Rotation

I. Kinematics
 - Angular analogs

II. Dynamics
 - Torque and Moment of Inertia
 - Fixed-axis
 - Rolling, slipping

III. Work and Energy
 - Fixed-axis, rolling

IV. Angular Momentum
 - Bodies and particles

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Translation		Rotation	
position	$\vec{r}$	angular position	$\vec{\theta}$
velocity	$\vec{v} = \dfrac{d\vec{r}}{dt}$	angular velocity	$\vec{\omega} = \dfrac{d\vec{\theta}}{dt}$
acceleration	$\vec{a} = \dfrac{d\vec{v}}{dt}$	angular acceleration	$\vec{\alpha} = \dfrac{d\vec{\omega}}{dt}$

Translation and rotation are two types of motion that can be seen to have a series of analogous concepts and quantities. Inspect the following equations and note the similarities…

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Translation		Rotation	
position	$\vec{r}$	angular position	$\vec{\theta}$
velocity	$\vec{v} = \frac{d\vec{r}}{dt}$	angular velocity	$\vec{\omega} = \frac{d\vec{\theta}}{dt}$
acceleration	$\vec{a} = \frac{d\vec{v}}{dt}$	angular acceleration	$\vec{\alpha} = \frac{d\vec{\omega}}{dt}$

constant acceleration:

$$\begin{cases} x = x_0 + v_0 t + \tfrac{1}{2} a t^2 \\ v = v_0 + at \\ v^2 = v_0^2 + 2a(x - x_0) \end{cases}$$

constant angular acceleration:

$$\begin{cases} \theta = \theta_0 + \omega_0 t + \tfrac{1}{2} \alpha t^2 \\ \omega = \omega_0 + \alpha t \\ \omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0) \end{cases}$$

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Translation	Rotation
$\Sigma\vec{F} = m\vec{a}$	$\Sigma\vec{\tau} = I\vec{\alpha}$
$W = \int \vec{F} \cdot d\vec{r}$	$W = \int \vec{\tau} \cdot d\vec{\theta}$
$K = \frac{1}{2}mv^2$	$K = \frac{1}{2}I\omega^2$
$\vec{p} = m\vec{v}$	$\vec{L} = I\vec{\omega}$
$\vec{J} = \int \vec{F}\,dt$	$\vec{A} = \int \vec{\tau}\,dt$

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Translation	Rotation
mass	rotational inertia (moment of inertia)
m	$I = \Sigma r_i^2 m_i = \int r^2 dm$ $I = I_{CM} + Mh^2$
force	torque
$\vec{F}$	$\vec{\tau} = \vec{r} \times \vec{F} = r_\perp F = rF_\perp$

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Translation | Rotation

"Linking" Equations:

$$s = r\theta$$

$$v = r\omega$$

$$a_\theta = r\alpha$$

$$a_r = r\omega^2$$

$$\vec{l} = \vec{r} \times \vec{p} = r_\perp p = r p_\perp$$

This set of equations "connects" some of the quantities from the two types of motion translation and rotation.

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	The student will be able to:	HW:
1	State and apply the relations between angular position, angular displacement, angular speed, angular velocity, and angular acceleration to solve related problems.	1 – 3
2	State and apply the relations between the angular (or rotational) motion of a body or system and the linear (or translational) motion of a point on the body or system.	4 – 7
3	Determine the torque of an applied force and solve related problems.	8 – 12
4	Determine the moment of inertia for a system of masses or sold body and solve related problems.	13 – 18
5	State and apply Newton's 2^{nd} Law for fixed-axis rotation to solve related problems.	19 – 21
6	Apply work and energy to solve fixed-axis rotation problems.	22 – 25
7	State and apply Newton's 2^{nd} Law for rolling (rotation and translation) to solve related problems (including those with slipping and without slipping)	26 – 33
8	Apply work and energy to solve rolling problems.	34 – 36
9	Determine angular momentum for a particle, system, or rotating body and relate to torque and angular impulse to solve problems.	37 – 42
10	Apply conservation of angular momentum to solve related problems.	43 – 49

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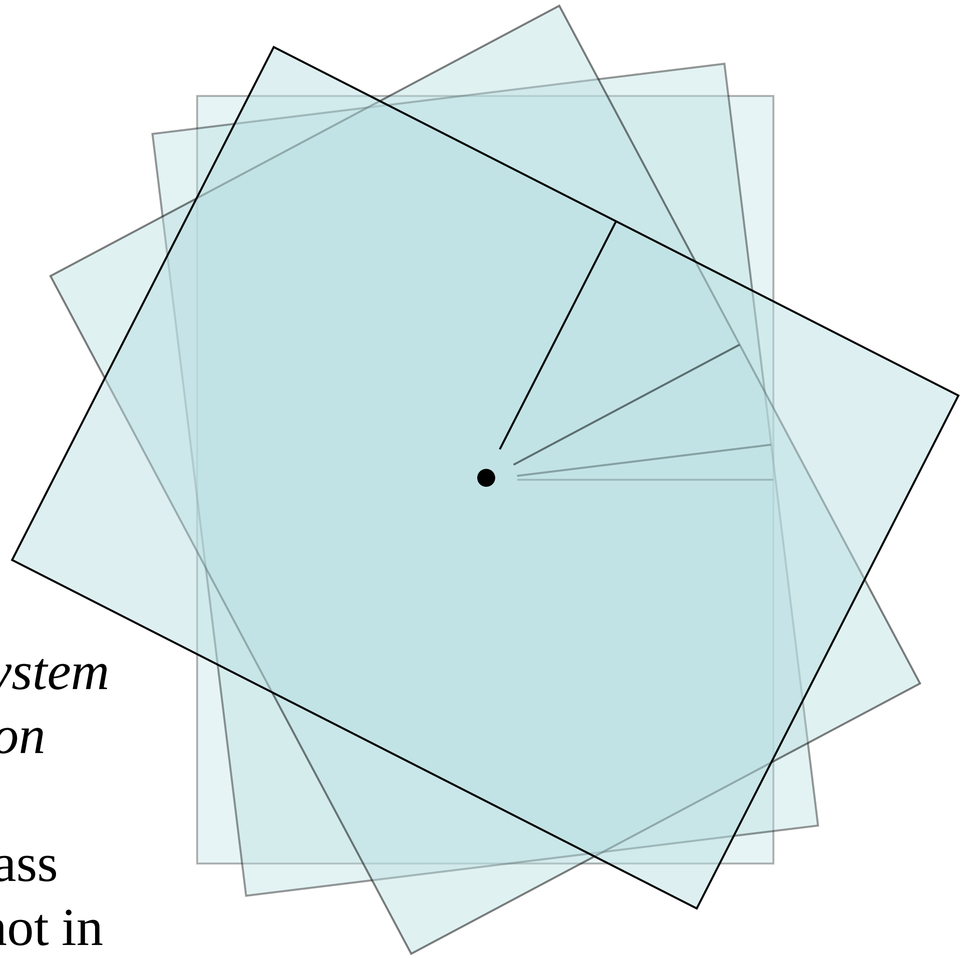
How much does this rotating object move? How fast is it moving? Is it accelerating?



Taking the object as a *system* its *speed* and *acceleration* are both *zero*! This is because the center of mass of the object/system is not in motion and therefore one could say that the object's position is not changing.

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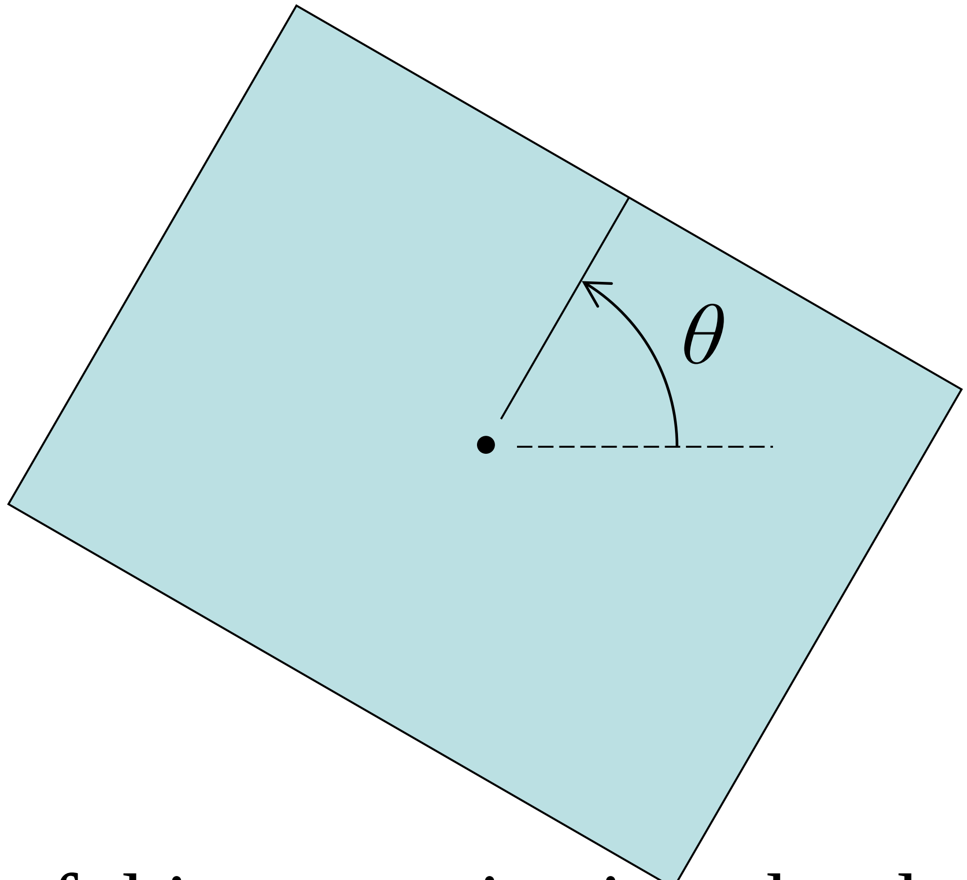
Angular position is an indicator of the orientation of an object relative to a reference. Symbol: θ



The direction of this vector is given by the right hand rule. Often it is simply described as clockwise (–) or counterclockwise (+).

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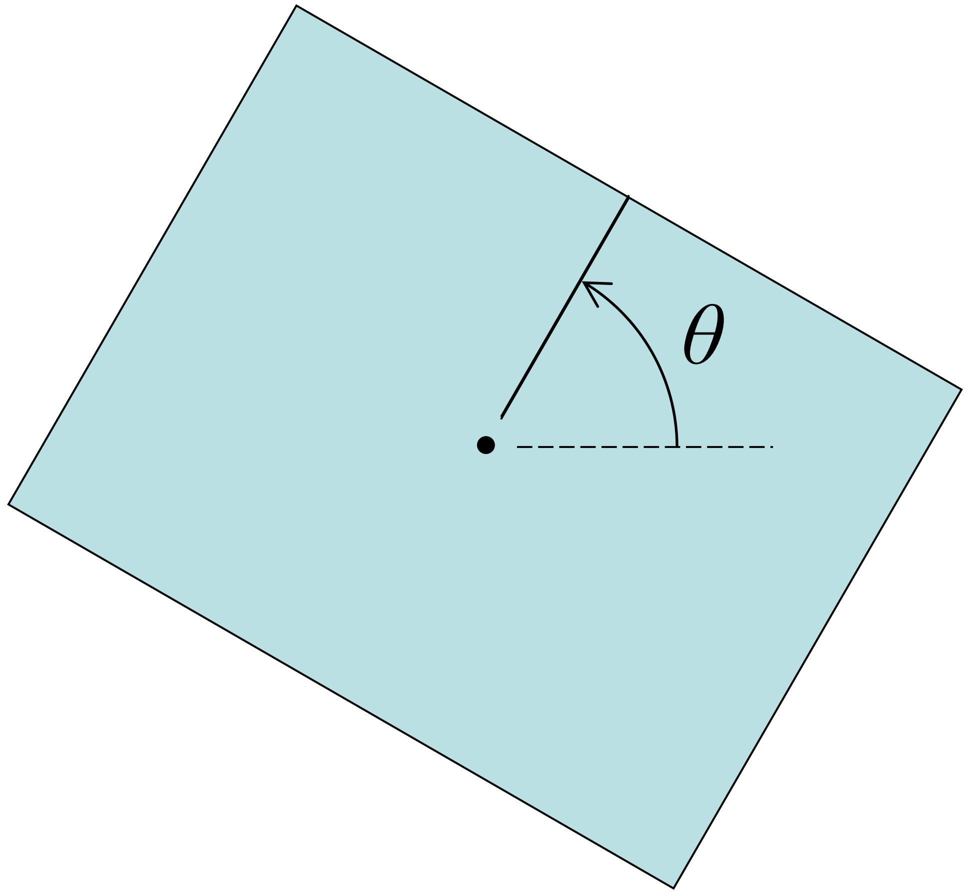
Angular displacement is the net change in angular position.



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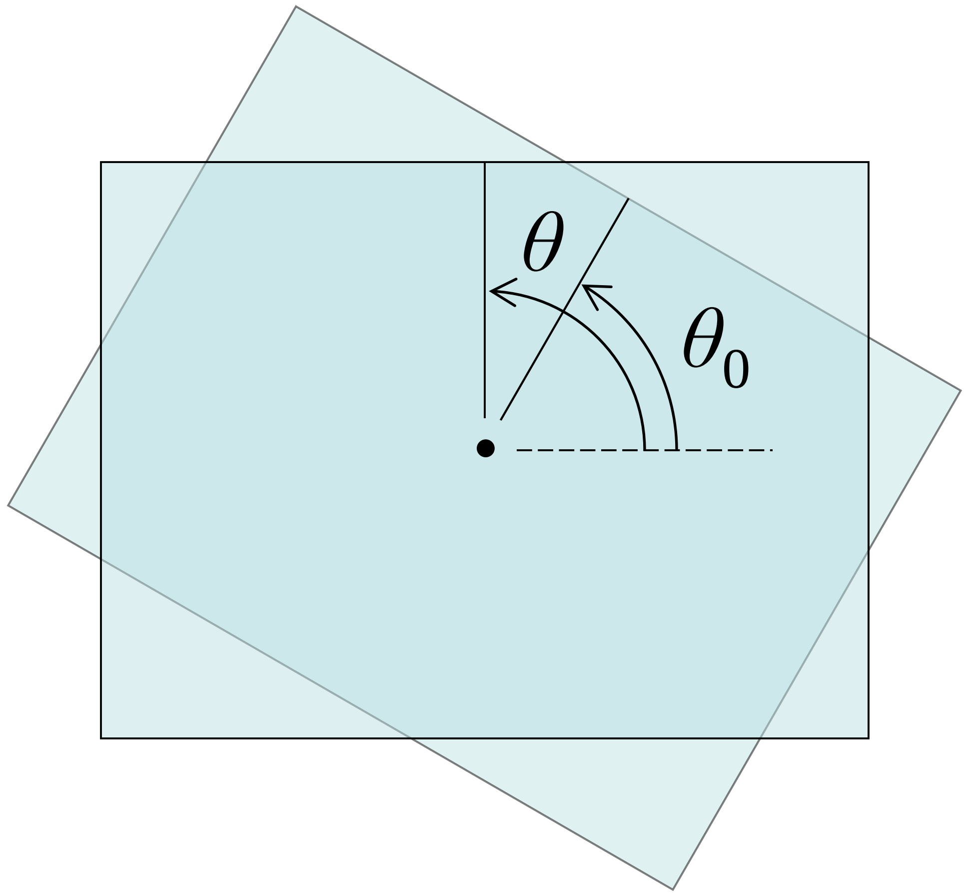
Angular displacement is the net change in angular position. $\Delta\theta = \theta - \theta_0$



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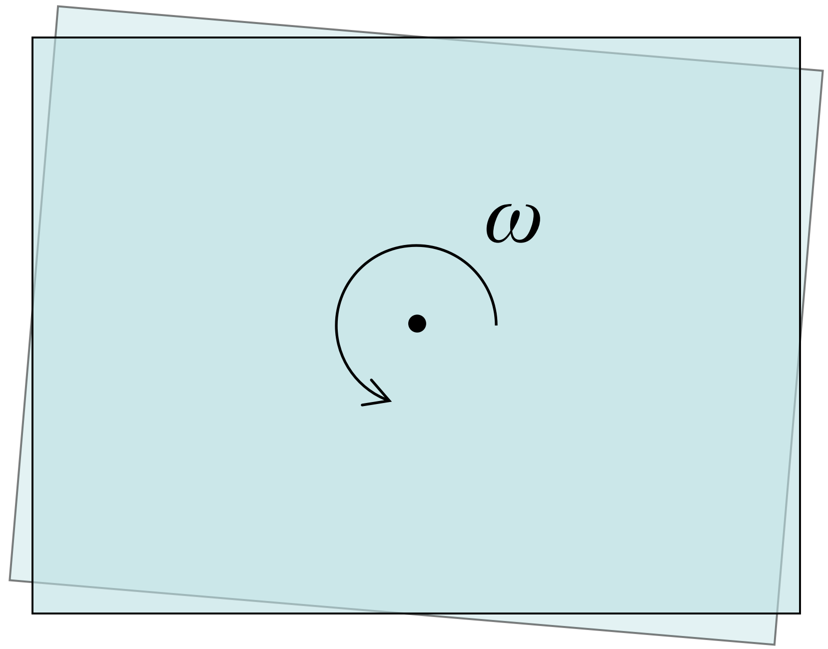
Angular velocity is the rate of change in angular position. Symbol: ω



Angular speed is the magnitude of angular velocity. Symbol: ω

Angular speed describes how rapidly an object is spinning or rotating. The greater the value the more rapid the rate of change in the angular orientation.

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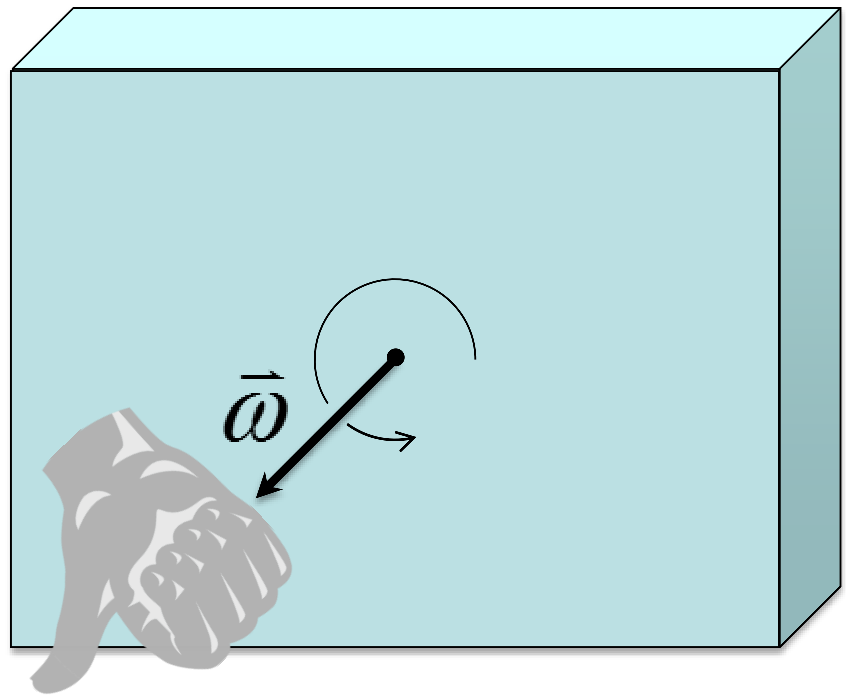
Angular velocity is the rate of change in angular position. Symbol: $\vec{\omega}$

Angular speed is the magnitude of angular velocity. Symbol: ω



Shown here is a 3-D perspective. The angular velocity *vector* is defined as an arrow pointing along or parallel to the axis of rotation. The direction is given by the "right hand rule": curl the fingers of the right hand in the direction of the rotation and the thumb points in the direction of the vector.

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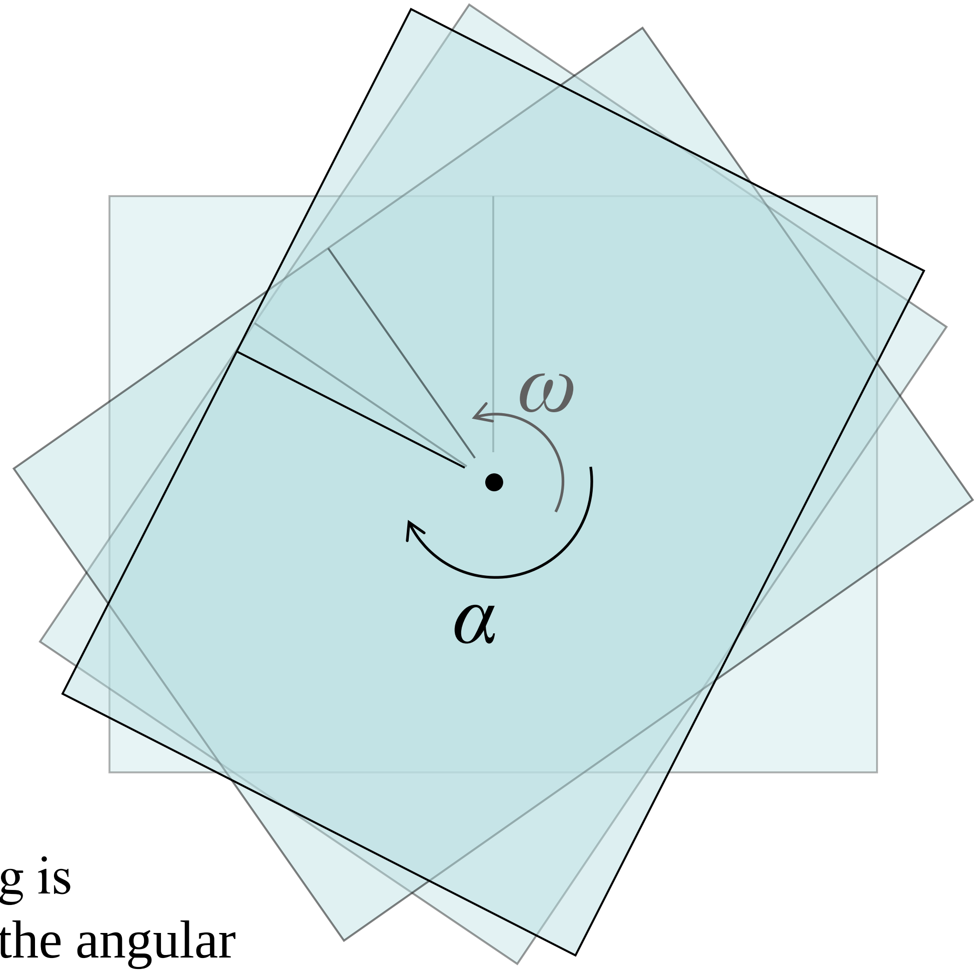
Angular acceleration is the rate of change in angular velocity. Symbol: α



In this example the object's rate of spinning is decreasing – therefore the angular acceleration is in the opposite direction (because the change in angular speed is negative).

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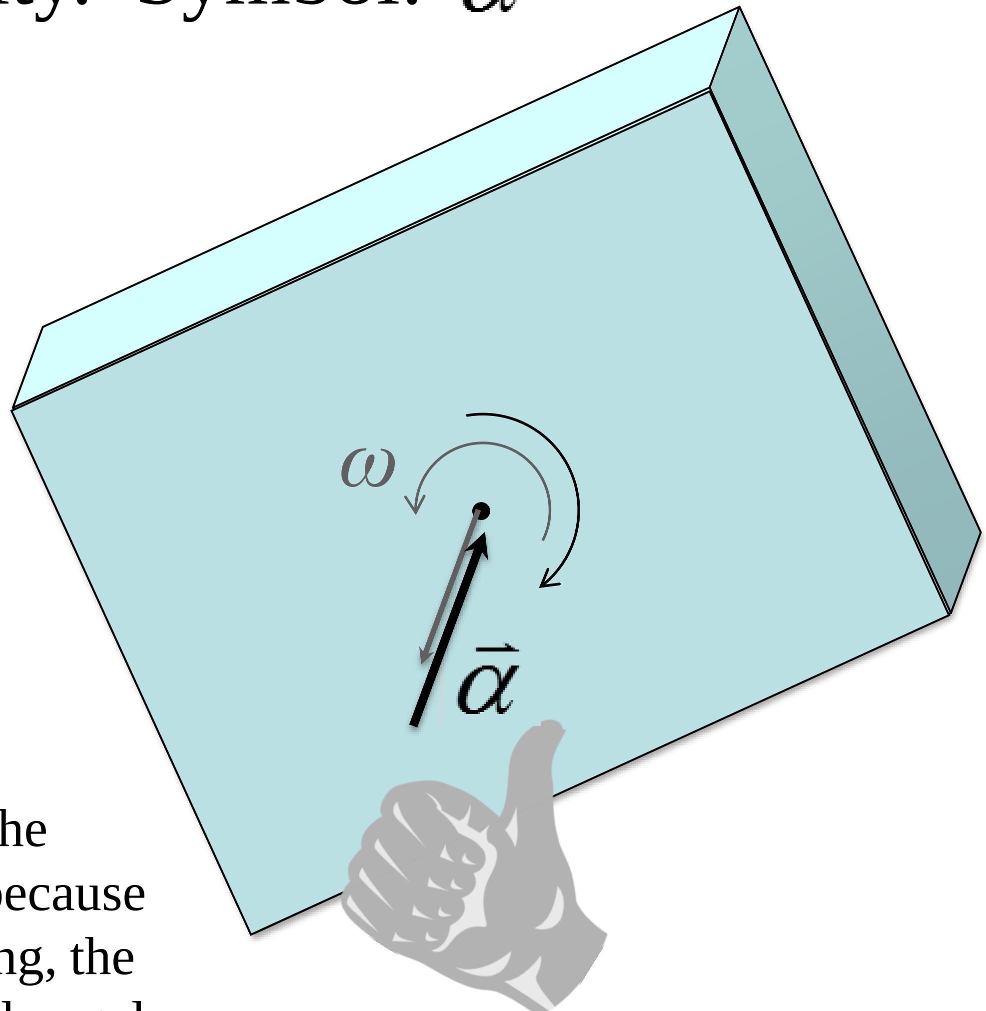
Angular acceleration is the rate of change in angular velocity. Symbol: $\vec{\alpha}$



Another illustration of the right hand rule. Here, because the spin rate is decreasing, the angular acceleration and angular velocity vectors point in opposite directions along the axis of rotation.

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constant acceleration:

$$\begin{cases} x = x_0 + v_0 t + \tfrac{1}{2} a t^2 \\ v = v_0 + a t \\ v^2 = v_0^2 + 2a\left(x - x_0\right) \end{cases}$$

constant angular acceleration:

$$\begin{cases} \theta = \theta_0 + \omega_0 t + \tfrac{1}{2} \alpha t^2 \\ \omega = \omega_0 + \alpha t \\ \omega^2 = \omega_0^2 + 2\alpha\left(\theta - \theta_0\right) \end{cases}$$

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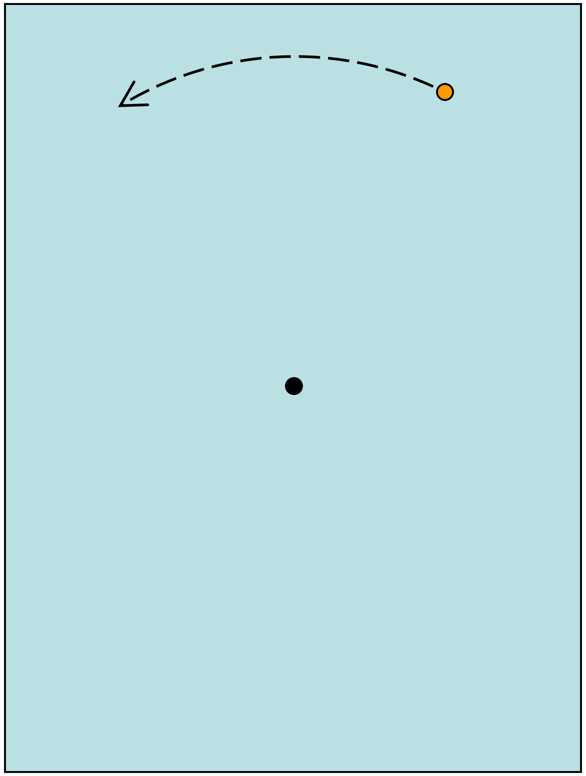
How does the *translation* of an individual particle relate to the *rotation* of the whole?



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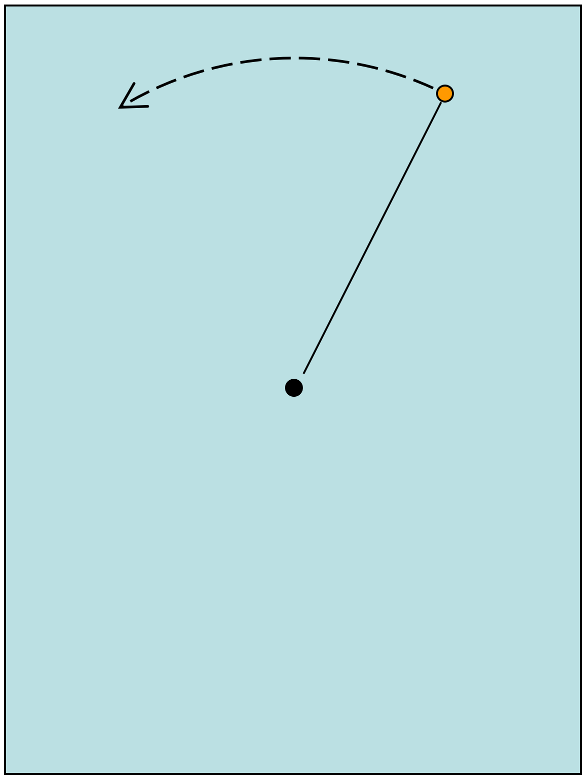

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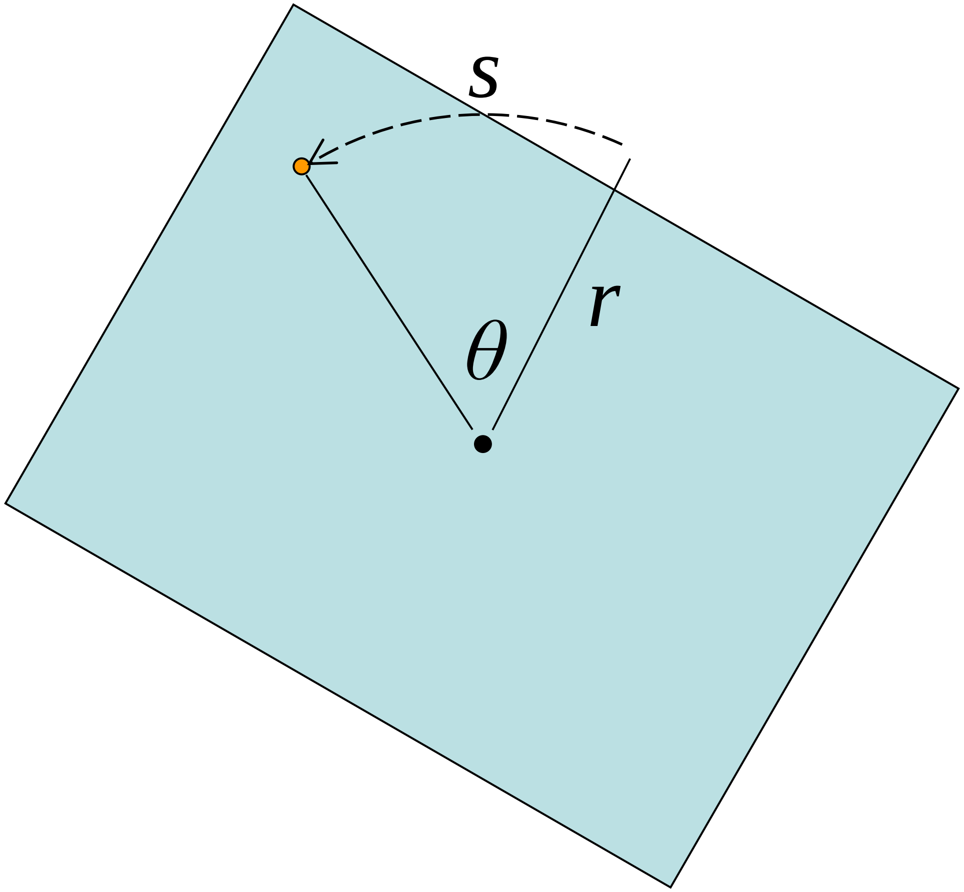
Arc length, *s*, relates to angular displacement:

$$s = r\theta$$



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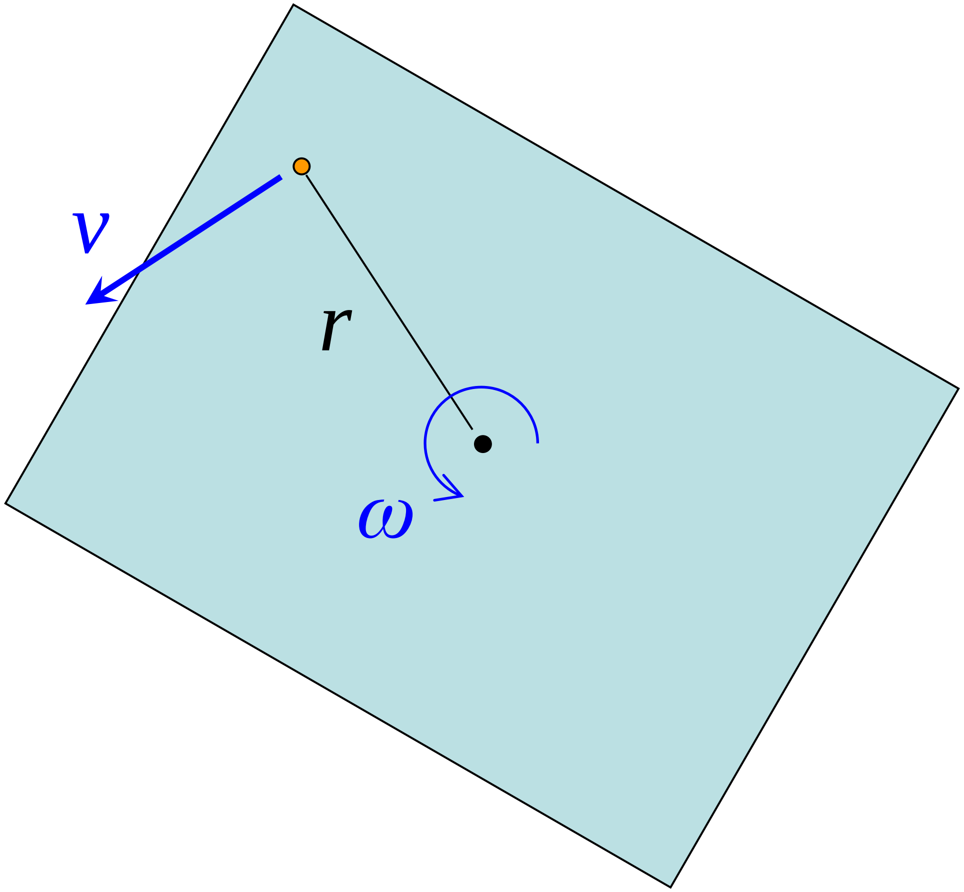
Speed and velocity relate to angular speed and angular velocity:

$$s = r\theta$$

$$v = r\omega$$



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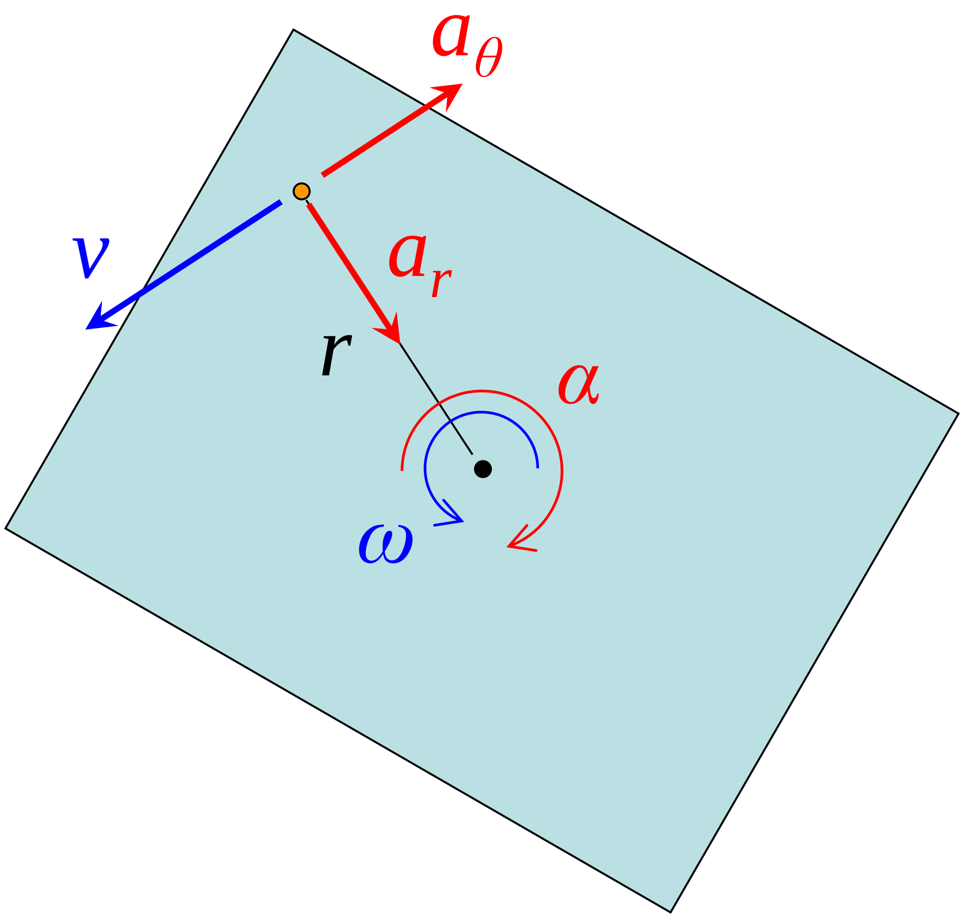
Radial and tangential acceleration relate to angular velocity and angular acceleration:

$$s = r\theta$$

$$v = r\omega$$

$$a_\theta = r\alpha$$

$$a_r = r\omega^2$$



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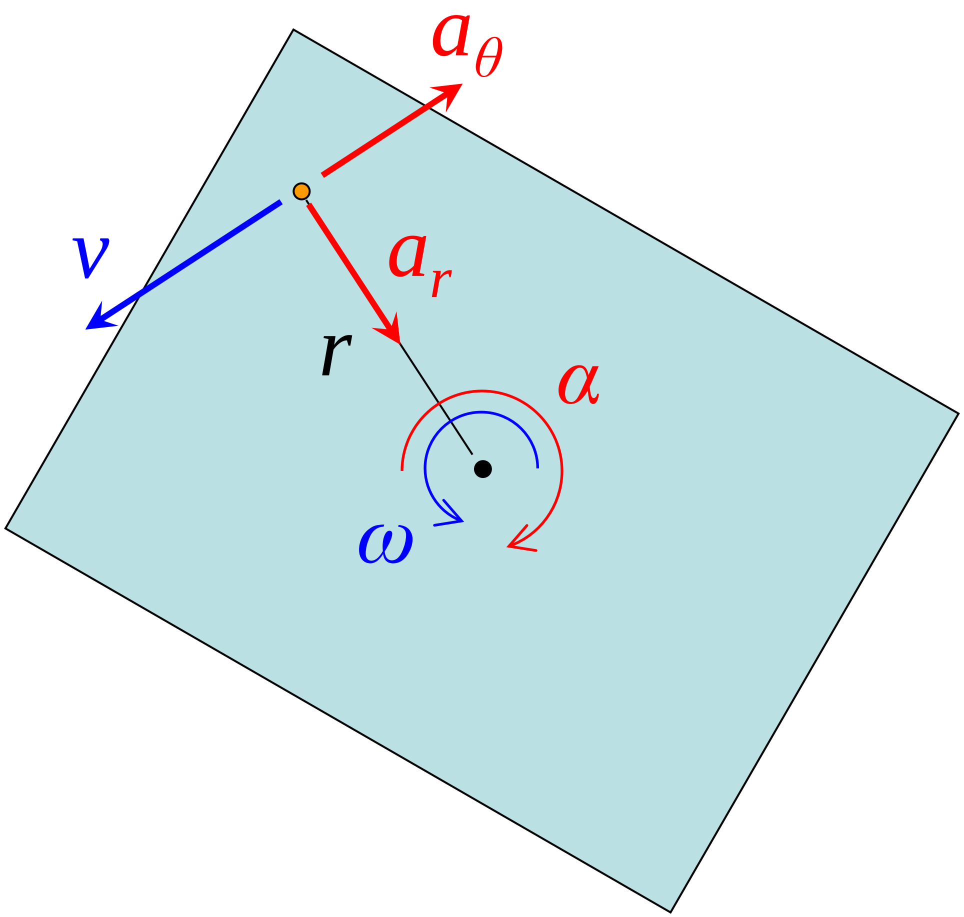
Radial and tangential acceleration relate to angular velocity and angular acceleration:

$$s = r\theta$$

$$v = r\omega$$

$$a_\theta = r\alpha$$

$$a_r = r\omega^2$$



Important note: all of these "linking equations" require the use of **radian** angle measures!

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