Electric Field vs. Potential

Interconnections

Electric Flux and Potential

I. Electric Flux
- flux defined
- Gauss's Law

II. Electric Potential
- work and energy of charge
- potential defined
- potential of discrete charge(s)
- potential of charge distributions
- **field related to potential**

III. Conductors

	The student will be able to:	HW:
1	Define and apply the concept of electric flux and solve related problems.	✓ 1 – 5
2	State and apply Gauss's Law and solve related problems using Gaussian surfaces.	✓ 6 – 17
3	Calculate work and potential energy for discrete charges and solve related problems including work to assemble or disassemble.	✓ 18 – 25
4	Define and apply the concept of electric potential and solve related problems for a discrete set of point charges and/or a continuous charge distribution.	✓ 26 – 32
5	Use the electric field to determine potential or potential difference and solve related problems.	33 – 36
6	Use potential to determine electric field and solve related problems.	37 – 39
7	State the properties of conductors in electrostatic equilibrium and solve related problems.	40 – 46

Potential Difference

As with all forms of potential energy we are often interested in the *difference* between two positions:

$$V_B - V_A = -\int_A^B \vec{E} \cdot d\vec{r}$$

where: V = electric potential associated with E, a particular electric field

A and B = arbitrary positions within the electric field

Potential Differential

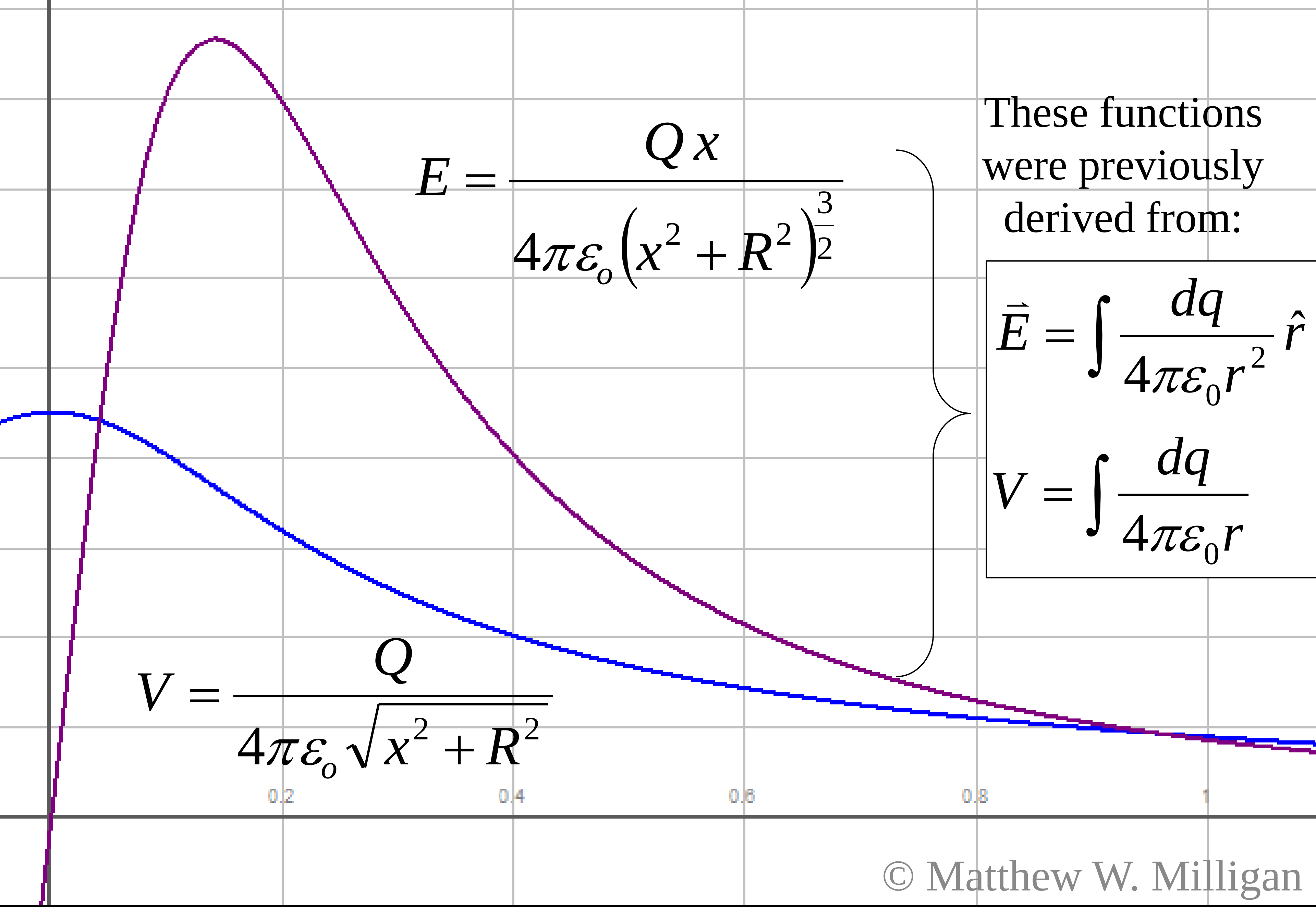
The converse of the previous equation allows one to find the electric field given electric potential

$$E_r = -\frac{dV}{dr}$$

where: E_r = component of electric field in the direction of r

V = potential as a function of position r

ring (along axis)



$$E = \frac{Q\,x}{4\pi\varepsilon_o\left(x^2 + R^2\right)^{\frac{3}{2}}}$$

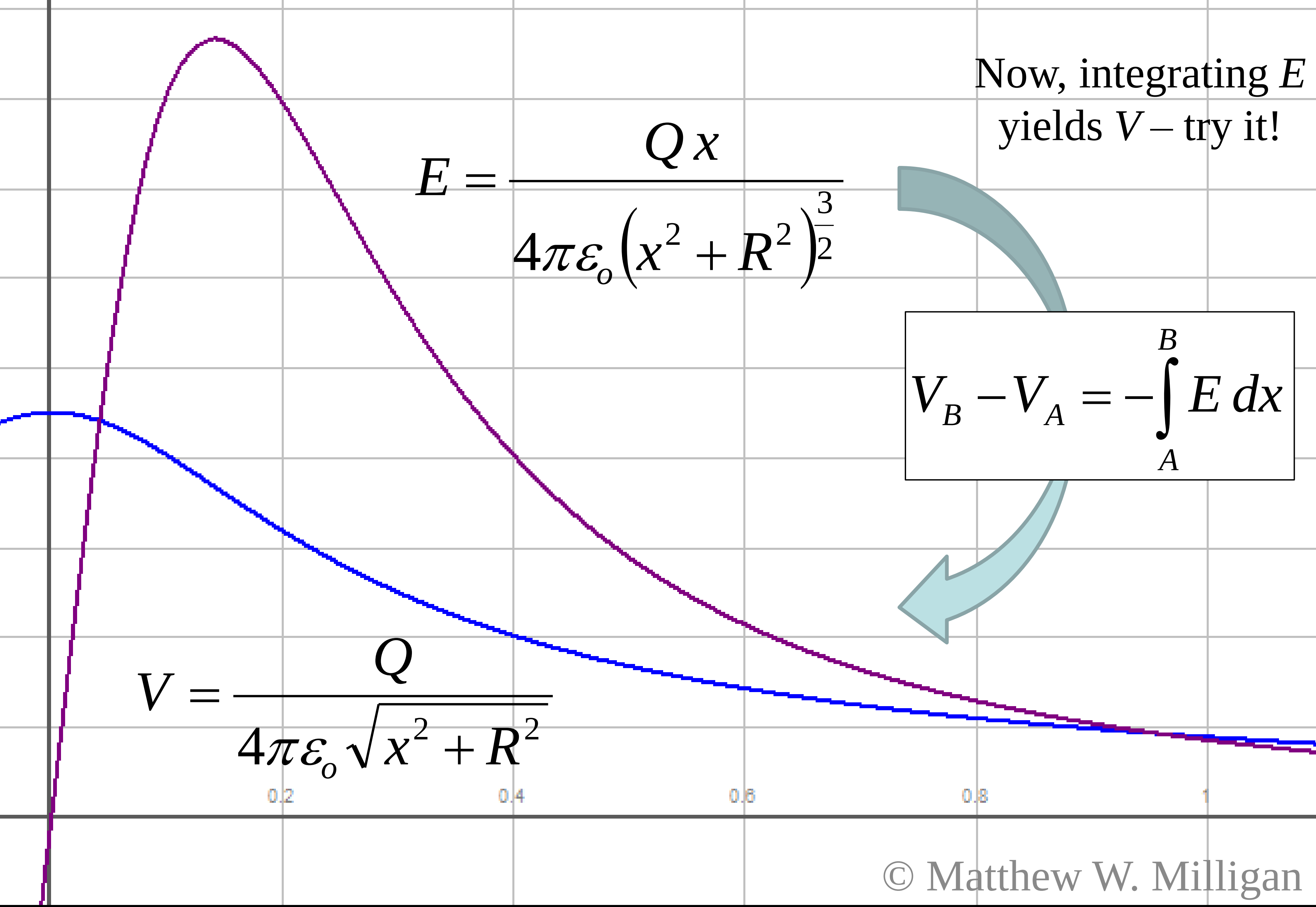
$$V = \frac{Q}{4\pi\varepsilon_o\sqrt{x^2 + R^2}}$$

These functions were previously derived from:

$$\vec{E} = \int \frac{dq}{4\pi\varepsilon_0 r^2}\hat{r}$$

$$V = \int \frac{dq}{4\pi\varepsilon_0 r}$$

ring (along axis)

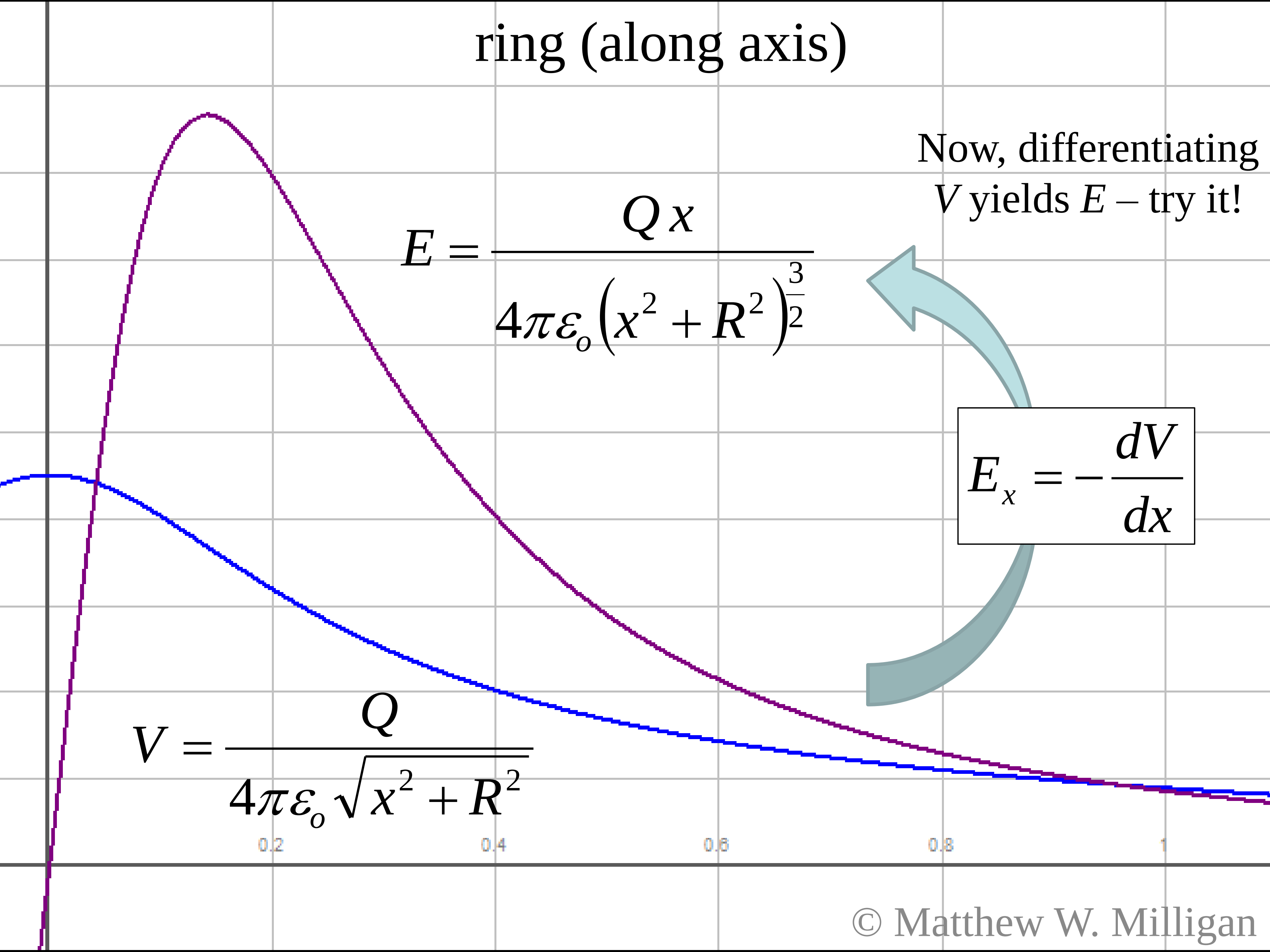


$$E = \frac{Q\,x}{4\pi\varepsilon_o\left(x^2 + R^2\right)^{\frac{3}{2}}}$$

Now, integrating *E* yields *V* – try it!

$$V_B - V_A = -\int_A^B E\,dx$$

$$V = \frac{Q}{4\pi\varepsilon_o\sqrt{x^2 + R^2}}$$


ring (along axis)
Now, differentiating
V yields E – try it!
$E = \frac{Q\,x}{4\pi\varepsilon_o\left(x^2 + R^2\right)^{\frac{3}{2}}}$
$E_x = -\frac{dV}{dx}$
$V = \frac{Q}{4\pi\varepsilon_o\sqrt{x^2 + R^2}}$
0.2
0.4
0.6
0.8
1
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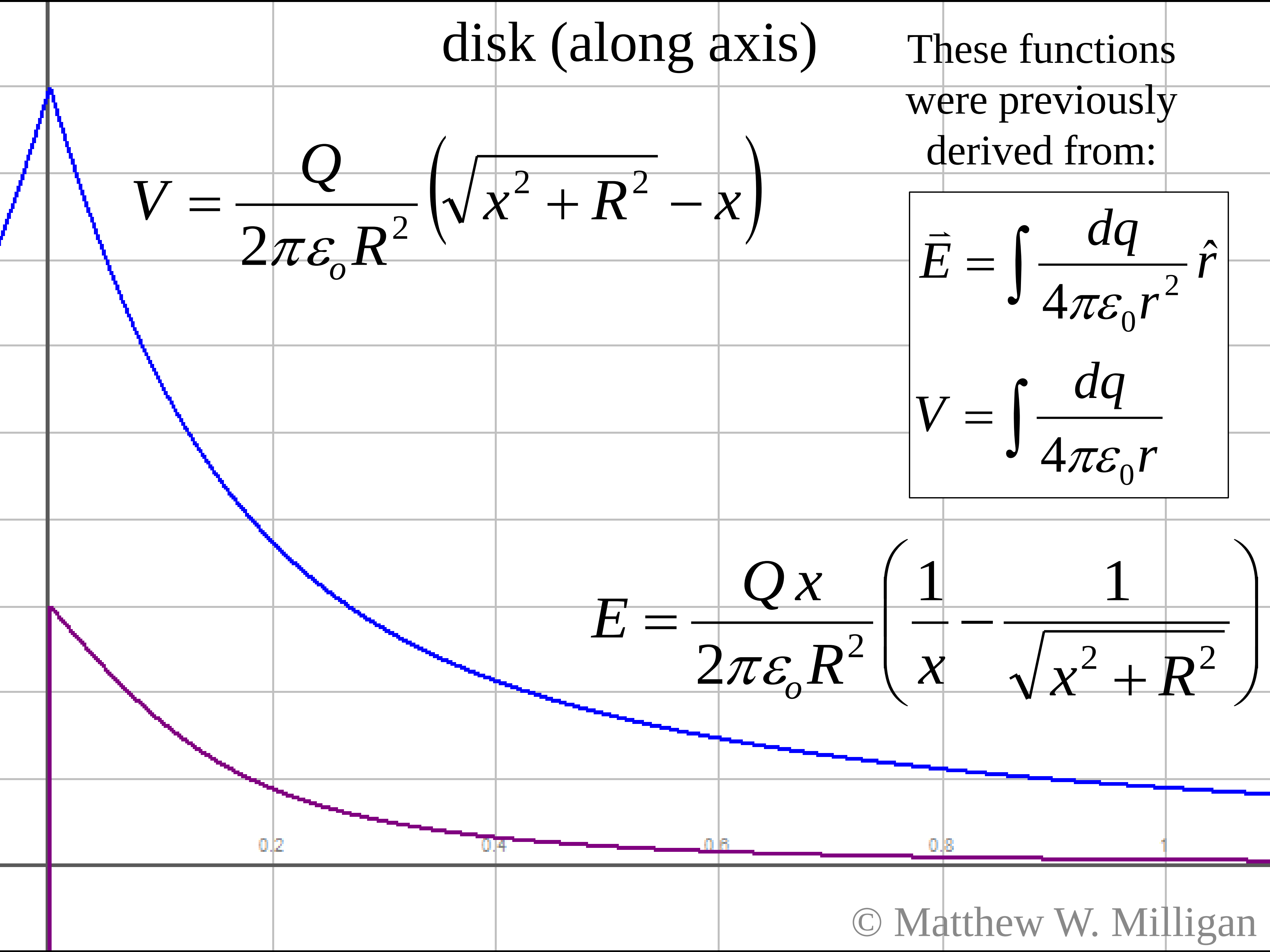
disk (along axis)



$$V = \frac{Q}{2\pi\varepsilon_o R^2}\left(\sqrt{x^2 + R^2} - x\right)$$

$$E = \frac{Q\,x}{2\pi\varepsilon_o R^2}\left(\frac{1}{x} - \frac{1}{\sqrt{x^2 + R^2}}\right)$$

These functions were previously derived from:

$$\vec{E} = \int \frac{dq}{4\pi\varepsilon_0 r^2}\hat{r}$$

$$V = \int \frac{dq}{4\pi\varepsilon_0 r}$$

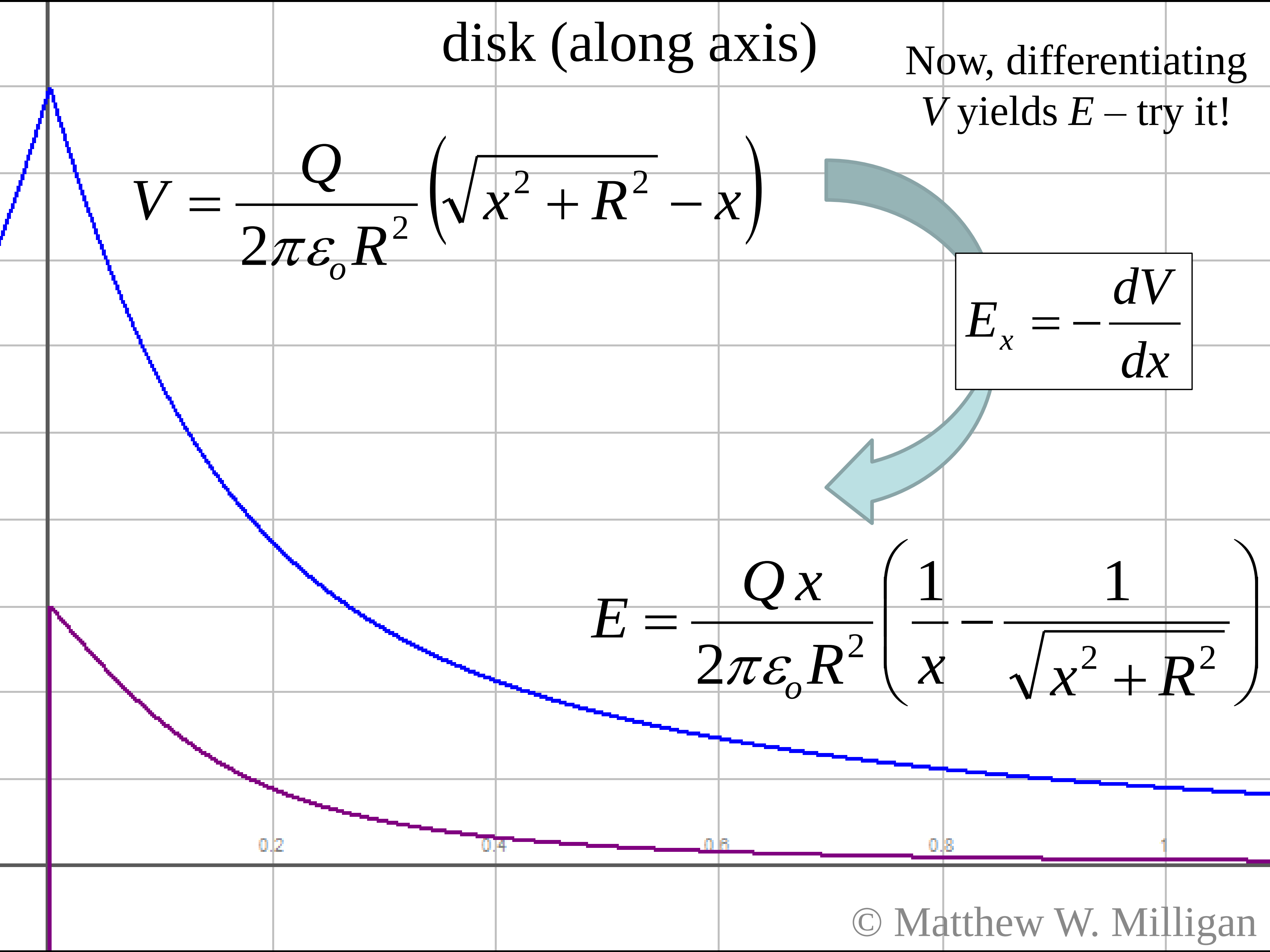
disk (along axis)

Now, differentiating V yields E – try it!

$$V = \frac{Q}{2\pi\varepsilon_o R^2}\left(\sqrt{x^2 + R^2} - x\right)$$

$$E_x = -\frac{dV}{dx}$$

$$E = \frac{Q\,x}{2\pi\varepsilon_o R^2}\left(\frac{1}{x} - \frac{1}{\sqrt{x^2 + R^2}}\right)$$



disk (along axis)

Now, integrating E yields V – try it!

$$V = \frac{Q}{2\pi\varepsilon_o R^2}\left(\sqrt{x^2+R^2}-x\right)$$

$$V_B - V_A = -\int_A^B E\,dx$$

$$E = \frac{Q\,x}{2\pi\varepsilon_o R^2}\left(\frac{1}{x}-\frac{1}{\sqrt{x^2+R^2}}\right)$$