Equilibrium & Oscillation

I. Equilibrium
- conditions
- stable vs. unstable

II. Oscillation
- Simple Harmonic Motion
- Mass and Spring
- Simple Pendulum
- Physical Pendulum

	The student will be able to:	HW:
1	State and apply the conditions for a particle or rigid body to be in equilibrium and solve related problems.	1 – 12
2	State and apply the condition for stable equilibrium and contrast with unstable equilibrium and solve related problems.	13, 14
3	Solve problems involving Simple Harmonic Motion including those concerning: conditions for occurrence, relation between period and the force constant k, relation between period and angular frequency, analyses of position, velocity, and acceleration using sine and cosine.	15 – 24
4	Solve problems involving simple pendulums.	25 – 27
5	Solve problems involving physical pendulums.	28 – 30

Equilibrium

- A rigid body is said to be in equilibrium if its linear and angular acceleration are both zero.
- An object at rest and not rotating is in equilibrium. But also motion with a constant velocity and/or constant angular velocity is considered to be equilibrium.

Conditions of Equilibrium:

Any rigid body in equilibrium must have zero net force and zero net torque!

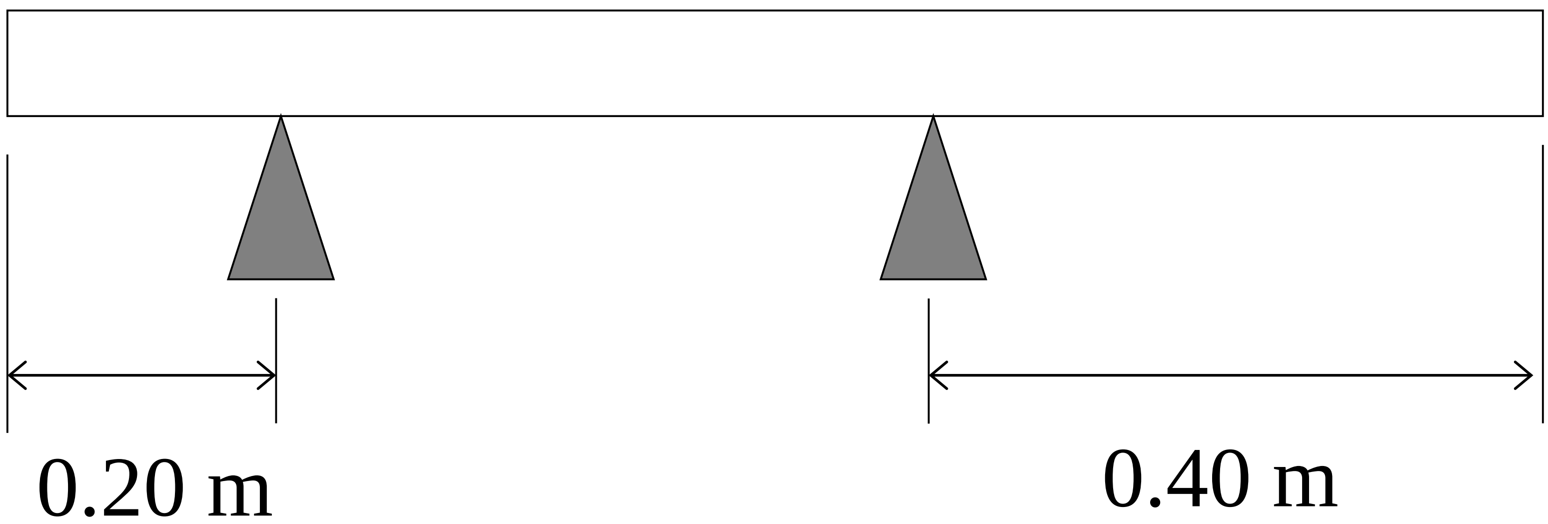
$$\Sigma \vec{F} = 0 \begin{cases} \Sigma F_x = 0 \\ \Sigma F_y = 0 \end{cases}$$

$$\Sigma \vec{\tau} = 0$$

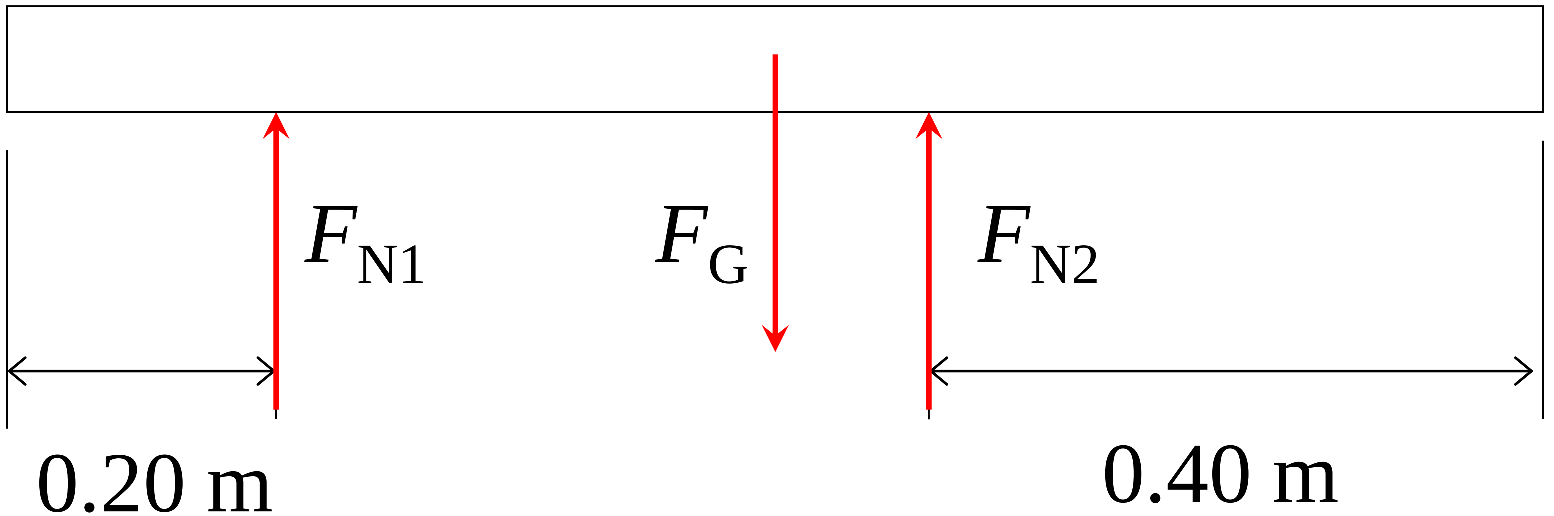
About Torque and Equilibrium . . .

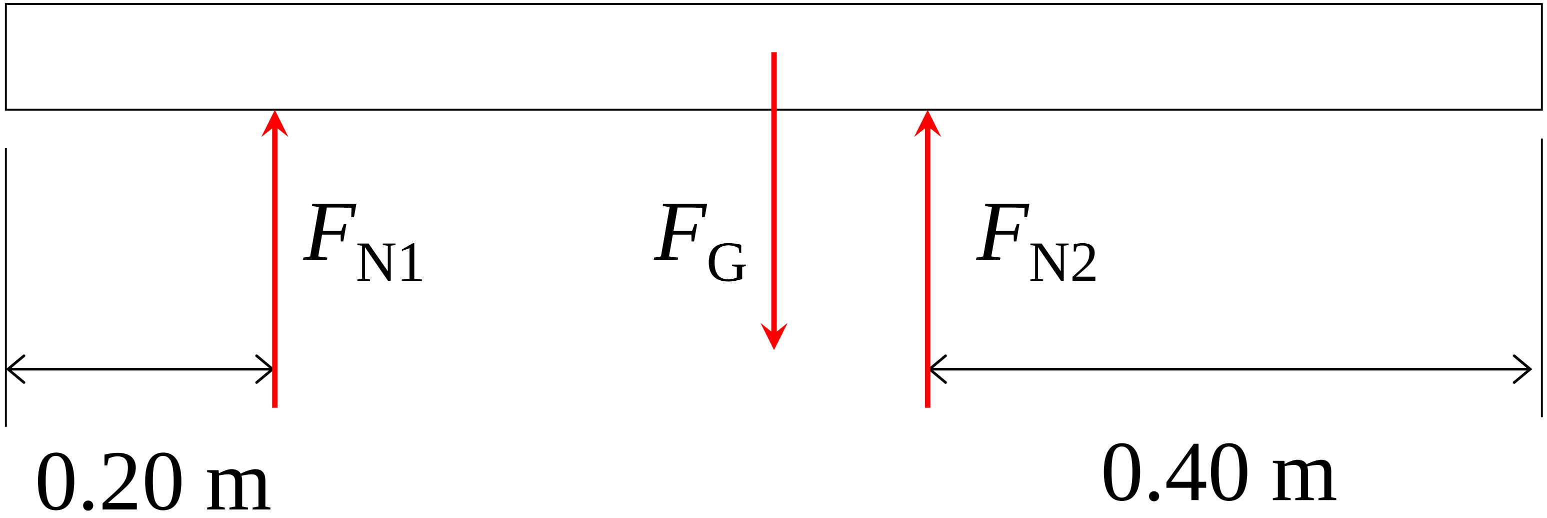
- In order to determine torque it is necessary to know the moment arm, **r**, relative to an axis of rotation. ($\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$)
- Because we are basically assuming there is *no* rotation, it is not important *where* the axis is located or *what* axis is used to determine **r**.
- Put another way, you can choose *any* convenient axis because the object is not rotating about *any* axis.

A uniform horizontal beam of mass 10<u>0</u> grams and length 1.00 m rests on two supports as shown. Determine the normal force at each support.



Draw a free body diagram and apply the conditions of equilibrium . . .



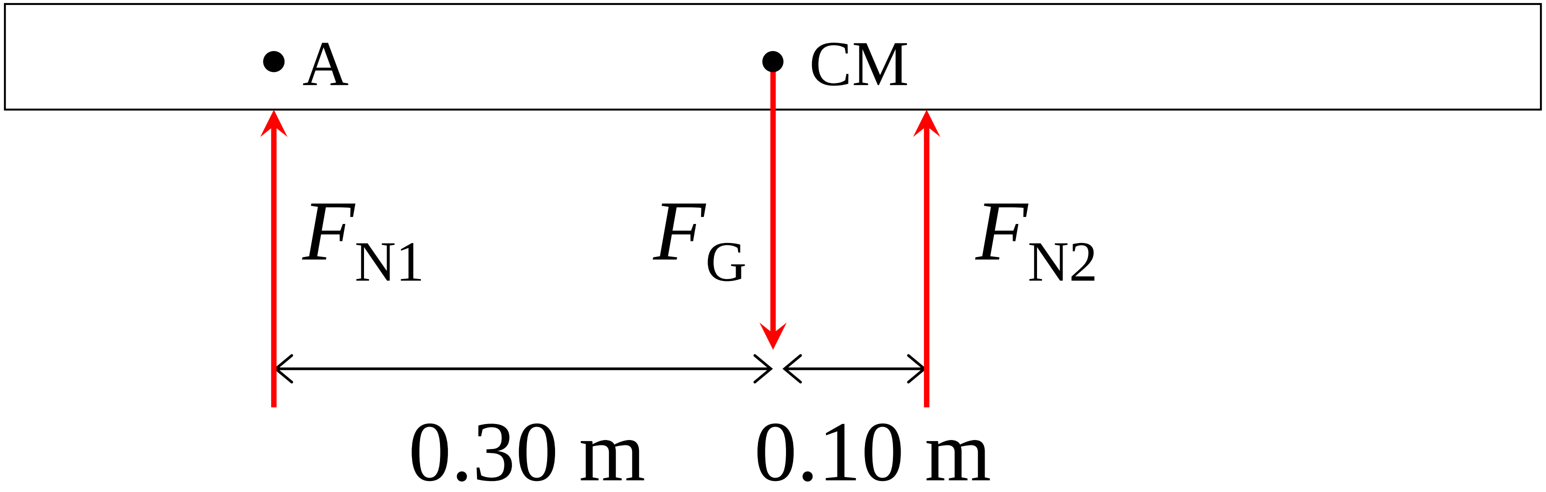


Net force equals zero:

$$\sum F_y = 0 = F_{N1} + F_{N2} - F_G$$

$$F_{N1} + F_{N2} - mg = 0$$

$$F_{N1} + F_{N2} = mg$$



Net torque about the center of mass equals zero:

$$\sum \tau_{CM} = 0 = -0.3F_{N1} + 0.1F_{N2}$$

OR, net torque about point A equals zero:

$$\sum \tau_{A} = 0 = -0.3F_{G} + 0.4F_{N2}$$

Either will work BUT, 2nd equation is *easier to use*!

Solving the equations for force and torque yields the following solution:

$$\mathbf{F}_{\mathrm{N1}} = 0.25\, mg = 0.245\text{ N, upward}$$

$$\mathbf{F}_{\mathrm{N2}} = 0.75\, mg = 0.735\text{ N, upward}$$

Note that these values satisfy all of the equations that were listed – if it is not moving and not rotating then there must be zero net force and zero net torque.

It is usually best to take the torque about a point through which an unknown force acts – thus eliminating a variable in the resulting equation.

Summarizing . . .

- To analyze a rigid body that is in equilibrium a series of equations involving force can be written.
- Set the sum of all *x*-forces equal to zero, set the sum of all *y*-forces equal to zero.
- Set the net torque about a convenient axis equal to zero.
- Solve the resulting system of equations.

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Stable vs. Unstable Equilibrium

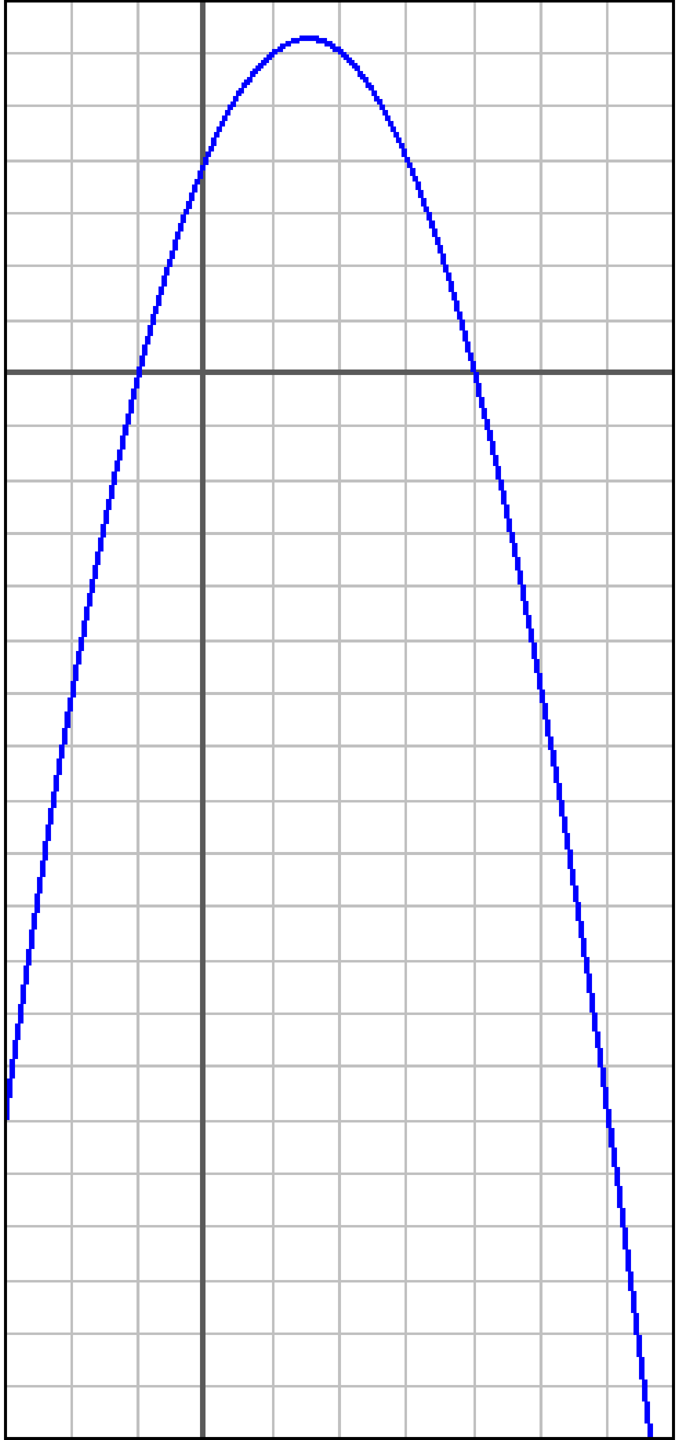
- A stable equilibrium "resists change" and there is a natural tendency to return to equilibrium when perturbed.
- There is a "restoring force" that is directed toward any position of stable equilibrium.
- A force acts in a direction away from any position of unstable equilibrium.

Equilibrium for One-dimensional Motion

For *any* position of equilibrium: $$F = 0$$

Stable equilibrium: $$\frac{dF}{dx} < 0$$

Unstable equilibrium: $$\frac{dF}{dx} > 0$$

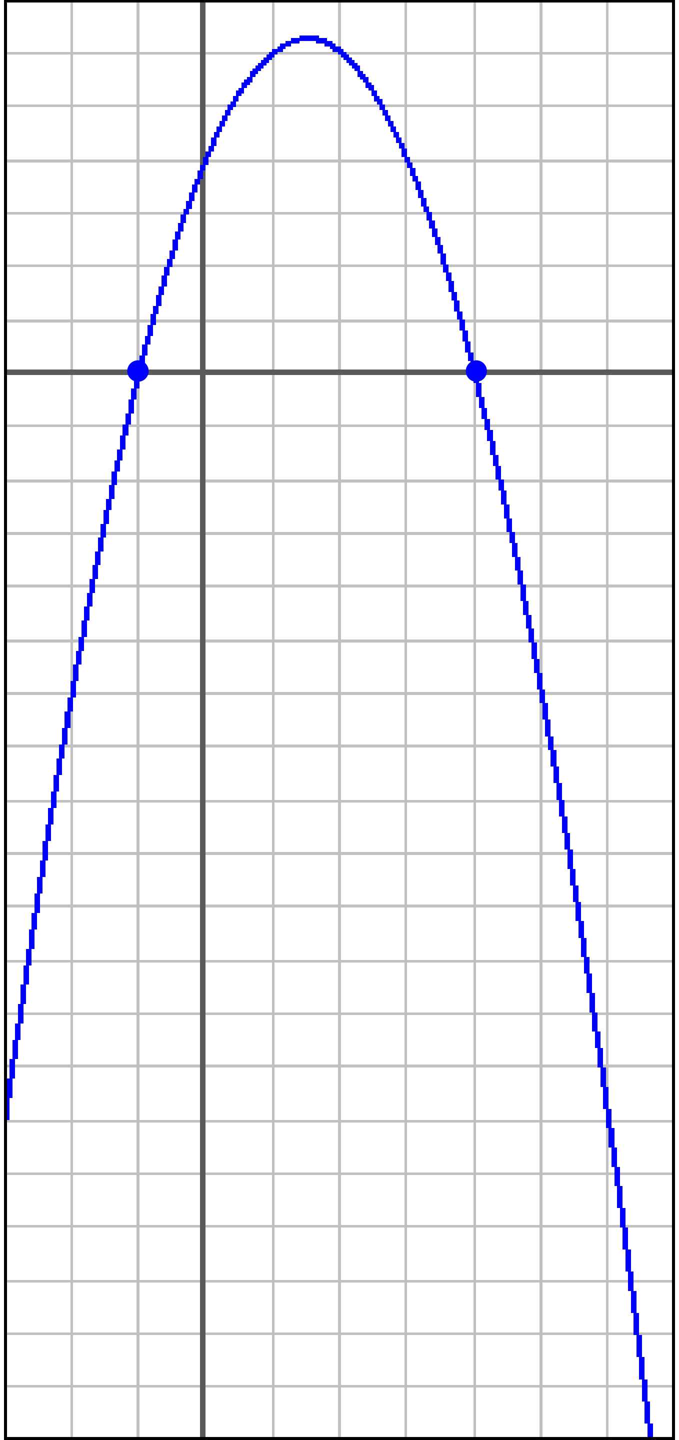


$$F = 4 + 3x - x^2$$

$$F = 0$$

$$\frac{dF}{dx} < 0$$

$$\frac{dF}{dx} > 0$$

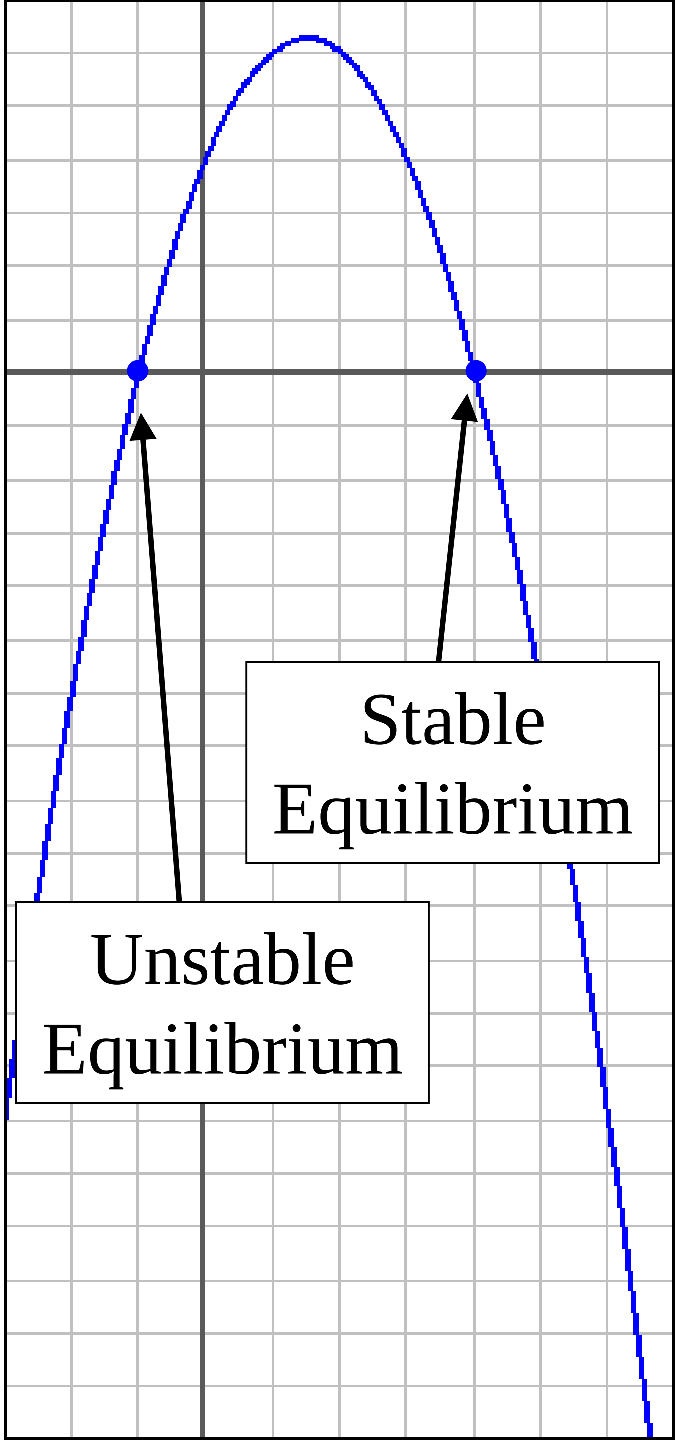


$$F = 4 + 3x - x^2$$

$$F = 0 \quad x = -1, 4$$

$$\frac{dF}{dx} < 0 \qquad x = 4$$

$$\frac{dF}{dx} > 0 \qquad x = -1$$



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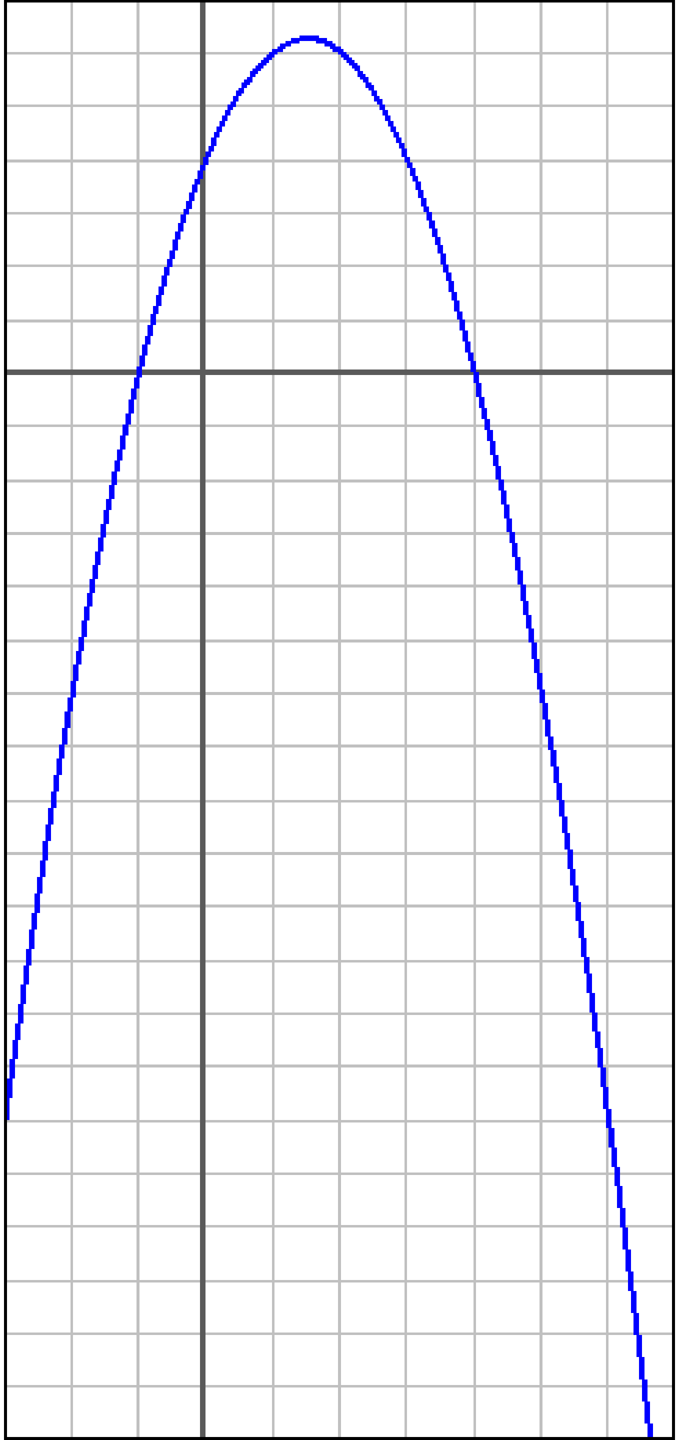

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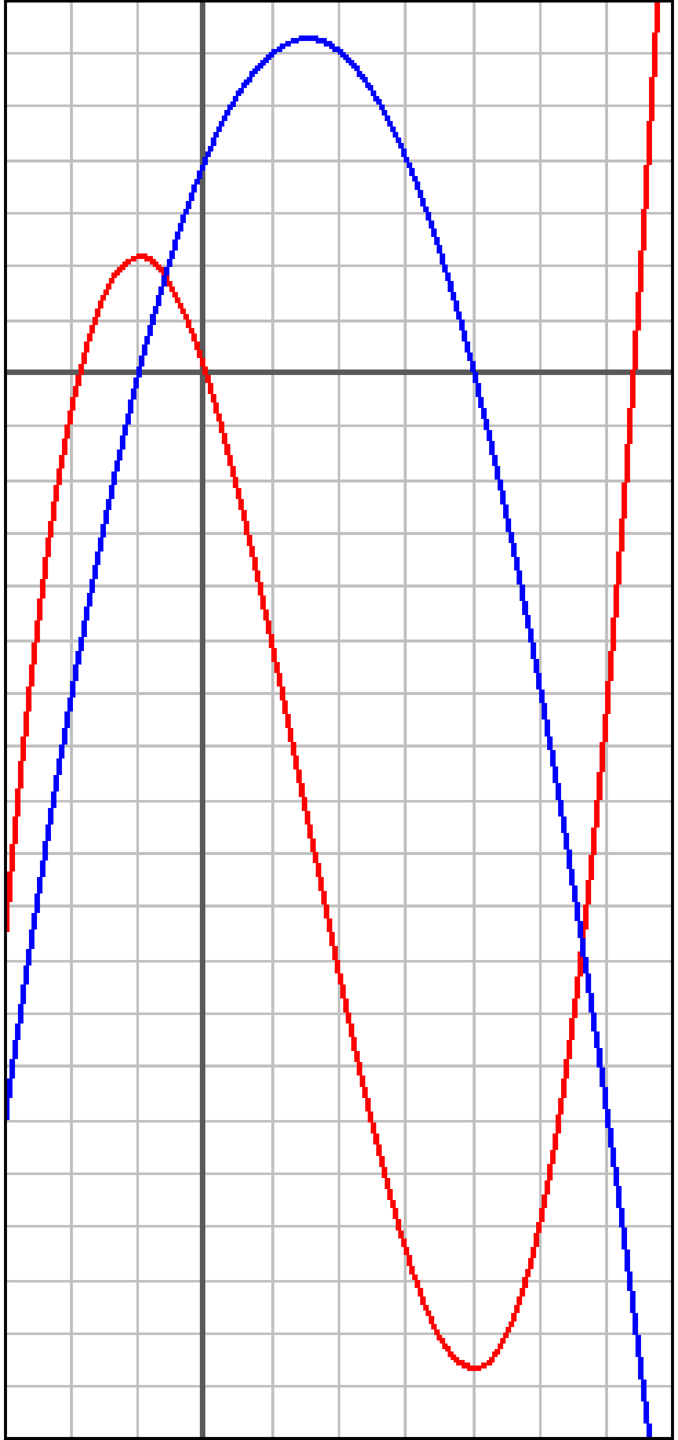


$$F = 4 + 3x - x^2$$

$$U = -\int F dx$$

$$F = -\frac{dU}{dx}$$

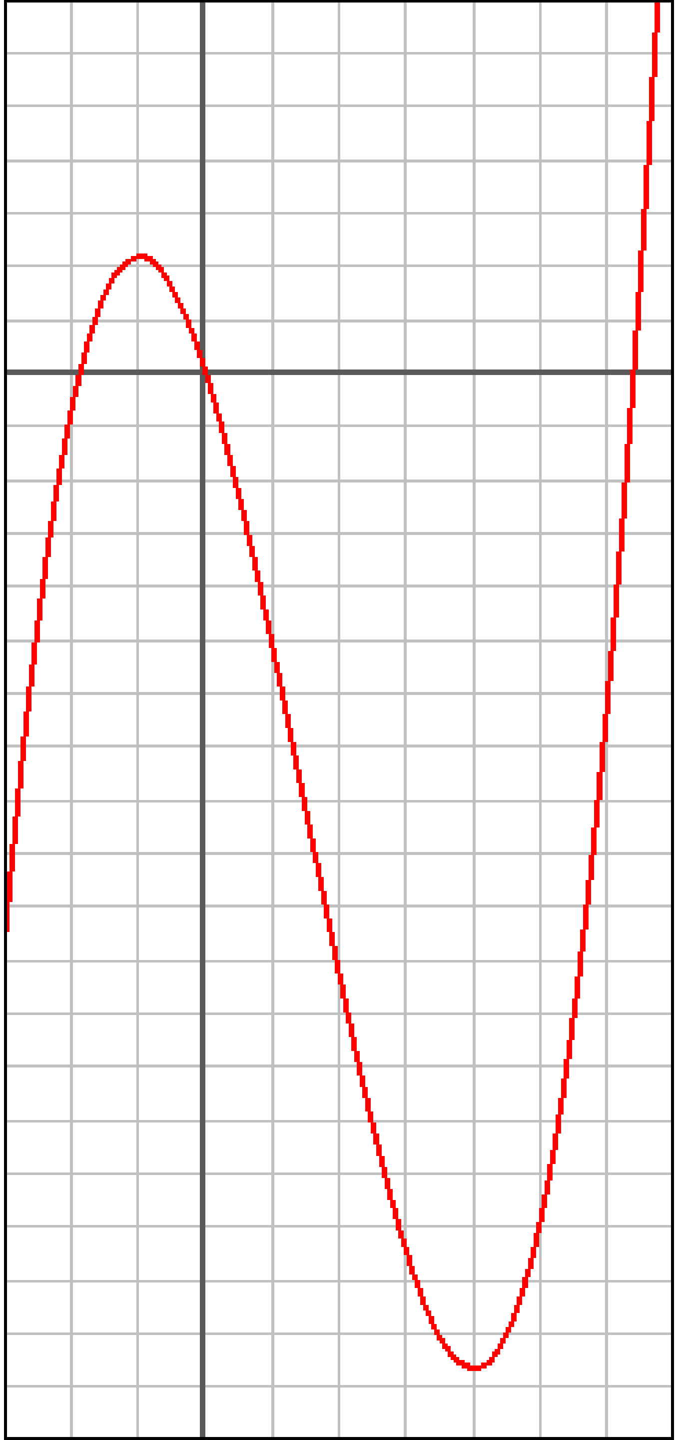
$$U = -4x - \frac{3x^2}{2} + \frac{x^3}{3}$$

Equilibrium for One-dimensional Motion

For *any* position of equilibrium: $$\frac{dU}{dx} = 0$$

Stable equilibrium: $$\frac{d^2U}{dx^2} > 0$$

Unstable equilibrium: $$\frac{d^2U}{dx^2} < 0$$



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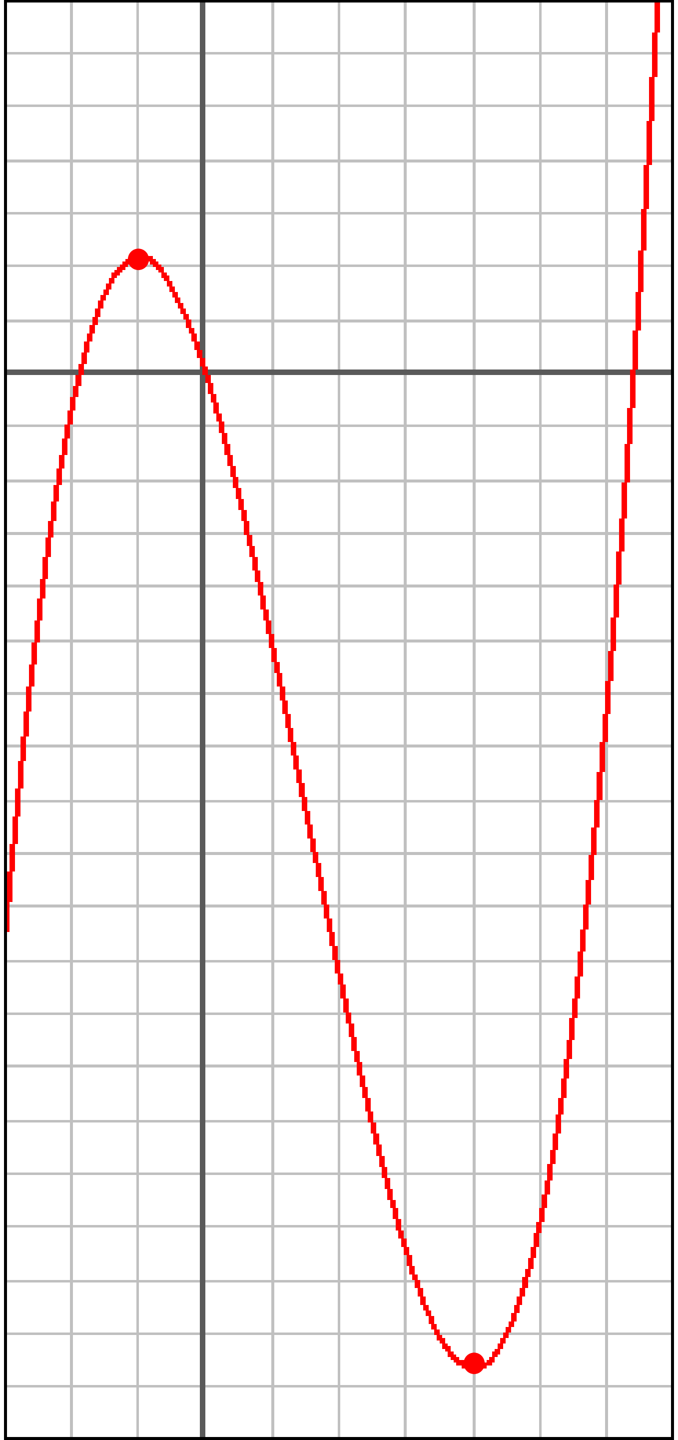

$$U = -4x - \frac{3x^2}{2} + \frac{x^3}{3}$$

$$\frac{dU}{dx} = 0 \qquad x = -1, 4$$

$$\frac{d^2U}{dx^2} > 0 \qquad x = 4$$

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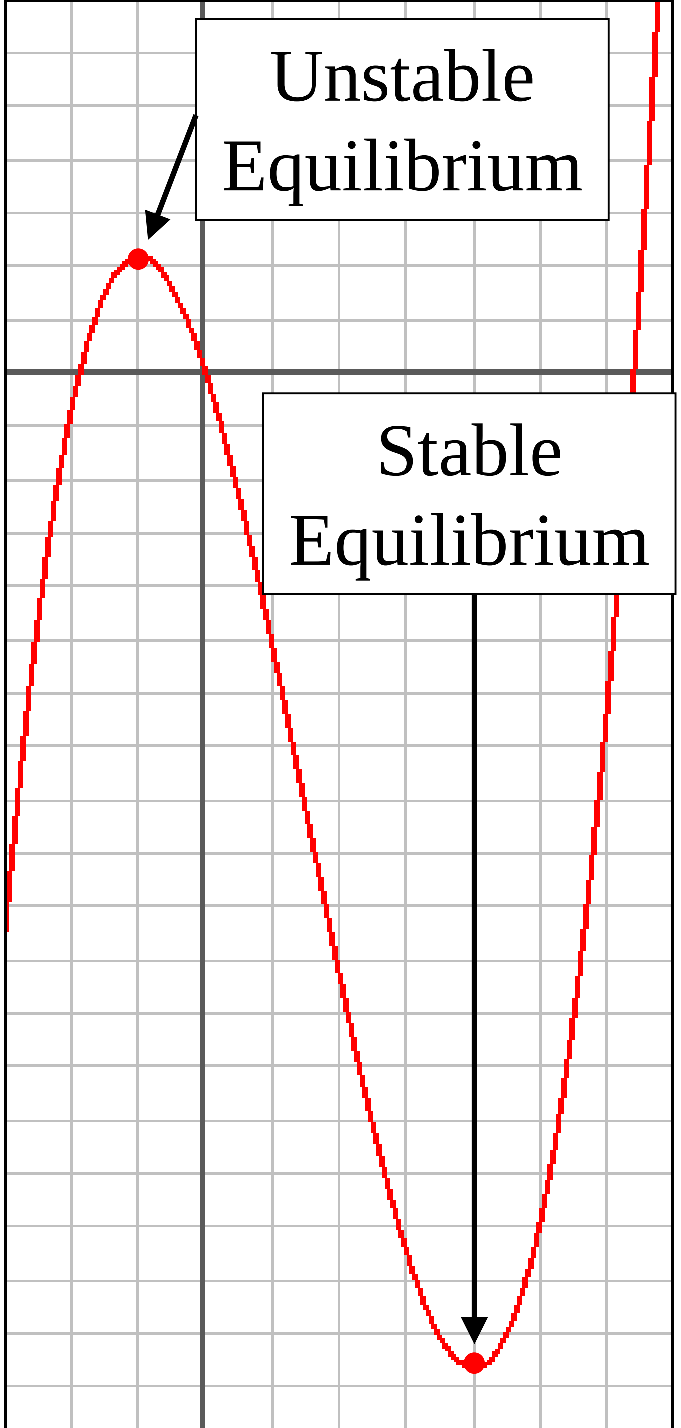

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