Advanced Kinematics

I. Vector addition/subtraction

II. Components

III. Relative Velocity

IV. Projectile Motion

V. Use of Calculus (nonuniform acceleration)

VI. Parametric Equations

	The student will be able to:	HW:
1	Calculate the components of a vector given its magnitude and direction. ✓	1 – 2
2	Calculate the magnitude and direction of a vector given its components. ✓	3 – 4
3	Use vector components as a means of analyzing/solving 2-D motion problems. ✓	5 – 6
4	Add or subtract vectors analytically (using trigonometric calculations). ✓	7 – 9
5	Use vector addition or subtraction as a means of solving relative motion problems.	10 – 15
6	State the horizontal and vertical relations for projectile motion and use the same to solve projectile problems.	16 – 24
7	Use derivatives to determine speed, velocity, or acceleration and solve for extrema and/or zeros.	25 – 27
8	Use integrals to determine distance, displacement, change in speed or velocity and solve for functions thereof given initial conditions.	28 – 31
9	Solve problems involving parametric equations that describe motion components	32 – 34

Frame of Reference

- Any quantity of motion must be measured *relative* to a certain frame of reference.
- A *frame of reference* is something with which a moving object is compared.
- The values of position, velocity, acceleration, etc. depend upon the chosen frame of reference.

  (The values could be *anything*, depending on *which* frame of reference is used!)

Relative Velocity Equation

The value of object *A*'s velocity in reference frame *C* differs from that in reference frame *B* by the velocity of *B* relative to *C*:

$$\vec{v}_{AC} = \vec{v}_{AB} + \vec{v}_{BC}$$

Relative Kinematics Equations

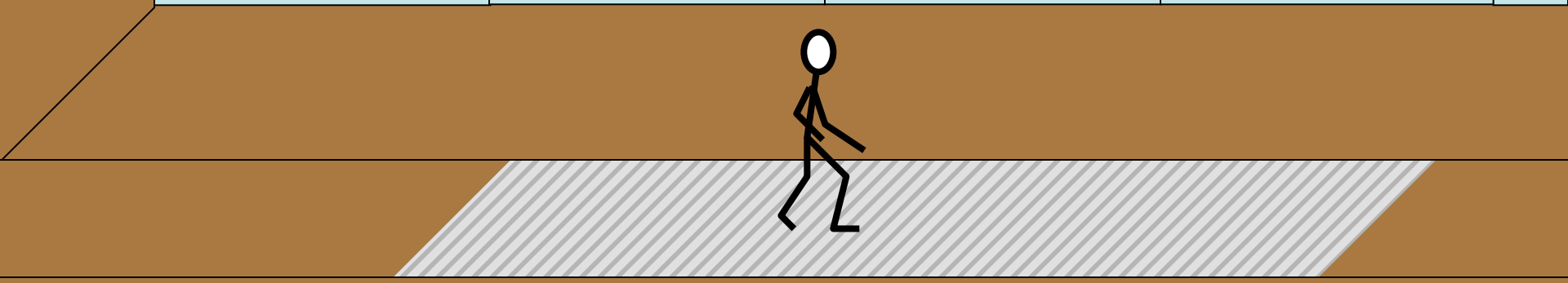
The same type of relationship exists for position and acceleration:

$$\vec{r}_{AC} = \vec{r}_{AB} + \vec{r}_{BC}$$

$$\vec{v}_{AC} = \vec{v}_{AB} + \vec{v}_{BC}$$

$$\vec{a}_{AC} = \vec{a}_{AB} + \vec{a}_{BC}$$

Consider a person walking on a moving sidewalk (often found in an airport). The net velocity is the vector sum of the person's "walking velocity walking" plus the velocity of the sidewalk.



$$\vec{v}_{PE} = \vec{v}_{PS} + \vec{v}_{SE}$$

person relative to earth	person relative to sidewalk	sidewalk relative to earth

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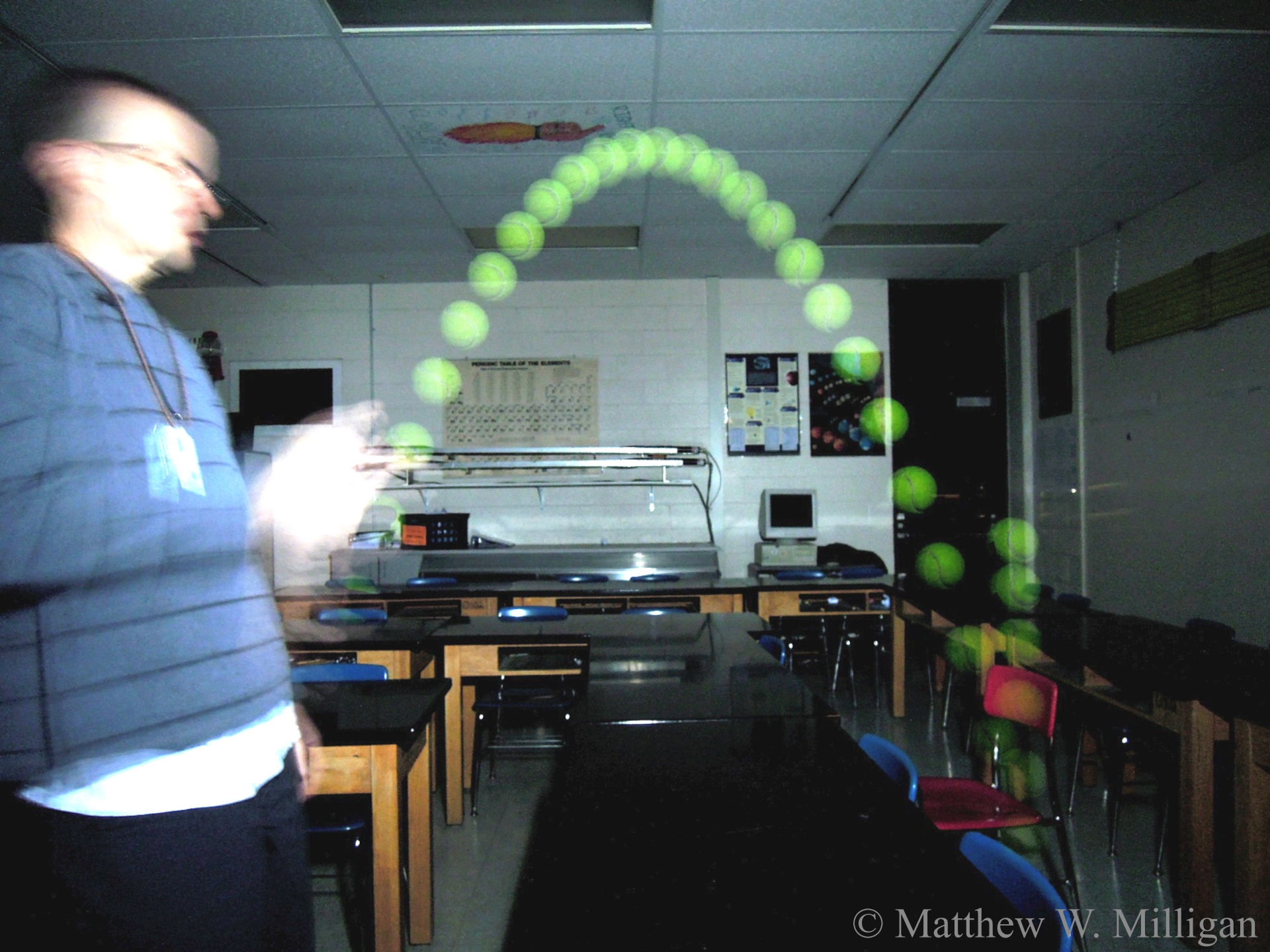
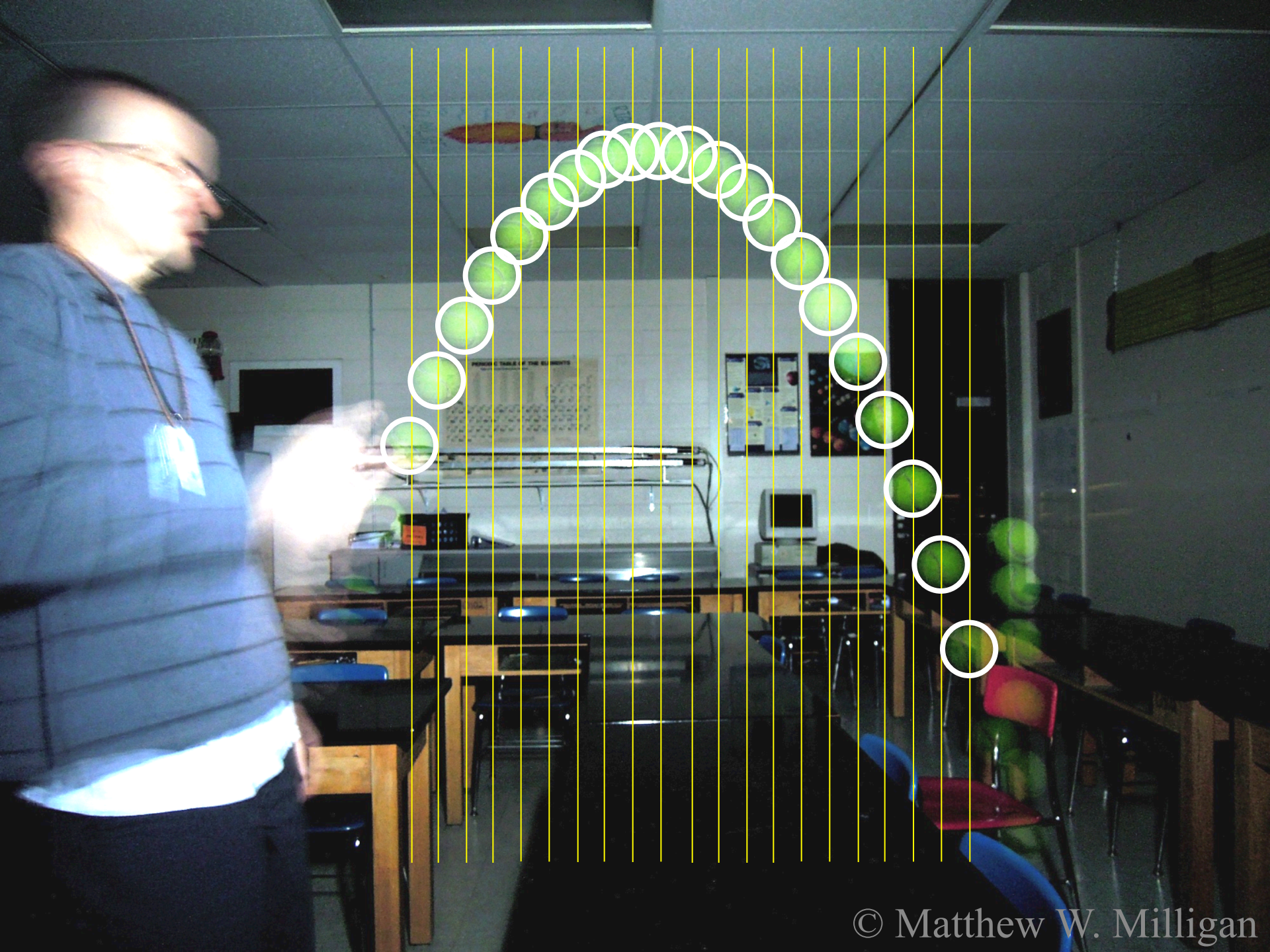
What is a "projectile"?

- A projectile is an object launched by some initial force, which then proceeds under the sole influence of gravity.
- The equations used to model this motion apply to the subsequent motion of the object *after* launch and *before* impact.

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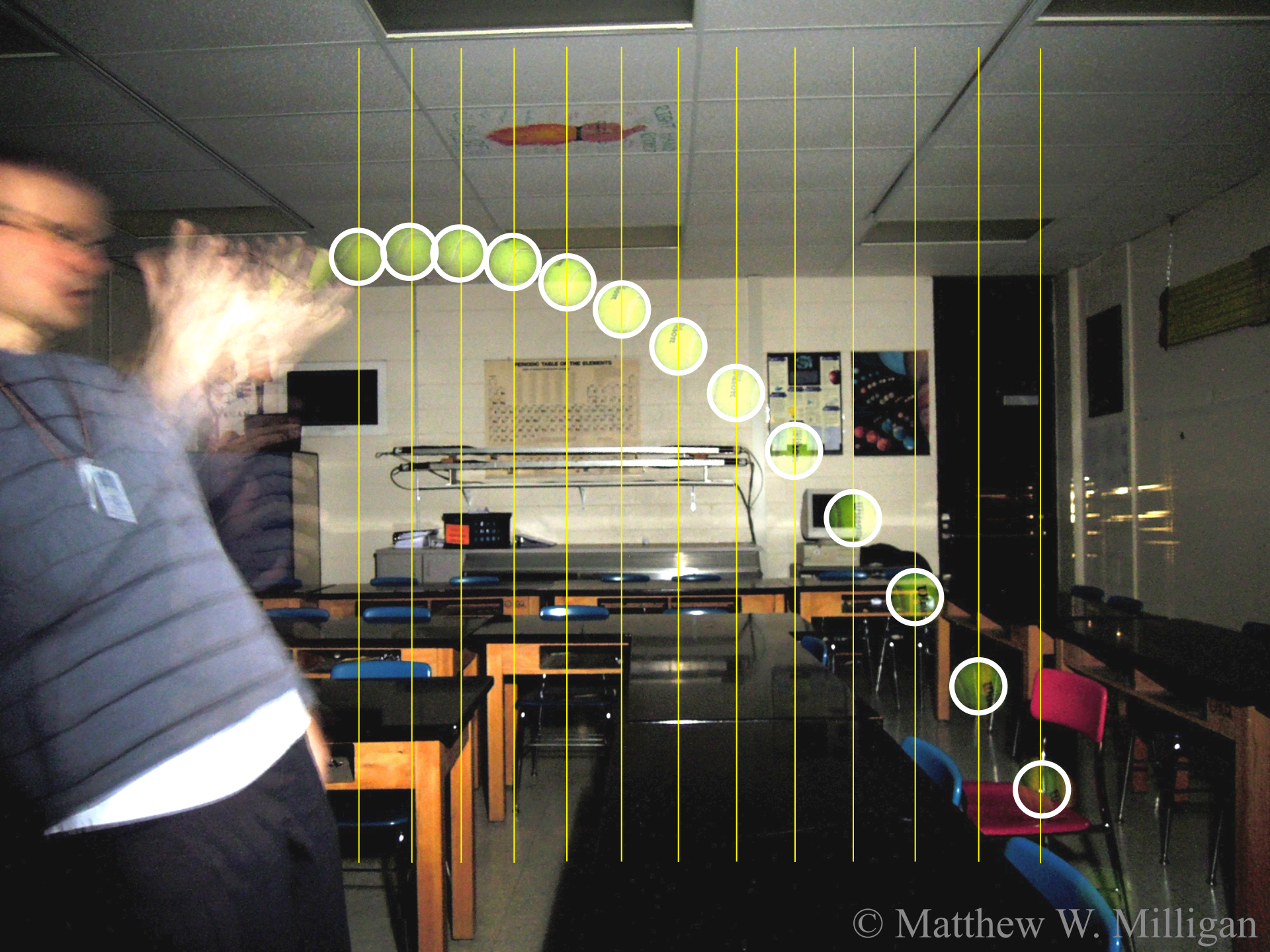
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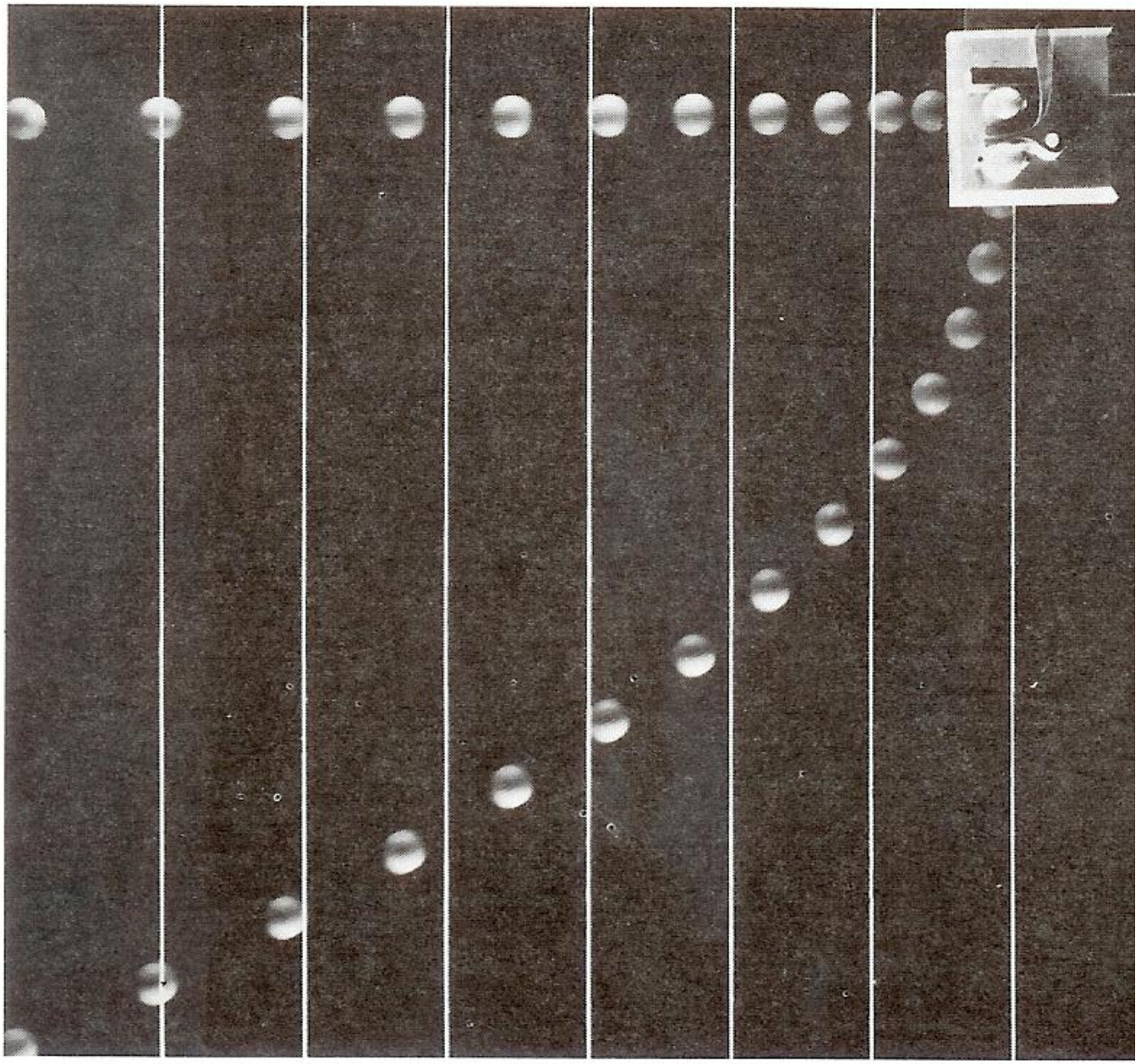
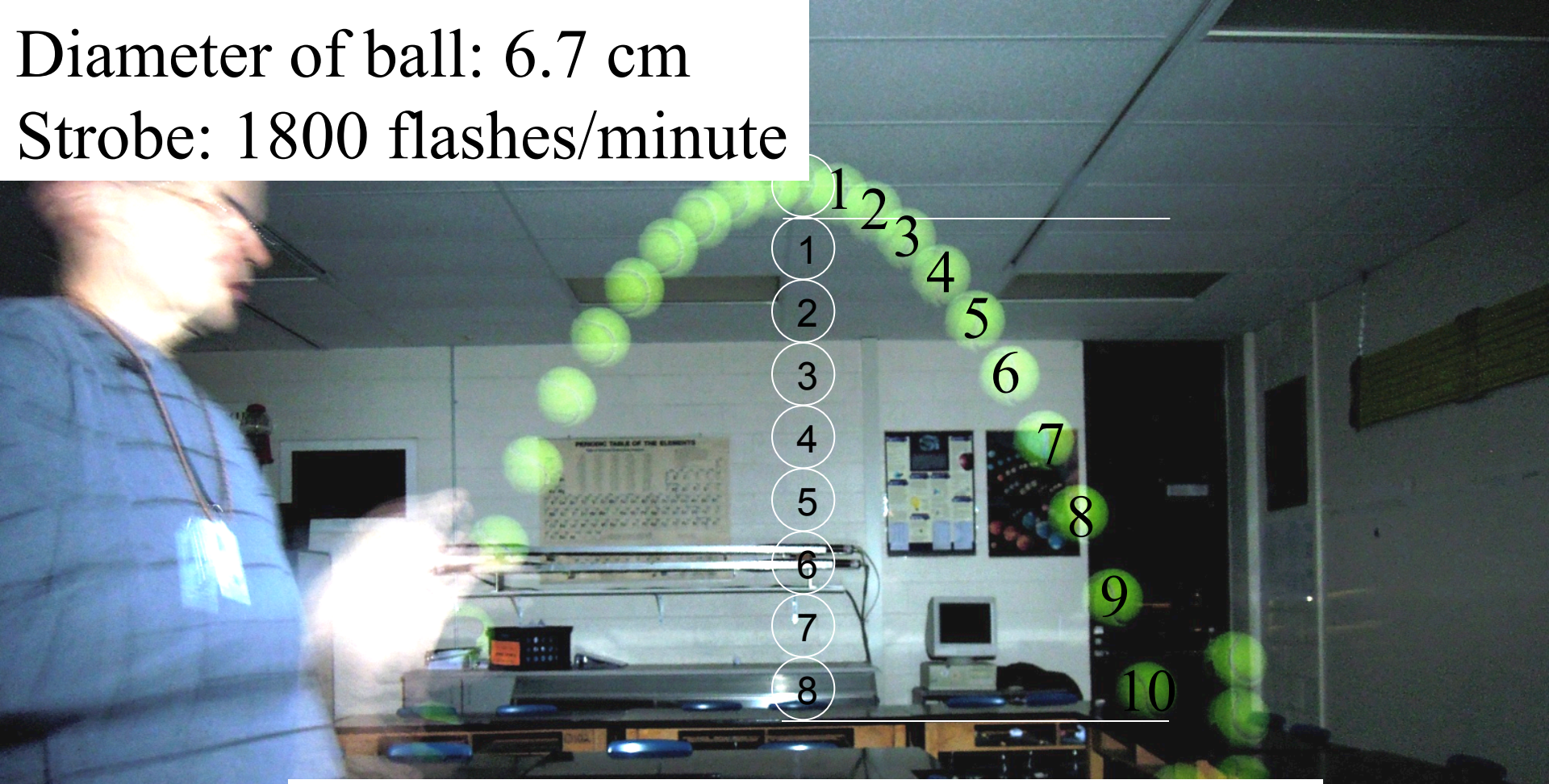
Projectile x and y

- Horizontally a projectile moves forward at a constant rate. The x-component of a projectile's velocity is constant.



Diameter of ball: 6.7 cm
Strobe: 1800 flashes/minute



The ball has fallen 8 diameters after 10 flashes of the strobe light.

What is its vertical acceleration?

Projectile x and y

- Horizontally a projectile moves forward at a constant rate. The x-component of a projectile's velocity is constant.
- Vertically a projectile moves with constant acceleration caused by gravity. The y-component of a projectile's acceleration is $-g$.
- The forward motion of a projectile does not alter the effect of gravity – nor does the force of gravity alter the forward motion of a projectile.

$$\vec{v} = \vec{v}_0 + \vec{a}t$$

$$\vec{d} = \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$$

$$a_x = ? \qquad a_y = ?$$

$$v_x = v_{0x} + a_x t \qquad v_y = v_{0y} + a_y t$$

$$d_x = v_{0x} t + \frac{1}{2} a_x t^2 \qquad d_y = v_{0y} t + \frac{1}{2} a_y t^2$$

$$\vec{v} = \vec{v}_0 + \vec{a}t$$

$$\vec{d} = \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$$

$$a_x = 0 \qquad a_y = -g$$

$$v_x = v_{0x} + \cancel{a_x t} \qquad v_y = v_{0y} + a_y t$$

$$d_x = v_{0x} t + \cancel{\frac{1}{2} a_x t^2} \qquad d_y = v_{0y} t + \frac{1}{2} a_y t^2$$

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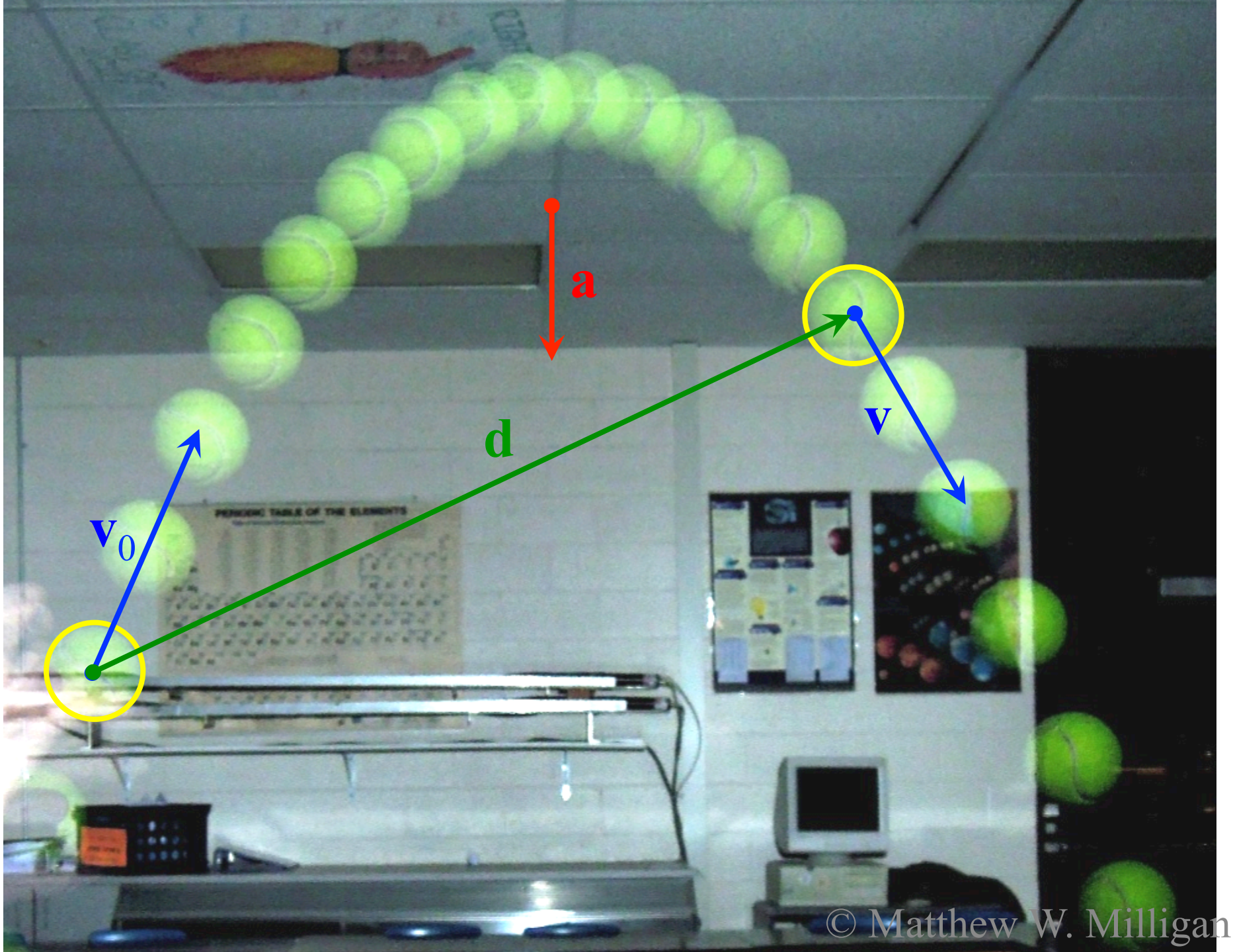
$$a_x = 0 \qquad a_y = -g$$

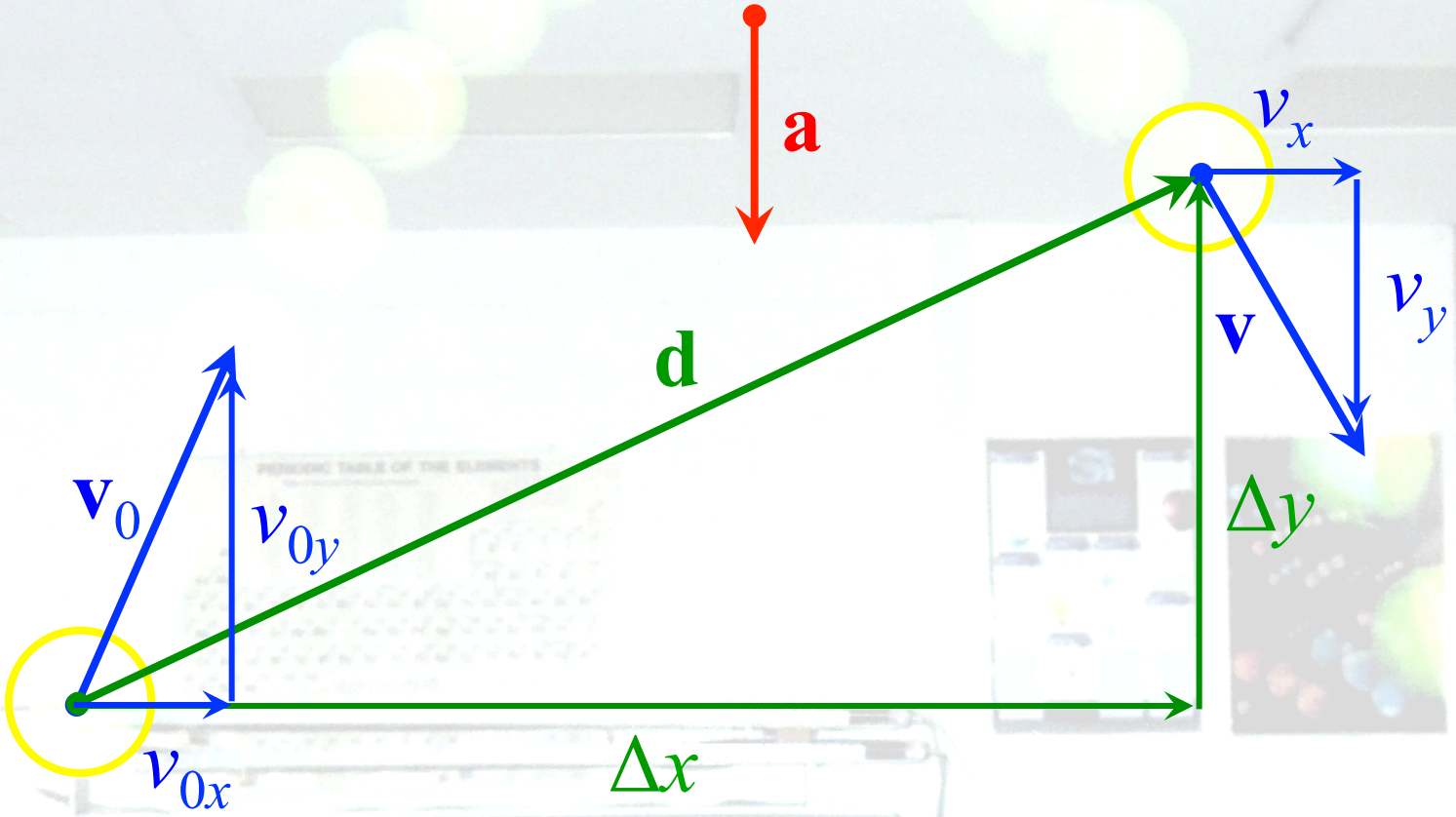
$$v_x = v_{0x} \qquad v_y = v_{0y} + a_y t$$

$$\Delta x = v_{0x} t \qquad \Delta y = v_{0y} t + \tfrac{1}{2} a_y t^2$$

Components of Projectile Motion

Horizontal	Vertical
$a_x = 0$	$a_y = -g$
$v_x = v_{0x}$	$v_y = v_{0y} + a_y t$
$\Delta x = v_{0x} t$	$\Delta y = v_{0y} t + \frac{1}{2} a_y t^2$





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$a_x = 0$	$a_y = -g$
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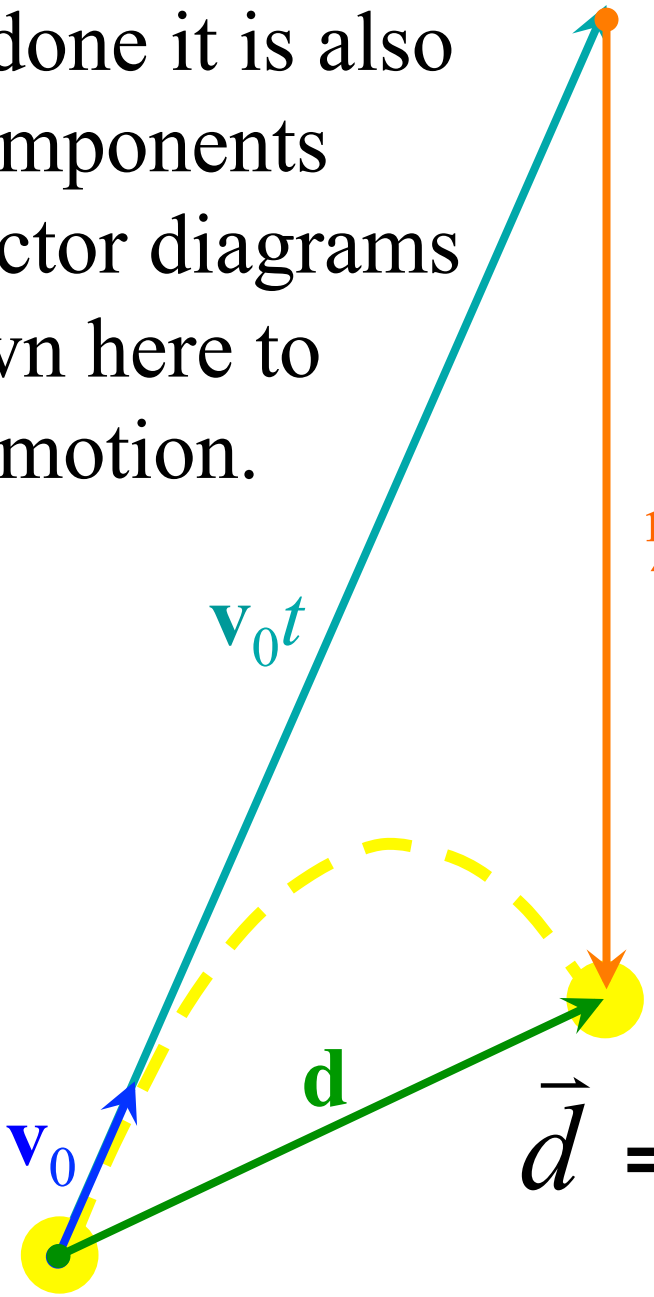
Although seldom done it is also valid to *not* use components and instead use vector diagrams such as those shown here to analyze projectile motion.



$$\vec{v} = \vec{v}_0 + \vec{a}t$$

$$\vec{d} = \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$$