Rotation

I. Kinematics
- Angular analogs

II. Dynamics
- **Torque** and Moment of Inertia
- Fixed-axis
- Rolling, slipping

III. Work and Energy
- Fixed-axis, rolling

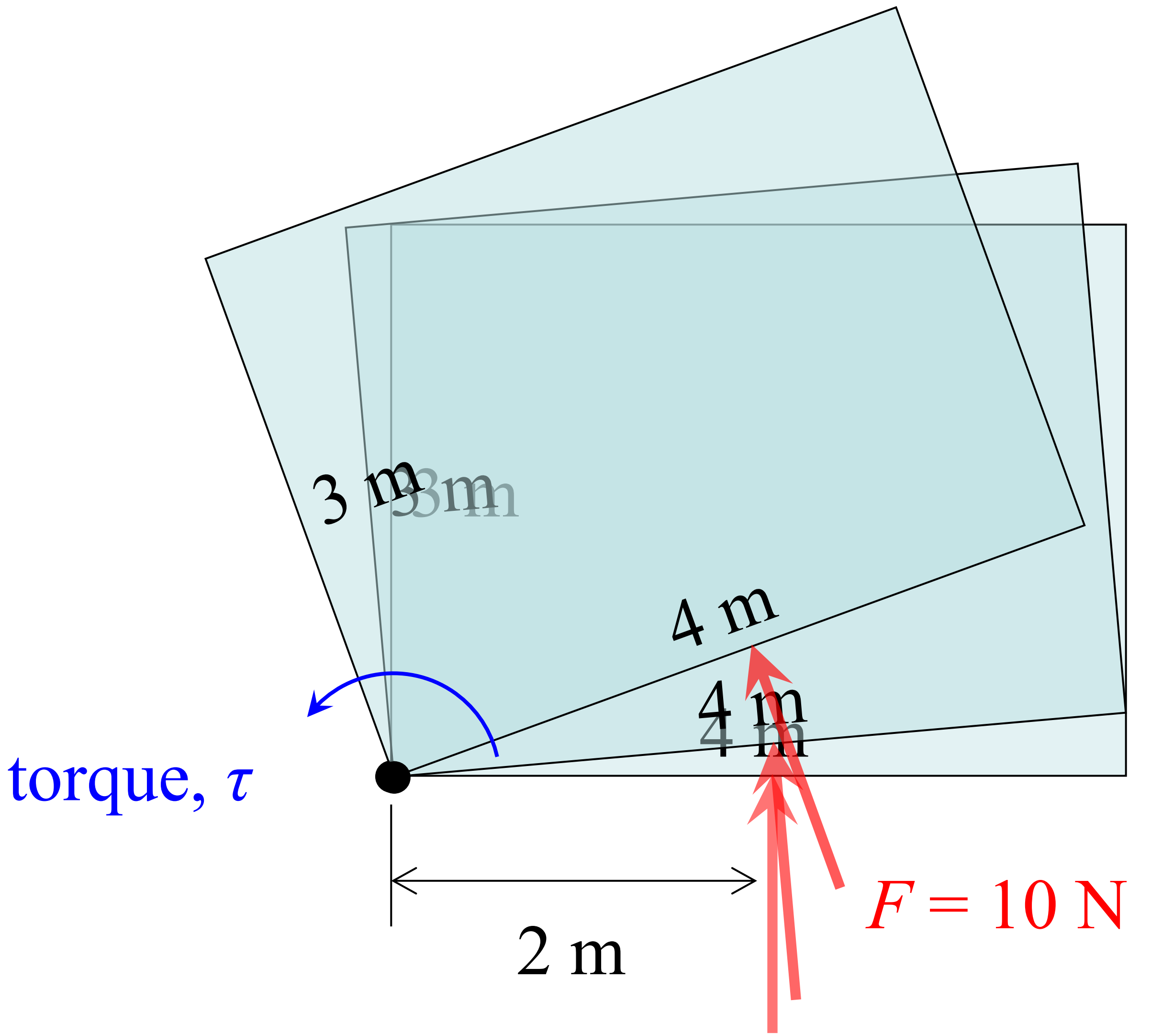
IV. Angular Momentum
- Bodies and particles

	The student will be able to:	HW:
1	State and apply the relations between angular position, angular displacement, angular speed, angular velocity, and angular acceleration to solve related problems.	✓ 1 – 3
2	State and apply the relations between the angular (or rotational) motion of a body or system and the linear (or translational) motion of a point on the body or system.	4 – 7 ✓
3	Determine the torque of an applied force and solve related problems.	8 – 12
4	Determine the moment of inertia for a system of masses or sold body and solve related problems.	13 – 18
5	State and apply Newton's 2^{nd} Law for fixed-axis rotation to solve related problems.	19 – 21
6	Apply work and energy to solve fixed-axis rotation problems.	22 – 25
7	State and apply Newton's 2^{nd} Law for rolling (rotation and translation) to solve related problems (including those with slipping and without slipping)	26 – 33
8	Apply work and energy to solve rolling problems.	34 – 36
9	Determine angular momentum for a particle, system, or rotating body and relate to torque and angular impulse to solve problems.	37 – 42
10	Apply conservation of angular momentum to solve related problems.	43 – 49

Torque

- A torque is something that can cause angular acceleration.
- A force acting on an object can create torque on the object.
- The resulting torque depends not only on the force applied but also on the position at which the force is applied.
- If a force is a "push or pull", then a torque is a "twist" or a "torsion".
- A torque is also sometimes referred to as a "moment" – especially in engineering.

Torque quantifies "how effectively" a force causes angular acceleration. The same force applied at twice the distance from the axis causes twice the angular acceleration and constitutes twice the torque.



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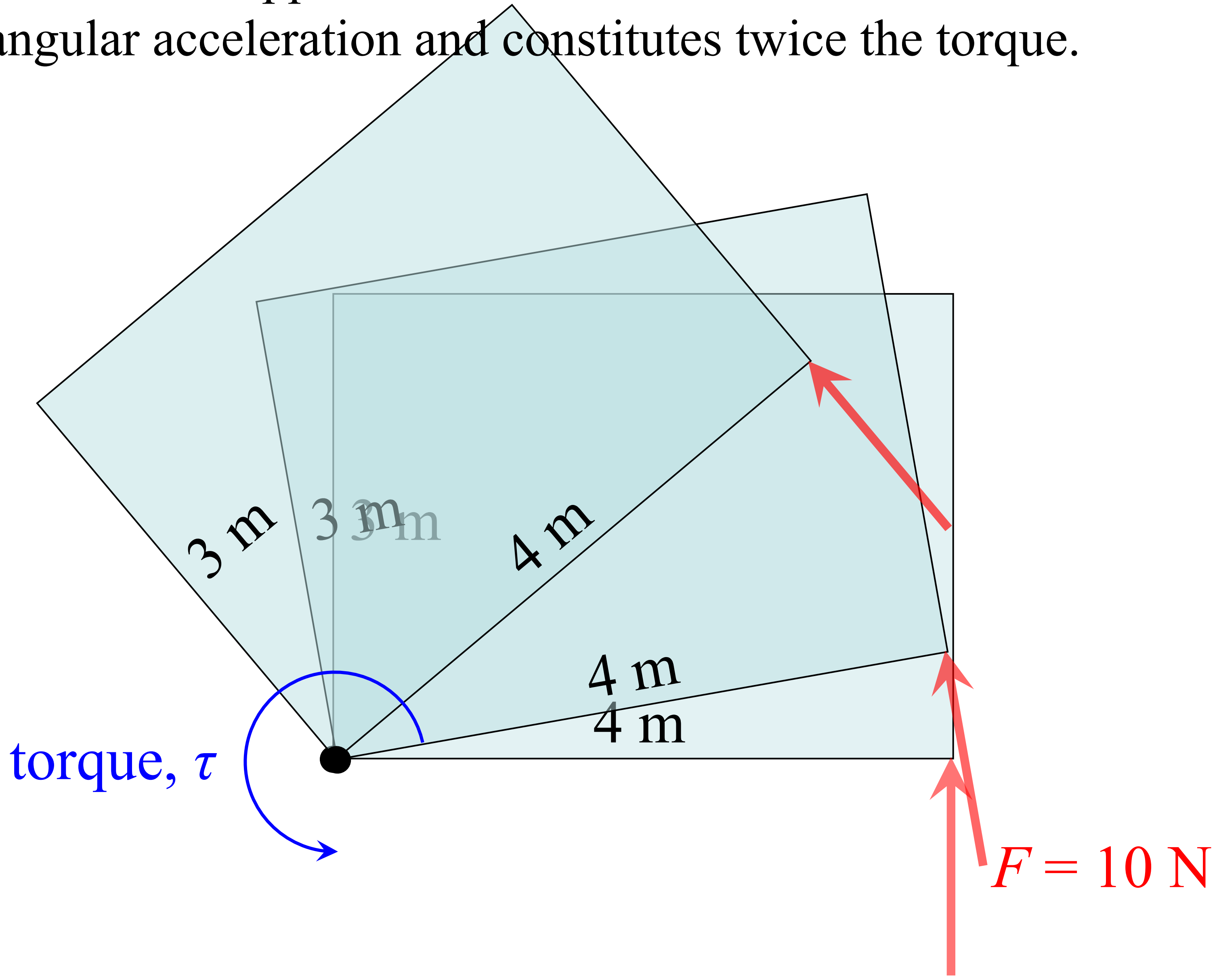


Torque

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\tau = r_{\perp} F$$

$$\tau = r F_{\perp}$$

$$\tau = rF \sin\theta$$

where: F = force

r = position at which force is applied relative to axis of rotation.

Torque

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\tau = r_{\perp} F$$

$$\tau = r F_{\perp}$$

$$\tau = \left| r_x F_y - r_y F_x \right|$$

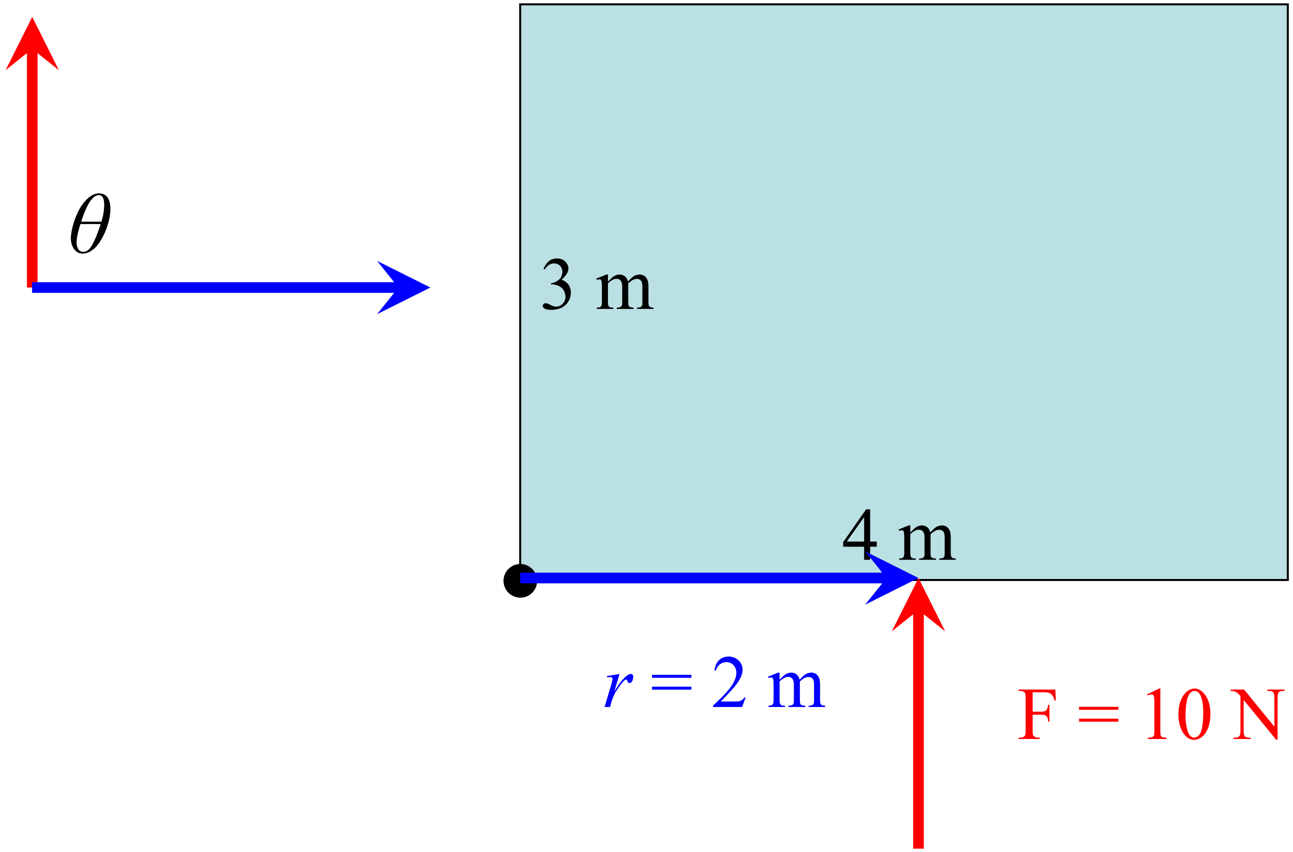
where: F = force

r = position at which force is applied relative to axis of rotation.

$$\tau = rF\sin\theta$$
$$\tau = 2 \cdot 10 \sin 90°$$

$$\tau = r_{\perp}F$$
$$\tau = 2 \cdot 10$$

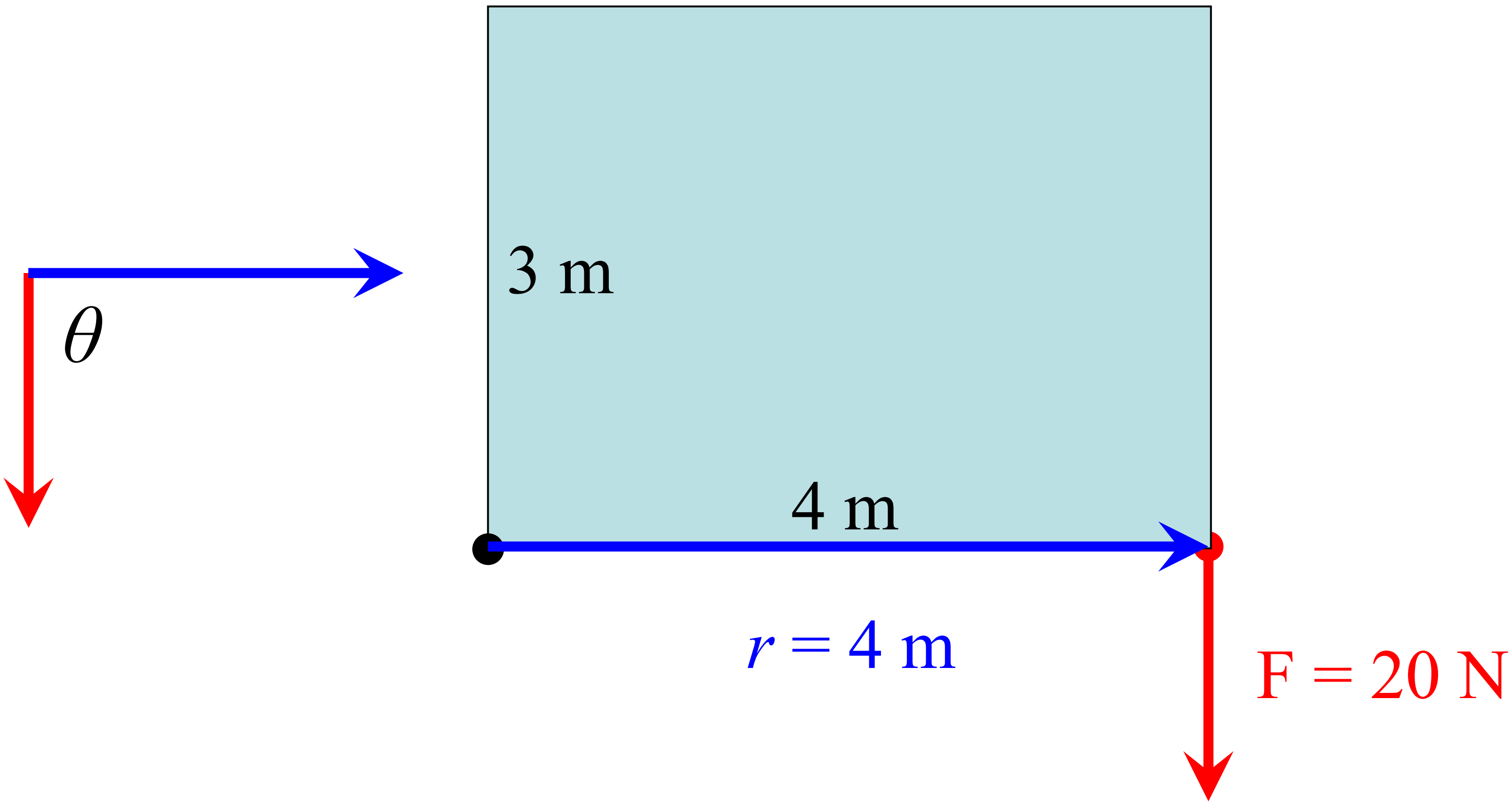
$$\tau = 20 \text{ Nm, CCW}$$



$$\tau = rF\sin\theta$$
$$\tau = 4 \cdot 20 \sin 90°$$

$$\tau = r_{\perp}F$$
$$\tau = 4 \cdot 20$$

$\tau = 80$ Nm, CW



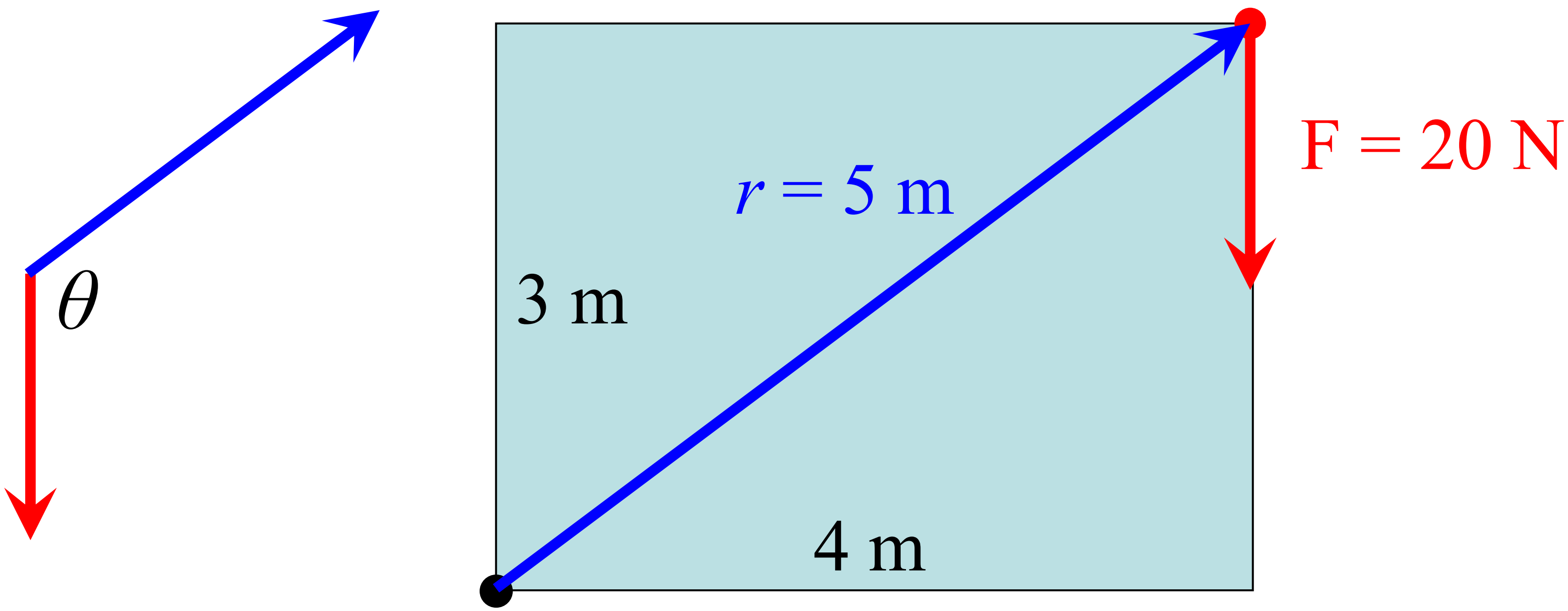
$$\tau = rF\sin\theta$$

$$\tau = 5 \cdot 20 \sin 127°$$

$$\tau = r_{\perp} F$$

$$\tau = 4 \cdot 20$$

τ = 80 Nm, CW



$$\tau = rF\sin\theta$$

$$\tau = 3 \cdot 10 \sin 127^\circ$$

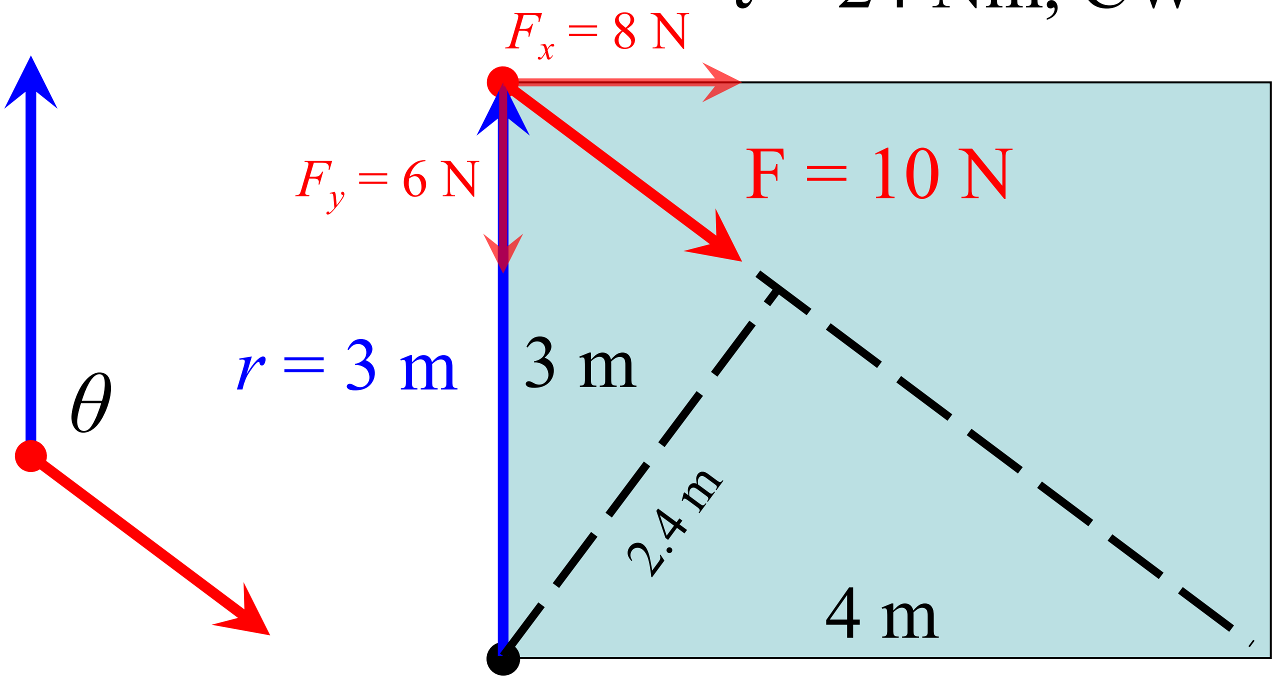
$$\tau = rF_\perp$$

$$\tau = 4 \cdot 20$$

$$\tau = 24 \text{ Nm, CW}$$

$$\tau = r_\perp F$$

$$\tau = 2.4 \cdot 10$$



As shown in this example there are multiple approaches to determine the torque. Note that the perpendicular distance 2.4 m is an example of what is sometimes called a "moment arm", "torque arm", or "leverage arm".

Mini-Lab: Torque

- Use a loop of string and a spring scale to lift and rotate meter stick, pivoting about one end.
- Note the point of application, r, the amount of force, F, and the direction, θ.
- Repeat with different values for each parameter.
- Calculate torque for each trial and compare – what should be observed?

r	F	θ	τ

Use your hand to prevent one end of the stick from moving, thus forming a pivot.

Apply just enough force to barely lift the free end of the stick.