Rotation

I. Kinematics
- Angular analogs

II. Dynamics
- Torque and **Moment of Inertia**
- Fixed-axis
- Rolling, slipping

III. Work and Energy
- Fixed-axis, rolling

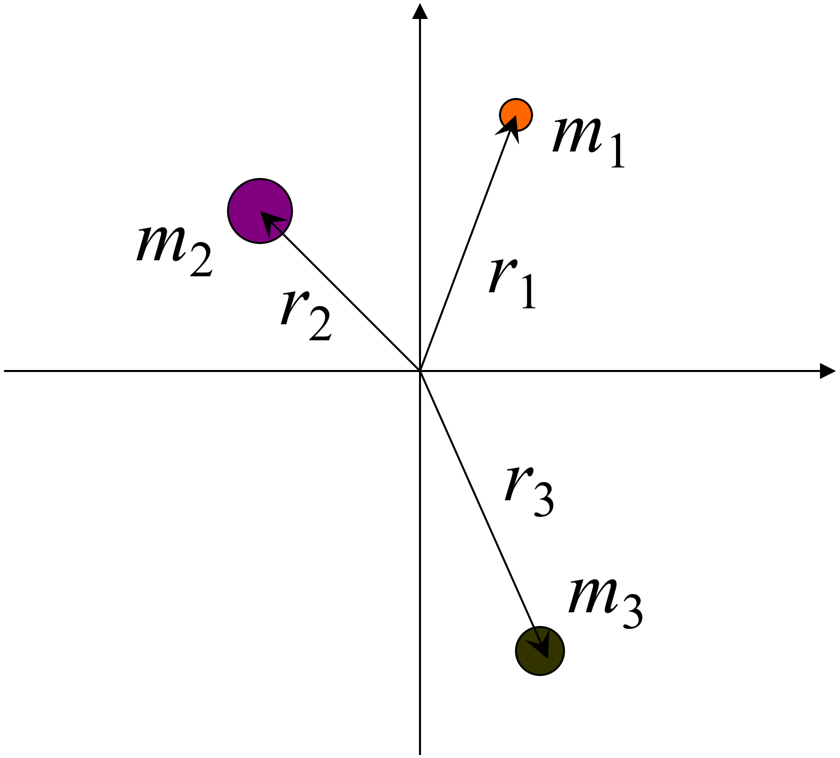
IV. Angular Momentum
- Bodies and particles

	The student will be able to:	HW:
1	State and apply the relations between angular position, angular displacement, angular speed, angular velocity, and angular acceleration to solve related problems.	1 – 3
2	State and apply the relations between the angular (or rotational) motion of a body or system and the linear (or translational) motion of a point on the body or system.	4 – 7
3	Determine the torque of an applied force and solve related problems.	8 – 12
4	Determine the moment of inertia for a system of masses or solid body and solve related problems.	13 – 18
5	State and apply Newton's 2^{nd} Law for fixed-axis rotation to solve related problems.	19 – 21
6	Apply work and energy to solve fixed-axis rotation problems.	22 – 25
7	State and apply Newton's 2^{nd} Law for rolling (rotation and translation) to solve related problems (including those with slipping and without slipping)	26 – 33
8	Apply work and energy to solve rolling problems.	34 – 36
9	Determine angular momentum for a particle, system, or rotating body and relate to torque and angular impulse to solve problems.	37 – 42
10	Apply conservation of angular momentum to solve related problems.	43 – 49

Rotational Inertia

- As explained in Newton's Laws of Motion any object that has mass has inertia – tendency to maintain state of motion.
- For a rotating body this tendency also depends on the arrangement of mass relative to the axis.
- The quantity "moment of inertia" or "rotational inertia" is defined to satisfy a rotational version of Newton's 2^{nd} Law.

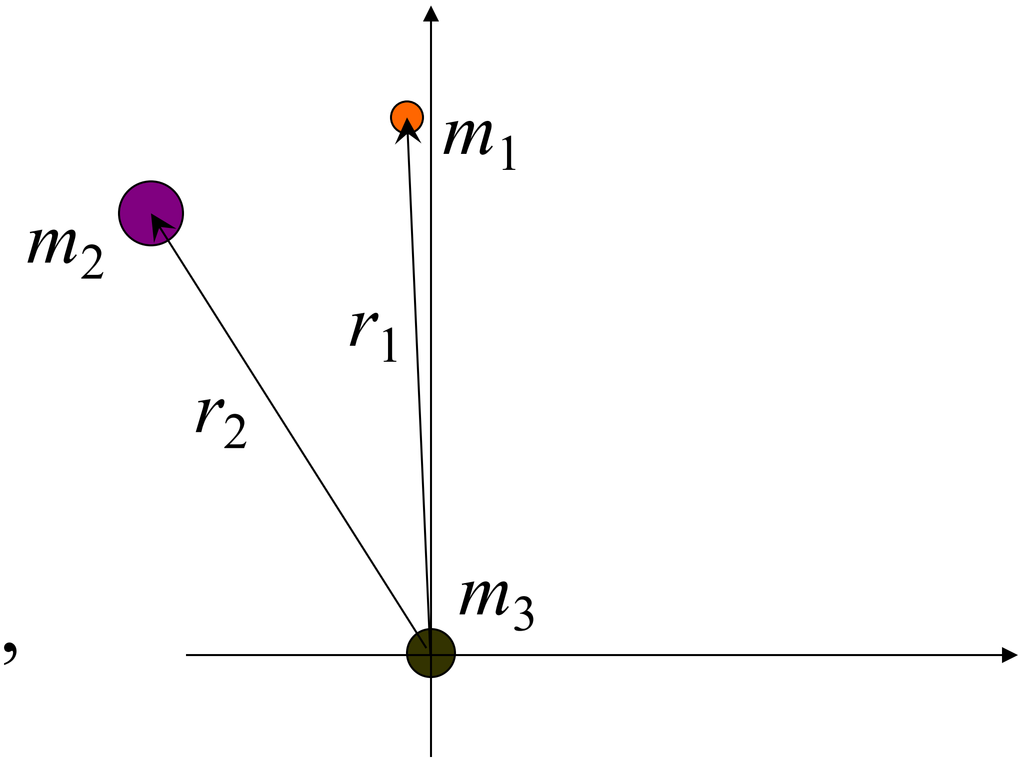
Moment of Inertia – System of Discrete Masses



$$I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \ldots$$

$$I = \sum m_i r_i^2$$

Moment of Inertia – System of Discrete Masses



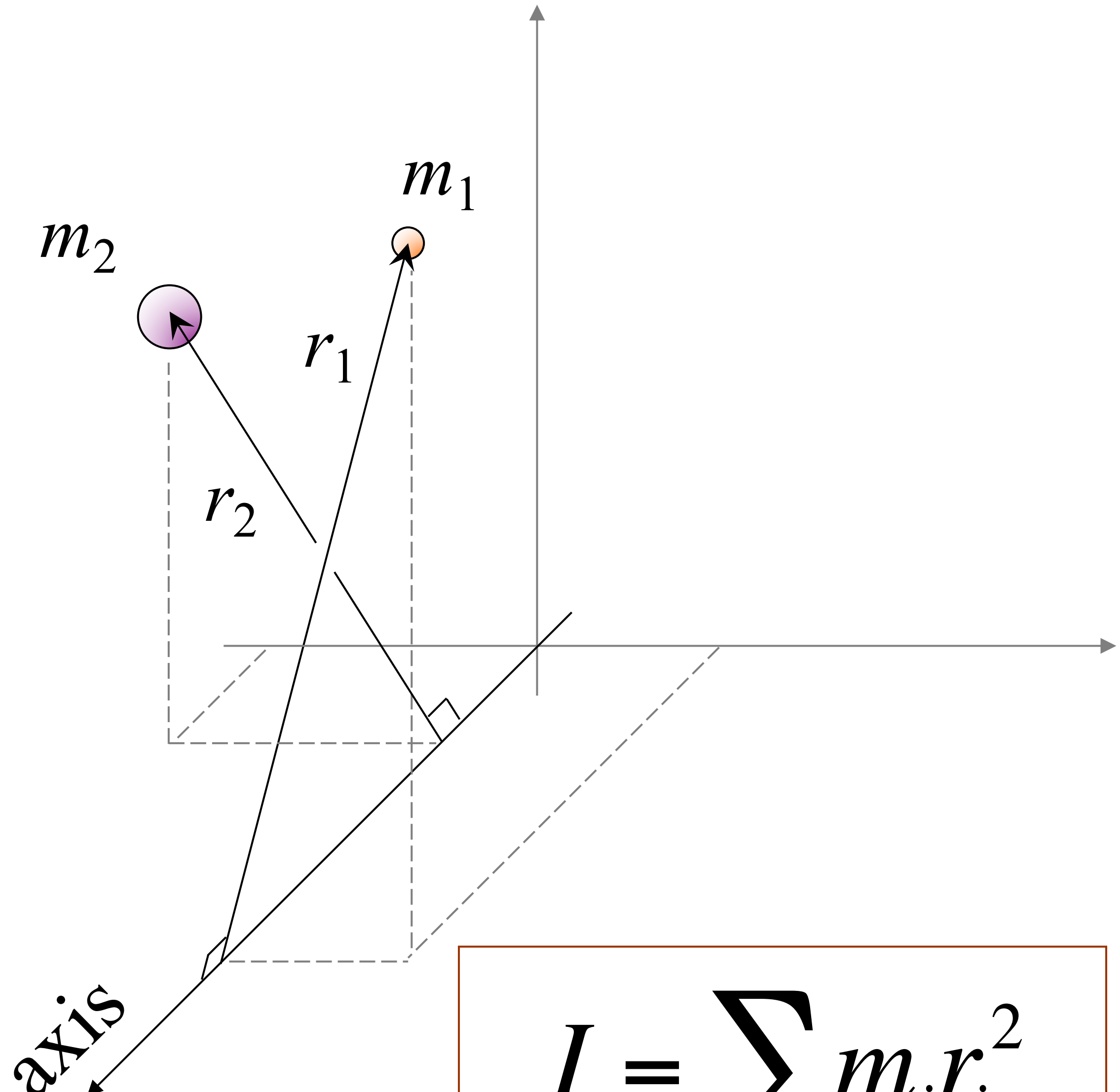
The exact same set of masses can have a different moment of inertia, depending on the axis location.

$$I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \ldots$$

$$I = \sum m_i r_i^2$$

Moment of Inertia – System of Discrete Masses

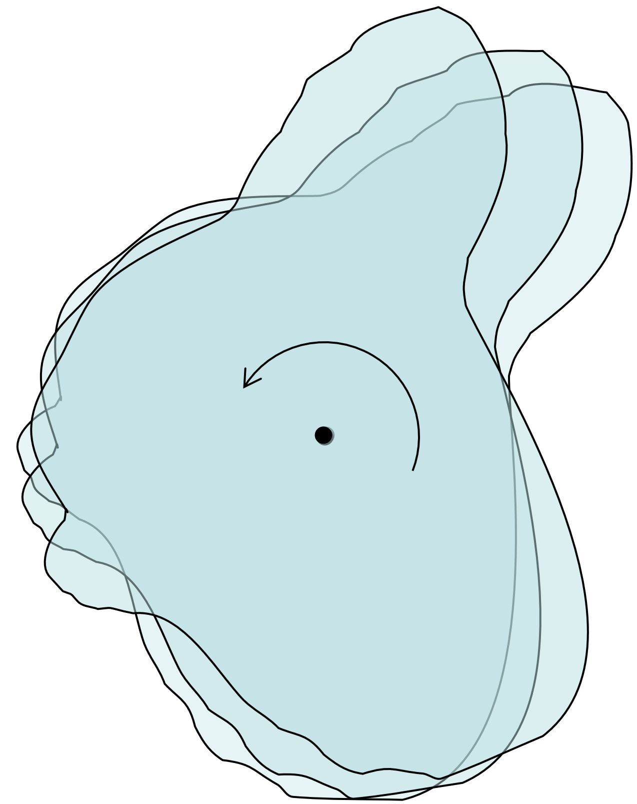
Note: the value of r is a *perpendicular distance* from an axis of rotation – the radius of a circle in which the particle moves when rotation occurs.



$$I = \sum m_i r_i^2$$

Moment of Inertia – Solid Body

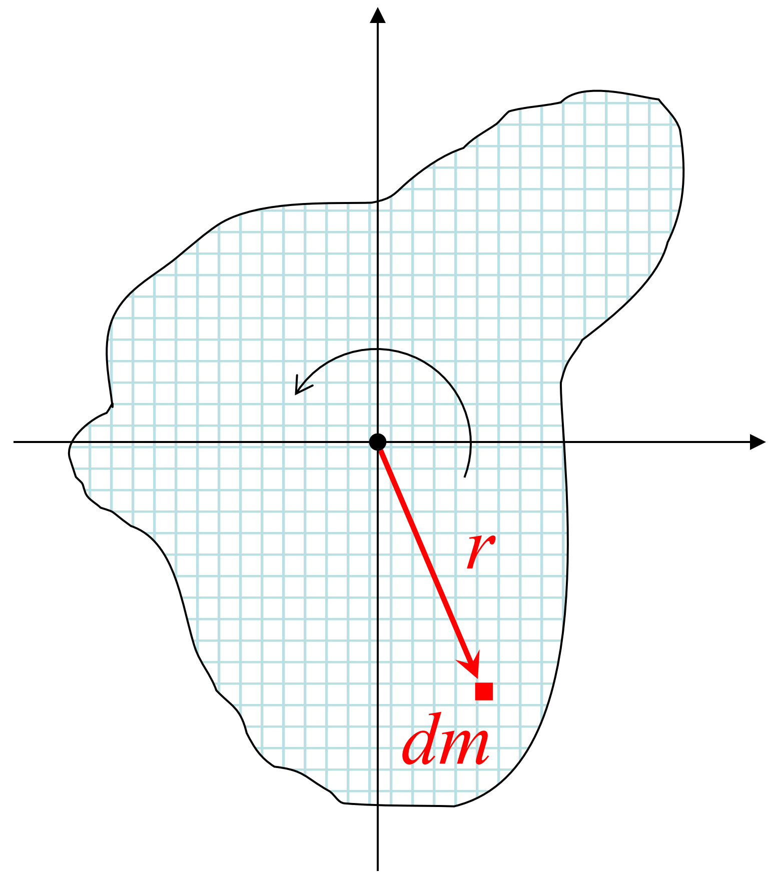
Suppose an arbitrary object rotates about a fixed axis as shown…



Moment of Inertia – Solid Body

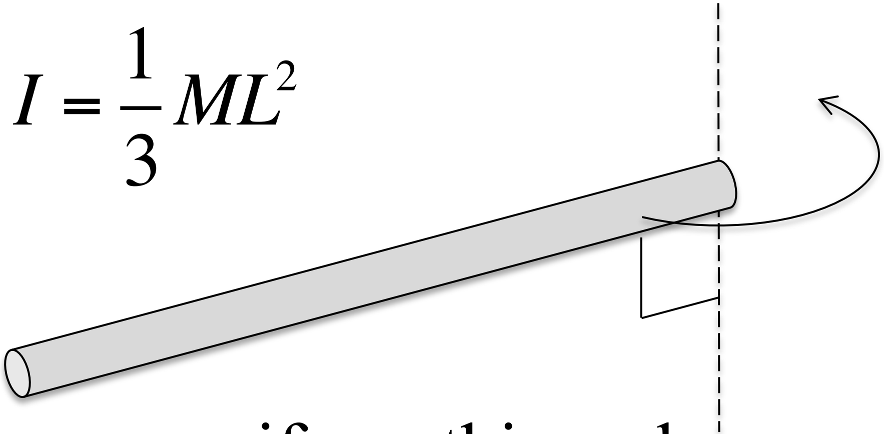
$$I = \int r^2 \, dm$$

where: dm = infinitesimal "piece" of mass
r = position of dm relative to axis of rotation



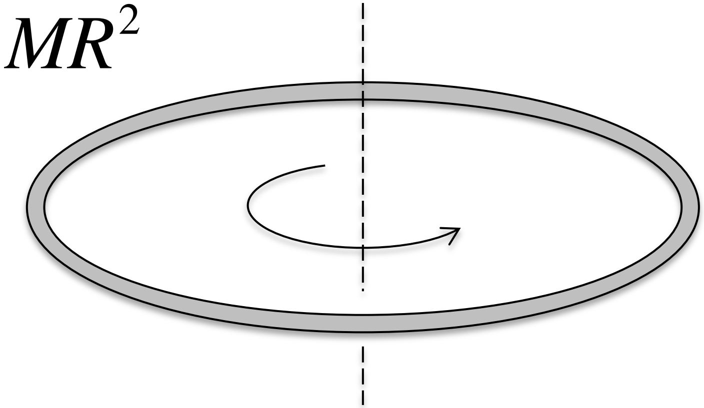
The rotational inertia is the infinite sum of mr^2 taking the object to be divided into tiny increments of mass denoted *dm,* each a particular distance *r* from the axis.

$$I = \frac{1}{3}ML^2$$



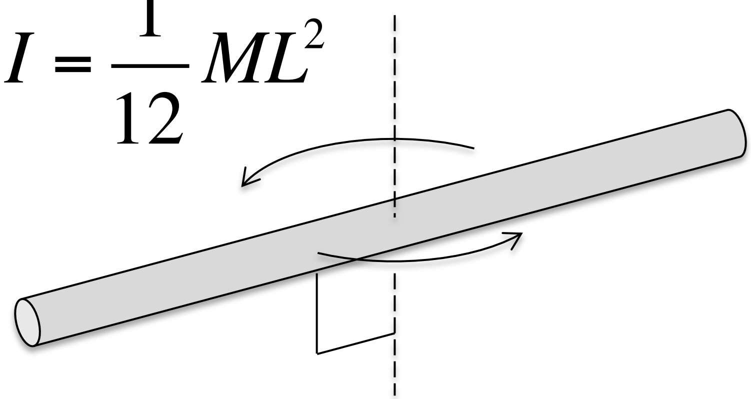
uniform thin rod,
perpendicular axis at end

$$I = MR^2$$



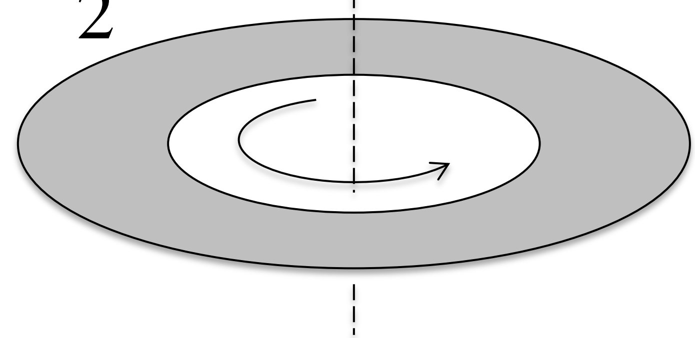
uniform thin ring,
perpendicular axis at center

$$I = \frac{1}{12}ML^2$$



uniform thin rod,
perpendicular axis at center

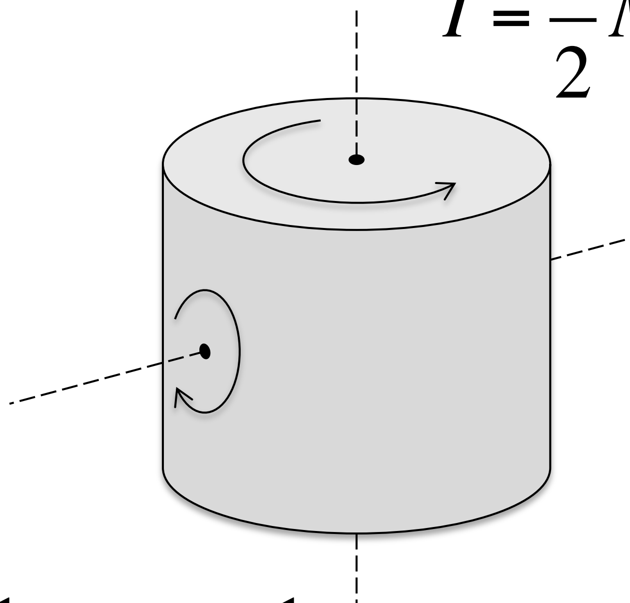
$$I = M\frac{R_1^2 + R_2^2}{2}$$



uniform "washer",
perpendicular axis at center

uniform solid cylinder,
axis through center
parallel to side

$$I = \frac{1}{2} MR^2$$

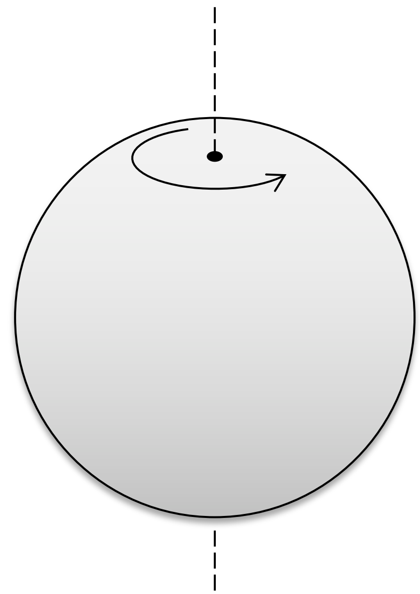


$$I = \frac{1}{4} MR^2 + \frac{1}{12} Mh^2$$

uniform solid cylinder,
axis through center
parallel to ends

uniform solid sphere,
axis through center

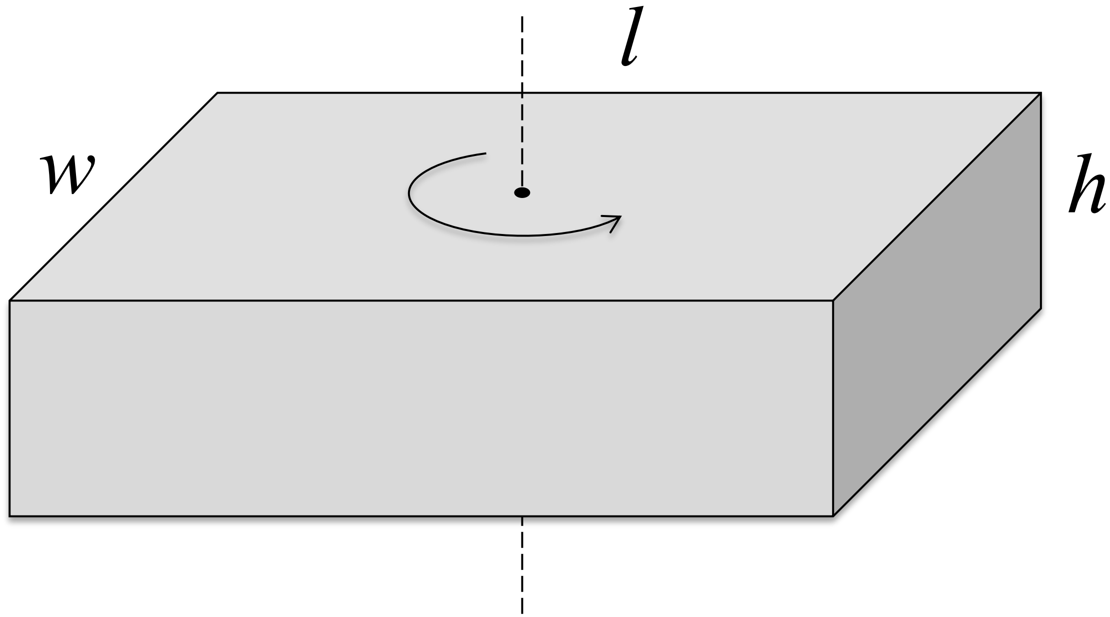
$$I = \frac{2}{5} MR^2$$



$$I = \frac{2}{3} MR^2$$

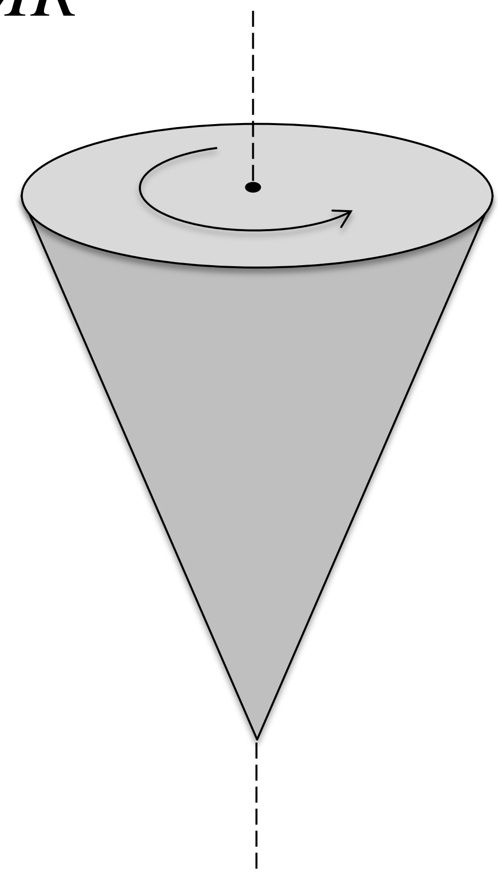
hollow spherical shell,
axis through center

$$I = M\frac{w^2 + l^2}{12}$$



uniform rectangular solid,
axis through center
parallel to *h*

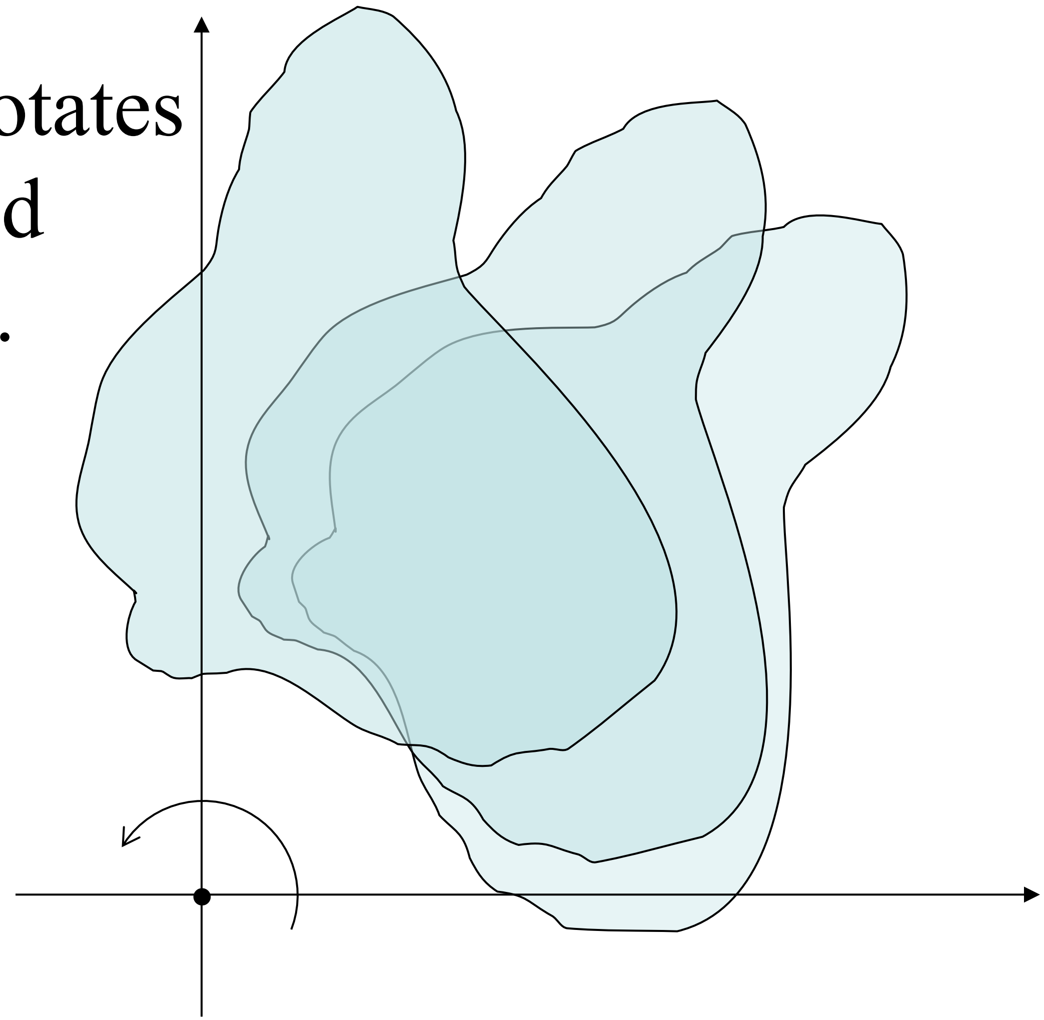
$$I = \frac{3}{10}MR^2$$



uniform solid cone
base radius *R*,
axis of symmetry

Moment of Inertia – Parallel Axis Theorem

Suppose the object rotates about a *different* fixed axis as shown here…

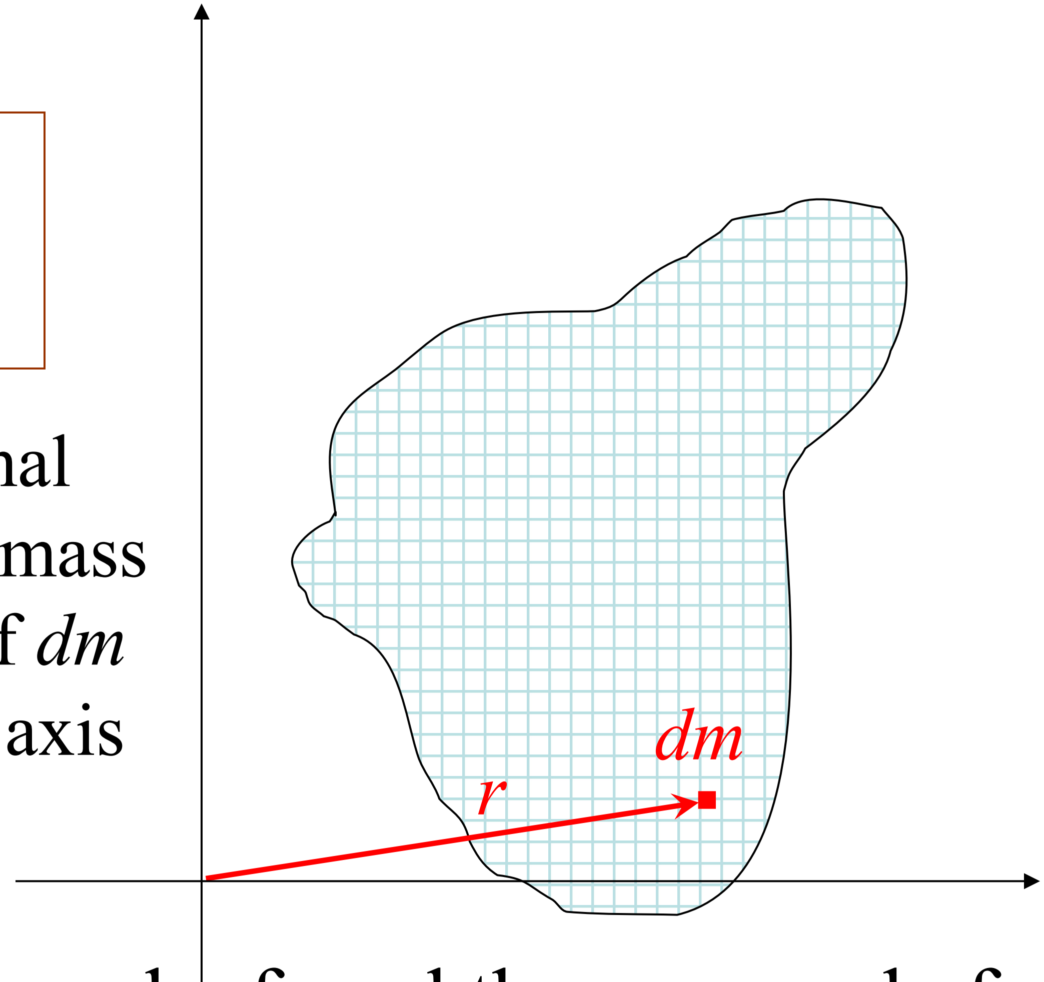


Moment of Inertia – Parallel Axis Theorem

$$I = \int r^2\, dm$$

where: dm = infinitesimal "piece" of mass

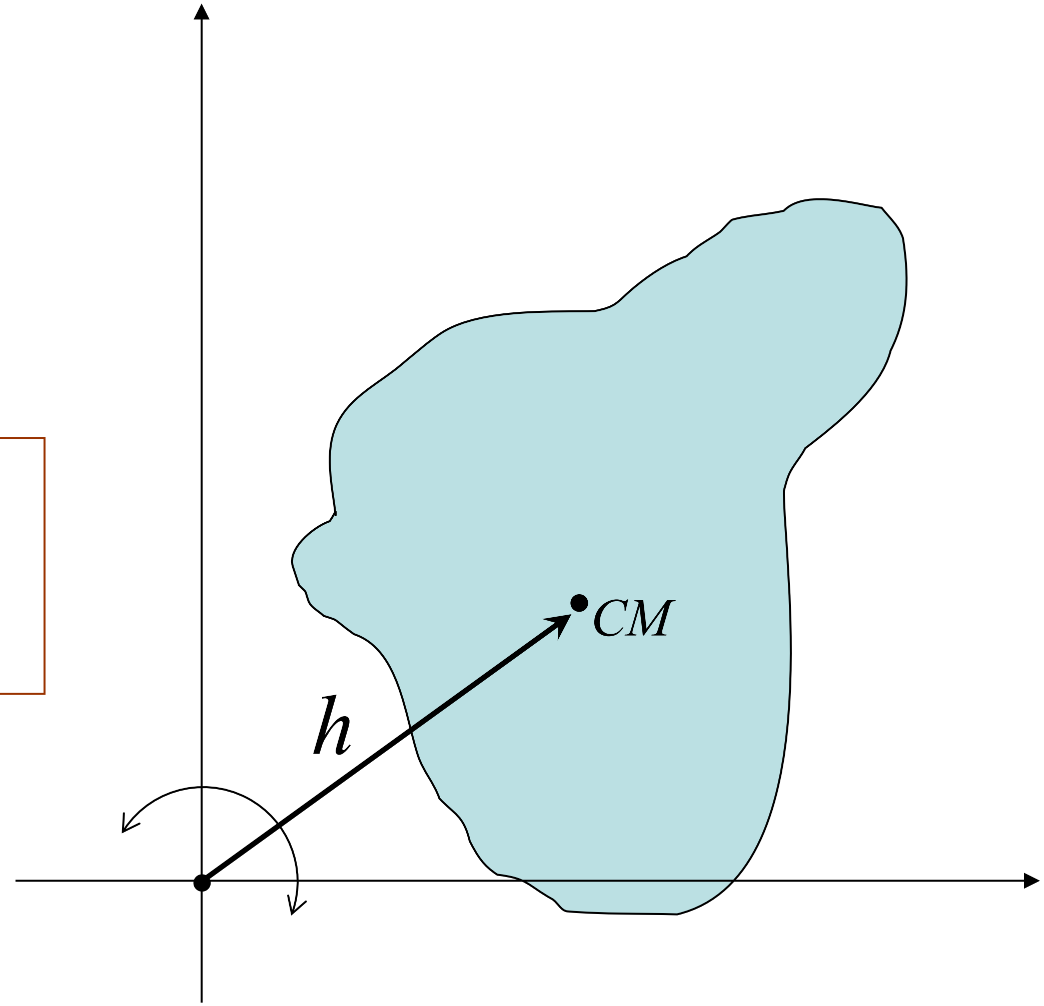
r = position of dm relative to axis of rotation



The rotational inertia can be found the same as before but the values of r are different. Or the parallel axis theorem may be applied…

Moment of Inertia – Parallel Axis Theorem

$$I = mh^2 + I_{CM}$$



where:

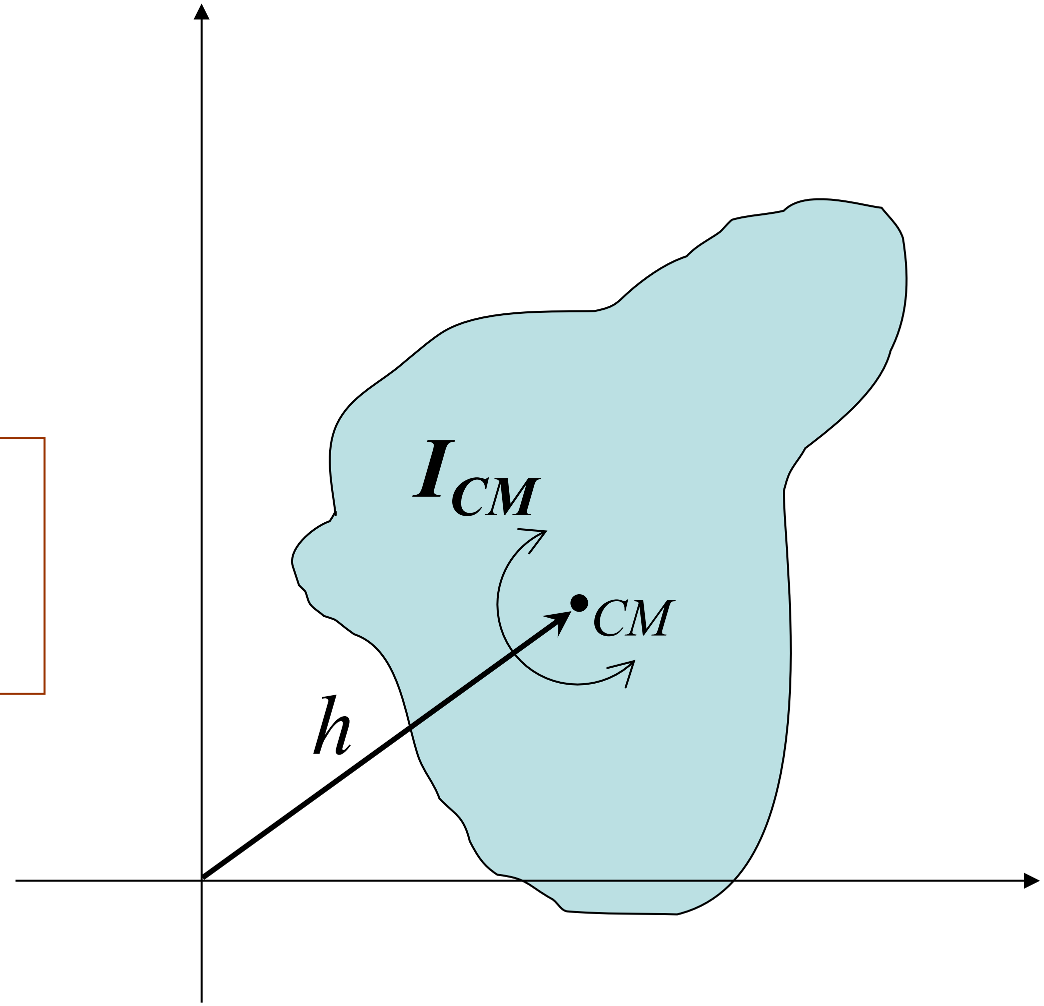
m = total mass

h = perpendicular distance between two parallel axes

I_{CM} = moment of inertia about a parallel axis passing through the center of mass

Moment of Inertia – Parallel Axis Theorem

$$I = mh^2 + \boldsymbol{I_{CM}}$$



where:

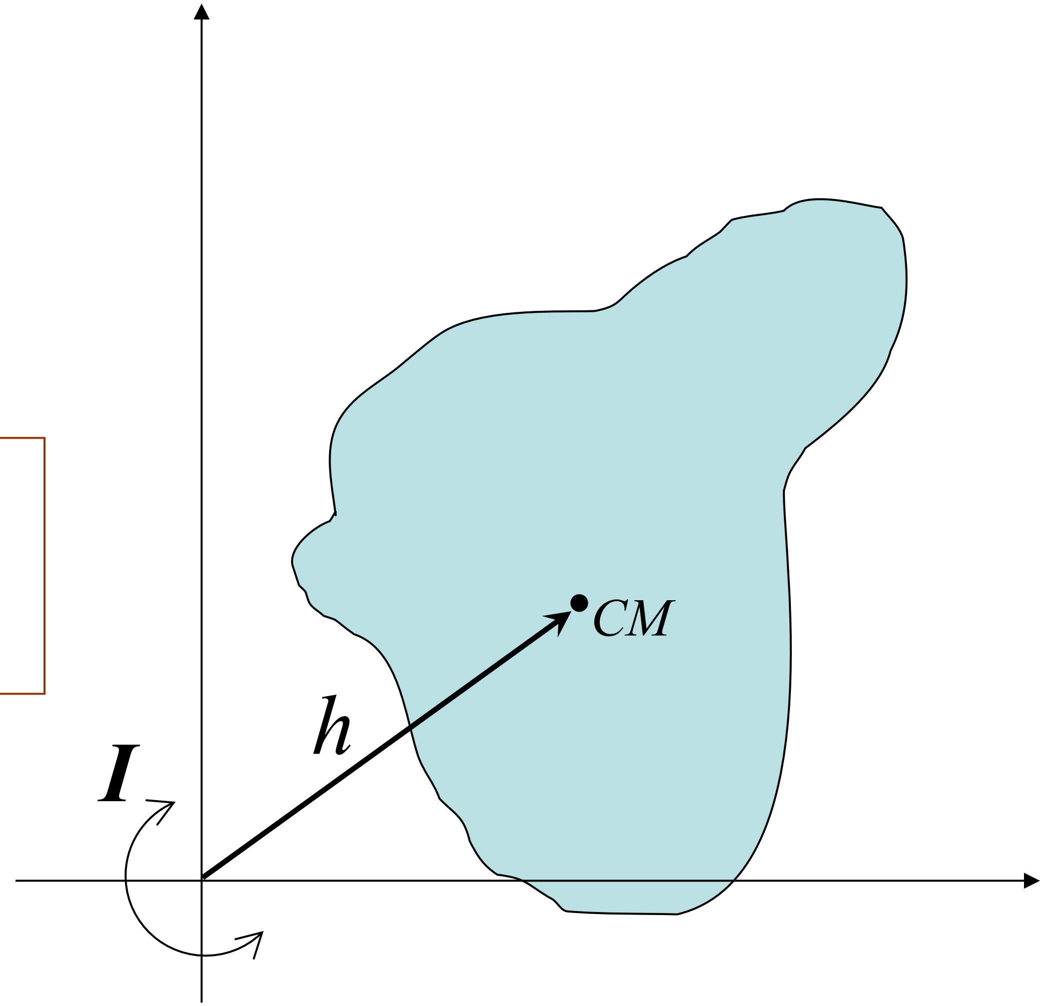
m = total mass

h = perpendicular distance

I_{CM} = moment of inertia about a parallel axis passing through the center of mass

Moment of Inertia – Parallel Axis Theorem

$$\boldsymbol{I} = mh^2 + I_{CM}$$

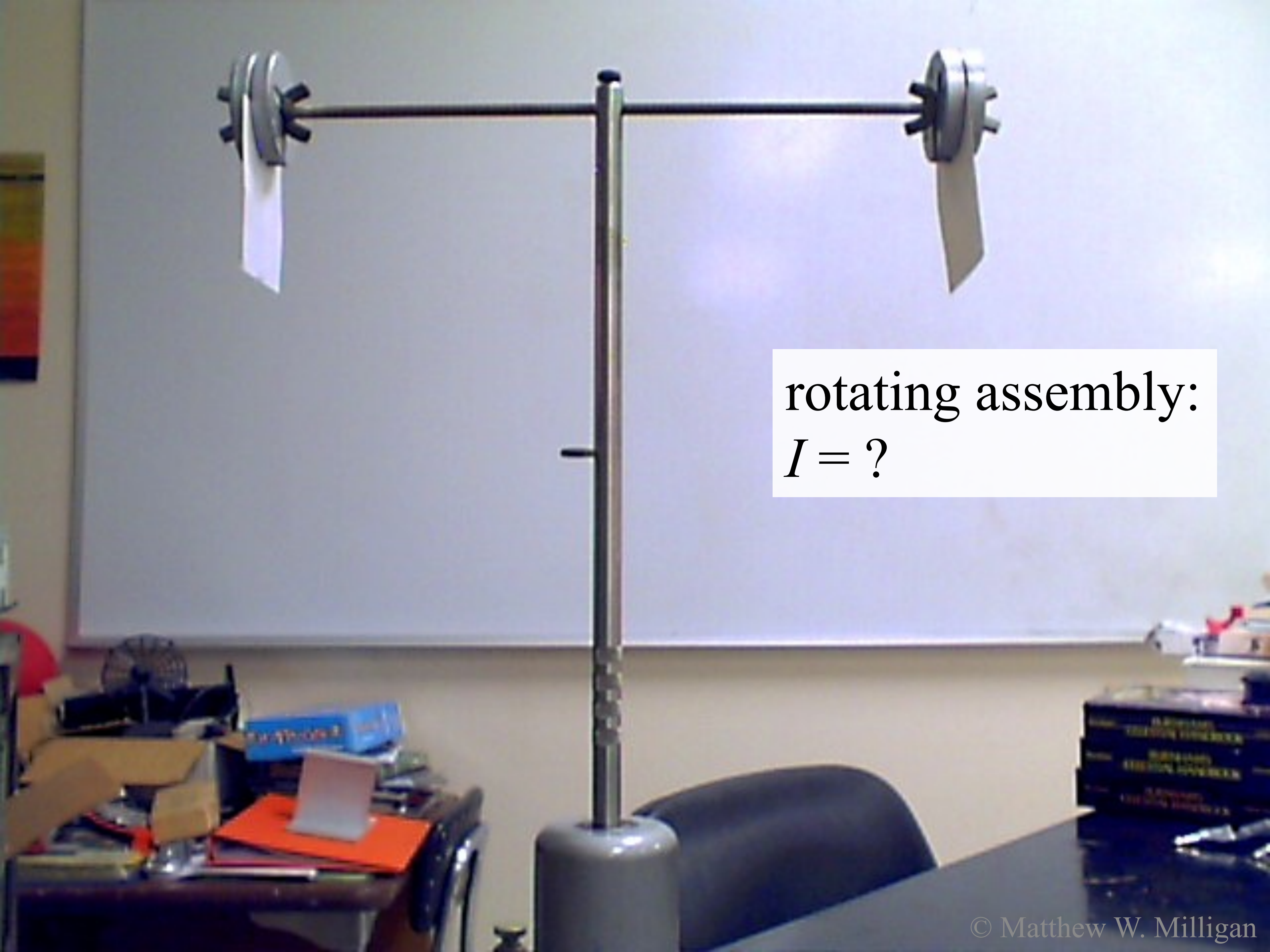


where:

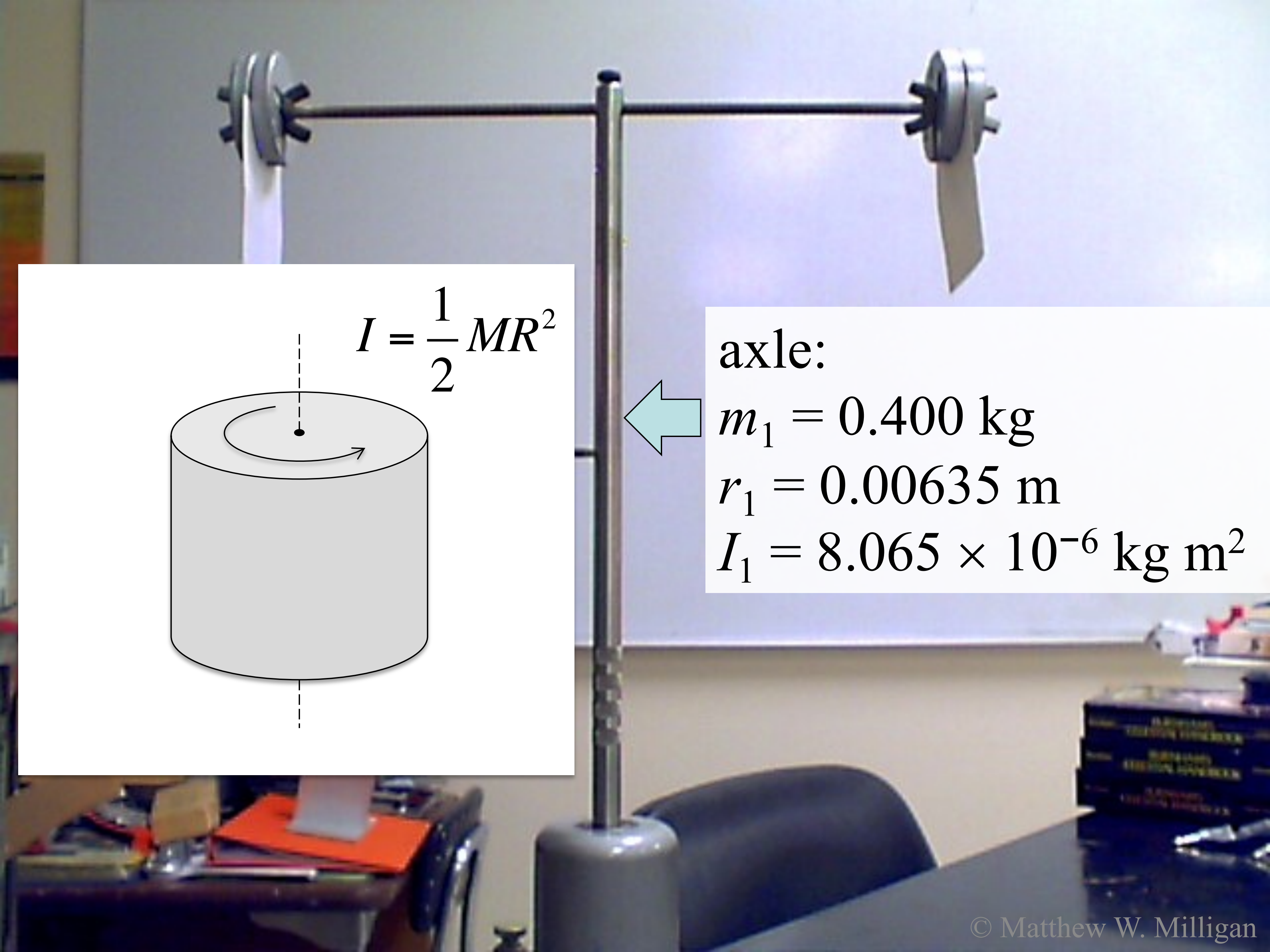
m = total mass

h = perpendicular distance between axes

I_{CM} = moment of inertia about a parallel axis passing through the center of mass



rotating assembly:
$I = ?$



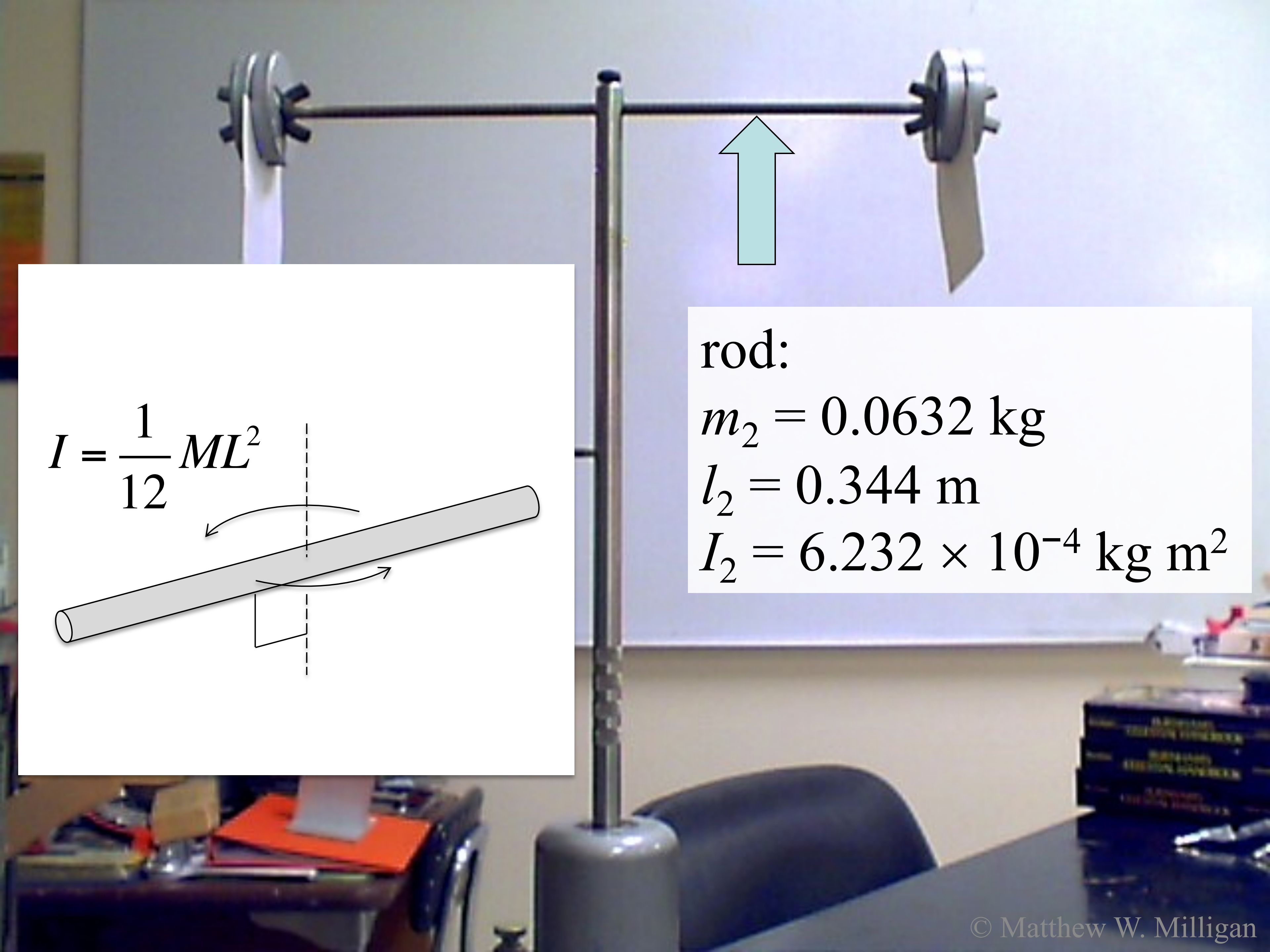
$$I = \frac{1}{2}MR^2$$

axle:

$m_1 = 0.400$ kg

$r_1 = 0.00635$ m

$I_1 = 8.065 \times 10^{-6}$ kg m^2



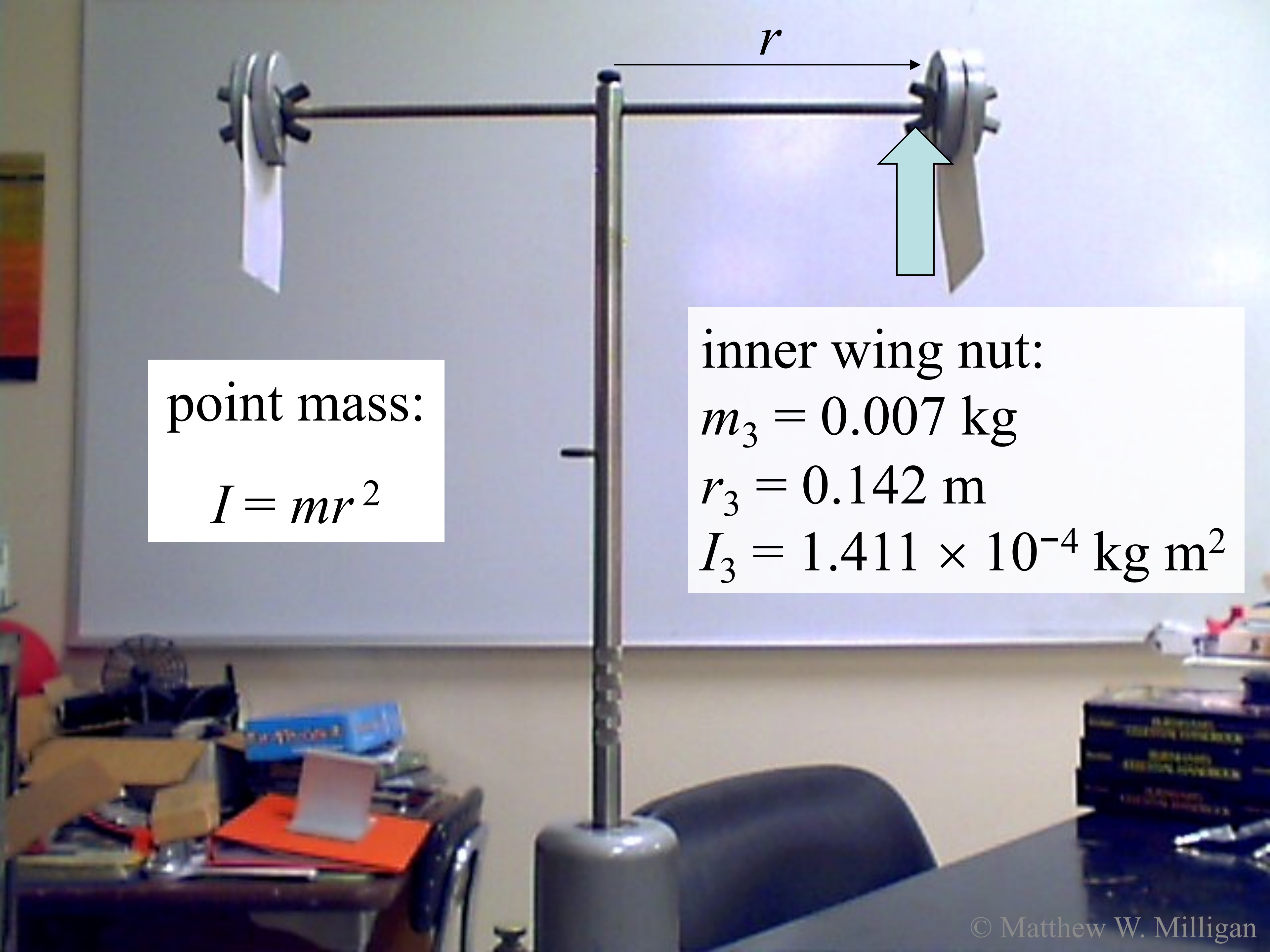
$$I = \frac{1}{12}ML^2$$

rod:

$m_2 = 0.0632\ \text{kg}$

$l_2 = 0.344\ \text{m}$

$I_2 = 6.232 \times 10^{-4}\ \text{kg m}^2$



r
point mass:
I = mr 2
inner wing nut:
m3 = 0.007 kg
r3 = 0.142 m
I3 = 1.411 × 10−4 kg m2
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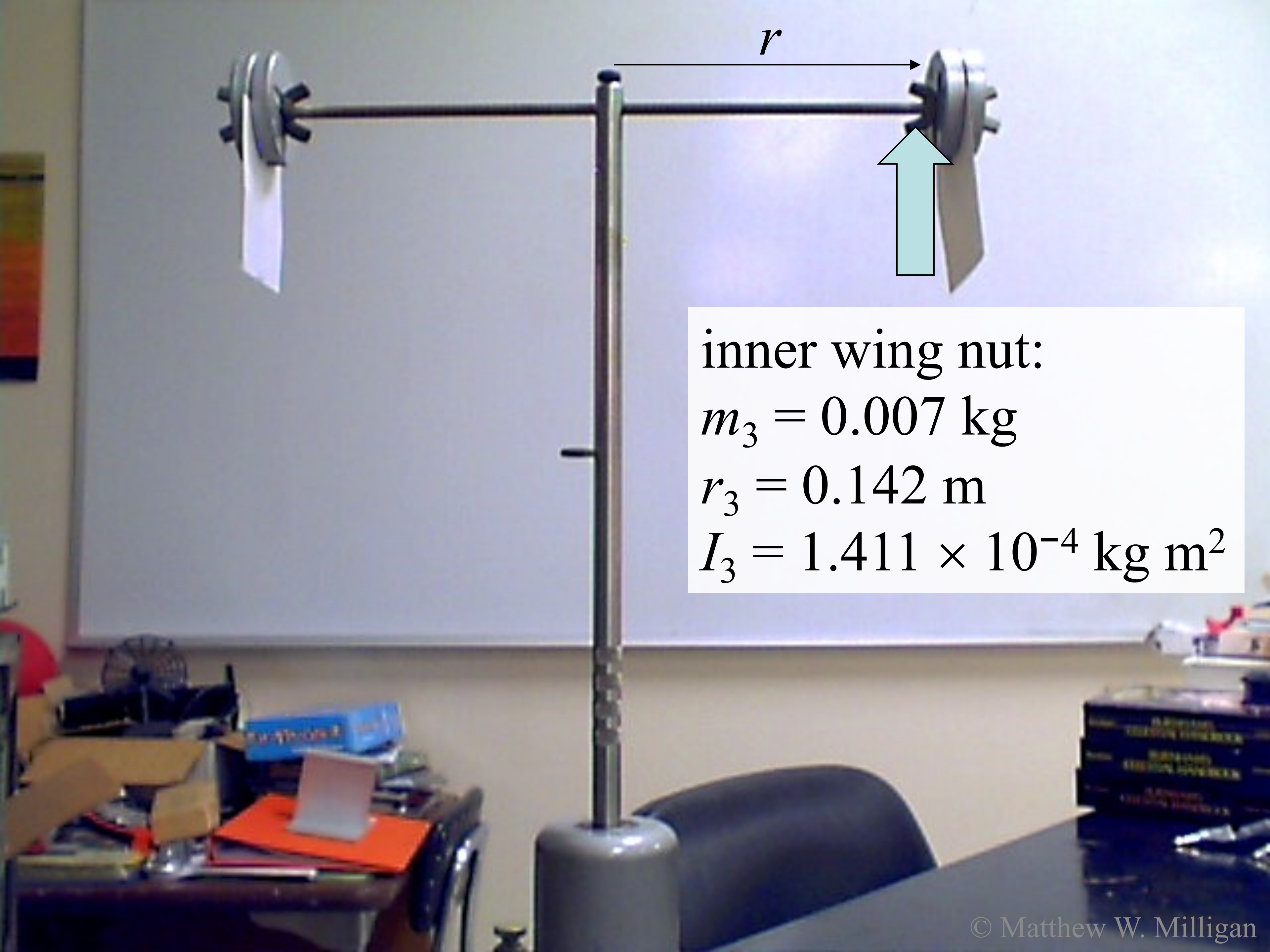
r
inner wing nut:
$m_3 = 0.007$ kg
$r_3 = 0.142$ m
$I_3 = 1.411 \times 10^{-4}$ kg m^2
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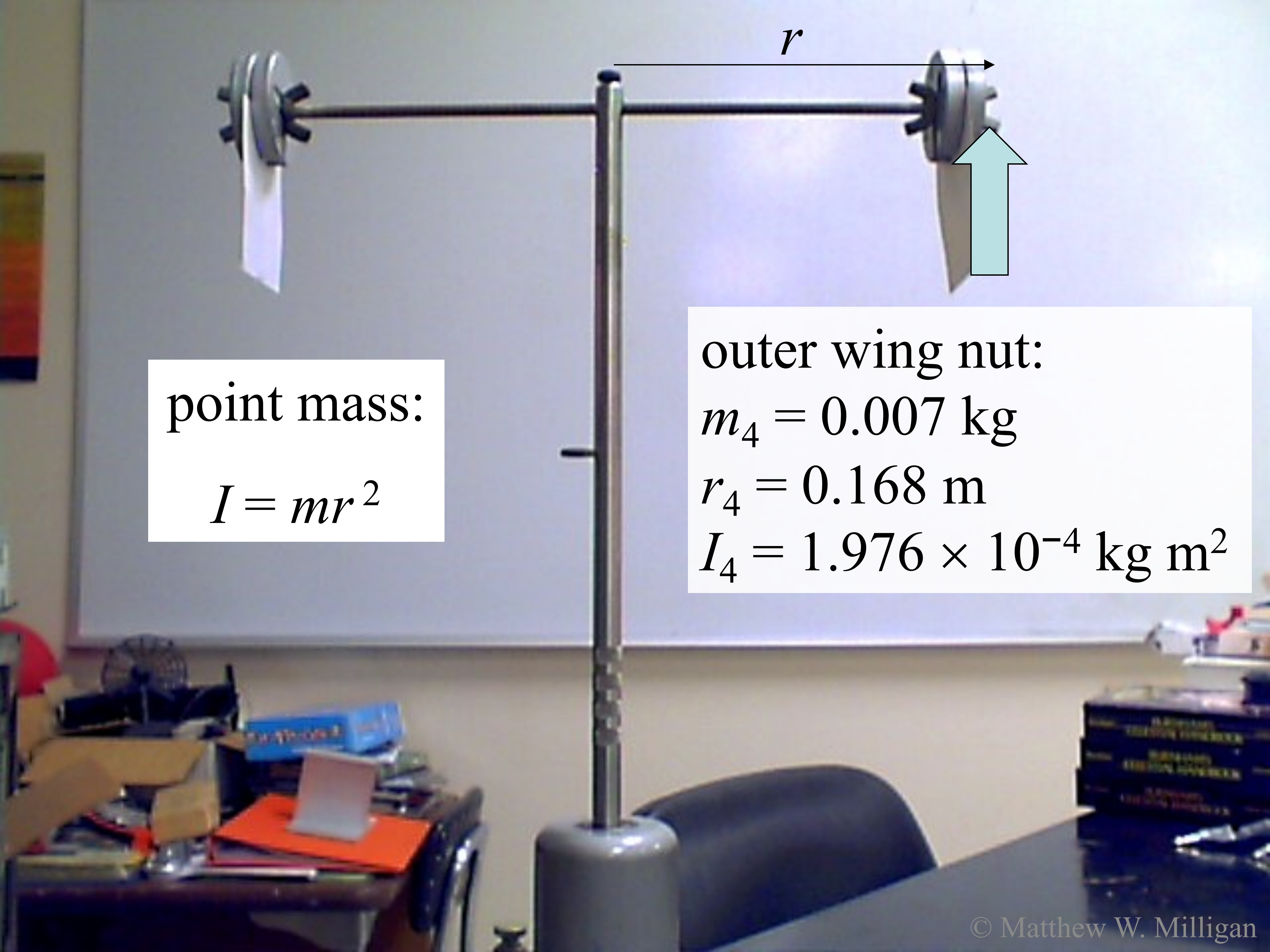
r
point mass:
$I = mr^2$
outer wing nut:
$m_4 = 0.007$ kg
$r_4 = 0.168$ m
$I_4 = 1.976 \times 10^{-4}$ kg m^2
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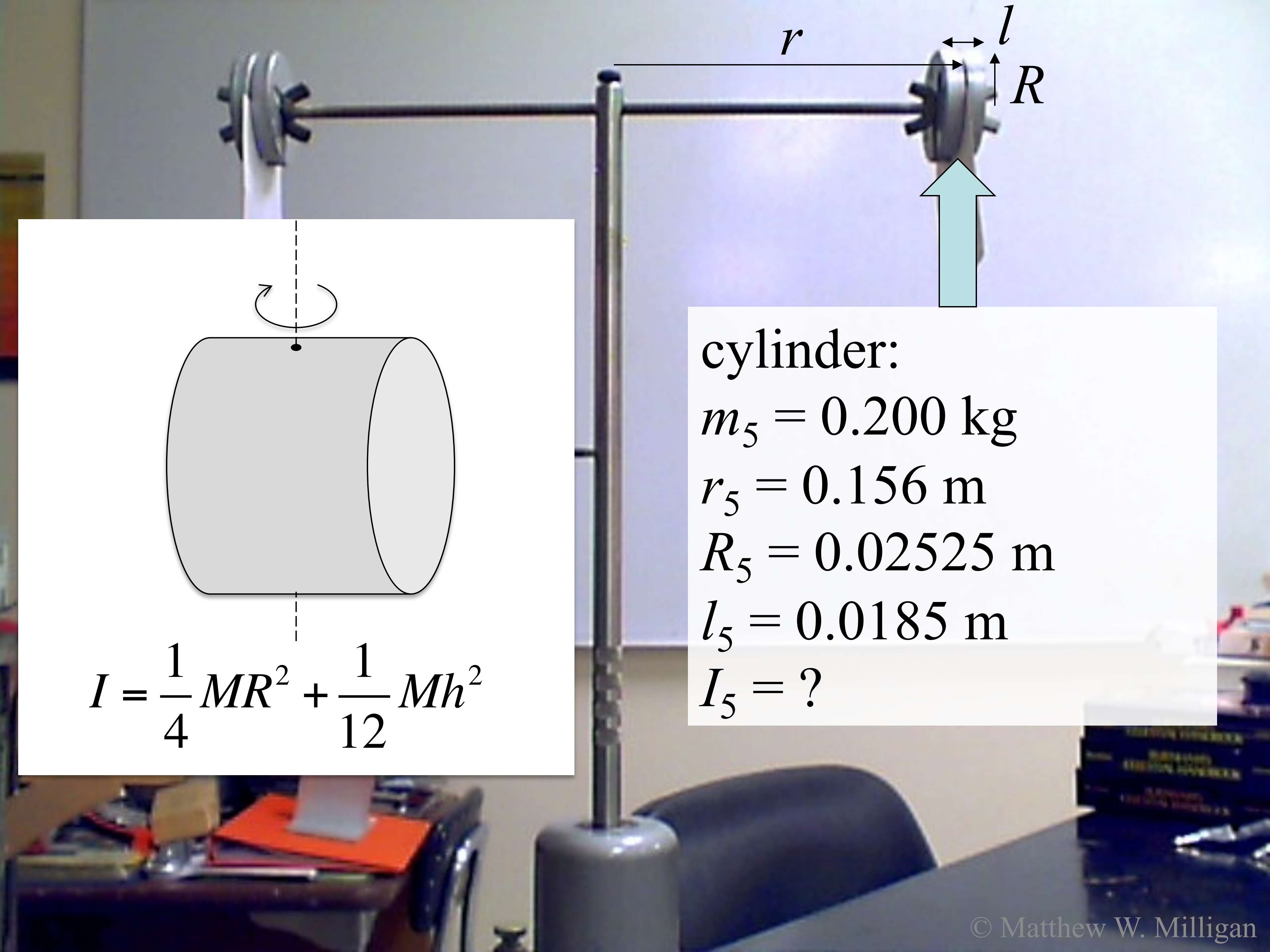


$$I = \frac{1}{4}MR^2 + \frac{1}{12}Mh^2$$

cylinder:
$m_5 = 0.200$ kg
$r_5 = 0.156$ m
$R_5 = 0.02525$ m
$l_5 = 0.0185$ m
$I_5 = ?$

Parallel Axis Theorem:

$$I = Mh^2 + I_{CM}$$

$$I = m_5 r_5^{\ 2} + \left(\frac{m_5 R_5^{\ 2}}{4} + \frac{m_5 l_5^{\ 2}}{12} \right)$$

cylinder:

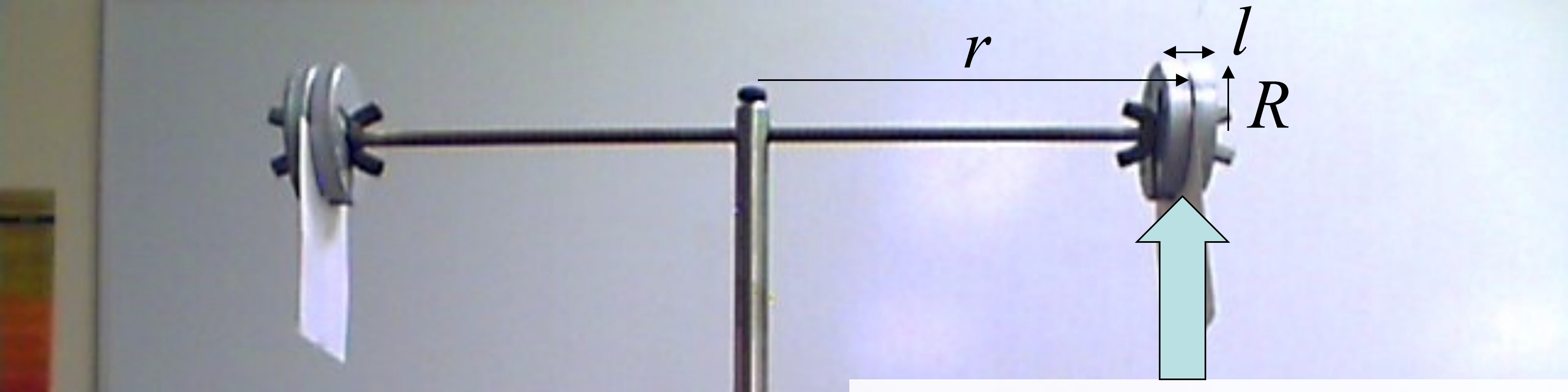
$m_5 = 0.200$ kg

$r_5 = 0.156$ m

$R_5 = 0.02525$ m

$l_5 = 0.0185$ m

$I_5 = ?$



Parallel Axis Theorem:

$$I = Mh^2 + I_{CM}$$

$$I = m_5 r_5^{\ 2} + \left(\frac{m_5 R_5^{\ 2}}{4} + \frac{m_5 l_5^{\ 2}}{12} \right)$$

cylinder:

$m_5 = 0.200$ kg

$r_5 = 0.156$ m

$R_5 = 0.02525$ m

$l_5 = 0.0185$ m

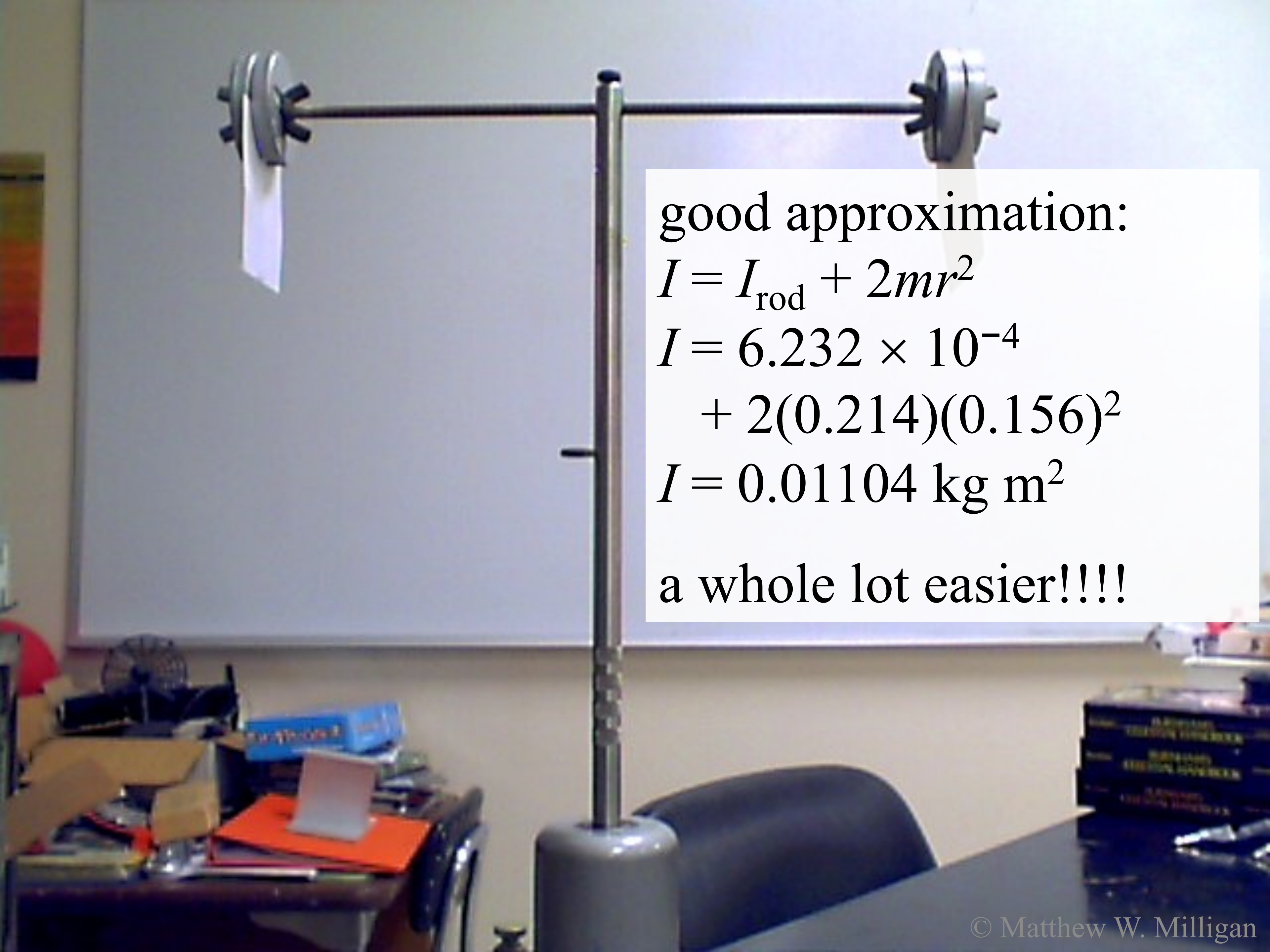
$I_5 = 4.9048 \times 10^{-3}$ kg m^2

rotating assembly:

$$I = I_1 + I_2 + 2I_3 + 2I_4 + 2I_5$$

$$\begin{aligned} I = {} & 8.065 \times 10^{-6} \\ & + 6.232 \times 10^{-4} \\ & + 2(1.411 \times 10^{-4}) \\ & + 2(1.976 \times 10^{-4}) \\ & + 2(4.9048 \times 10^{-3}) \end{aligned}$$

$$I = 0.01112 \text{ kg m}^2$$



good approximation:

$I = I_{\text{rod}} + 2mr^2$

$I = 6.232 \times 10^{-4}$

$\quad + 2(0.214)(0.156)^2$

$I = 0.01104 \text{ kg m}^2$

a whole lot easier!!!!