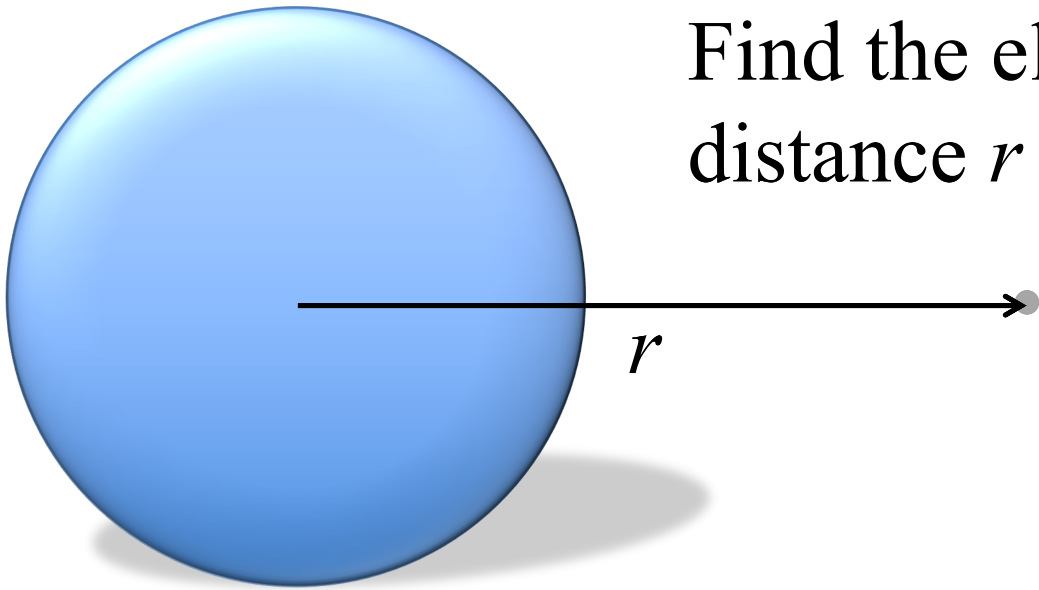
Electric Potential of a Charged Shell

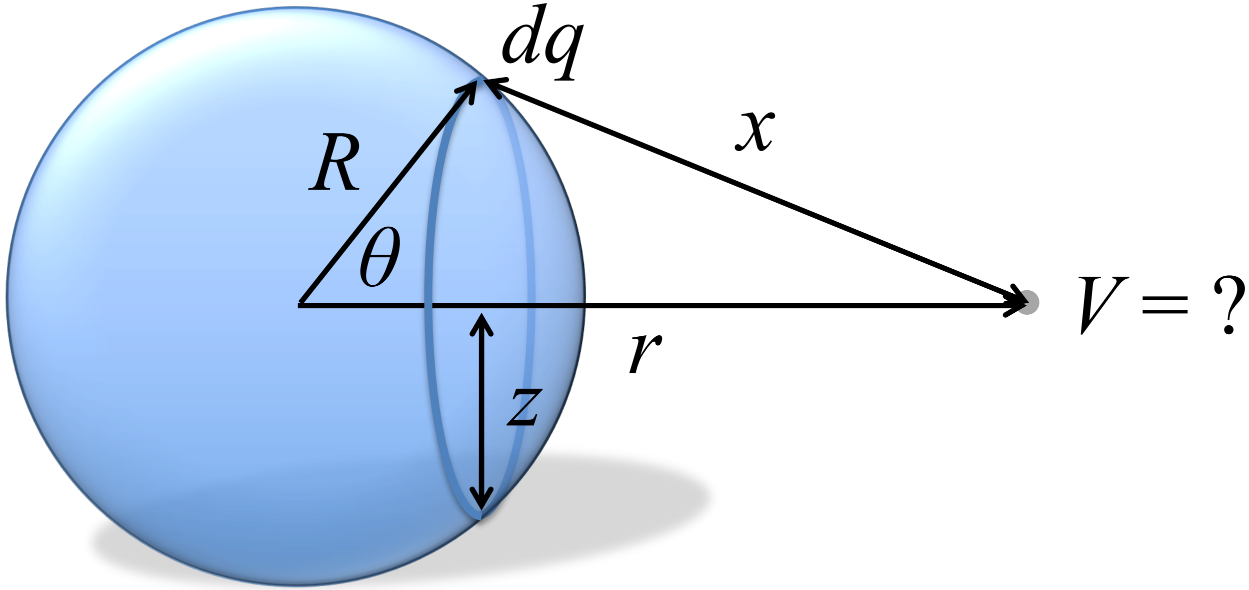
The Hard Way

Find the electric potential V at distance r from the center…

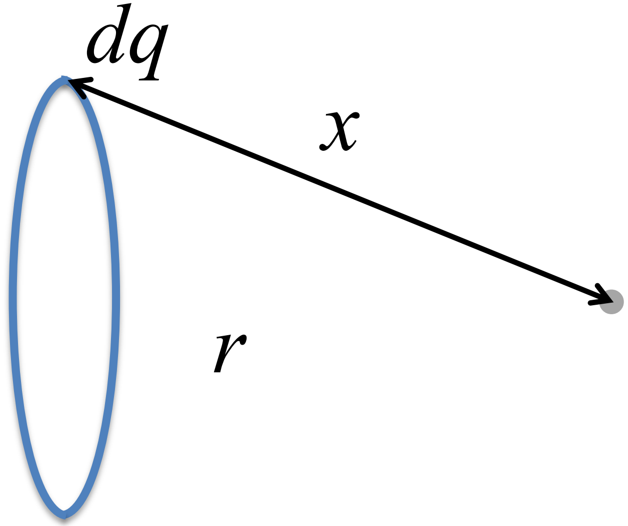


Spherical surface of radius R and uniform charge Q.

Charge per area: $$\sigma = \frac{Q}{4\pi R^2}$$

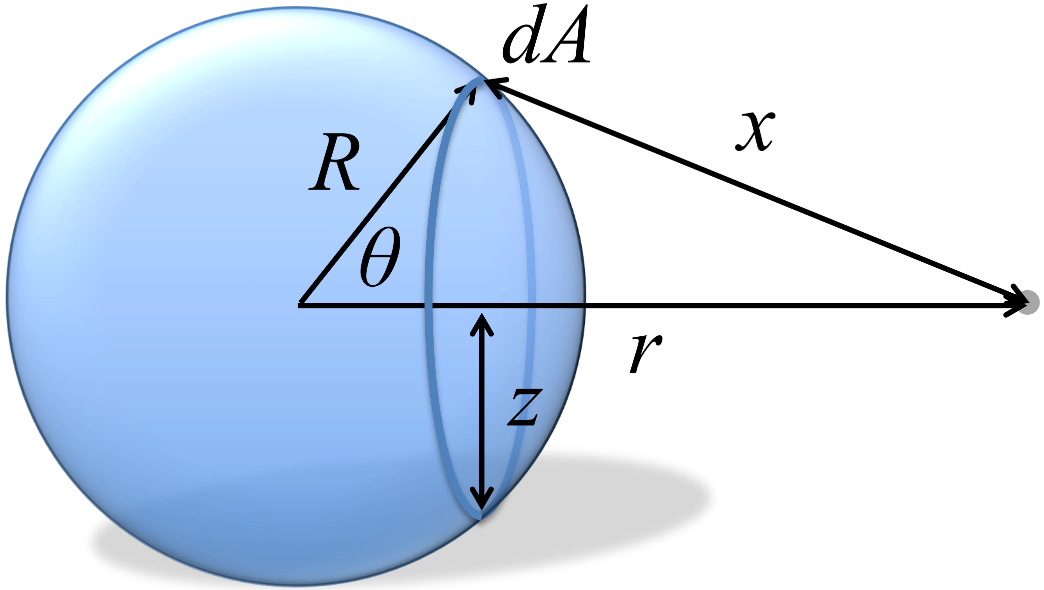


Slice sphere into rings of radius z and charge dq, all points on which are at distance x from the point in question…



$$dV = \frac{dq}{4\pi\varepsilon_0 x}$$

Potential contribution, dV, of each ring (previously discussed)



$$dV = \frac{dq}{4\pi\varepsilon_0 x}$$

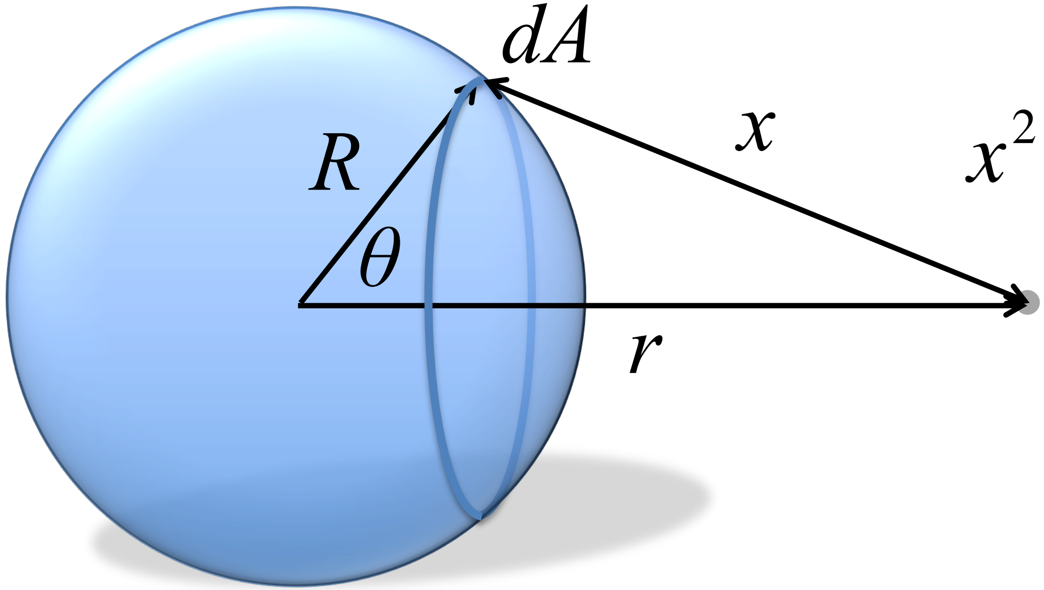
$$dV = \frac{Q\sin\theta d\theta}{8\pi\varepsilon_0 x}$$

Area of each ring is given by: $dA = 2\pi zRd\theta$

or $dA = 2\pi(R\sin\theta)Rd\theta$

and therefore dq is:

$dq = \sigma 2\pi R^2 \sin\theta d\theta = ½\ Q\sin\theta d\theta$



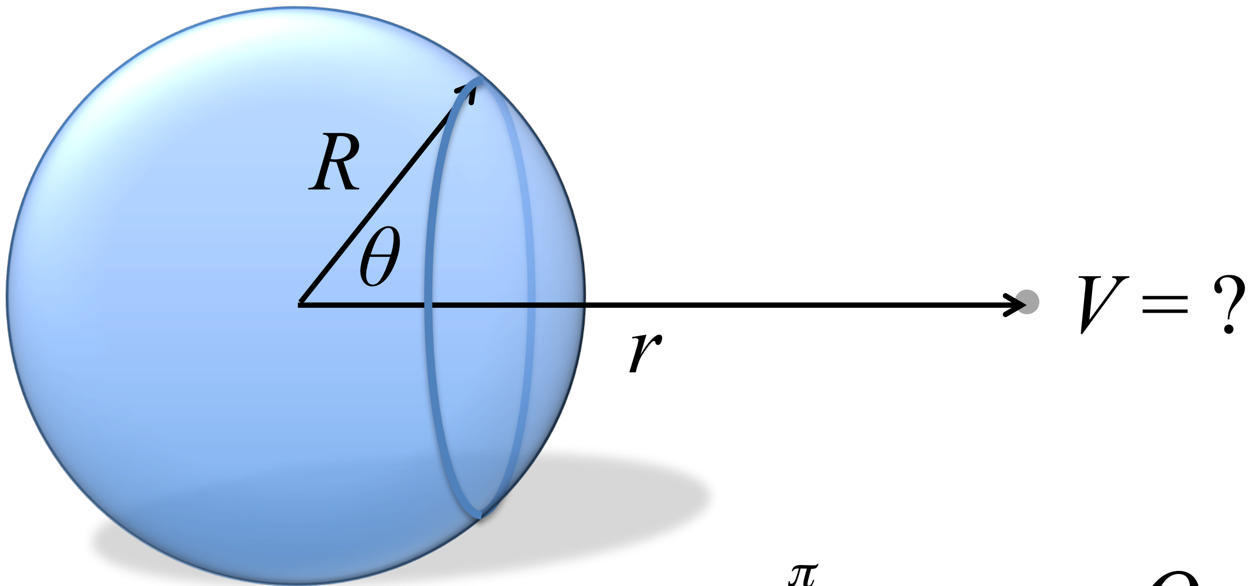
Geometry:

$$x^2 = R^2 + r^2 - 2Rr\cos\theta$$

$$dV = \frac{dq}{4\pi\varepsilon_0 x}$$

$$dV = \frac{Q\sin\theta d\theta}{8\pi\varepsilon_0 x}$$

$$dV = \frac{Q\sin\theta d\theta}{8\pi\varepsilon_0\sqrt{R^2 + r^2 - 2Rr\cos\theta}}$$



$$V=\int_0^{\pi}\frac{Q\sin\theta d\theta}{8\pi\varepsilon_0\left(R^2+r^2-2Rr\cos\theta\right)^{\frac{1}{2}}}$$

$$V = \int_0^{\pi} \frac{Q \sin\theta \, d\theta}{8\pi\varepsilon_0 \left(R^2 + r^2 - 2Rr\cos\theta\right)^{\frac{1}{2}}}$$

$$u = R^2 + r^2 - 2Rr\cos\theta$$

$$\frac{du}{d\theta} = 2Rr\sin\theta$$

$$V = \frac{Q}{16\pi\varepsilon_0 Rr} \int_{R^2+r^2-2Rr}^{R^2+r^2+2Rr} u^{-\frac{1}{2}} \, du$$

$$\frac{du}{2Rr} = \sin\theta \, d\theta$$

$$V = \frac{Q}{16\pi\varepsilon_0 Rr} \left(2\sqrt{u}\right)_{u=R^2+r^2-2Rr}^{u=R^2+r^2+2Rr}$$

$$V = \frac{Q}{8\pi\varepsilon_0 Rr} \left(\sqrt{R^2 + 2Rr + r^2} - \sqrt{R^2 - 2Rr + r^2}\right)$$

$$V = \frac{Q}{8\pi\varepsilon_0 Rr}\left(\sqrt{R^2 + 2Rr + r^2} - \sqrt{R^2 - 2Rr + r^2}\right)$$

$$\pm(r - R)$$

$$V = \frac{Q}{8\pi\varepsilon_0 Rr}\left(r + R - \left(r - R\right)\right)$$

$$V = \frac{Q}{8\pi\varepsilon_0 Rr}\left(2R\right)$$

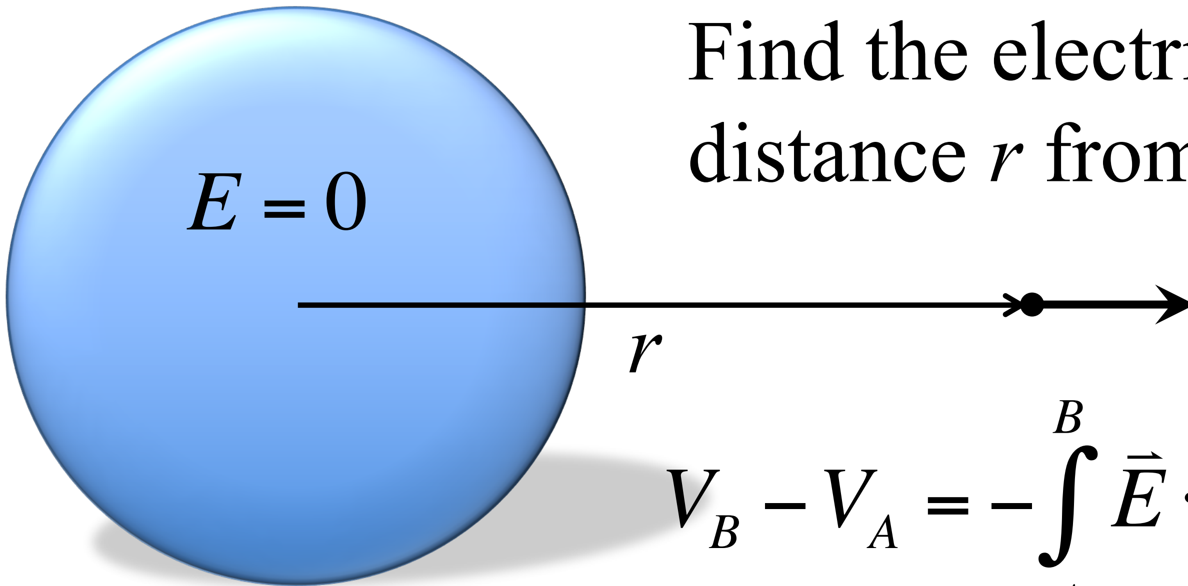
$$V = \frac{Q}{4\pi\varepsilon_0 r} \quad r > R$$

$$V = \frac{Q}{8\pi\varepsilon_0 Rr}\left(r + R - \left(R - r\right)\right)$$

$$V = \frac{Q}{8\pi\varepsilon_0 Rr}\left(2r\right)$$

$$V = \frac{Q}{4\pi\varepsilon_0 R} \quad r \leq R$$

But, what is the easy way?!



Find the electric potential V at distance r from the center…

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

$$V_B - V_A = -\int_A^B \vec{E} \cdot d\vec{r}$$

The "easy way" is to use the fundamental connection between field and potential. The field *outside* the sphere is the same as a point charge, therefore so is the potential: $V = kq/r$, $r \geq R$. The field *inside* the sphere is zero, therefore there is no difference in potential from that at the surface: $V = kq/R$, $r \leq R$.