Electric Flux and Potential

I. Electric Flux
- flux defined
- **Gauss's Law**

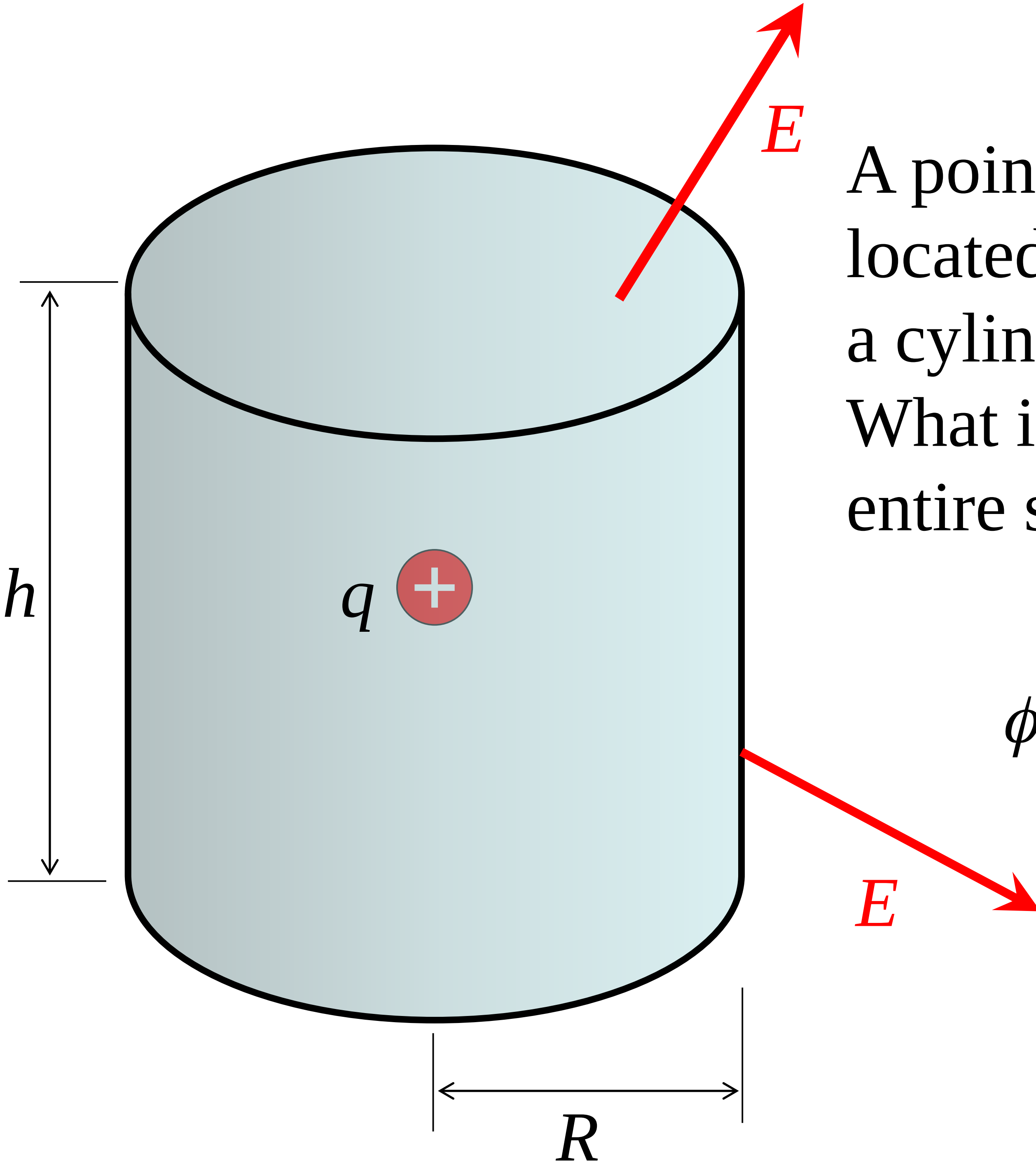
II. Electric Potential
- work and energy of charge
- potential defined
- potential of discrete charge(s)
- potential of charge distributions
- field related to potential

III. Conductors

© Matthew W. Milligan

	The student will be able to:	HW:
1	Define and apply the concept of electric flux and solve related problems.	✓ 1 – 5
2	State and apply Gauss's Law and solve related problems using Gaussian surfaces.	6 – 17
3	Calculate work and potential energy for discrete charges and solve related problems including work to assemble or disassemble.	18 – 25
4	Define and apply the concept of electric potential and solve related problems for a discrete set of point charges and/or a continuous charge distribution.	26 – 32
5	Use the electric field to determine potential or potential difference and solve related problems.	33 – 36
6	Use potential to determine electric field and solve related problems.	37 – 39
7	State the properties of conductors in electrostatic equilibrium and solve related problems.	40 – 46

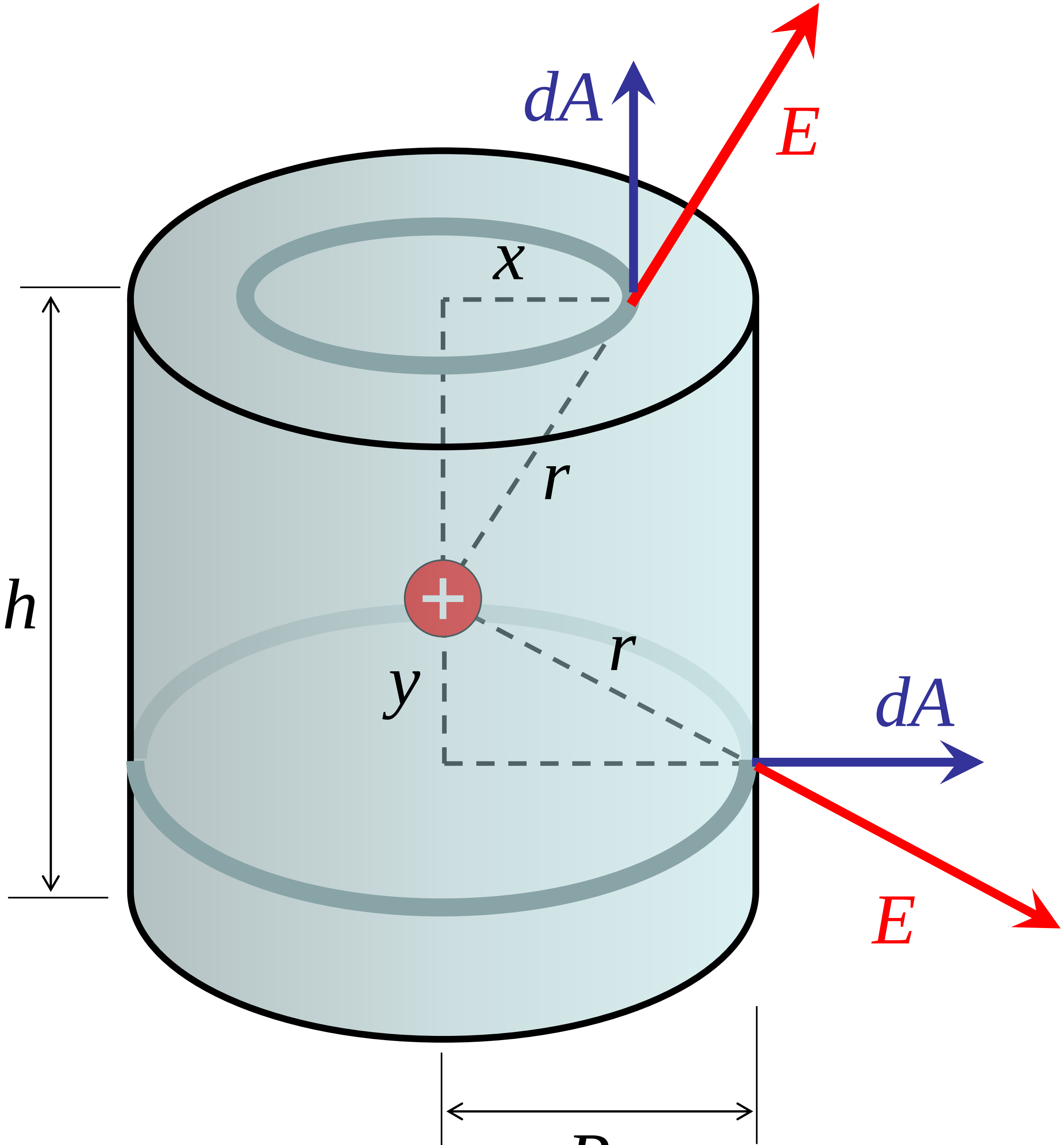
© Matthew W. Milligan



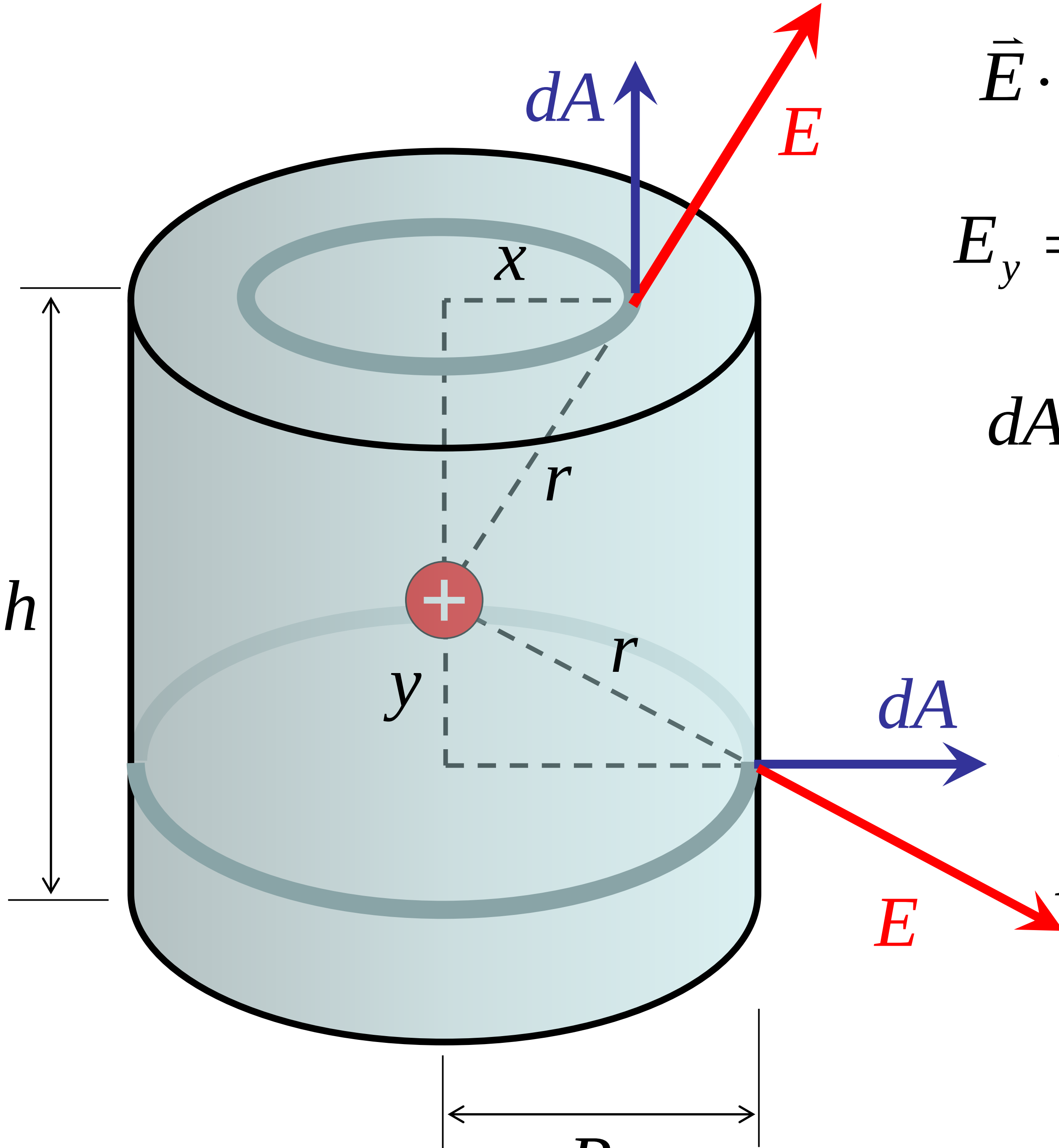
A point charge q is located at the center of a cylindrical surface. What is the flux for the entire surface?

$$\phi = \oint \vec{E} \cdot d\vec{A}$$

© Matthew W. Milligan



© Matthew W. Milligan



$$\vec{E}\cdot d\vec{A} = E_y dA$$

$$E_y = \frac{h}{2}\cdot\frac{1}{r}\cdot\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}$$

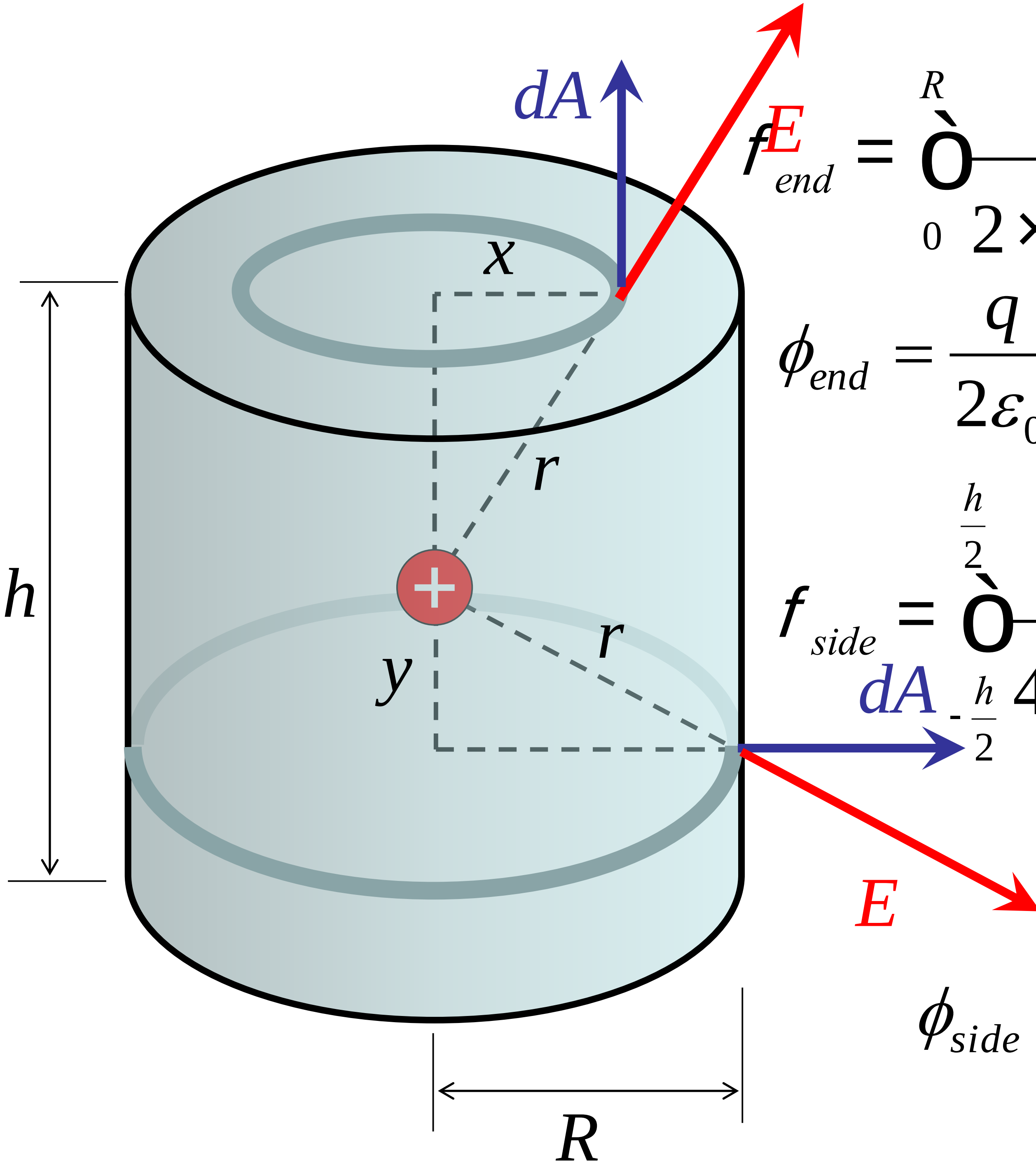
$$dA = 2\pi\, x\, dx$$

$$\vec{E}\cdot d\vec{A} = E_x dA$$

$$E_x = \frac{R}{r}\cdot\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}$$

$$dA = 2\pi\, R dy$$

© Matthew W. Milligan



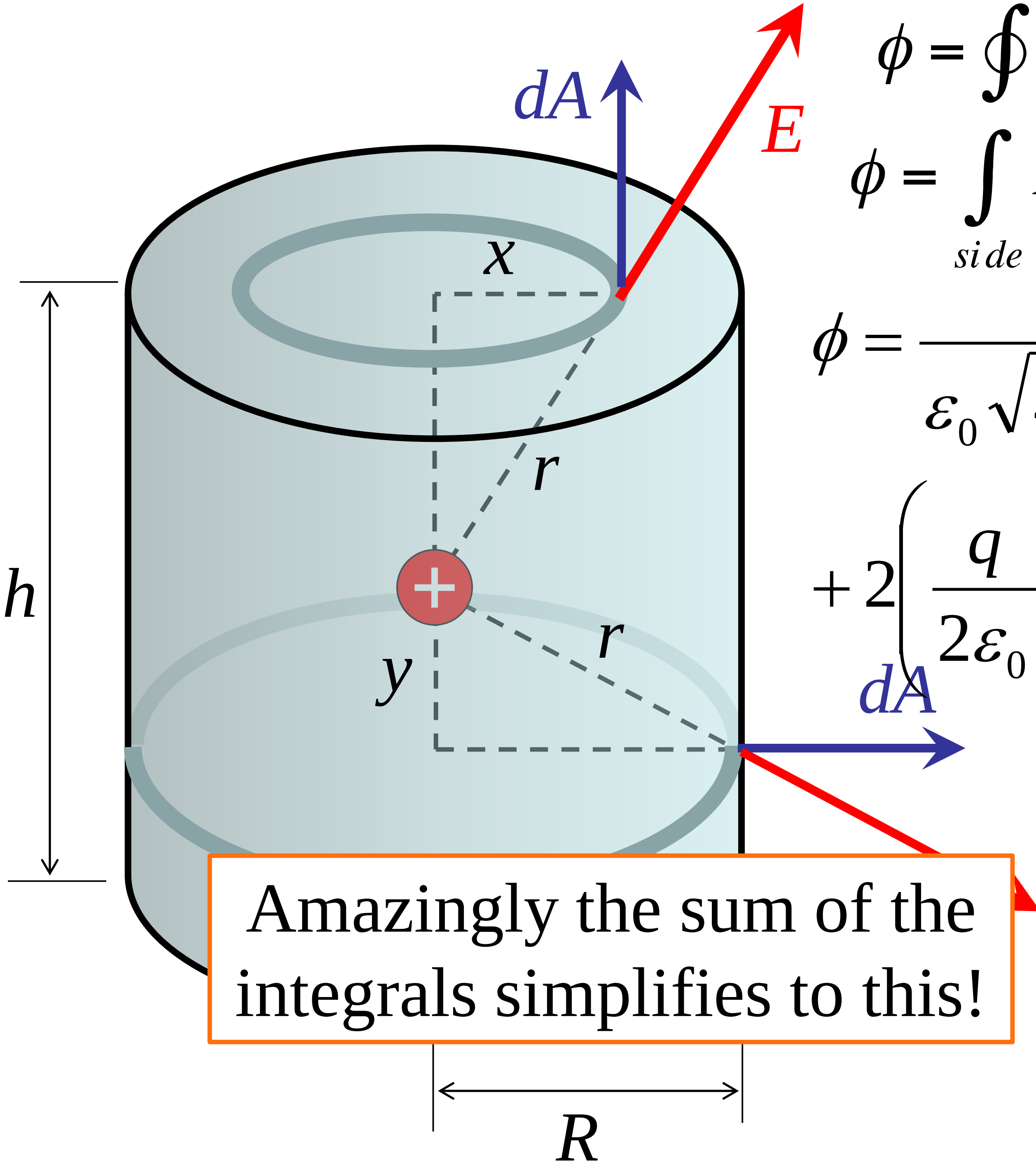
$$\phi_{end} = \int_0^R \frac{hq \cdot 2\pi x}{2 \cdot 4\pi\varepsilon_0 \left(\frac{h^2}{4} + x^2\right)^{\frac{3}{2}}} \cdot dx$$

$$\phi_{end} = \frac{q}{2\varepsilon_0} - \frac{qh}{2\varepsilon_0\sqrt{4R^2 + h^2}}$$

$$\phi_{side} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \frac{Rq \cdot 2\pi R}{4\pi\varepsilon_0 \left(R^2 + y^2\right)^{\frac{3}{2}}} \cdot dy$$

$$\phi_{side} = \frac{qh}{\varepsilon_0\sqrt{4R^2 + h^2}}$$

© Matthew W. Milligan



$$\phi = \oint \vec{E} \cdot d\vec{A}$$

$$\phi = \int_{side} \vec{E} \cdot d\vec{A} + 2\int_{end} \vec{E} \cdot d\vec{A}$$

$$\phi = \frac{qh}{\varepsilon_0 \sqrt{4R^2 + h^2}}$$

$$+2\left(\frac{q}{2\varepsilon_0} - \frac{qh}{2\varepsilon_0 \sqrt{4R^2 + h^2}} \right)$$

Amazingly the sum of the integrals simplifies to this!

$$\phi = \frac{q}{\varepsilon_0}$$

© Matthew W. Milligan

Gauss determined that a similar process integrating flux completely around a closed surface containing amount of charge q will *always simplify* to $\Phi = q/\varepsilon_0$, regardless of the shape of the surface or the configuration or amount of charge!

Gauss's Law

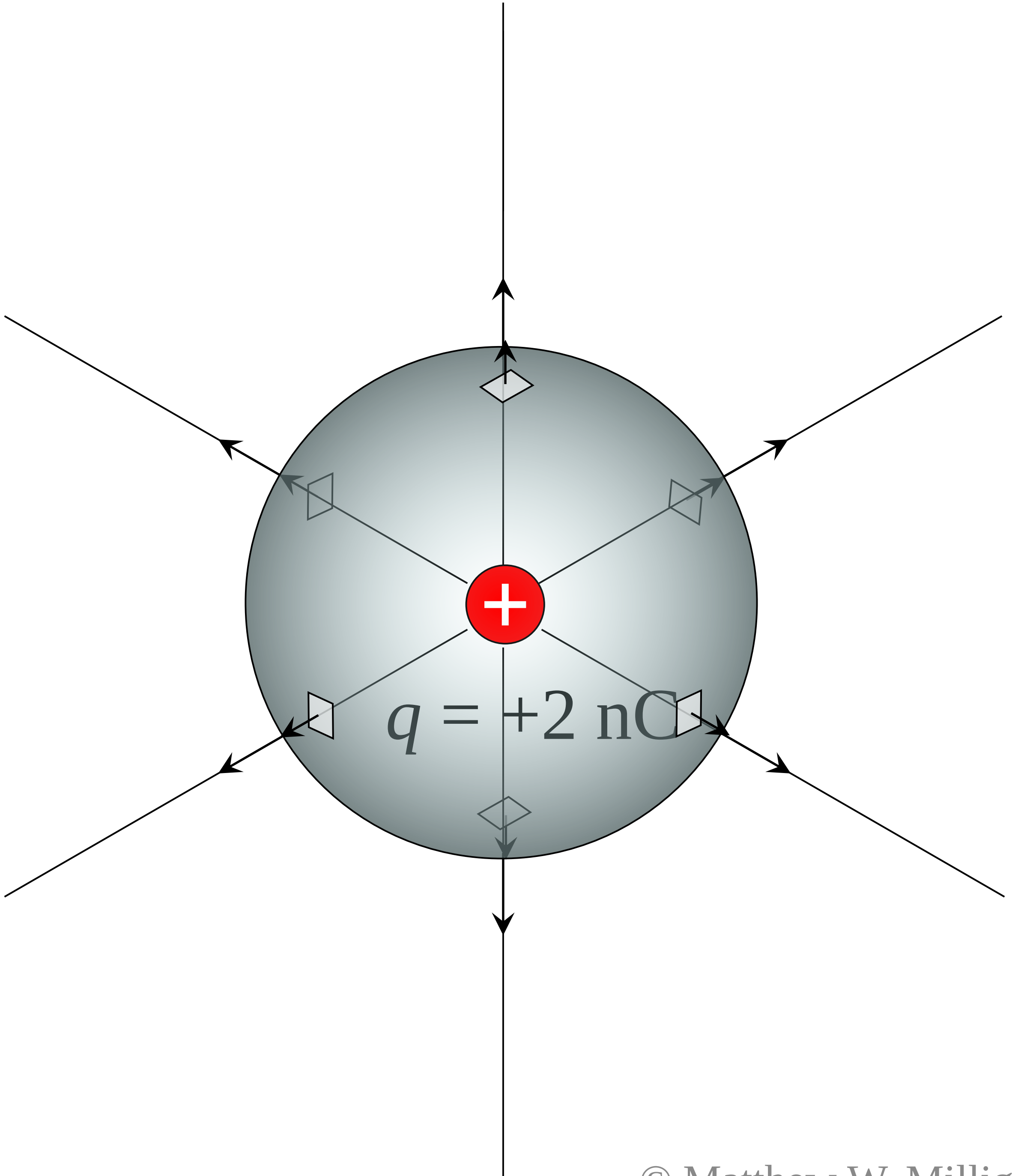
$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\varepsilon_0}$$

where: q_{enc} = total charge enclosed by surface
E = electric field
A = area vector (normal to surface)

note: The circle on the integral sign indicates integration entirely around a *closed surface* – *dA* is an incremental piece of this surface.

© Matthew W. Milligan

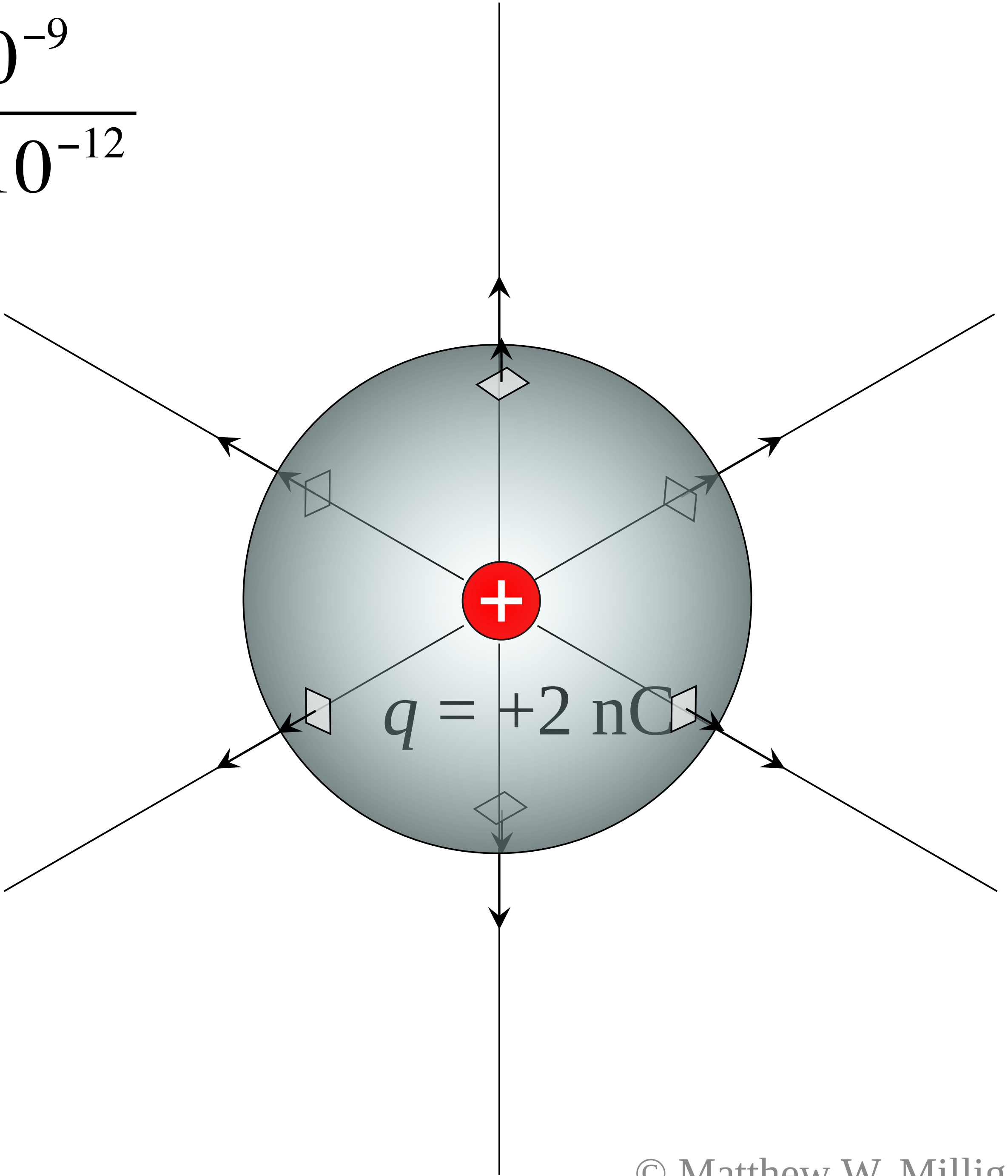
$\Phi_E = ?$



© Matthew W. Milligan

$$\oint \vec{E} \cdot d\vec{A} = \frac{2 \times 10^{-9}}{8.85 \times 10^{-12}}$$

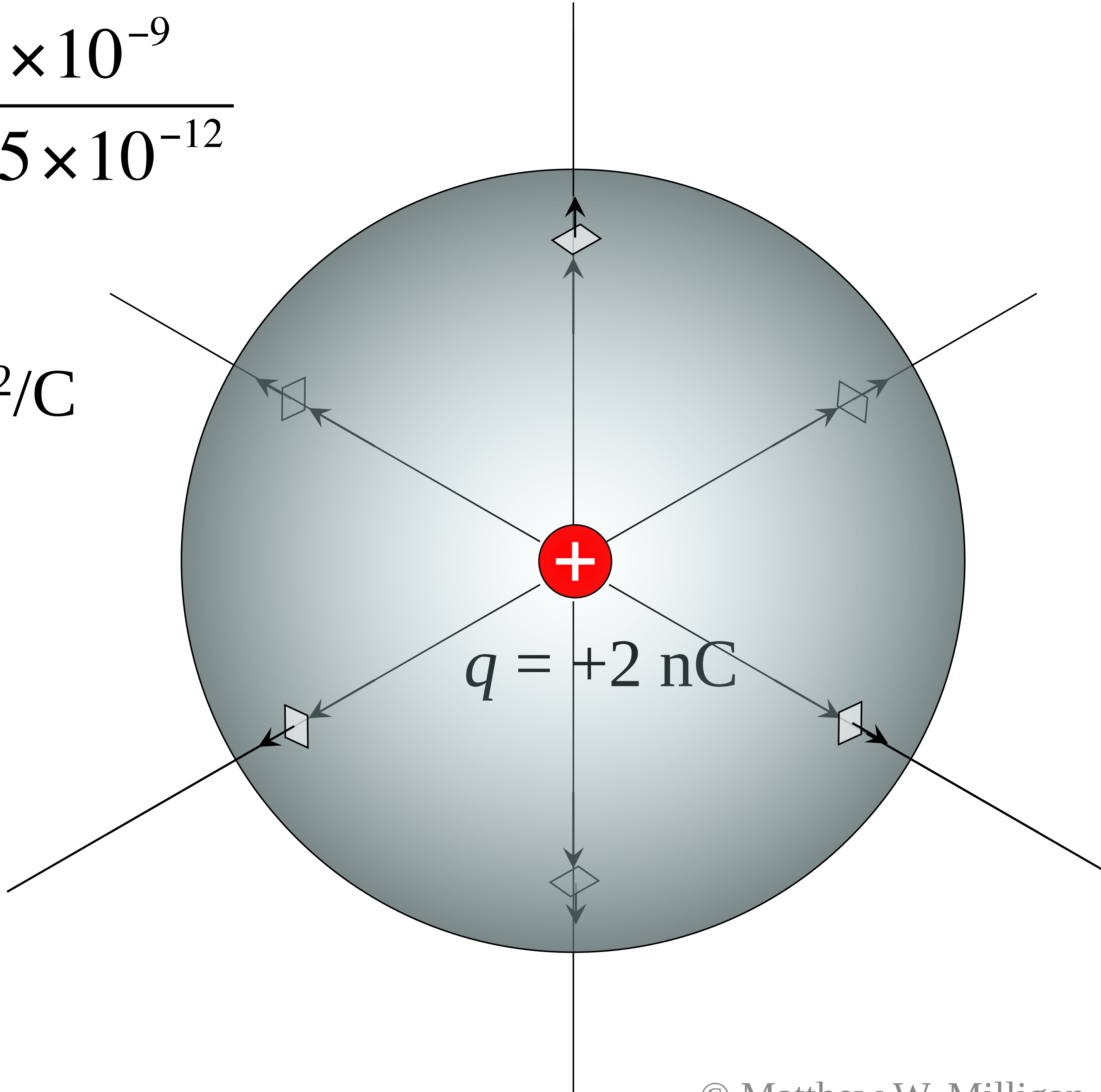
$\Phi_E = 226\ \text{Nm}^2/\text{C}$



© Matthew W. Milligan

$$\oint \vec{E} \cdot d\vec{A} = \frac{2 \times 10^{-9}}{8.85 \times 10^{-12}}$$

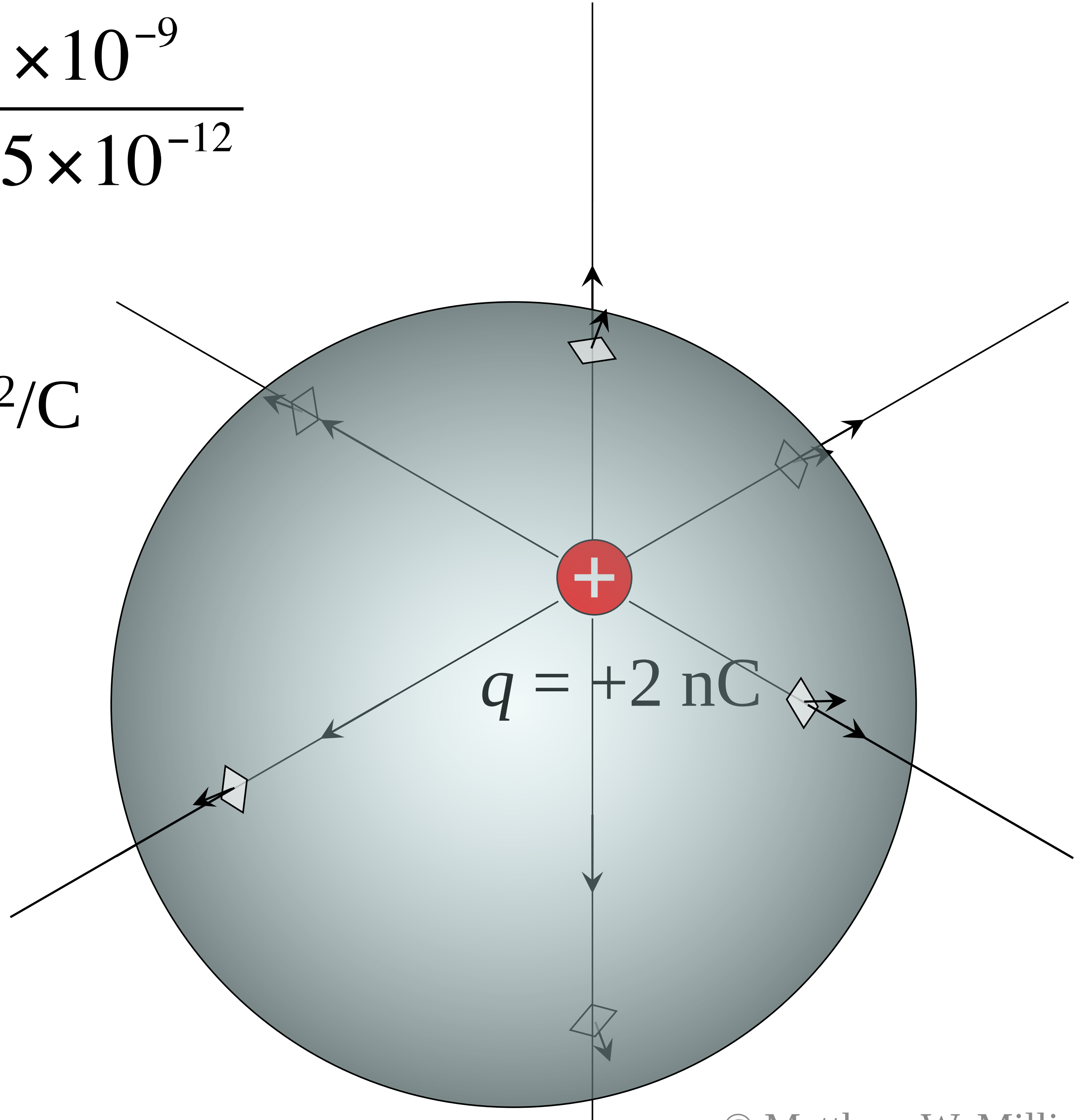
$$\Phi_E = 226 \text{ Nm}^2/\text{C}$$



© Matthew W. Milligan

$$\oint \vec{E} \cdot d\vec{A} = \frac{2 \times 10^{-9}}{8.85 \times 10^{-12}}$$

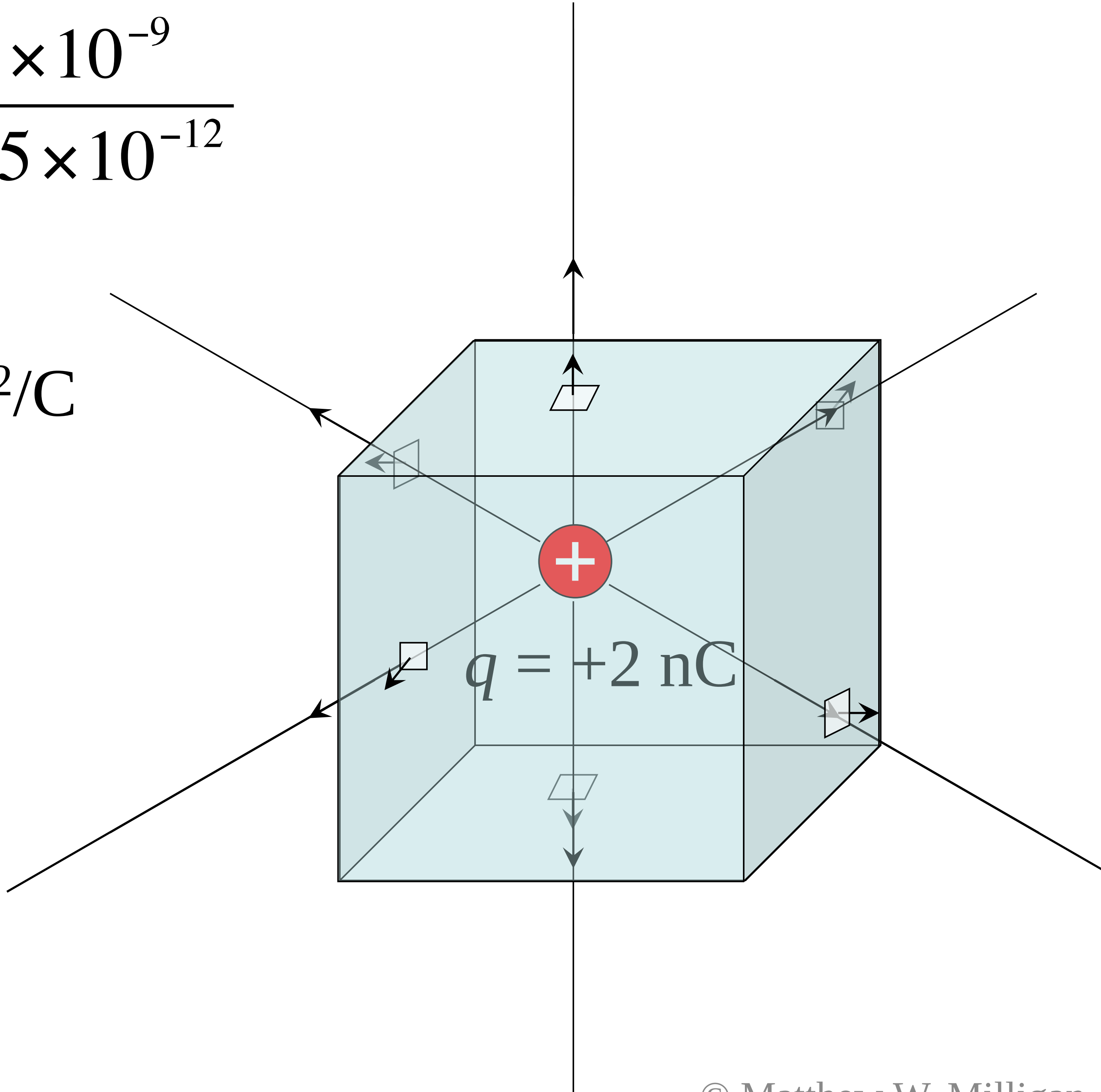
$\Phi_E = 226\ \text{Nm}^2/\text{C}$



© Matthew W. Milligan

$$\oint \vec{E} \cdot d\vec{A} = \frac{2 \times 10^{-9}}{8.85 \times 10^{-12}}$$

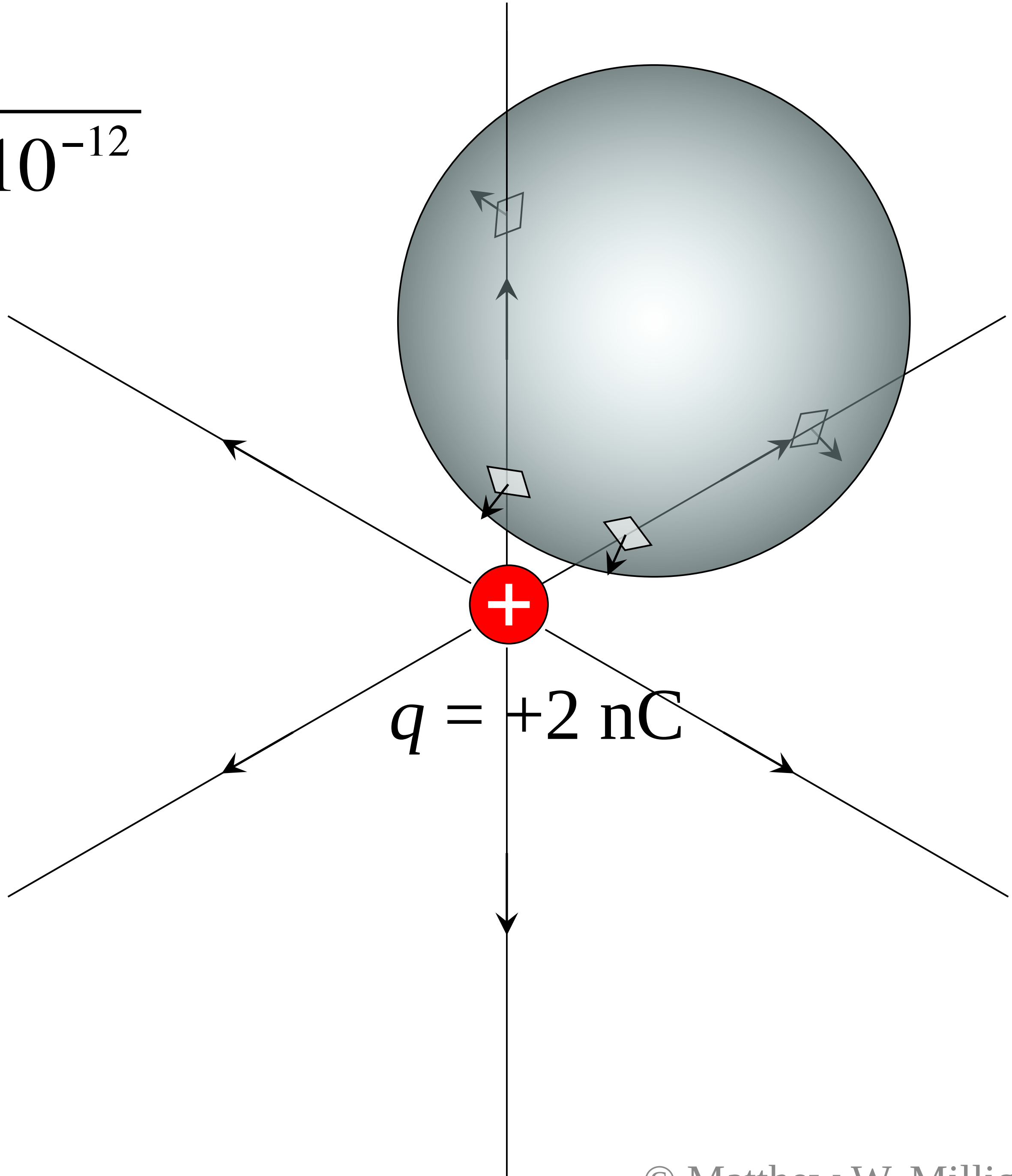
$\Phi_E = 226\ \mathrm{Nm^2/C}$



© Matthew W. Milligan

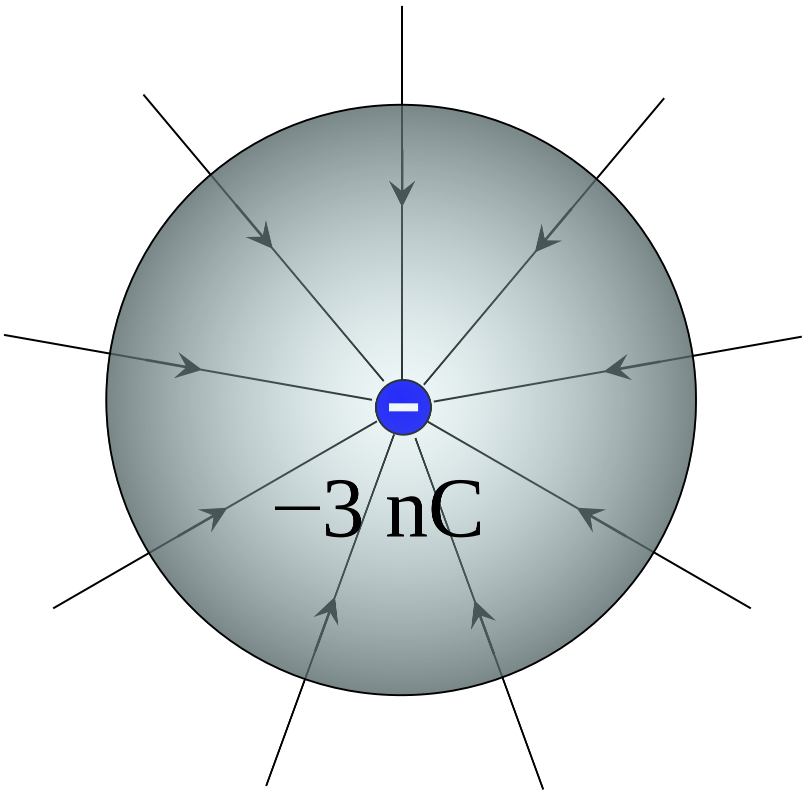
$$\oint \vec{E} \cdot d\vec{A} = \frac{0}{8.85 \times 10^{-12}}$$

$$\Phi_E = 0$$



© Matthew W. Milligan

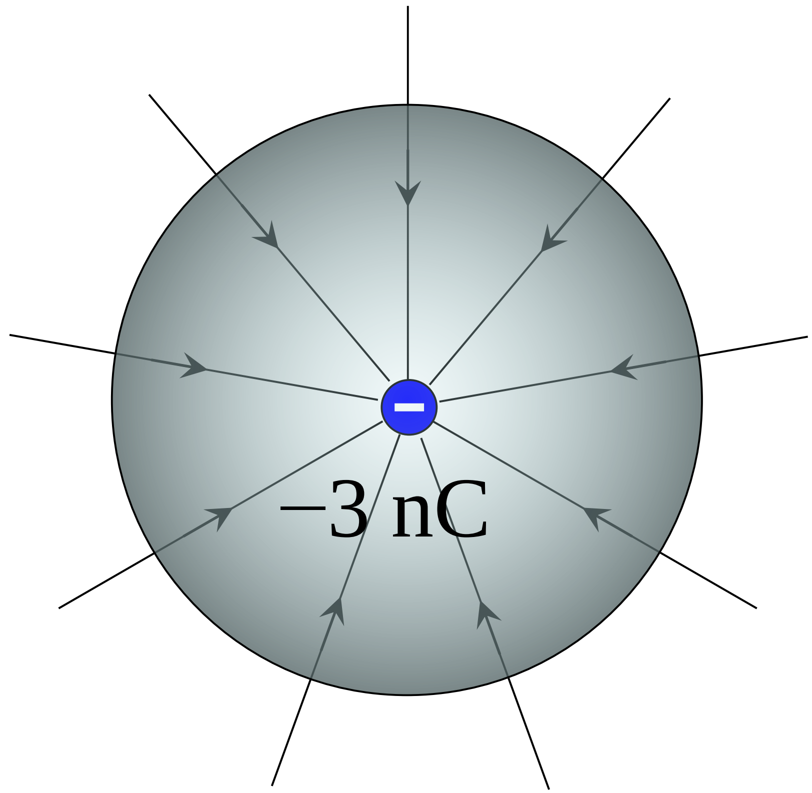
$\Phi_E = \ ?$



© Matthew W. Milligan

$$\oint \vec{E} \cdot d\vec{A} = \frac{-3 \times 10^{-9}}{8.85 \times 10^{-12}}$$

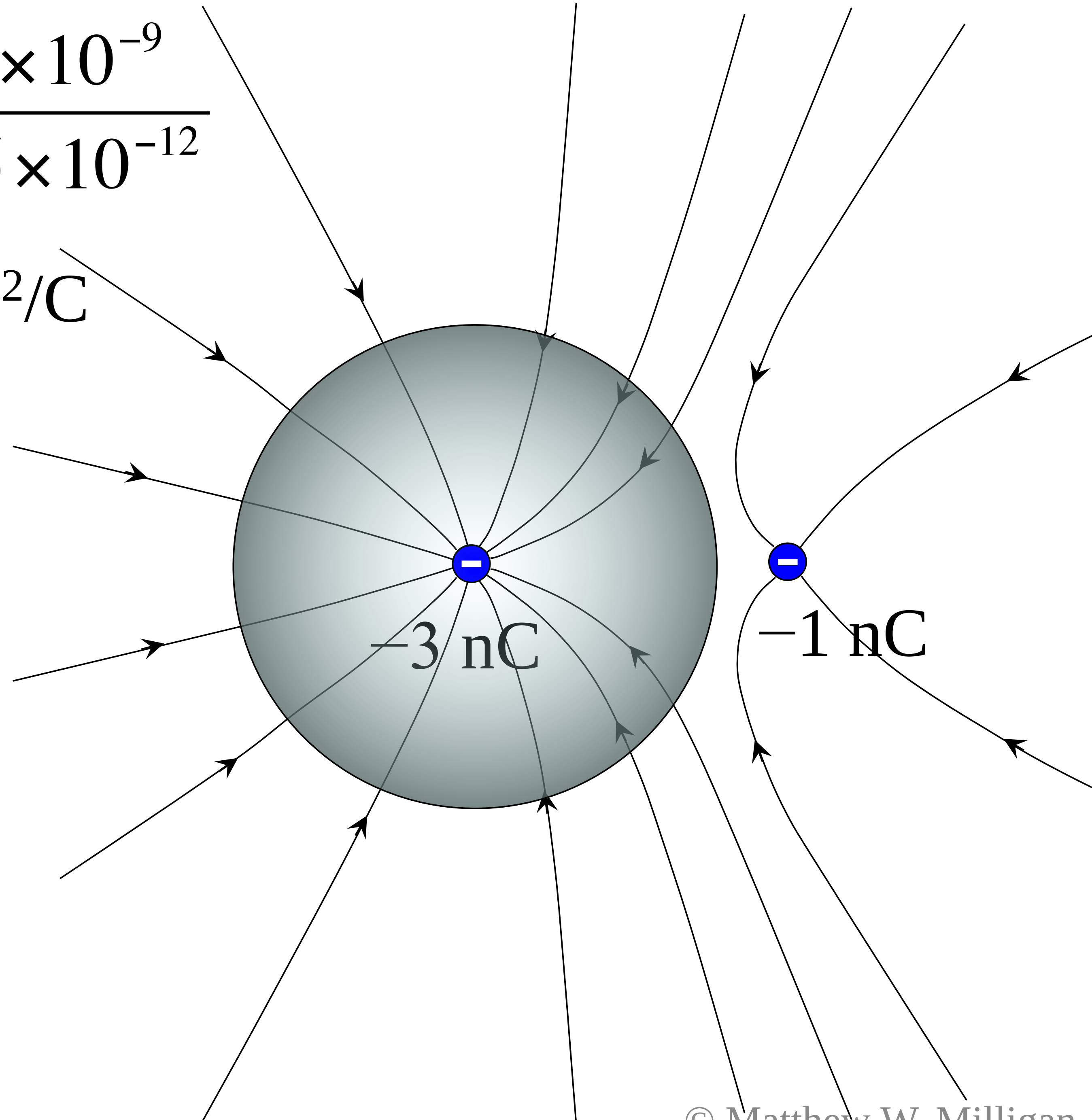
$$\Phi_E = -339 \text{ Nm}^2/\text{C}$$



© Matthew W. Milligan

$$\oint \vec{E} \cdot d\vec{A} = \frac{-3 \times 10^{-9}}{8.85 \times 10^{-12}}$$

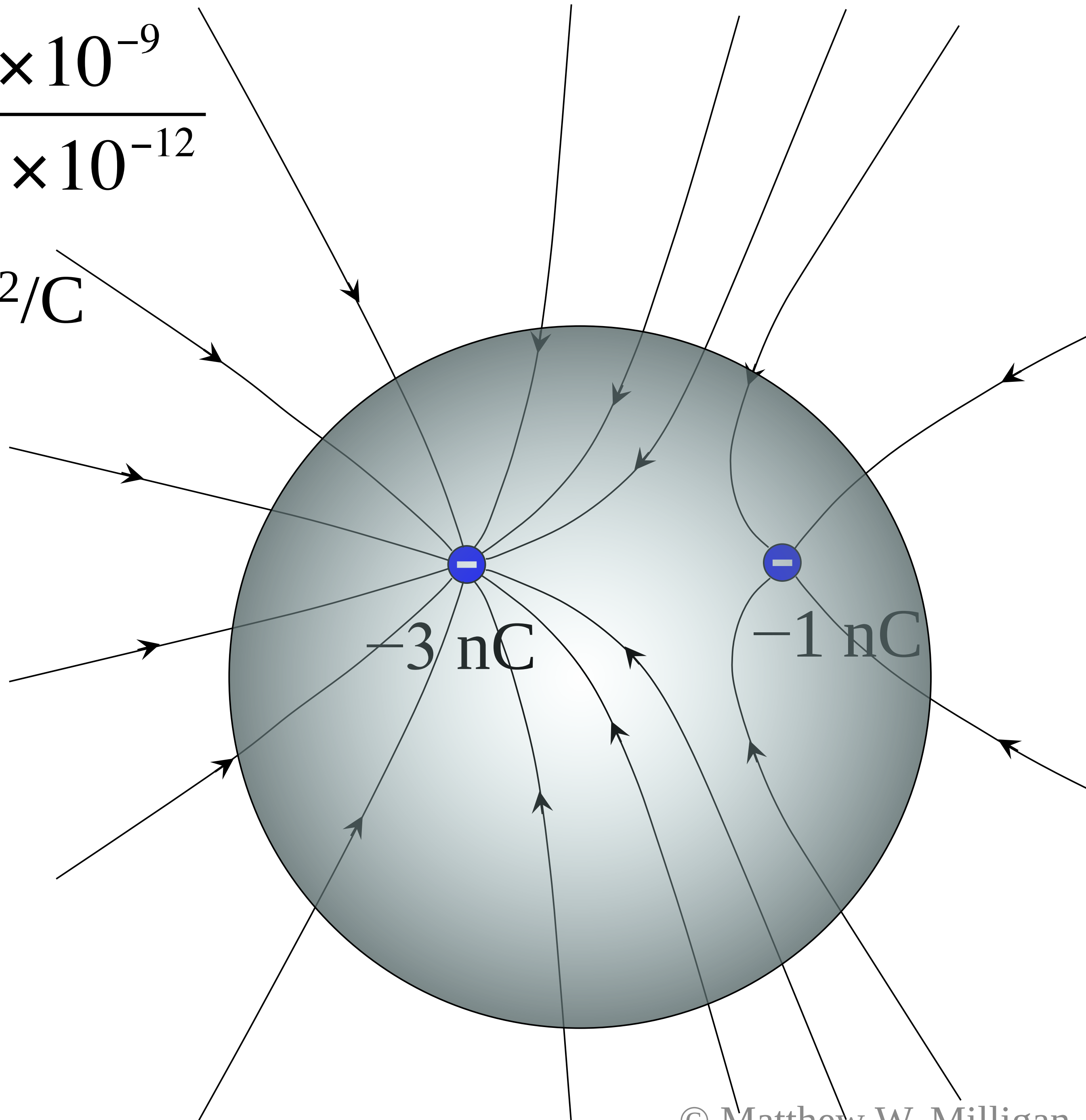
$$\Phi_E = -339 \text{ Nm}^2/\text{C}$$



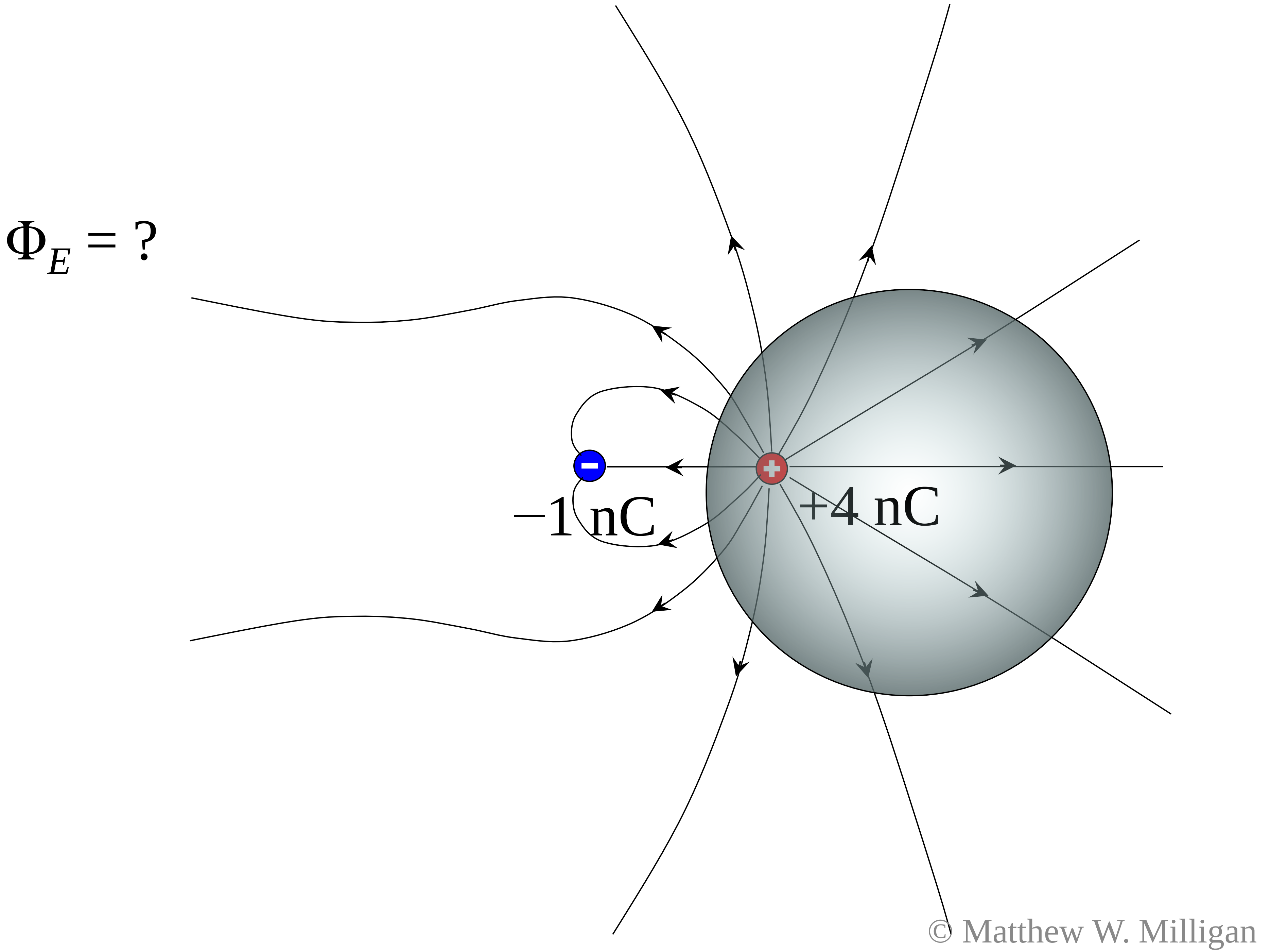
© Matthew W. Milligan

$$\oint \vec{E} \cdot d\vec{A} = \frac{-4 \times 10^{-9}}{8.85 \times 10^{-12}}$$

$$\Phi_E = -452 \text{ Nm}^2/\text{C}$$

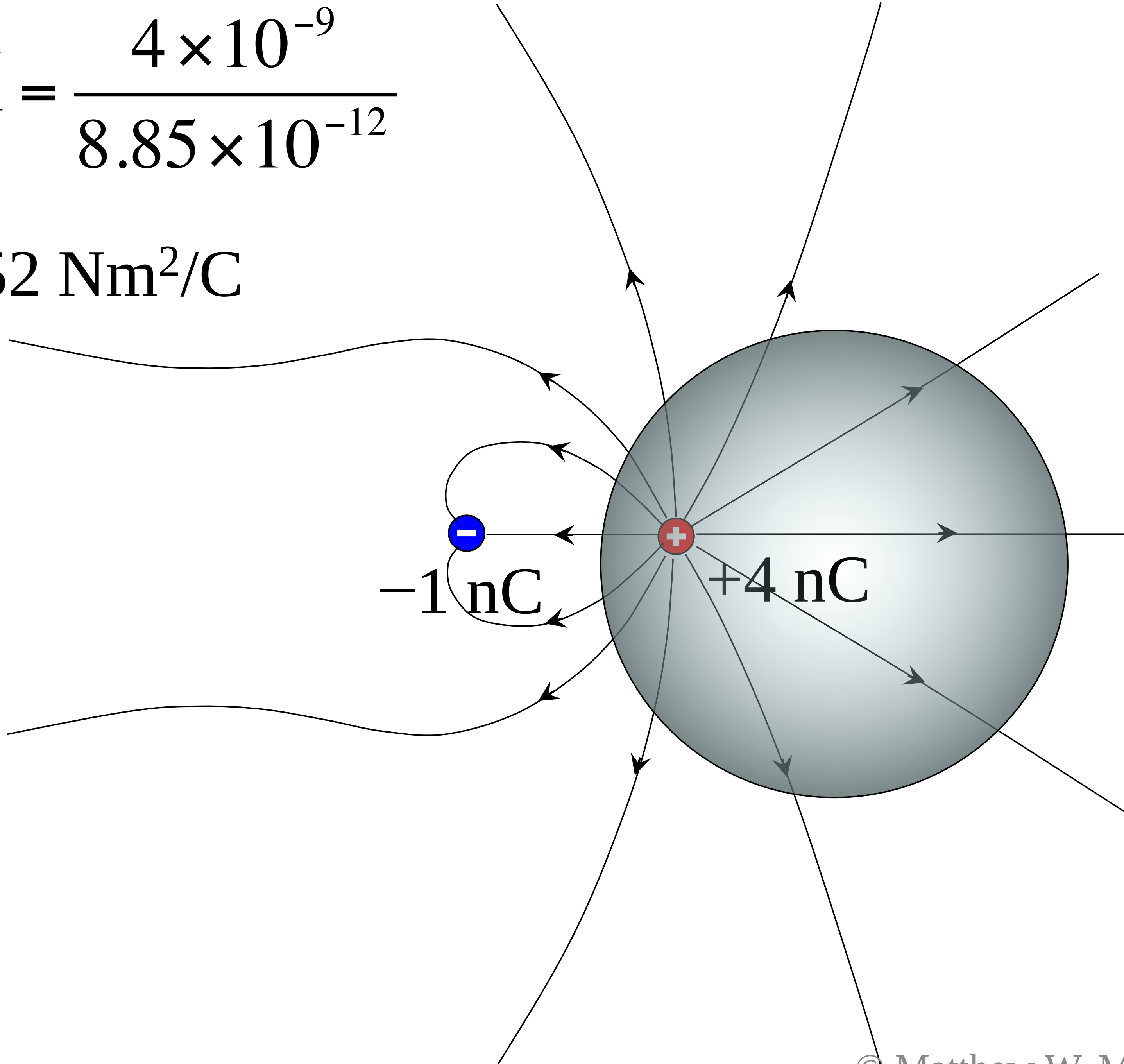


© Matthew W. Milligan


$\Phi_E = ?$
−1 nC
+4 nC
© Matthew W. Milligan

$$\oint \vec{E} \cdot d\vec{A} = \frac{4 \times 10^{-9}}{8.85 \times 10^{-12}}$$

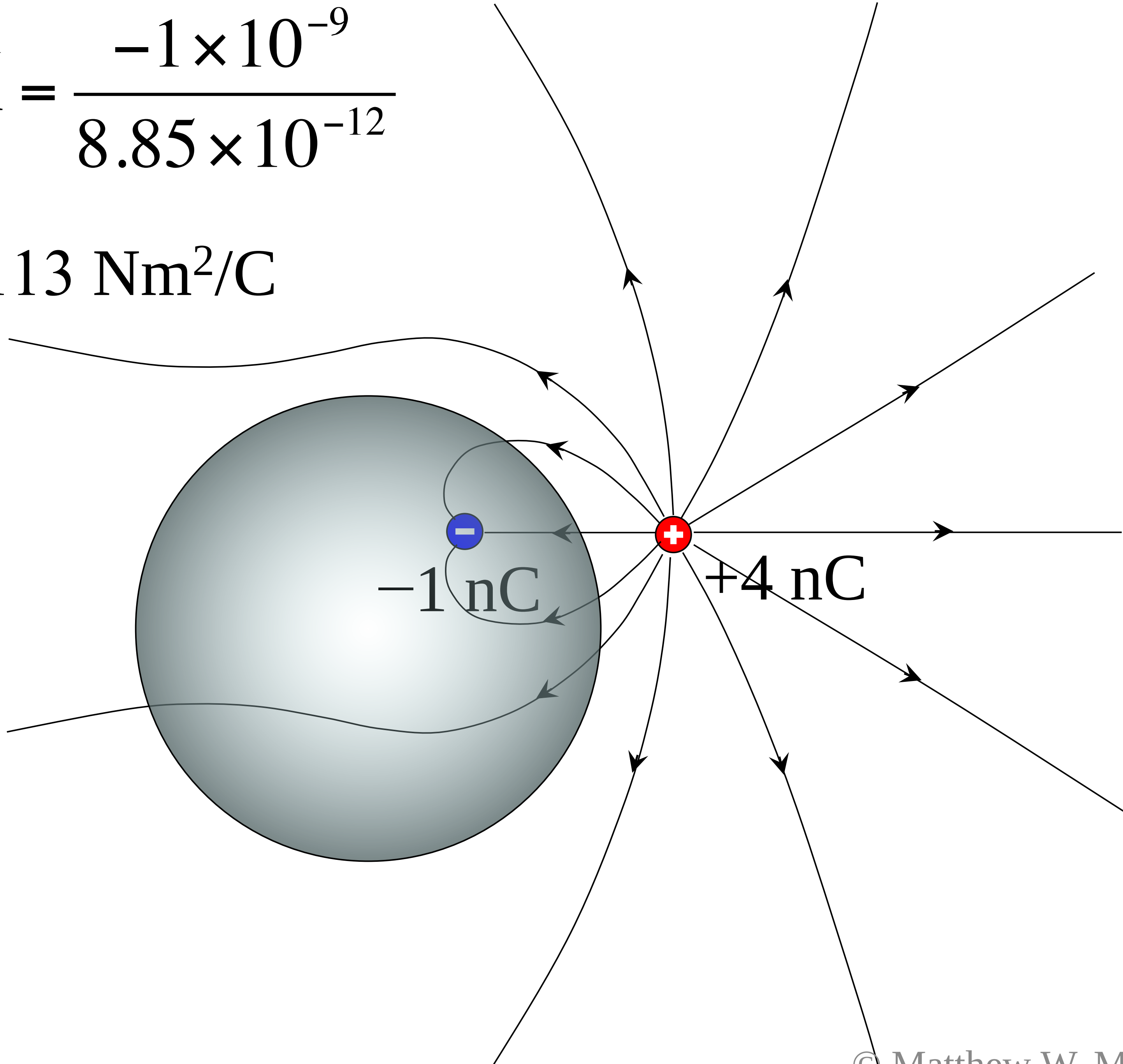
$$\Phi_E = 452 \text{ Nm}^2/\text{C}$$



© Matthew W. Milligan

$$\oint \vec{E} \cdot d\vec{A} = \frac{-1\times10^{-9}}{8.85\times10^{-12}}$$

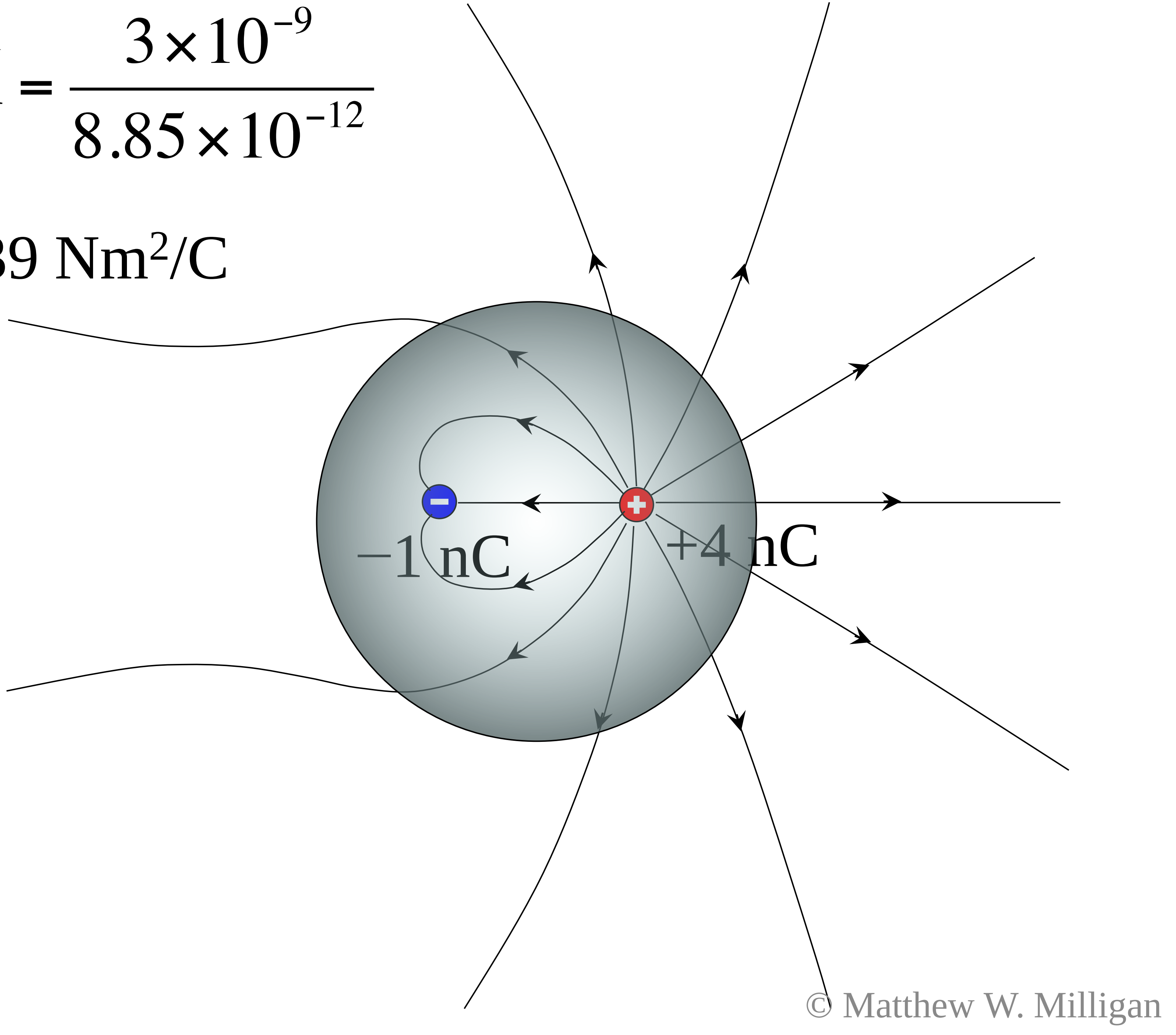
$$\Phi_E = -113 \text{ Nm}^2/\text{C}$$



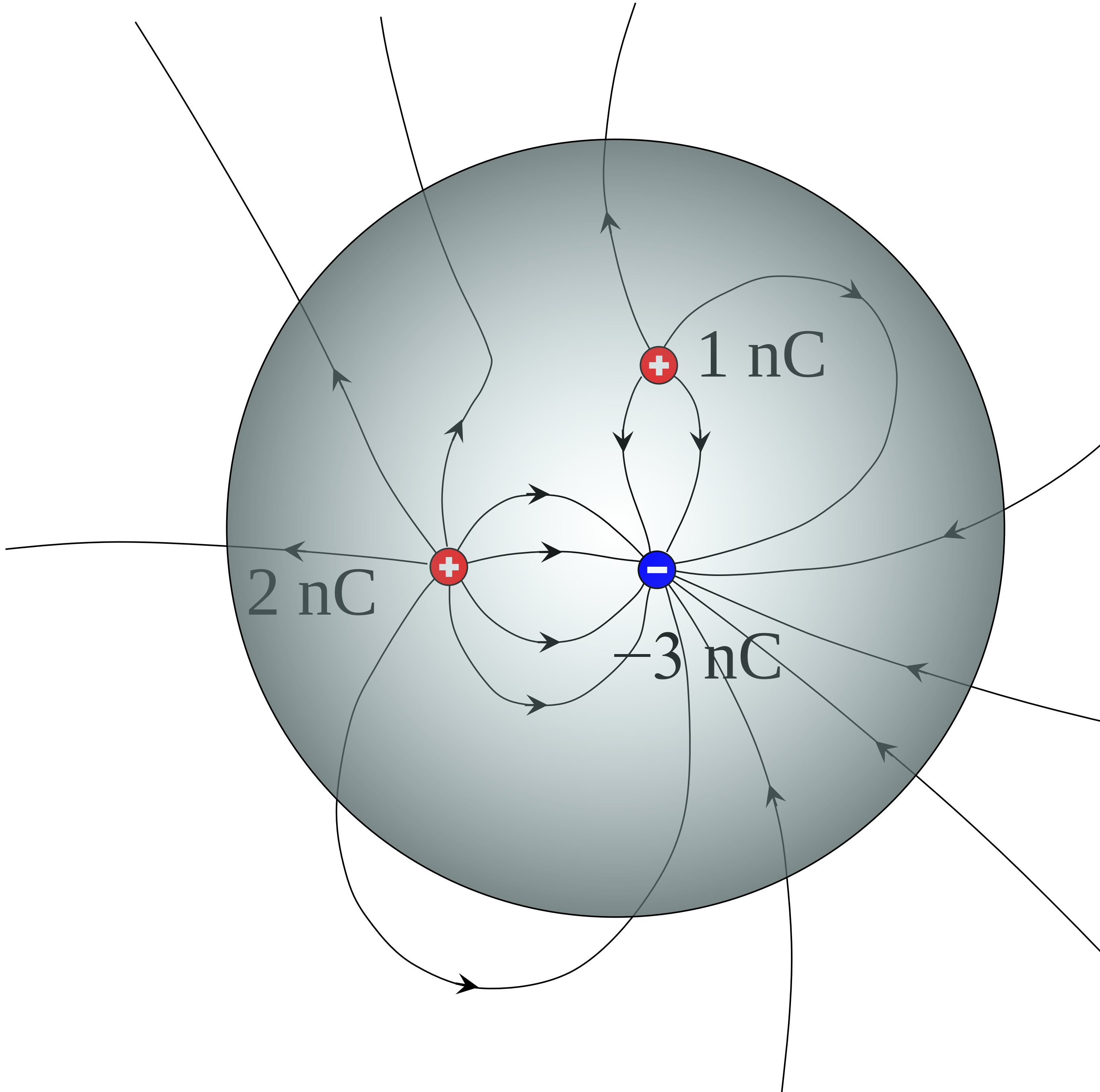
© Matthew W. Milligan

$$\oint \vec{E} \cdot d\vec{A} = \frac{3\times10^{-9}}{8.85\times10^{-12}}$$

$$\Phi_E = 339\ \text{Nm}^2/\text{C}$$



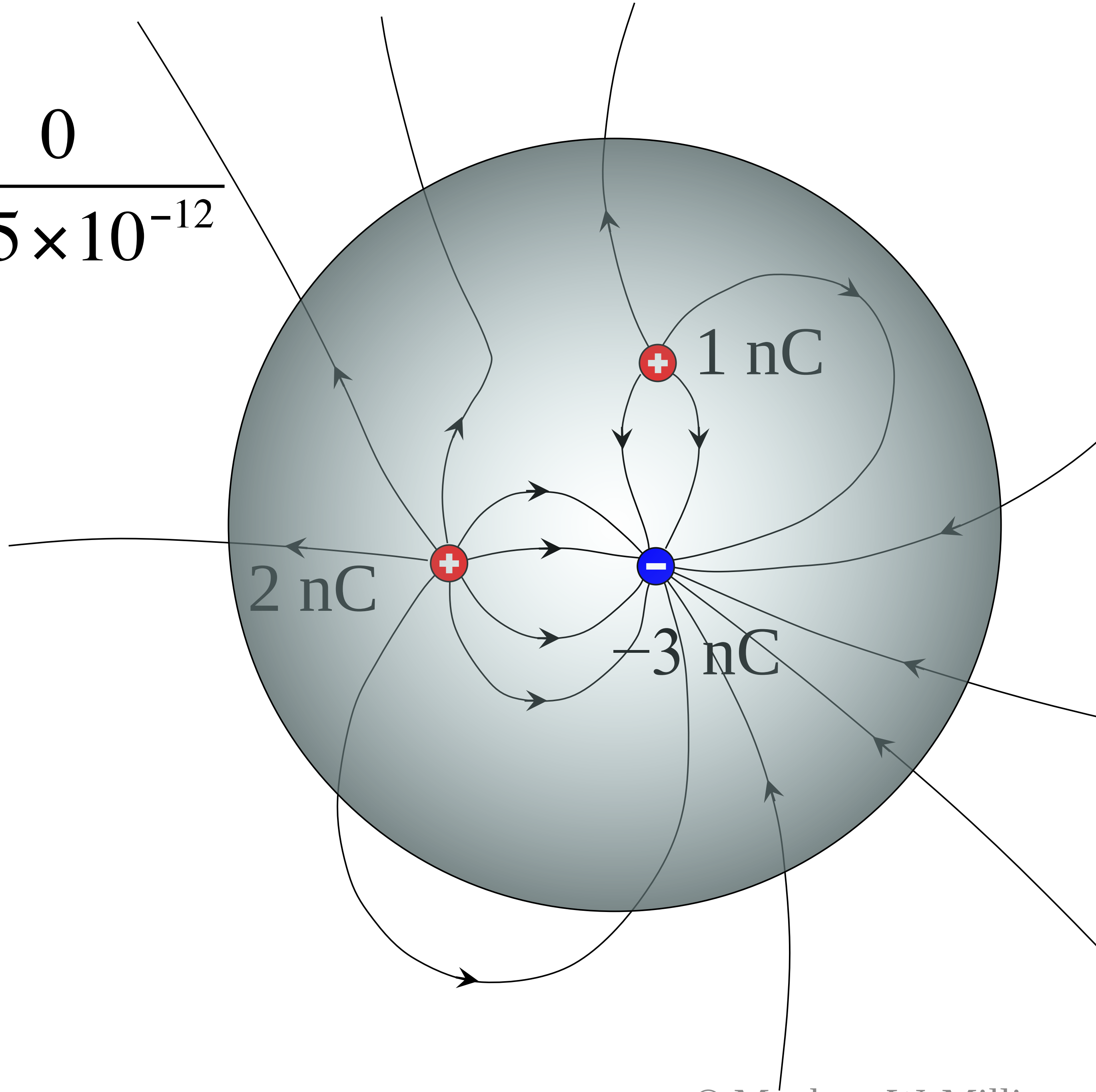
$\Phi_E = ?$



© Matthew W. Milligan

$$\oint \vec{E} \cdot d\vec{A} = \frac{0}{8.85 \times 10^{-12}}$$

$$\Phi_E = 0$$



© Matthew W. Milligan

Maxwell's Equations

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

Gauss's Law

$$\oint \vec{B} \cdot d\vec{A} =$$

$$\oint \vec{E} \cdot d\vec{\ell} =$$

$$\oint \vec{B} \cdot d\vec{\ell} =$$

© Matthew W. Milligan

Using Gauss's Law to Solve for **E**

- In certain situations Gauss's law is handy for determining the electric field of a charge distribution.
- This only works in **highly symmetrical situations**!
- It is necessary to simplify the integral to the form: $\mathbf{E}\cdot\mathbf{A}$.
- In order to do this, one must imagine a "Gaussian surface" along which the ***field is uniform*** and the dot product may be found.

© Matthew W. Milligan