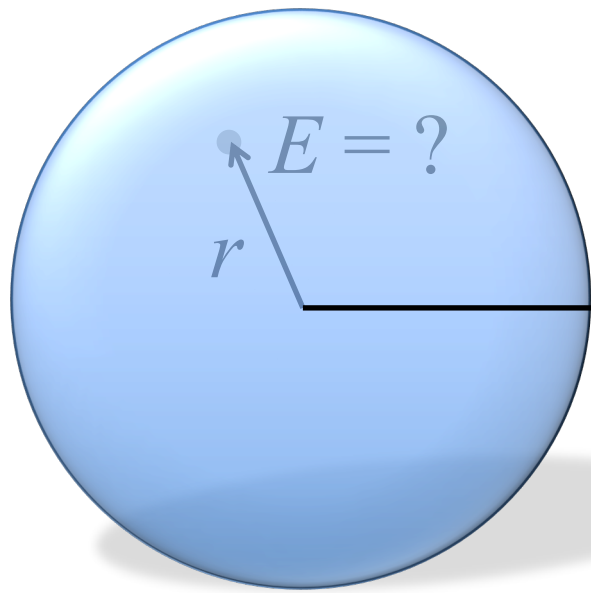
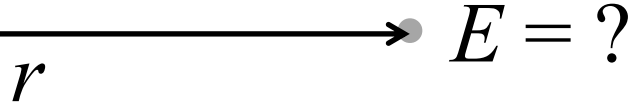


Gauss's Law vs. Coulomb's Law

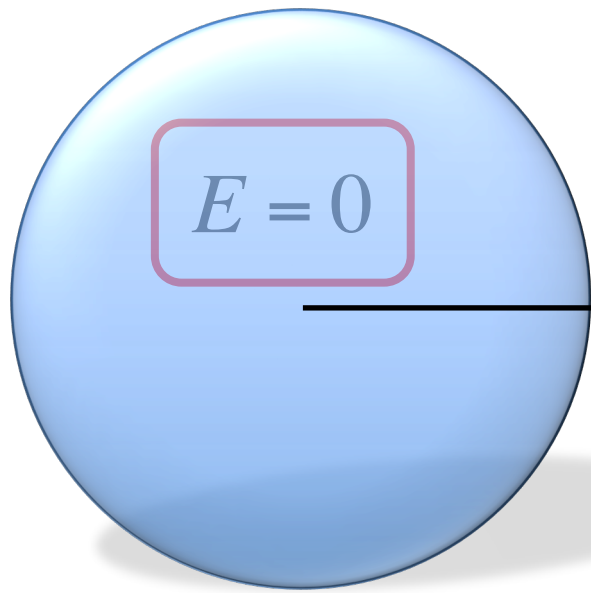
An Appreciation



Find the electric field E at distance r from the center...



Spherical surface
of radius R and
uniform charge Q .



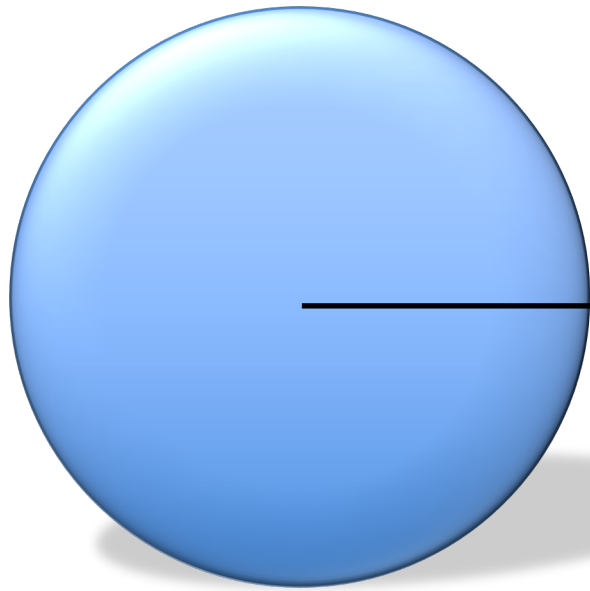
Spherical surface
of radius R and
uniform charge Q .

Find the electric field E at
distance r from the center...

$$E = \frac{Q}{4\pi\epsilon_0 r^2}$$

It is *easy* to derive these results using
Gauss's Law: The field anywhere
inside is zero and anywhere outside is
equivalent to that of a point charge.

BUT, what if you did *not* use Gauss's Law?! ...

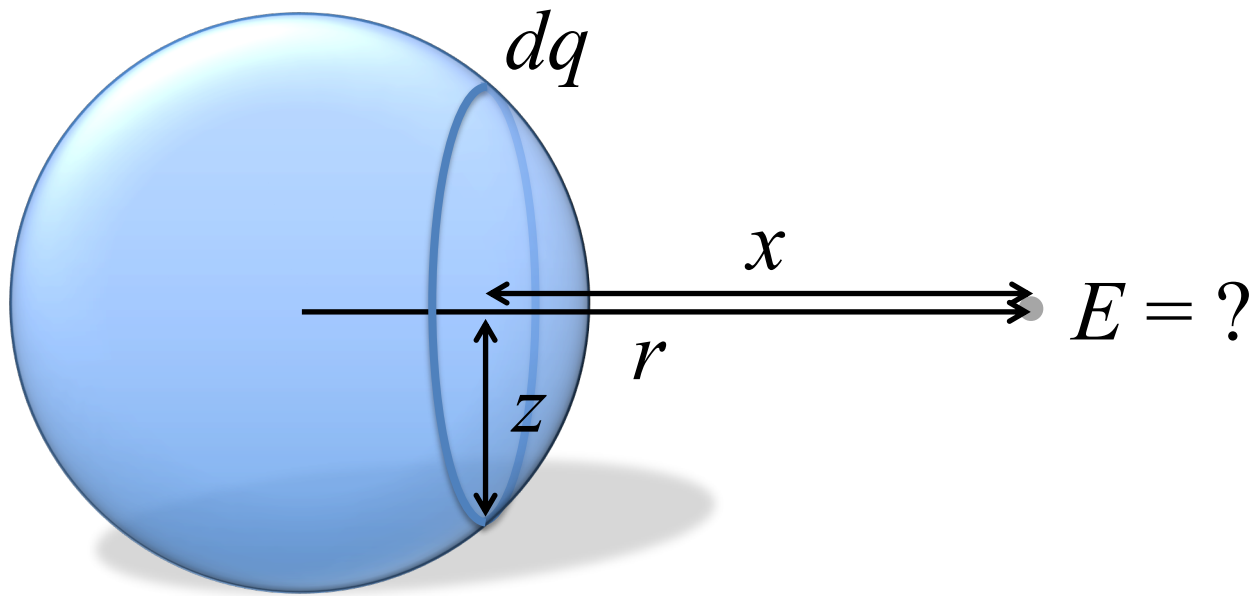


Find the electric field E at distance r from the center...

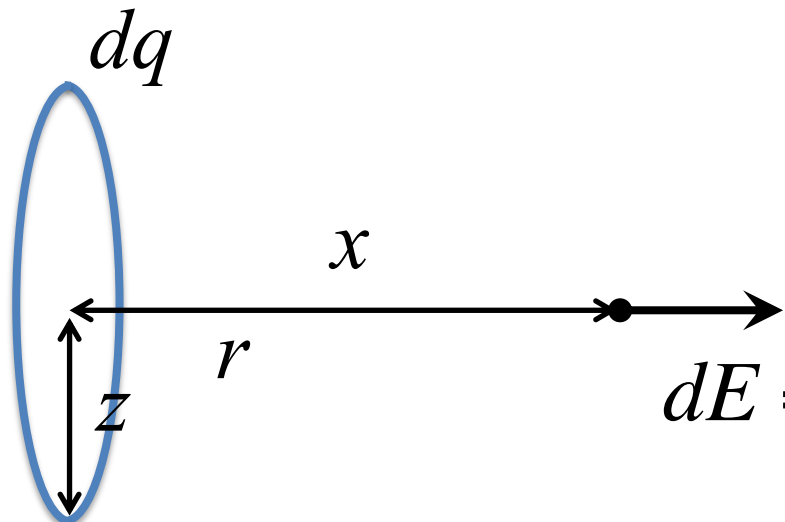
$$r \quad \rightarrow \quad E = ?$$

Spherical surface of radius R and uniform charge Q .

Charge per area: $\sigma = \frac{Q}{4\pi R^2}$



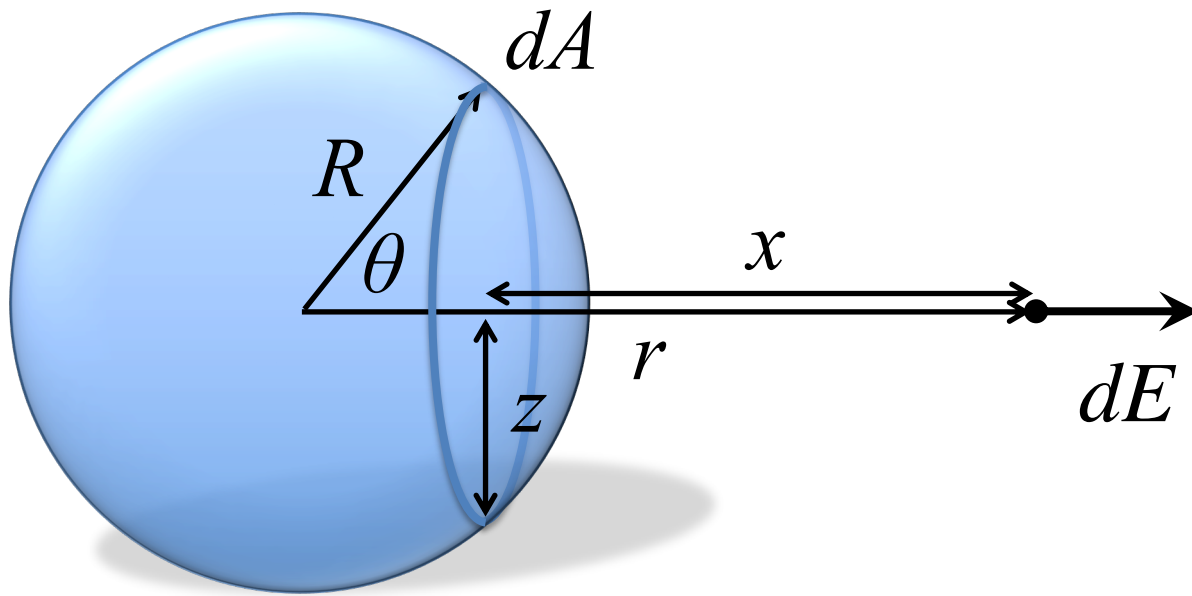
Slice sphere into rings of radius z and charge dq , each at distance x from the point in question...



The diagram shows a blue ring of charge dq on the left. A horizontal line with arrows at both ends represents the distance x from the ring to a point on the right. A vertical double-headed arrow represents the distance z from the center of the ring to the horizontal line. The distance from the ring to the point is labeled r . To the right of the point, the electric field contribution is given by the equation:

$$dE = \frac{xdq}{4\pi\epsilon_0 (z^2 + x^2)^{\frac{3}{2}}}$$

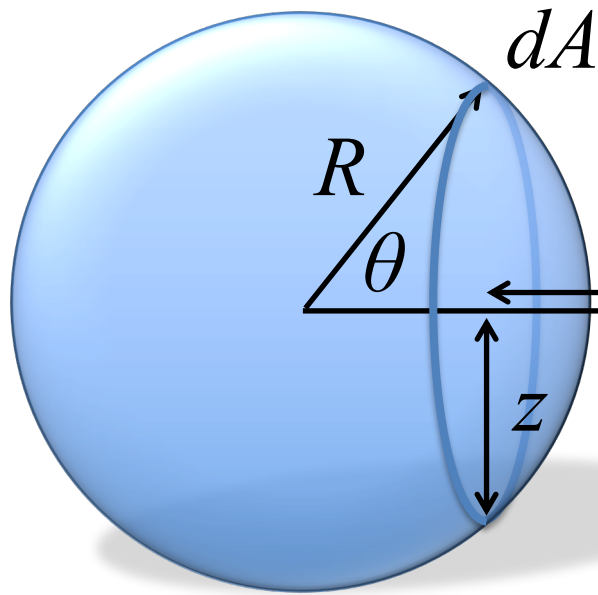
Field contribution, dE , of each ring (previously derived using Coulomb's Law and inverse square property of point charges)



$$dE = \frac{x dq}{4\pi\epsilon_0 (z^2 + x^2)^{\frac{3}{2}}}$$

Area of each ring is
 given by: $dA = 2\pi z R d\theta$
 and therefore dq is:
 $dq = \sigma 2\pi z R d\theta$

$$dE = \frac{x\sigma 2\pi z R d\theta}{4\pi\epsilon_0 (z^2 + x^2)^{\frac{3}{2}}}$$



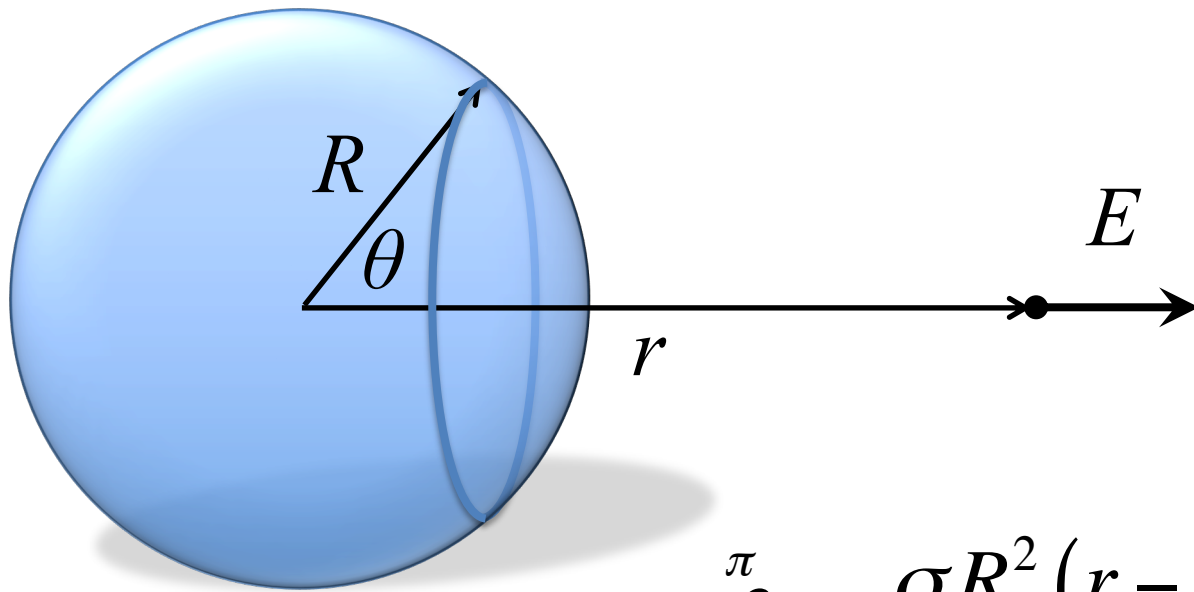
$$dE = \frac{x\sigma 2\pi z R d\theta}{4\pi\epsilon_0 (z^2 + x^2)^{\frac{3}{2}}}$$

Geometry:

$$z = R \sin \theta$$

$$x = r - R \cos \theta$$

$$dE = \frac{(r - R \cos \theta) \sigma 2\pi R^2 \sin \theta d\theta}{4\pi\epsilon_0 \left((R \sin \theta)^2 + (r - R \cos \theta)^2 \right)^{\frac{3}{2}}}$$



$$E = \int_0^{\pi} \frac{\sigma R^2 (r - R \cos \theta) \sin \theta d\theta}{2\epsilon_0 \left(R^2 \sin^2 \theta + (r - R \cos \theta)^2 \right)^{\frac{3}{2}}}$$

This should do it! All that remains is to evaluate...

$$E = \int_0^{\pi} \frac{\sigma R^2 (r - R \cos \theta) \sin \theta d\theta}{2\epsilon_0 \left(R^2 \sin^2 \theta + (r - R \cos \theta)^2 \right)^{\frac{3}{2}}}$$

$$E = \int_0^{\pi} \frac{\sigma R^2 (r - R \cos \theta) \sin \theta d\theta}{2\epsilon_0 \left(R^2 \sin^2 \theta + r^2 - 2Rr \cos \theta + R^2 \cos^2 \theta \right)^{\frac{3}{2}}}$$

$$E = \int_0^{\pi} \frac{\sigma R^2 (r - R \cos \theta) \sin \theta d\theta}{2\epsilon_0 \left(R^2 (\sin^2 \theta + \cos^2 \theta) + r^2 - 2Rr \cos \theta \right)^{\frac{3}{2}}}$$

$$E = \int_0^\pi \frac{\sigma R^2 (r - R \cos \theta) \sin \theta d\theta}{2\epsilon_0 (R^2 + r^2 - 2Rr \cos \theta)^{\frac{3}{2}}} \quad u = R^2 + r^2 - 2Rr \cos \theta$$

$$\cos \theta = \frac{R^2 + r^2 - u}{2Rr}$$

$$E = \frac{\sigma R^2}{2\epsilon_0} \int_0^\pi \frac{u^{-\frac{3}{2}}}{2R} - \frac{(R^2 + r^2) u^{-\frac{3}{2}}}{4Rr^2} + \frac{u^{-\frac{1}{2}}}{4Rr^2} du \quad \frac{du}{d\theta} = 2Rr \sin \theta$$

$$E = \frac{\sigma R^2}{2\epsilon_0} \int_0^{\theta=\pi} \frac{u^{-\frac{3}{2}}}{2R} - \frac{(R^2 + r^2) u^{-\frac{3}{2}}}{4Rr^2} + \frac{u^{-\frac{1}{2}}}{4Rr^2} du \quad \frac{du}{2Rr} = \sin \theta d\theta$$

$$E = \frac{\sigma R^2}{2\epsilon_0} \left(-\frac{1}{R\sqrt{u}} + \frac{R^2 + r^2}{2Rr^2\sqrt{u}} + \frac{\sqrt{u}}{2Rr^2} \right)_{\theta=0}^{\theta=\pi}$$

$$E = \frac{\sigma R^2}{2\varepsilon_0} \left(-\frac{1}{R\sqrt{u}} + \frac{R^2 + r^2}{2Rr^2\sqrt{u}} + \frac{\sqrt{u}}{2Rr^2} \right)_{\theta=0}^{\theta=\pi}$$

$$E = \frac{\sigma R^2}{2\varepsilon_0} \left(\frac{-2r^2 + R^2 + r^2 + u}{2Rr^2\sqrt{u}} \right)_{\theta=0}^{\theta=\pi} \quad u = R^2 + r^2 - 2Rr \cos \theta$$

$$E = \frac{\sigma R^2}{2\varepsilon_0} \left(\frac{-2r^2 + R^2 + r^2 + R^2 + r^2 - 2Rr \cos \theta}{2Rr^2\sqrt{u}} \right)_{\theta=0}^{\theta=\pi}$$

$$E = \frac{\sigma R^2}{2\varepsilon_0} \left(\frac{2R^2 - 2Rr \cos \theta}{2Rr^2\sqrt{u}} \right)_{\theta=0}^{\theta=\pi}$$

$$E = \frac{\sigma R^2}{2\epsilon_0} \left(\frac{2R^2 - 2Rr \cos\theta}{2Rr^2 \sqrt{u}} \right)_{\theta=0}^{\theta=\pi}$$

$$E = \frac{\sigma R^2}{2\epsilon_0} \left(\frac{R - r \cos\theta}{r^2 \sqrt{u}} \right)_{\theta=0}^{\theta=\pi}$$

$$E = \frac{\sigma R^2}{2\epsilon_0} \left(\frac{R - r \cos\theta}{r^2 \sqrt{R^2 + r^2 - 2Rr \cos\theta}} \right)_{\theta=0}^{\theta=\pi}$$

$$E = \frac{\sigma R^2}{2\epsilon_0} \left(\frac{R - r(-1)}{r^2 \sqrt{R^2 + r^2 - 2Rr(-1)}} - \frac{R - r(1)}{r^2 \sqrt{R^2 + r^2 - 2Rr(1)}} \right)$$

$$E = \frac{\sigma R^2}{2\epsilon_0} \left(\frac{R+r}{r^2 \sqrt{r^2 + 2Rr + R^2}} - \frac{R-r}{r^2 \sqrt{r^2 - 2Rr + R^2}} \right)$$

$$E = \frac{\sigma R^2}{2\epsilon_0} \left(\frac{R+r}{r^2 \sqrt{(r+R)^2}} + \frac{r-R}{r^2 \sqrt{(r-R)^2}} \right)$$

$$E = \frac{\sigma R^2}{2\epsilon_0} \left(\frac{1}{r^2} + \frac{1}{r^2} \right)$$

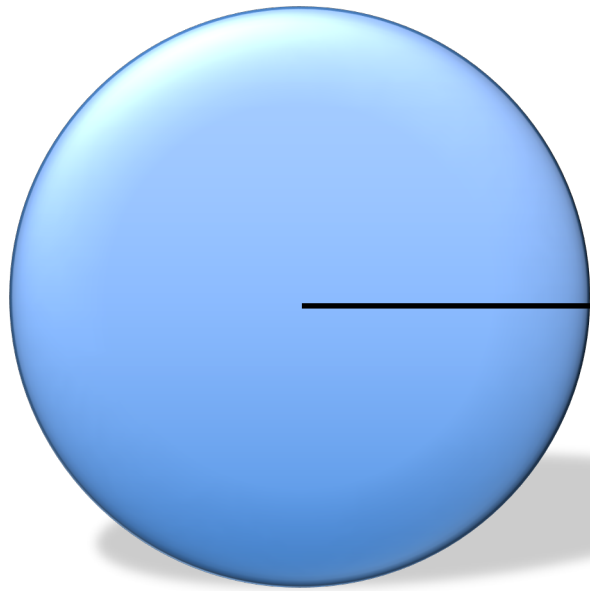
$$E = \frac{2\sigma R^2}{2\epsilon_0 r^2}$$

$$E = \frac{4\pi R^2}{\epsilon_0 r^2} Q$$

$$E = \frac{Q}{4\pi\epsilon_0 r^2}$$

If $r > R$

Find the electric field E at distance r from the center...



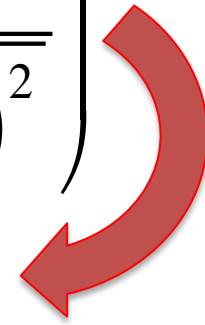
$$E = \frac{Q}{4\pi\epsilon_0 r^2}$$

Spherical surface
of radius R and
uniform charge Q .

$$E = \frac{\sigma R^2}{2\epsilon_0} \left(\frac{R+r}{r^2 \sqrt{r^2 + 2Rr + R^2}} - \frac{R-r}{r^2 \sqrt{r^2 - 2Rr + R^2}} \right)$$

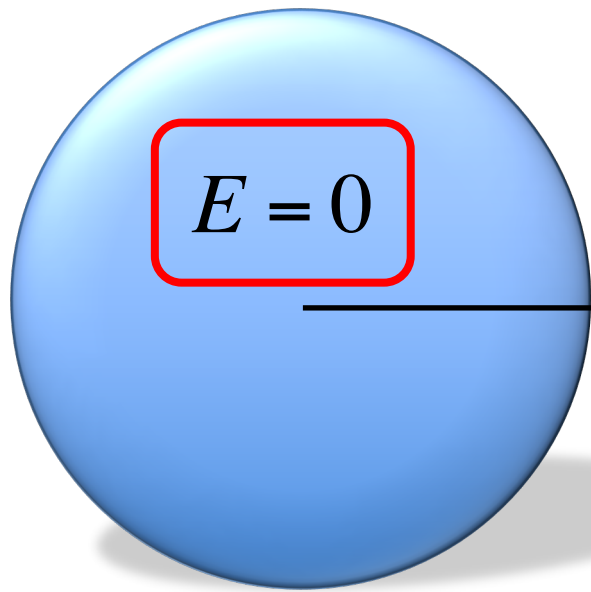
$$E = \frac{\sigma R^2}{2\epsilon_0} \left(\frac{R+r}{r^2 \sqrt{(r+R)^2}} + \frac{r-R}{r^2 \sqrt{(r-R)^2}} \right)$$

$$E = \frac{\sigma R^2}{2\epsilon_0} \left(\frac{1}{r^2} + \frac{-1}{r^2} \right)$$



If $r < R$

$$E = 0$$



$$E = 0$$

Find the electric field E at distance r from the center...

$$E = \frac{Q}{4\pi\epsilon_0 r^2}$$

Spherical surface of radius R and uniform charge Q .

The exact same result for both outside and inside can be derived based on Coulomb's Law – but just think how much easier it is using Gauss's Law! Thank you Mr. Gauss, I appreciate you!!!!