Rotation

I. Kinematics
 - Angular analogs

II. Dynamics
 - Torque and Moment of Inertia
 - Fixed-axis
 - Rolling, slipping

III. Work and Energy
 - Fixed-axis, rolling

IV. Angular Momentum
 - Bodies and particles

	The student will be able to:	HW:
1	State and apply the relations between angular position, angular displacement, angular speed, angular velocity, and angular acceleration to solve related problems.	✓ 1 – 3
2	State and apply the relations between the angular (or rotational) motion of a body or system and the linear (or translational) motion of a point on the body or system.	4 – 7 ✓
3	Determine the torque of an applied force and solve related problems.	✓ 8 – 12
4	Determine the moment of inertia for a system of masses or solid body and solve related problems.	✓ 13 – 18
5	State and apply Newton's 2nd Law for fixed-axis rotation to solve related problems.	19 – 21
6	Apply work and energy to solve fixed-axis rotation problems.	22 – 25
7	State and apply Newton's 2nd Law for rolling (rotation and translation) to solve related problems (including those with slipping and without slipping)	26 – 33
8	Apply work and energy to solve rolling problems.	34 – 36
9	Determine angular momentum for a particle, system, or rotating body and relate to torque and angular impulse to solve problems.	37 – 42
10	Apply conservation of angular momentum to solve related problems.	43 – 49

Newton's 2nd Law for Rotation

$$\vec{\tau}_{net} = I\vec{\alpha}$$

$$\Sigma\vec{\tau} = I\vec{\alpha}$$

where: τ = torque
I = rotational inertia
α = angular acceleration

Work, Energy, Power for Rotation

- The definitions and units for work, energy, and power do not change for rotational motion!
- Key difference is the relation of work to torque and the relation of kinetic energy to angular speed.
- The equations are just as expected using the analogous rotational quantities:

$$W = \int \tau d\theta$$

$$K = \frac{1}{2} I \omega^2$$

Work, Energy, Power for Rotation

- The Work-Energy Theorem and Conservation of Energy are exactly the same as before:

$$\Sigma W = \Delta K$$

$$\Sigma W_{NC} + U_1 + K_1 = U_2 + K_2$$

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  - **Rolling, slipping**

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Newton's 2nd Law for a system of particles

$$\Sigma\vec{F}_{ext} = \left(\Sigma m\right)\vec{a}_{CM}$$

Nothing new here – but now the rolling object *is* the "system of particles"!

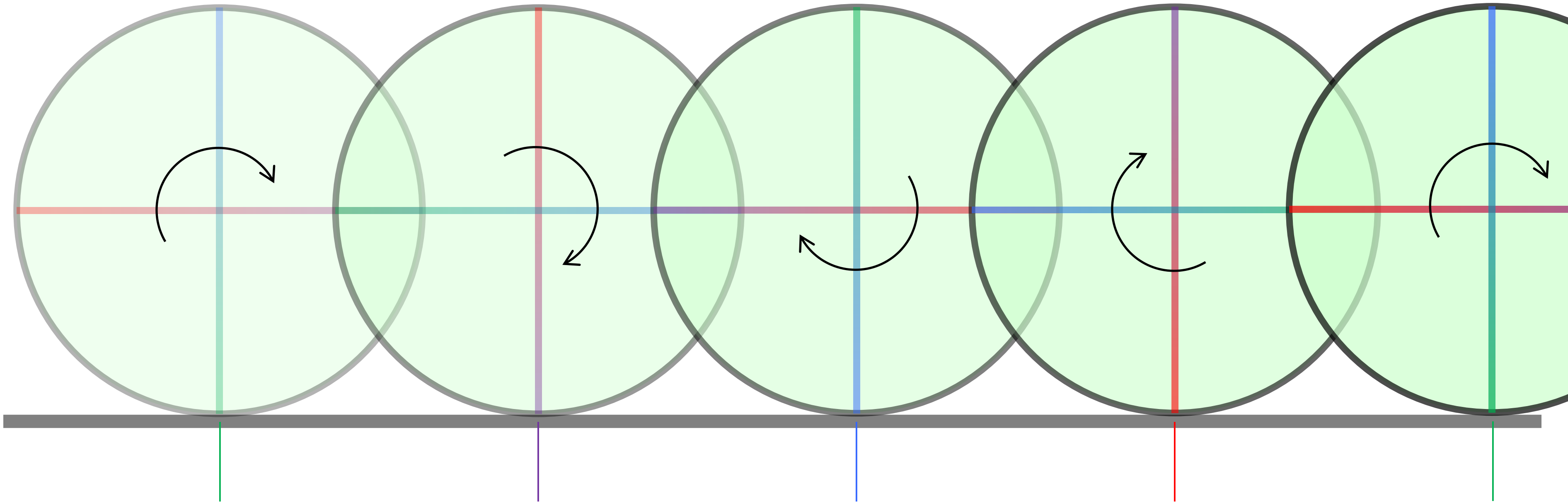
Newton's 2nd Law for rotation *without* a fixed axis:

$$St_{ext} = I_{CM}a$$

Similar to the systems of particles concepts – analyze based on the center of mass! The axis of rotation passes through the center of mass and moves with the object.

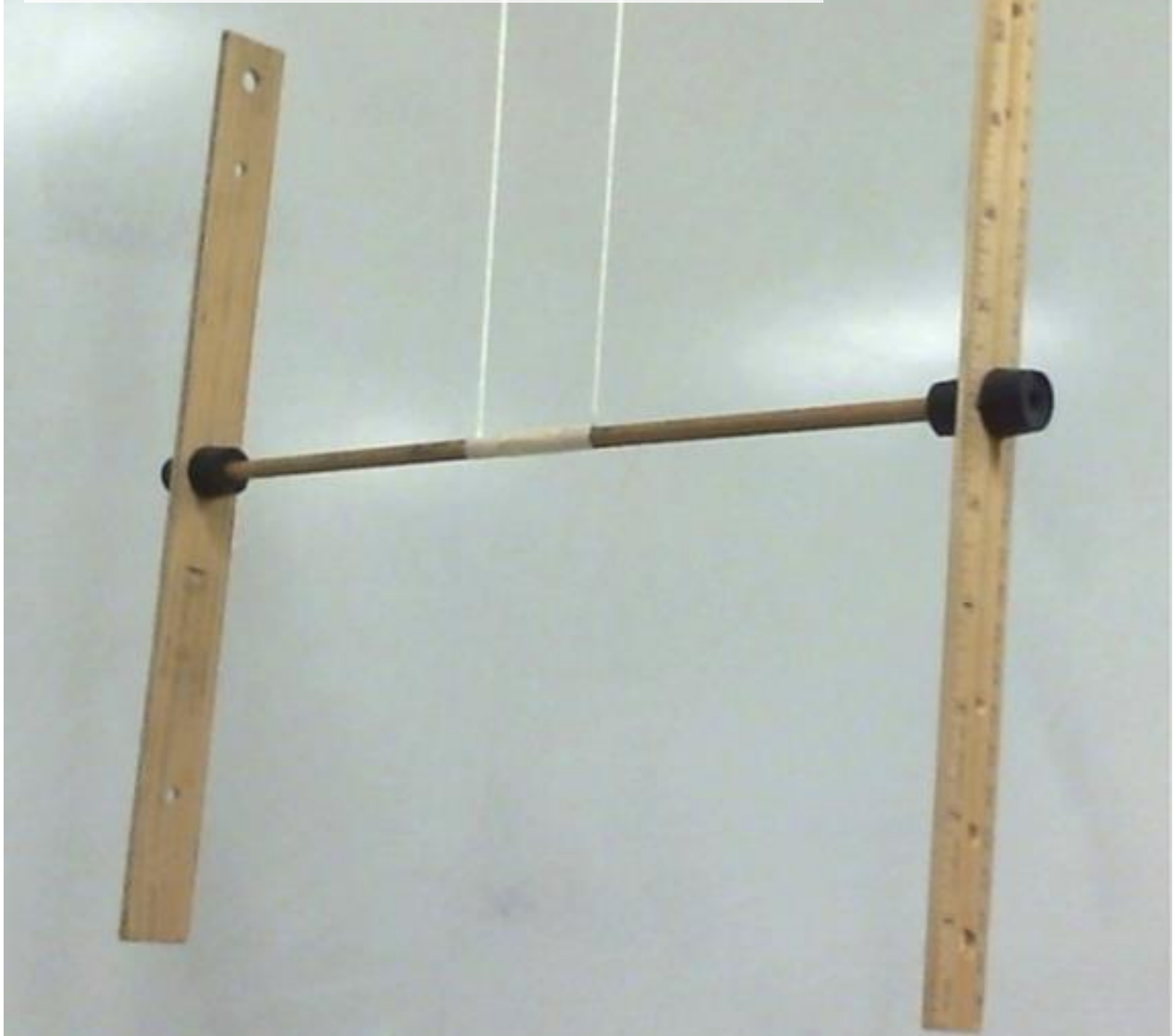
Rolling *Without* Slipping

Rolling across a surface – in what direction is the force of friction?



Note the tic marks are separated by one quarter the circumference. If the wheel rolls without slipping it moves forward a distance equal to its circumference every time it completes one revolution.

What is the acceleration of a Slo-Yo?!

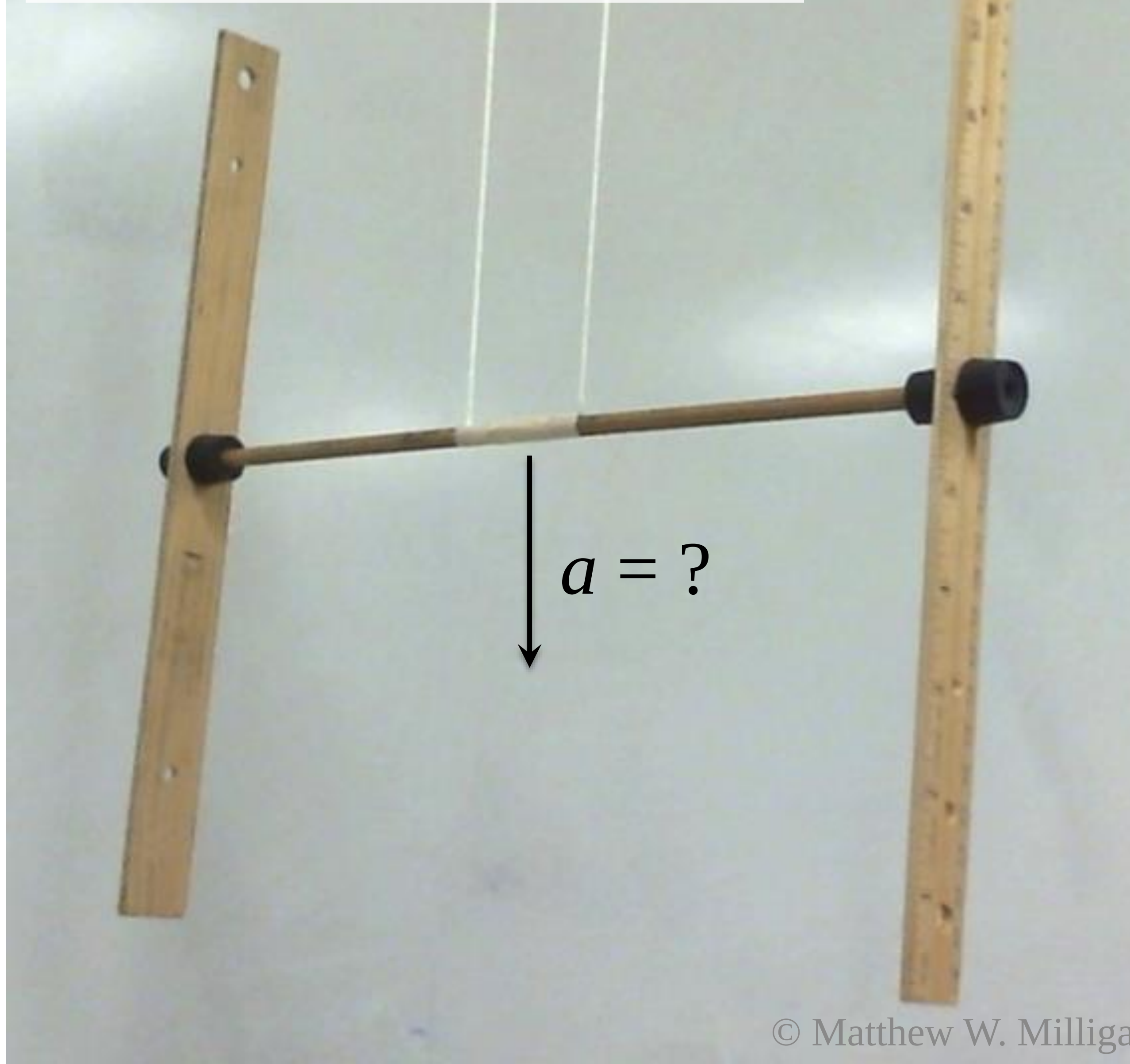


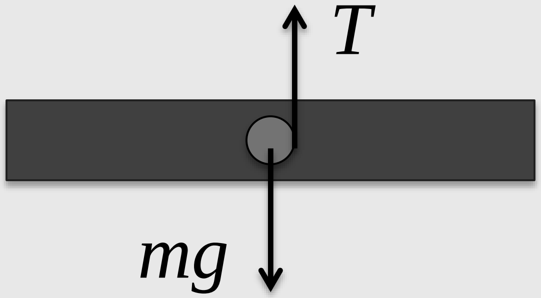
Total Mass:
axle + rulers
$m = 45.6$ g

Axle:
¼ inch dia.
$r = 0.3175$ cm
$l = 34.0$ cm

Each Ruler:
$M = 13$ g
$L = 30.8$ cm
$w = 2.6$ cm

What is the acceleration of a “Slo-Yo”?!





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$$SF = ma$$

$$mg - T = ma$$

$$mg - \frac{I}{r^2}a = ma$$

$$a = \frac{mg}{m + \frac{I}{r^2}}$$

$$a = \frac{45.6}{45.6 + 20535} \times 9.8$$

$$a = \frac{0.447\ \text{N}}{20.58\ \text{kg}}$$

$$a = 0.022\ \tfrac{\text{m}}{\text{s}^2}$$

$$St = Ia$$

$$Tr = I\frac{a}{r}$$

$$T = \frac{I}{r^2}a$$

$$I = 2\left(\frac{ML^2}{12} + \frac{Mw^2}{12}\right)$$

$$I = 2\left(1027.7 + 7.32\right)$$

$$I = 2070\ \text{g} \cdot \text{cm}^2$$

$$\frac{I}{r^2} = \frac{2070}{0.3175^2} = 20535\ \text{g}$$