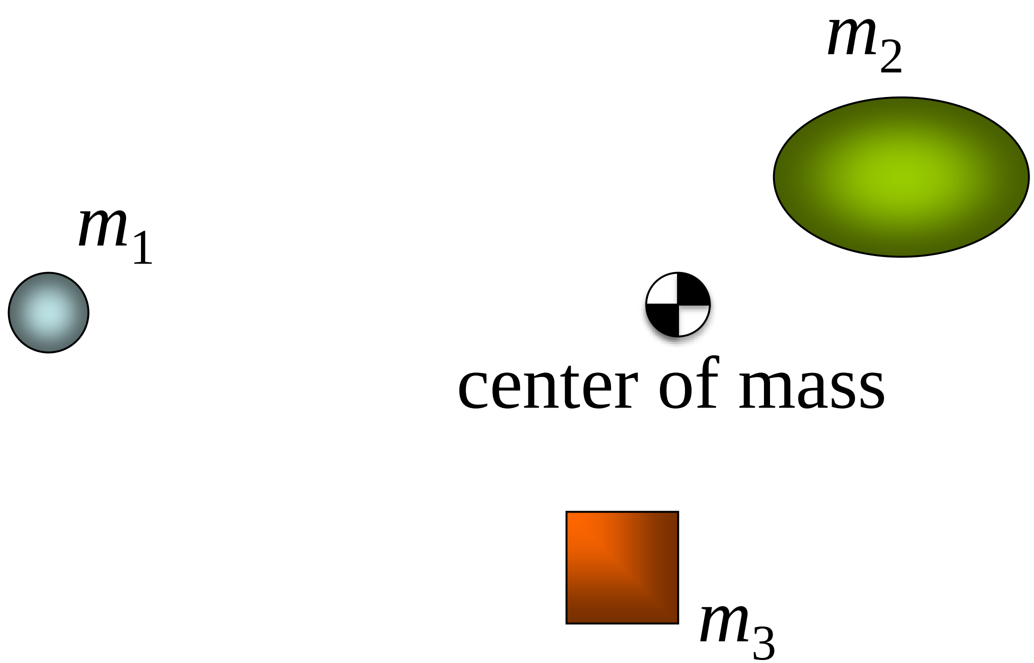
- A **system of particles** is simply an arbitrary set of objects.
- It is useful to analyze such a set in terms of **external forces**.
- An **external force** is an interaction between an object outside of the set with one or more of the objects in the set.
- An internal force is an interaction between two objects contained in the set.

Newton's 2nd Law for a system of particles

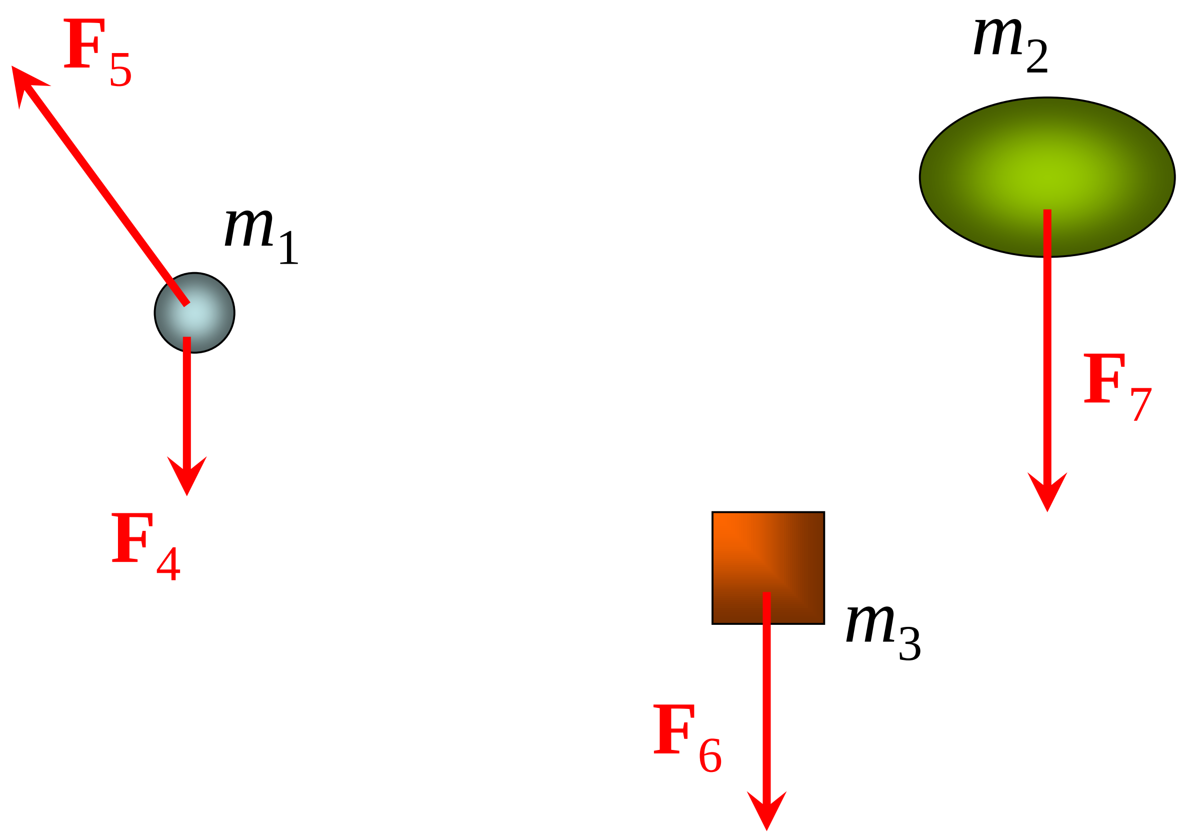
$$\Sigma \vec{F}_{ext} = \left(\Sigma m\right)\vec{a}_{cm}$$

The net external force equals the total mass times the acceleration of the center of mass of the system.

Consider a collection of three objects…



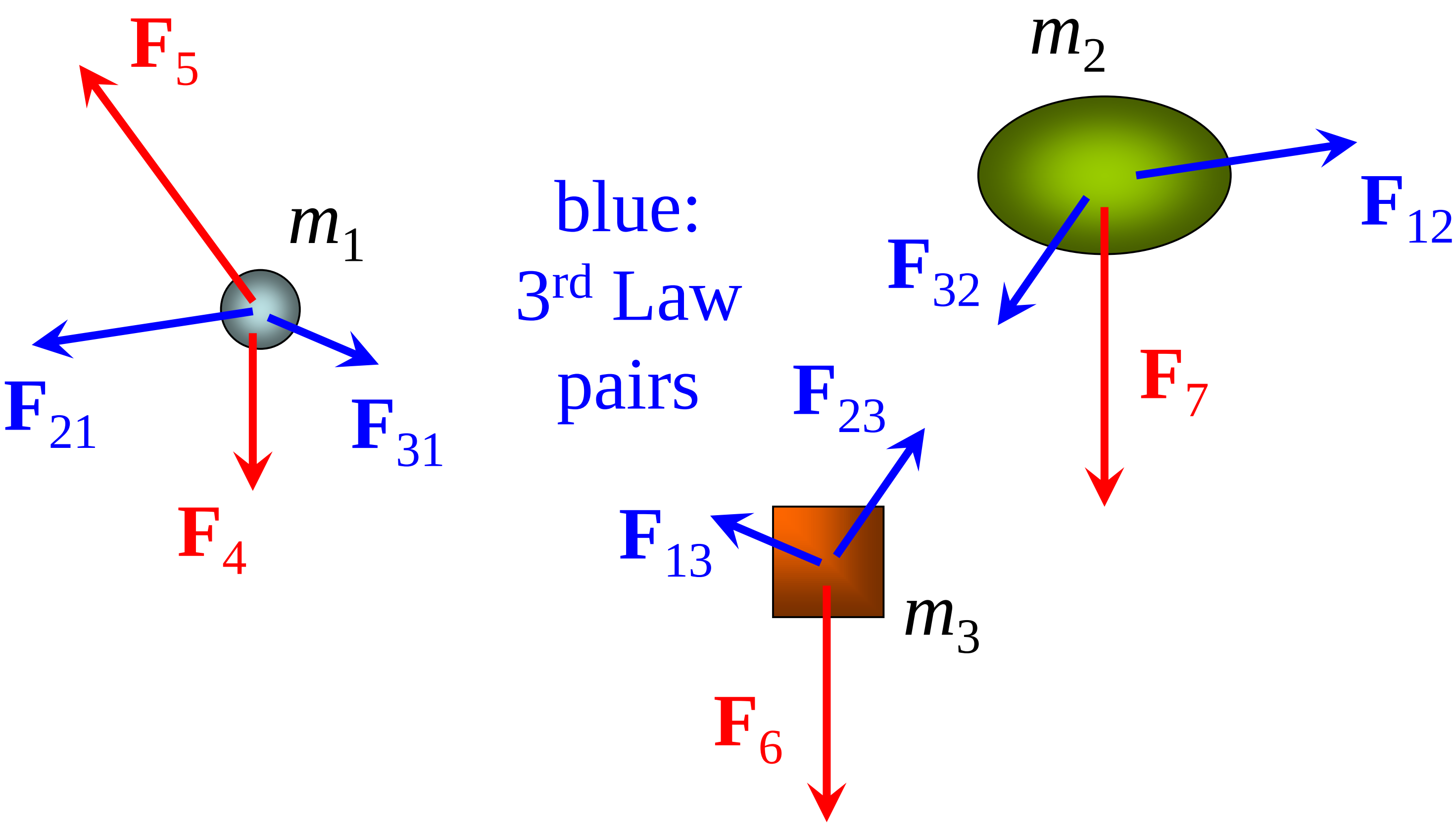
Consider a collection of three objects…



Suppose all three objects are affected by a force field (like gravity from a planet) – forces F_4, F_6, and F_7.

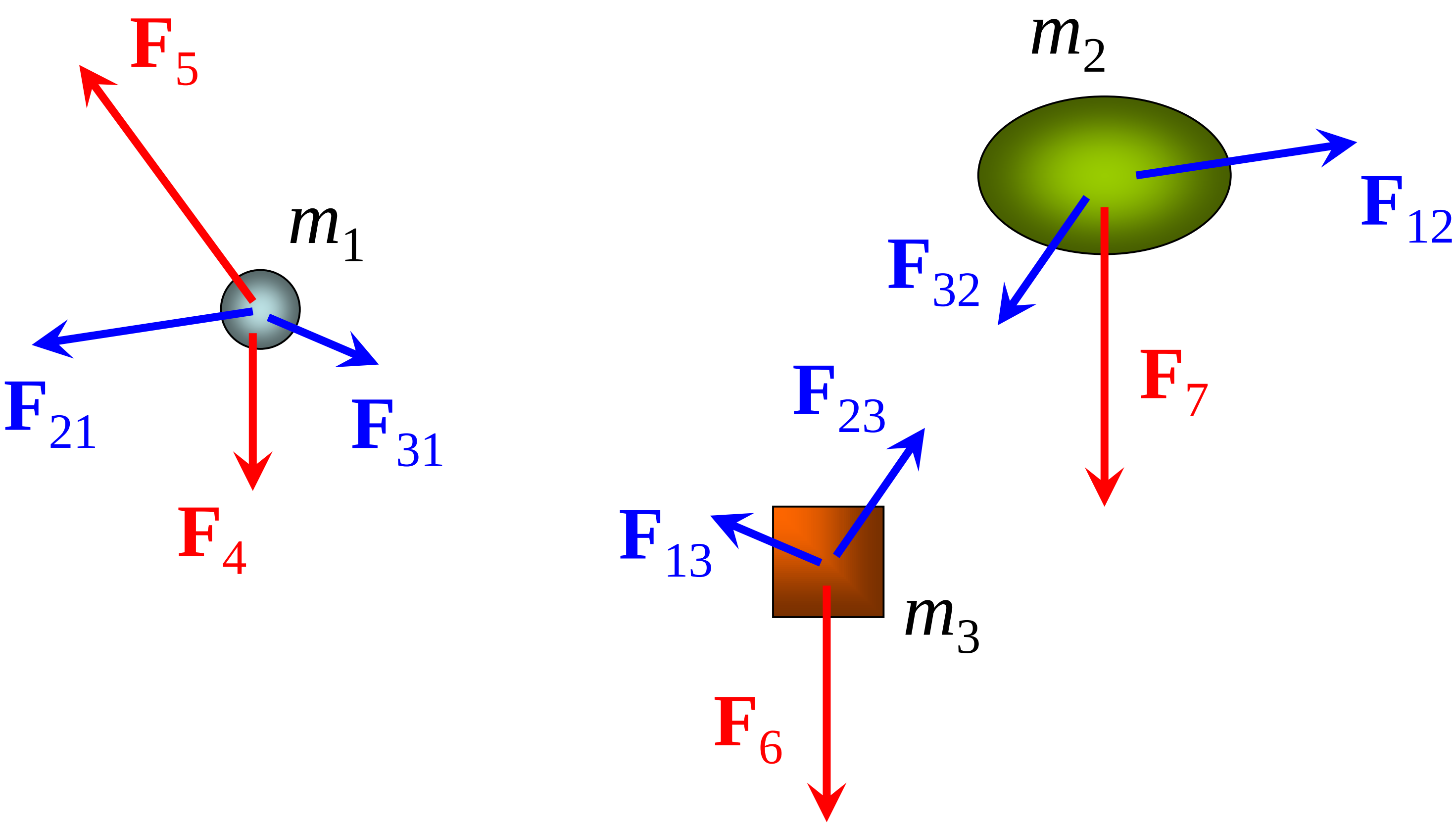
And object one is pulled by some forth object (like a person) – force F_5.

Consider a collection of three objects…



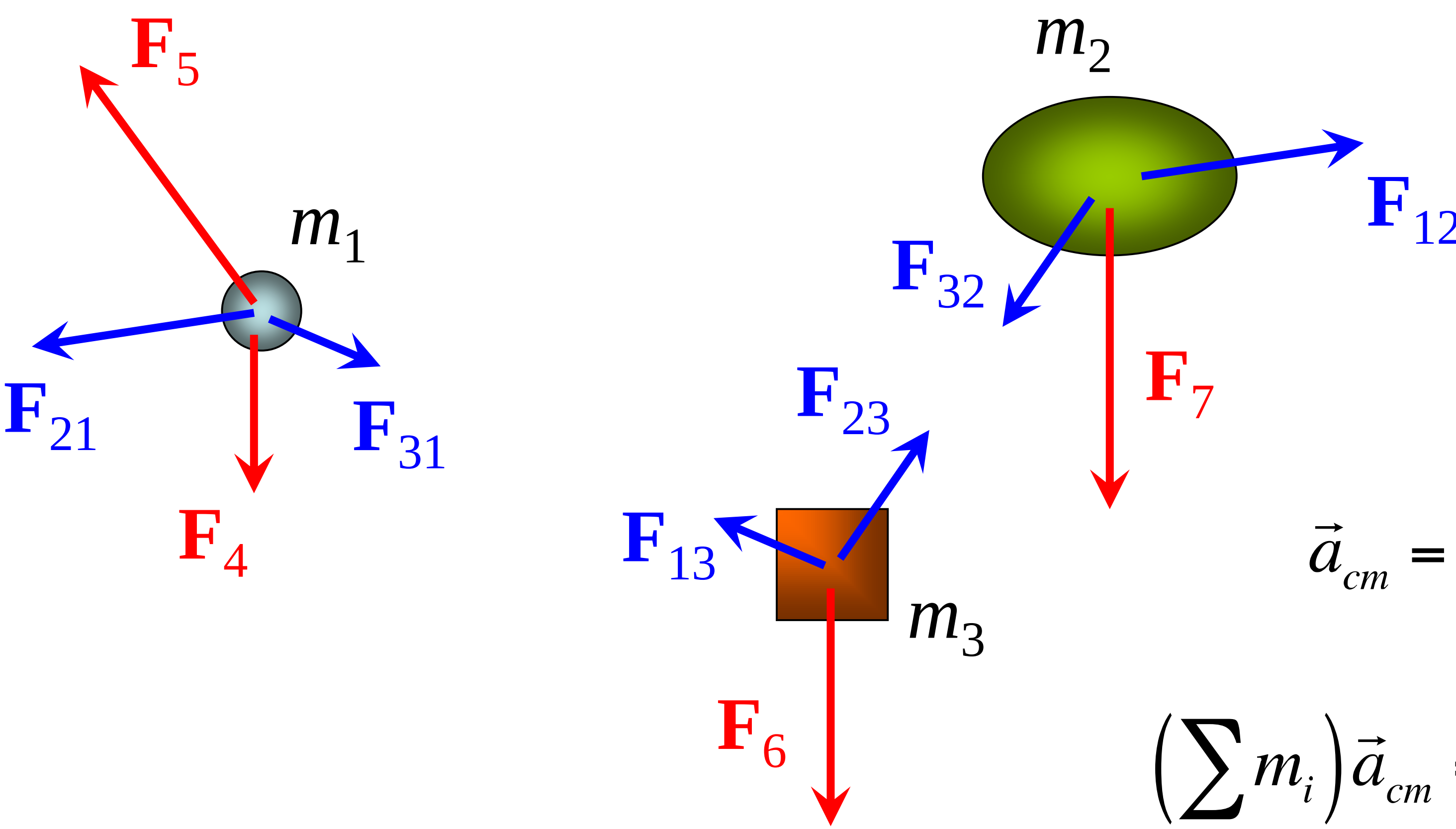
Suppose object one repels object two (like two positive charges that repel) – force pairs F_{12} & F_{21}.

And object three attracts one and two (like a magnet and pieces of steel) – force pairs F_{13} & F_{31} and F_{23} & F_{32}.



$$\Sigma\mathbf{F}_1 + \Sigma\mathbf{F}_2 + \Sigma\mathbf{F}_3 = m_1\mathbf{a}_1 + m_2\mathbf{a}_2 + m_3\mathbf{a}_3$$

$$\mathbf{F}_{21} + \mathbf{F}_{31} + \mathbf{F}_4 + \mathbf{F}_5 + \mathbf{F}_{12} + \mathbf{F}_{32} + \mathbf{F}_6 + \mathbf{F}_{13} + \mathbf{F}_{23} + \mathbf{F}_7 = (\Sigma\, m)\mathbf{a}_{CM}$$

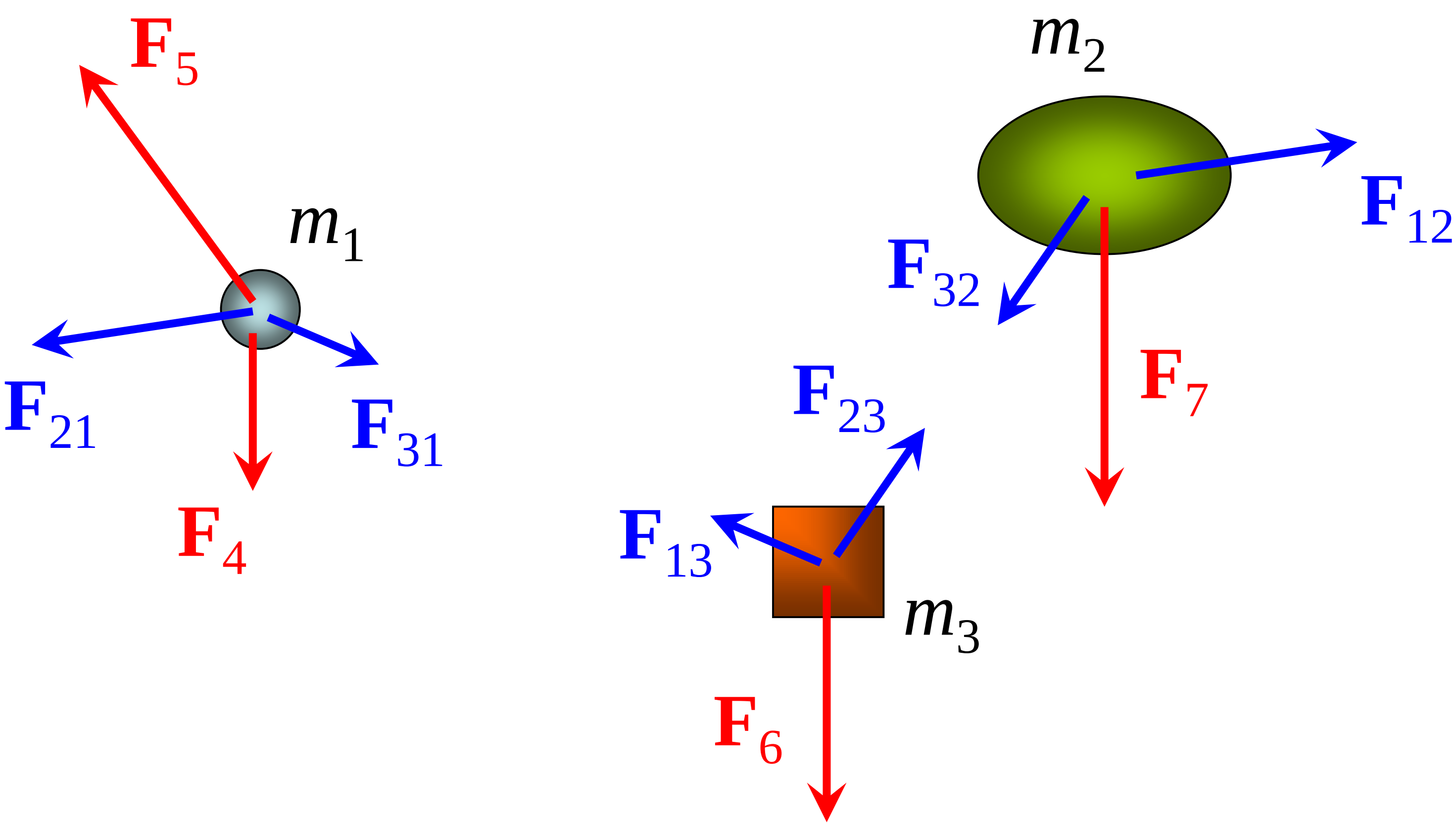


$$\vec{a}_{cm} = \frac{\sum m_i \vec{a}_i}{\sum m_i}$$

$$\left(\sum m_i\right)\vec{a}_{cm} = \sum m_i \vec{a}_i$$

$$\Sigma \mathbf{F}_1 + \Sigma \mathbf{F}_2 + \Sigma \mathbf{F}_3 = m_1 \mathbf{a}_1 + m_2 \mathbf{a}_2 + m_3 \mathbf{a}_3$$

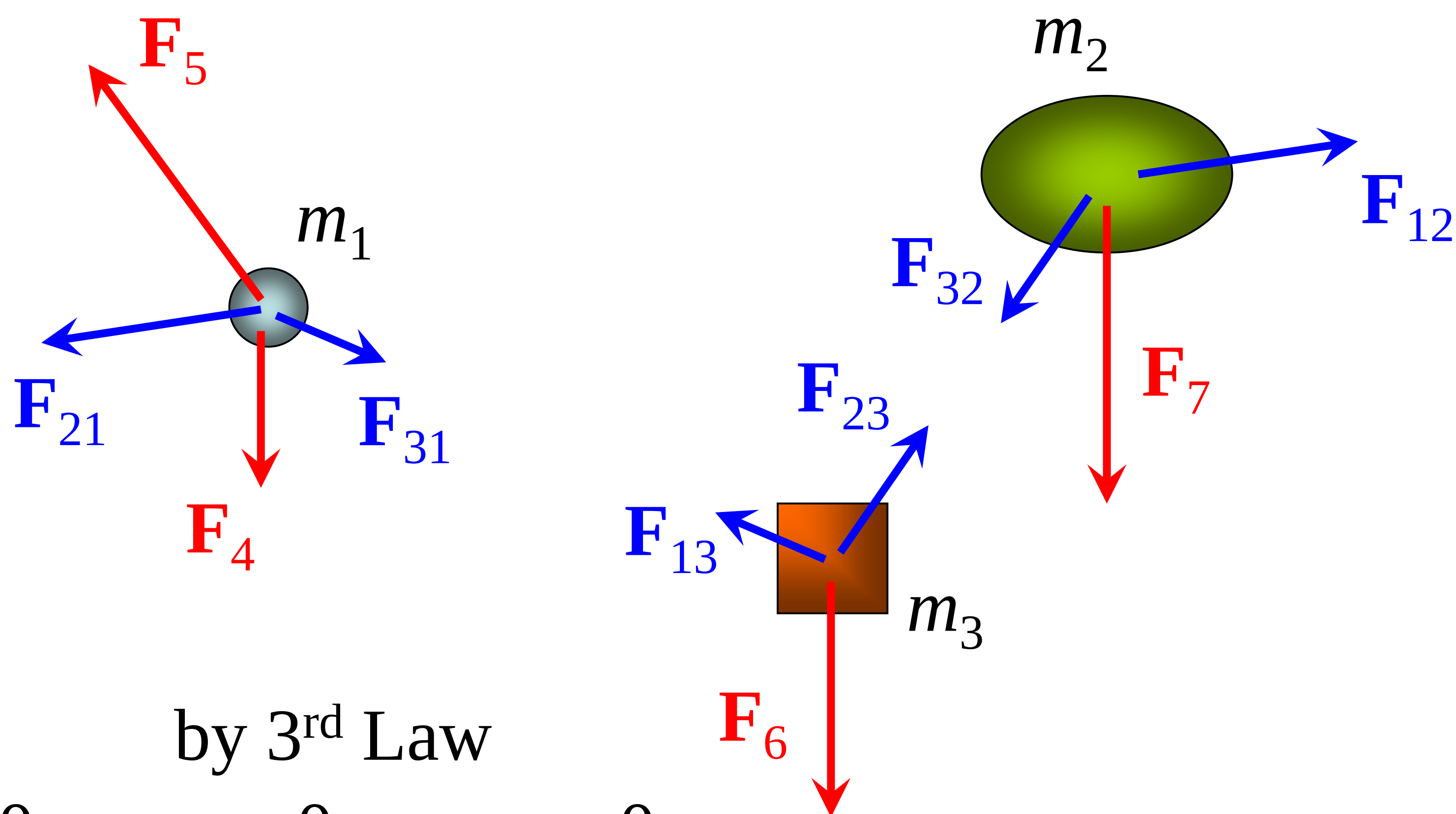
$$\mathbf{F}_{21} + \mathbf{F}_{31} + \mathbf{F}_4 + \mathbf{F}_5 + \mathbf{F}_{12} + \mathbf{F}_{32} + \mathbf{F}_6 + \mathbf{F}_{13} + \mathbf{F}_{23} + \mathbf{F}_7 = (\Sigma\, m)\mathbf{a}_{CM}$$



$$\Sigma\mathbf{F}_1 + \Sigma\mathbf{F}_2 + \Sigma\mathbf{F}_3 = m_1\mathbf{a}_1 + m_2\mathbf{a}_2 + m_3\mathbf{a}_3$$

$$\mathbf{F}_{21} + \mathbf{F}_{31} + \mathbf{F}_4 + \mathbf{F}_5 + \mathbf{F}_{12} + \mathbf{F}_{32} + \mathbf{F}_6 + \mathbf{F}_{13} + \mathbf{F}_{23} + \mathbf{F}_7 = (\Sigma\, m)\mathbf{a}_{\mathrm{CM}}$$

$$\mathbf{F}_{12} + \mathbf{F}_{21} + \mathbf{F}_{13} + \mathbf{F}_{31} + \mathbf{F}_{23} + \mathbf{F}_{32} + \mathbf{F}_4 + \mathbf{F}_5 + \mathbf{F}_6 + \mathbf{F}_7 = (\Sigma\, m)\mathbf{a}_{\mathrm{CM}}$$

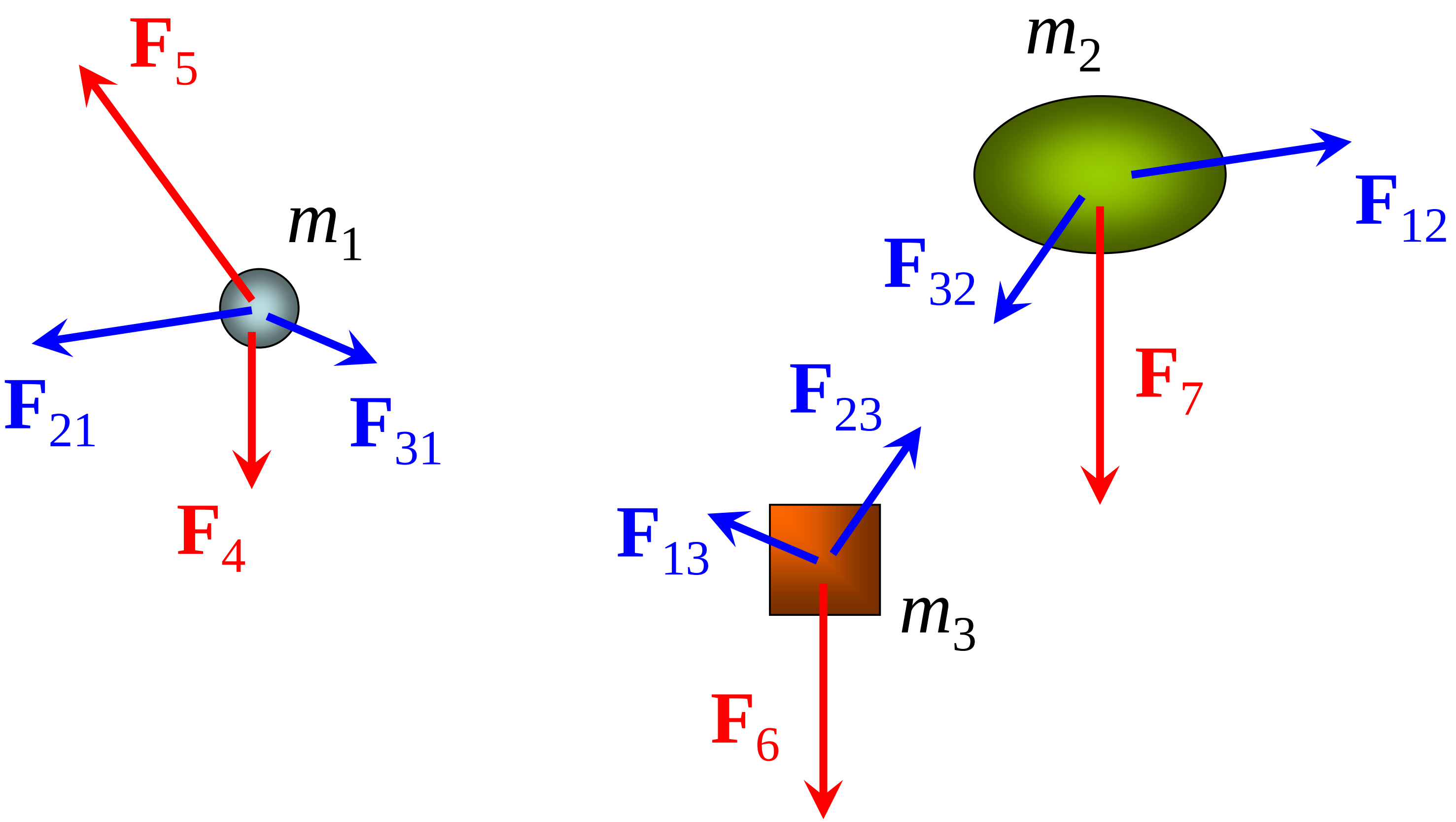


by 3rd Law

$$\overset{0}{\cancel{\mathbf{F}_{12}+\mathbf{F}_{21}}}+\overset{0}{\cancel{\mathbf{F}_{13}+\mathbf{F}_{31}}}+\overset{0}{\cancel{\mathbf{F}_{23}+\mathbf{F}_{32}}}+\mathbf{F}_4+\mathbf{F}_5+\mathbf{F}_6+\mathbf{F}_7=(\Sigma\, m)\mathbf{a}_{\mathrm{CM}}$$

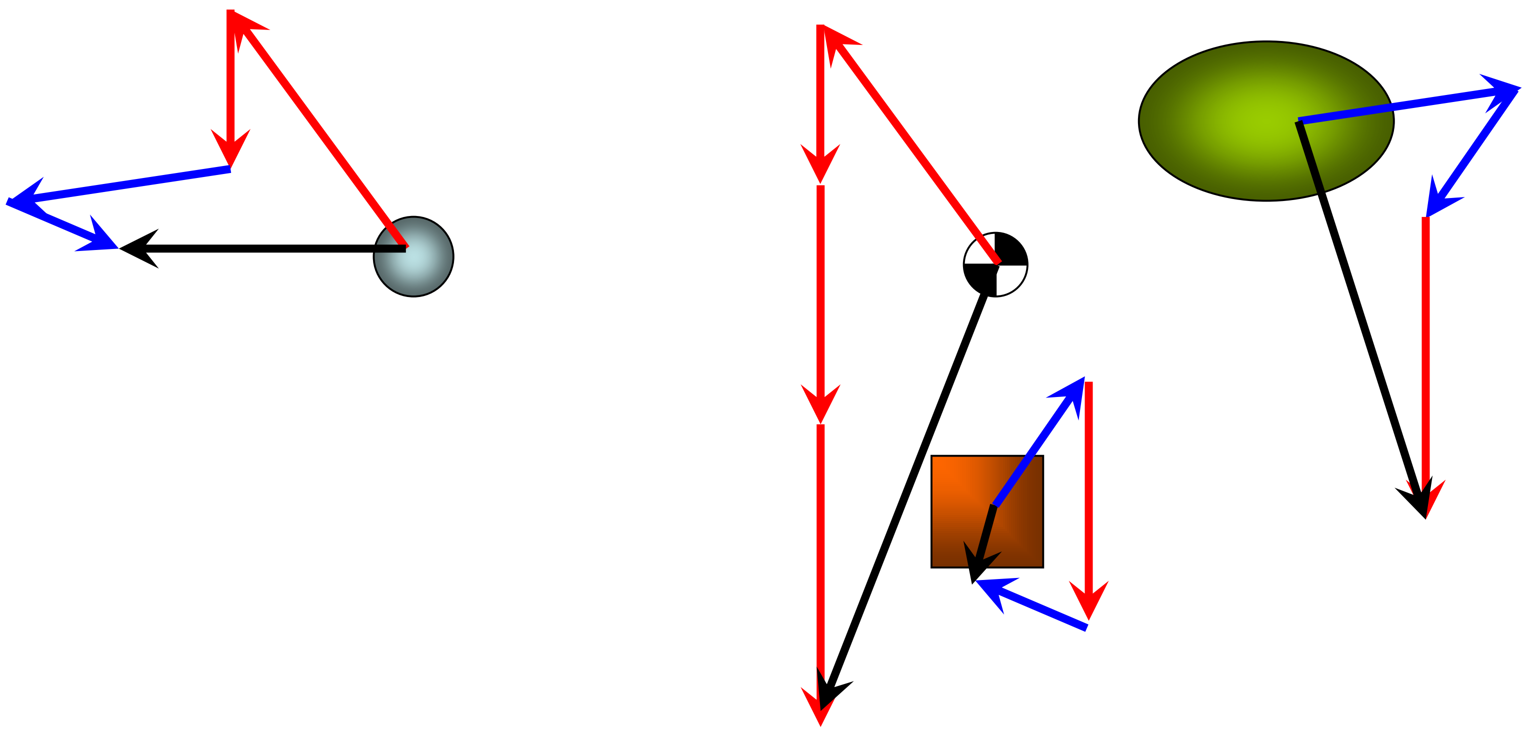
$$\cancel{\Sigma\mathbf{F}_{\mathrm{int}}}+\Sigma\mathbf{F}_{\mathrm{ext}}=(\Sigma\, m)\mathbf{a}_{\mathrm{CM}}$$

$$\Sigma\mathbf{F}_{\mathrm{ext}}=(\Sigma\, m)\mathbf{a}_{\mathrm{CM}}$$



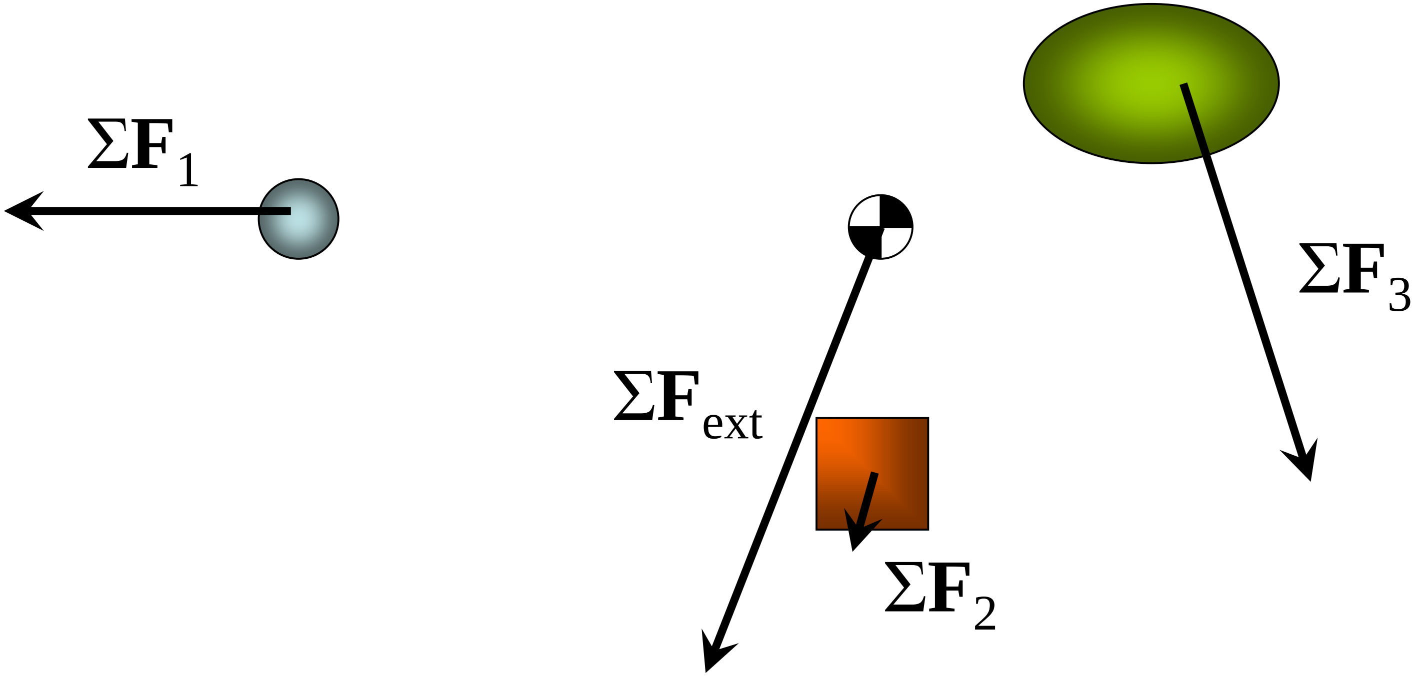
$$\Sigma \mathbf{F}_{ext} = (\Sigma\, m)\mathbf{a}_{CM}$$

Newton's 2nd Law for a system of particles



$$\Sigma \mathbf{F}_{\text{ext}} = (\Sigma\, m)\mathbf{a}_{\text{CM}}$$

Newton's 2nd Law for a system of particles



$$\Sigma \mathbf{F}_{ext} = (\Sigma\, m)\mathbf{a}_{CM}$$

Newton's 2nd Law for a system of particles

Consequences of $\Sigma \mathbf{F}_{ext} = (\Sigma\, m)\mathbf{a}_{CM}$

- The motion of the center of mass is completely controlled by external forces.
- Internal forces can have no effect on the motion of the center of mass!
- Can also be applied to a single body, such as a human body.
- The acceleration of the CM of your body is equal the net external force divided by your total mass. The forces of your muscles have no effect on the motion of your CM.

The Fosbury Flop

Dick Fosbury image credit: AP

The "Fosbury Flop" revolutionized the sport of the high jump. A person can only jump upward at a certain speed. Once you leave the ground your center of mass accelerates downward at g – freefall (because gravity is the net external force on the system). This means your center of mass can only go so high ($v^2/2g$) no matter how you move the parts of your body. *But*, by contorting the body as shown it may pass over a bar that is *higher* than the position of the body's center of mass. Thus it allows a higher bar to be hurdled without having to increase the initial upward speed of the jump.