Rotation

I. Kinematics
- Angular analogs

II. Dynamics
- Torque and Moment of Inertia
- Fixed-axis
- Rolling, slipping

III. Work and Energy
- Fixed-axis, rolling

IV. Angular Momentum
- Bodies and particles

	The student will be able to:	HW:
1	State and apply the relations between angular position, angular displacement, angular speed, angular velocity, and angular acceleration to solve related problems.	1 – 3
2	State and apply the relations between the angular (or rotational) motion of a body or system and the linear (or translational) motion of a point on the body or system.	4 – 7
3	Determine the torque of an applied force and solve related problems.	8 – 12
4	Determine the moment of inertia for a system of masses or solid body and solve related problems.	13 – 18
5	State and apply Newton's 2nd Law for fixed-axis rotation to solve related problems.	19 – 21
6	Apply work and energy to solve fixed-axis rotation problems.	22 – 25
7	State and apply Newton's 2nd Law for rolling (rotation and translation) to solve related problems (including those with slipping and without slipping)	26 – 33
8	Apply work and energy to solve rolling problems.	34 – 36
9	Determine angular momentum for a particle, system, or rotating body and relate to torque and angular impulse to solve problems.	37 – 42
10	Apply conservation of angular momentum to solve related problems.	43 – 49

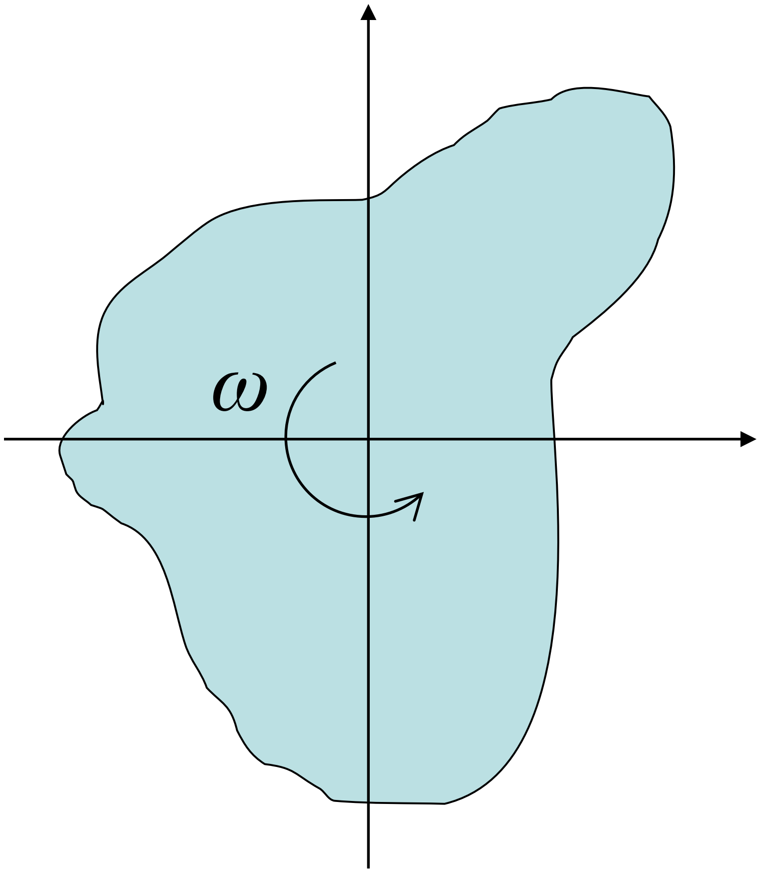
Angular Momentum – Solid Body

$$\vec{L} = I\vec{\omega}$$

where: L = angular momentum

I = moment of inertia

ω = angular velocity



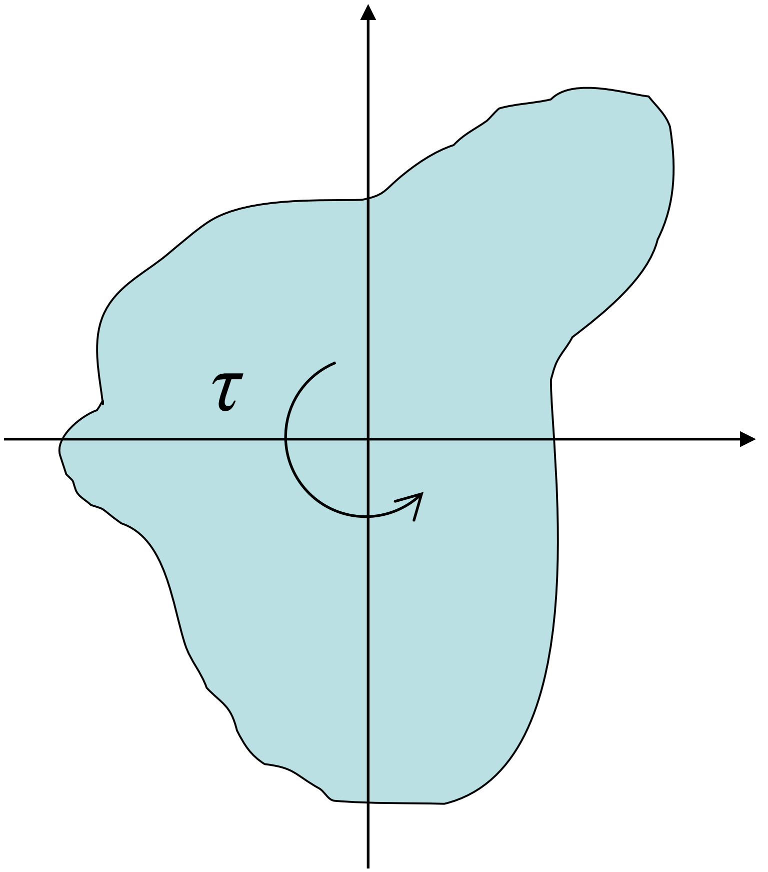
Angular Impulse

$$\vec{A} = \int \vec{\tau}\, dt$$

where: A = angular impulse

τ = torque

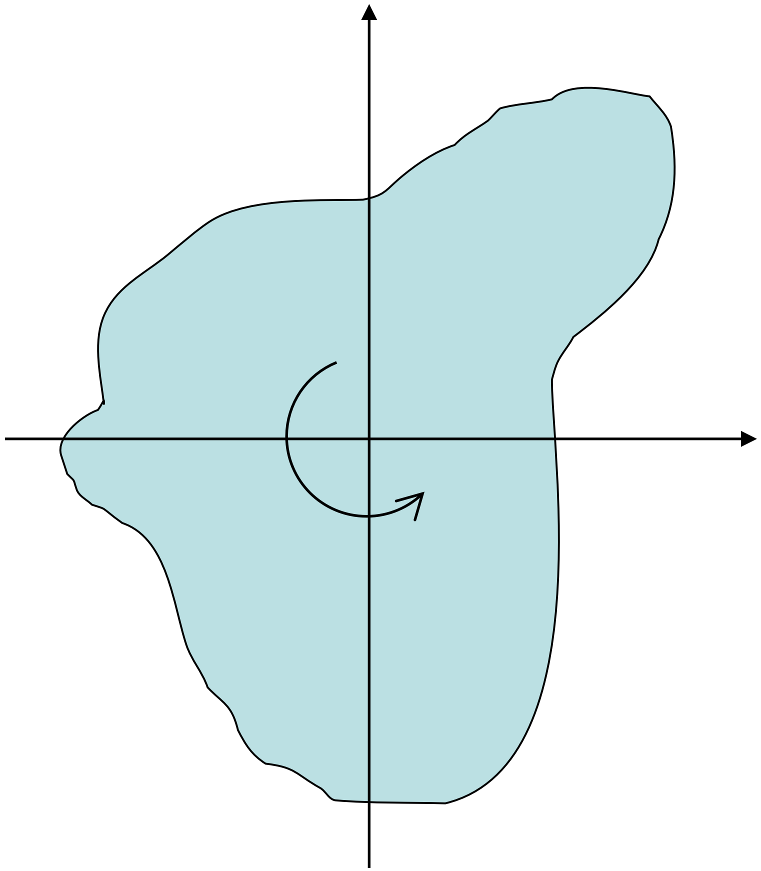
t = time



Angular Impulse vs Angular Momentum

$$\Sigma \vec{A} = \Delta \vec{L}$$

$$\Sigma \vec{\tau} = \frac{d\vec{L}}{dt}$$



Conservation of Angular Momentum

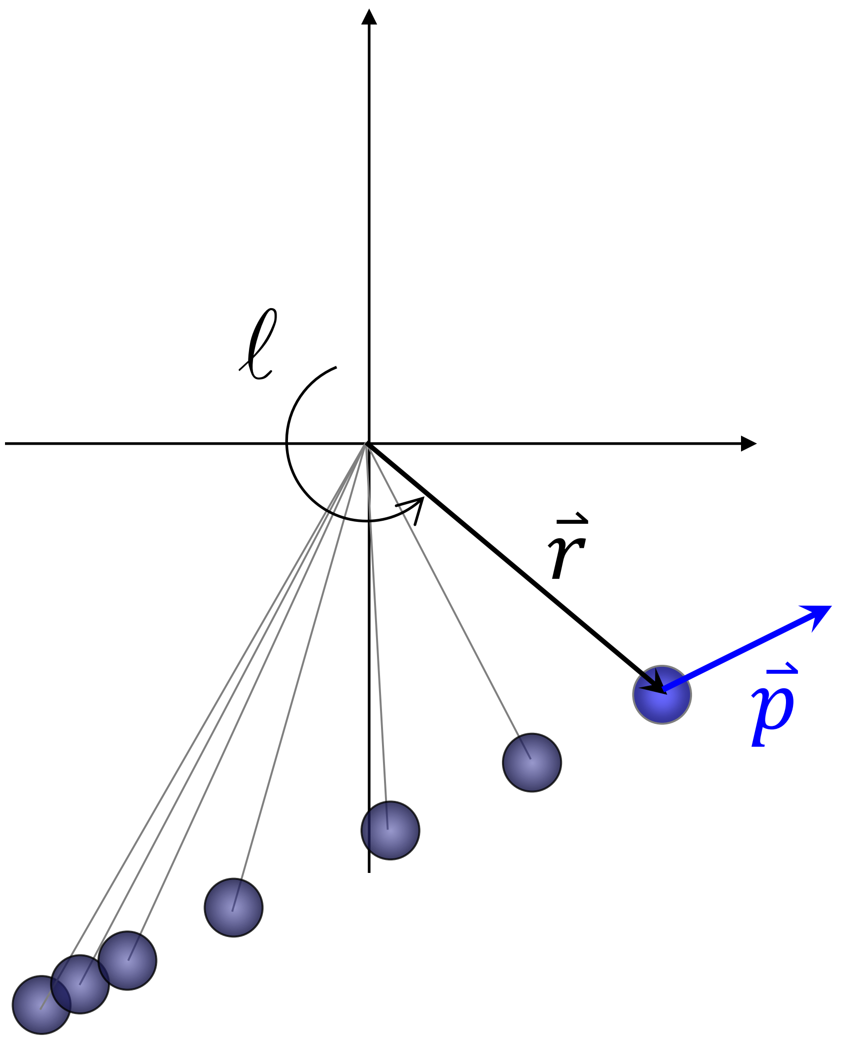
$$\Sigma\vec{L} = \Sigma\vec{L}'$$

- Angular momentum is a conserved quantity.
- The total angular momentum of a system is constant if the net external torque is zero.
- Internal torque have no effect on the total angular momentum of a system, but rather causes the transfer of angular momentum from one object to another.

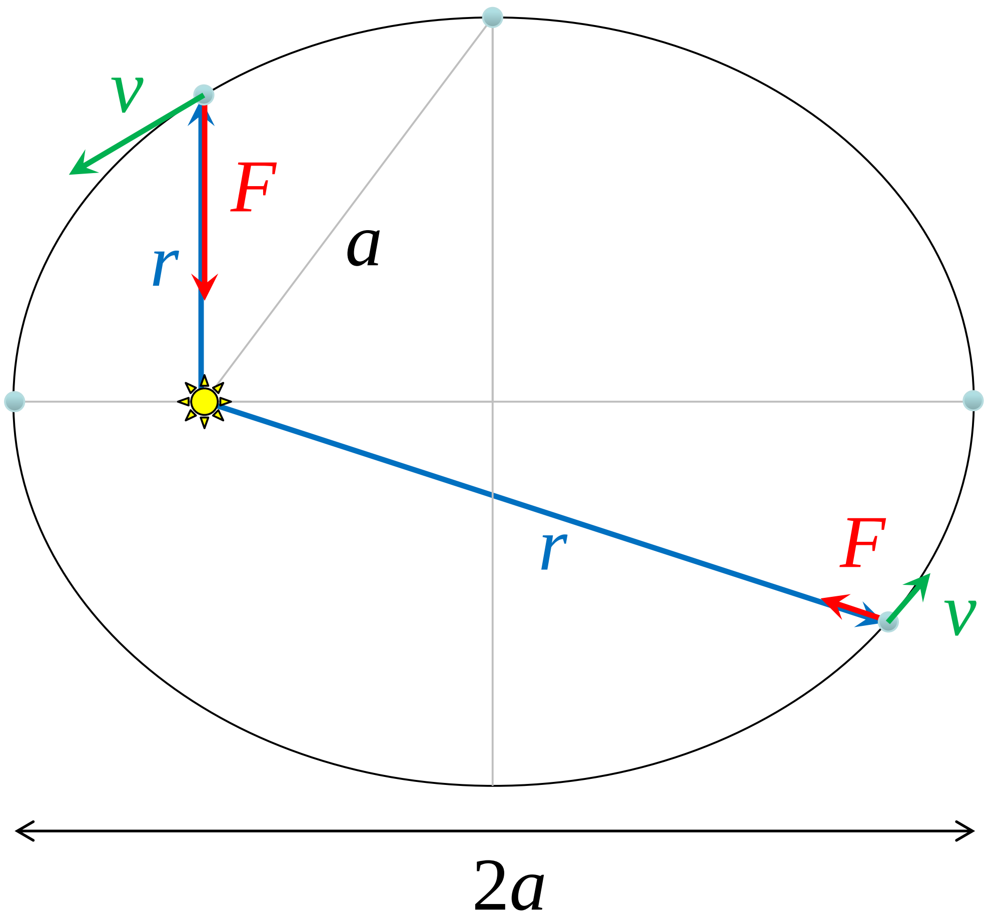
Angular Momentum – Particle

$$\vec{\ell} = \vec{r} \times \vec{p}$$

where: ℓ = angular momentum
r = position relative to axis
p = linear momentum



Special Focus: Elliptical Orbits



The position, *r*, velocity, *v*, and gravitational force, *F*, all vary in magnitude and direction as an object follows an elliptical orbit.

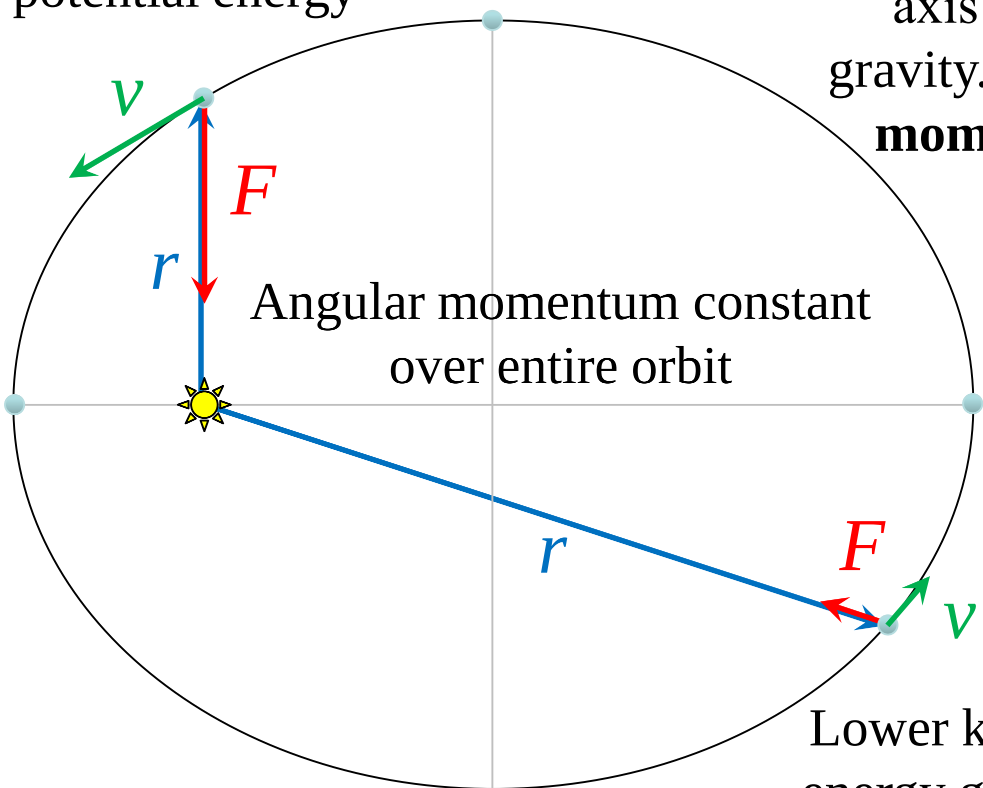
The diagram illustrates the major axis, 2*a*, and the semi-major axis, *a*, of the ellipse. The value of *a* is usually referred to as the average distance between the two objects. (the symbol "*a*" does not represent acceleration in this diagram!)

This type of motion can be analyzed by using conservation of angular momentum and conservation of energy…

Special Focus: Elliptical Orbits

As shown in the figure the force of gravity acting on the satellite is always directed toward the central body. Taking the central body to be the "axis of revolution" there is zero torque due to gravity. Because there is no torque, the **angular momentum of the system remains constant**.

Greater kinetic energy lower potential energy

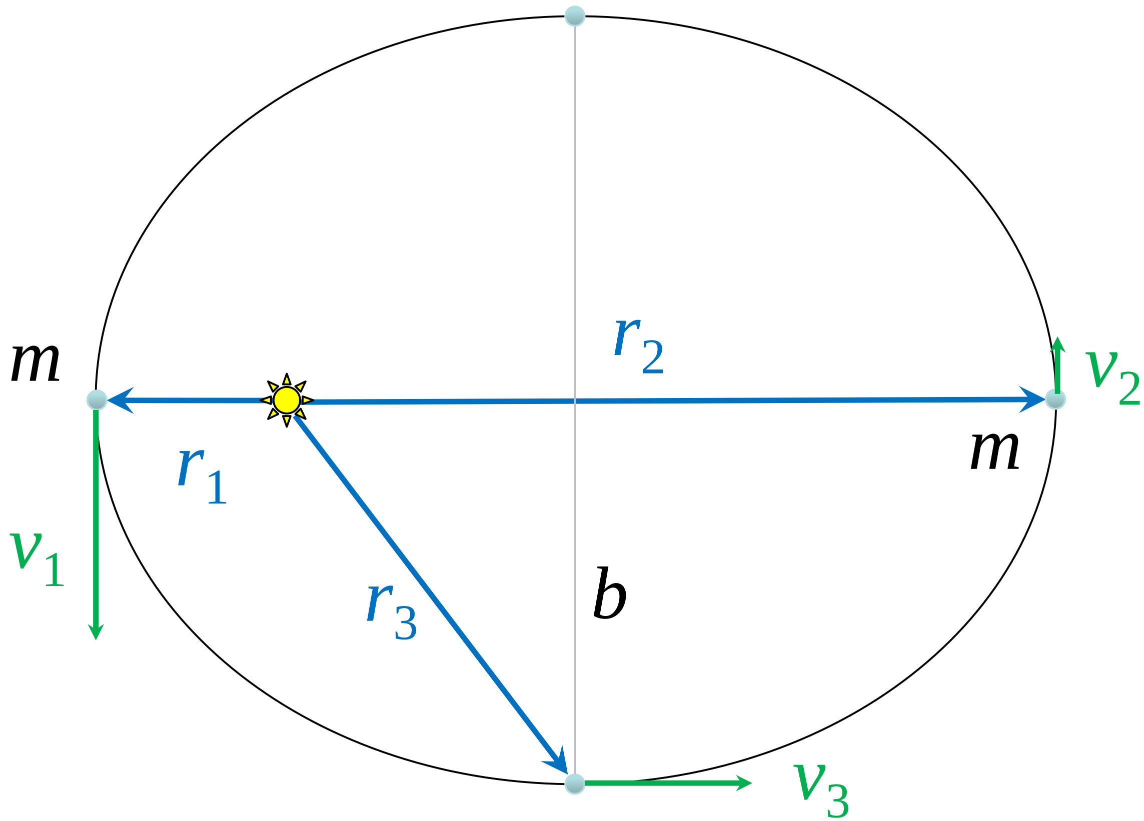


Lower kinetic energy greater potential energy

And because gravity is a conservative force and no other forces are significant, the **total mechanical energy of the system remains constant**. As the satellite nears the central body it is like "falling down" and potential energy is converted to kinetic energy.

Elliptical Orbits and Angular Momentum

The angular momentum should remain constant over the course of the entire orbit. Angular momentum is easiest to determine at the nearest and farthest points in the orbit.



$$\vec{\ell} = \vec{r} \times \vec{p}$$

Angular momentum of satellite at any point in the orbit

$$L = L'$$

$$r_1 m v_1 = r_2 m v_2$$

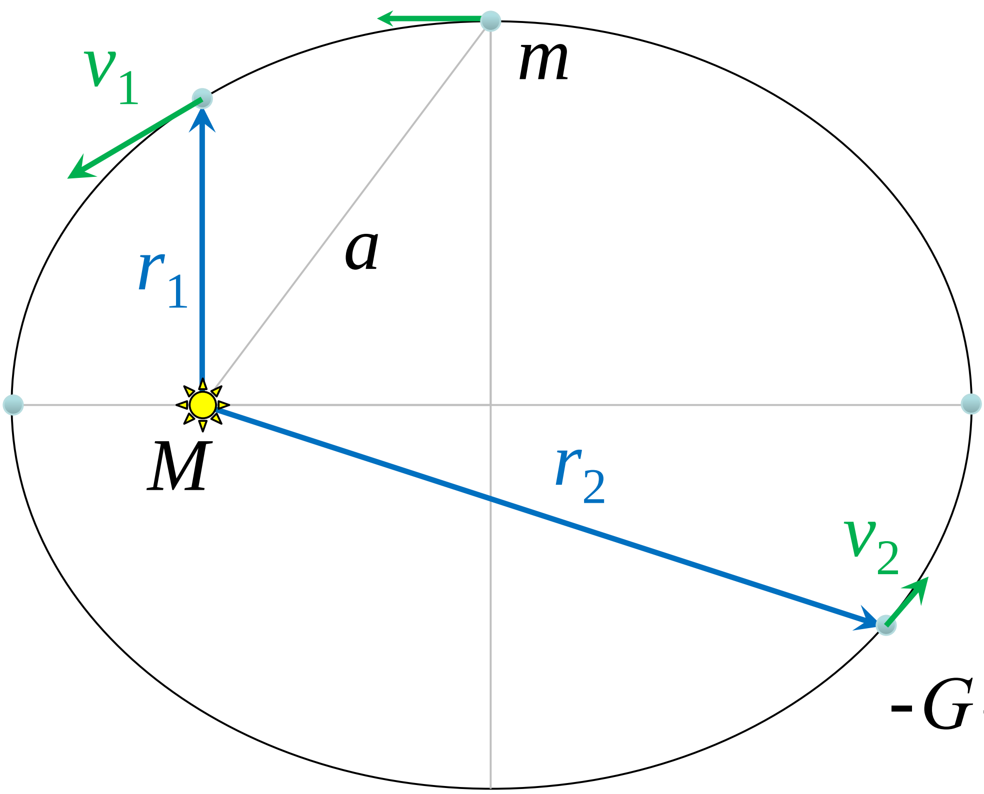
Angular momentum at periapsis

Angular momentum at apoapsis

Note: finding angular momentum at the extremes of the orbit is simple. But at other points it is complicated because *r* and *v* are not perpendicular. e.g. At the third point shown, angular momentum is given by: bmv_3 (b = semi-minor axis)

Elliptical Orbits and Energy

The total mechanical energy of the system remains constant because only the conservative force of gravity acts. There is a special formula for this potential energy.



Conservation of Mechanical Energy:

$$E = U_1 + K_1 = U_2' + K_2'$$

$$-G\frac{Mm}{r_1} + \frac{1}{2}mv_1^2 = -G\frac{Mm}{r_2} + \frac{1}{2}mv_2^2$$

$$E = -G\frac{Mm}{2a}$$

Total energy of the system