Simple Harmonic Motion

Understanding Oscillations

Equilibrium & Oscillation

I. Equilibrium
- conditions
- stable vs. unstable

II. Oscillation
- Simple Harmonic Motion
- Mass and Spring
- Simple Pendulum
- Physical Pendulum

	The student will be able to:	HW:
1	State and apply the conditions for a particle or rigid body to be in equilibrium and solve related problems. ✓	1 – 12
2	State and apply the condition for stable equilibrium and contrast with unstable equilibrium and solve related problems. ✓	13, 14
3	Solve problems involving Simple Harmonic Motion including those concerning: conditions for occurrence, relation between period and the force constant *k*, relation between period and angular frequency, analyses of position, velocity, and acceleration using sine and cosine.	15 – 24
4	Solve problems involving simple pendulums.	25 – 27
5	Solve problems involving physical pendulums.	28 – 30

Basic Ideas:

- **Simple Harmonic Motion** (SHM) is a special type of oscillation that occurs under certain conditions.
- In order for SHM to occur, there must be a restoring force proportional to displacement from a position of equilibrium.
- The oscillation in SHM is sinusoidal (can be modeled by the sine function).

Condition for SHM

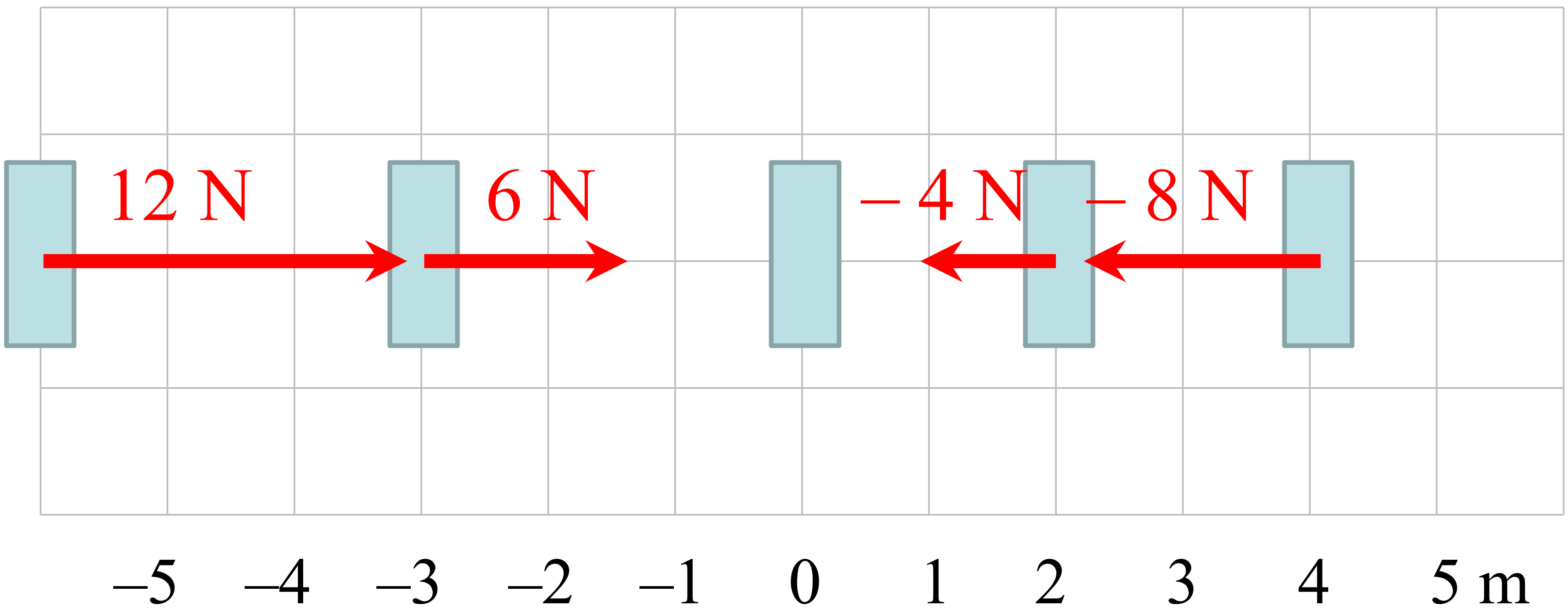
$$\Sigma\vec{F} = -k\vec{x}$$

Where: $\Sigma\vec{F}$ = net force acting on object

k = a positive constant

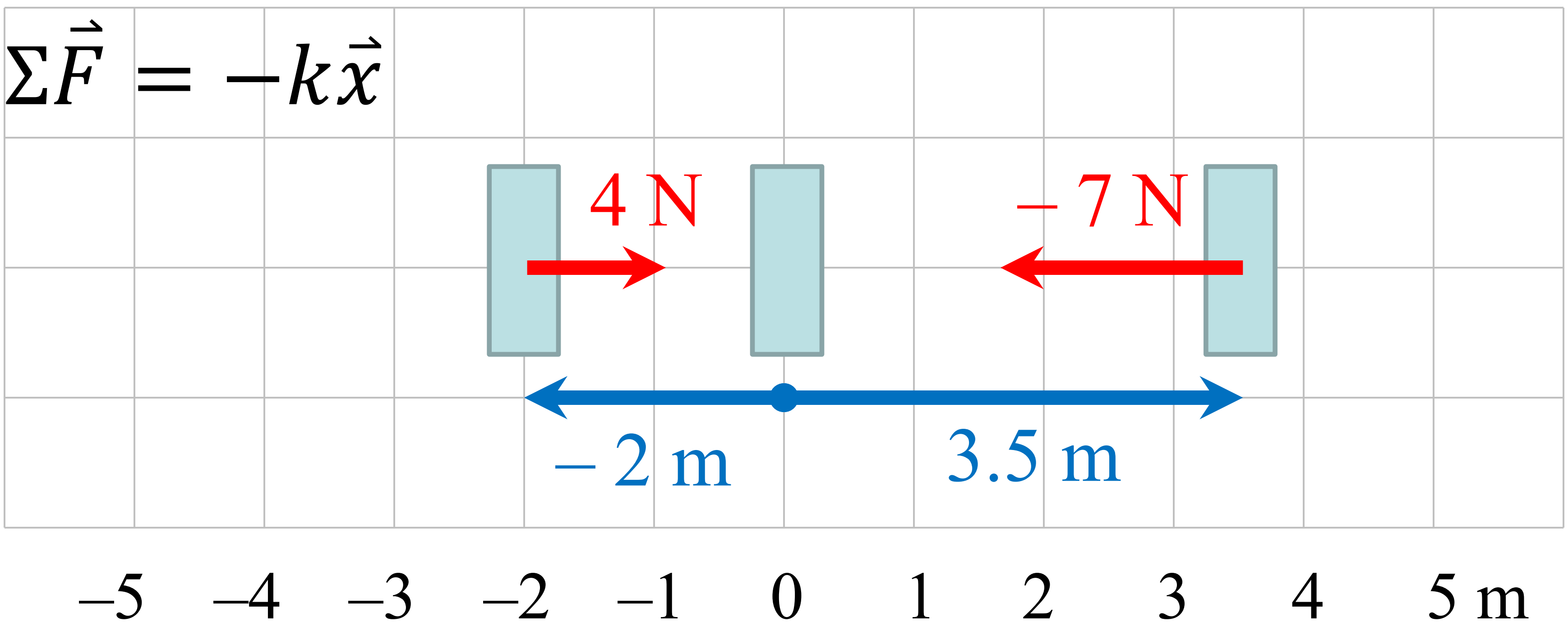
$\vec{x}$ = position relative to equilibrium

What would be the value of k for this example?



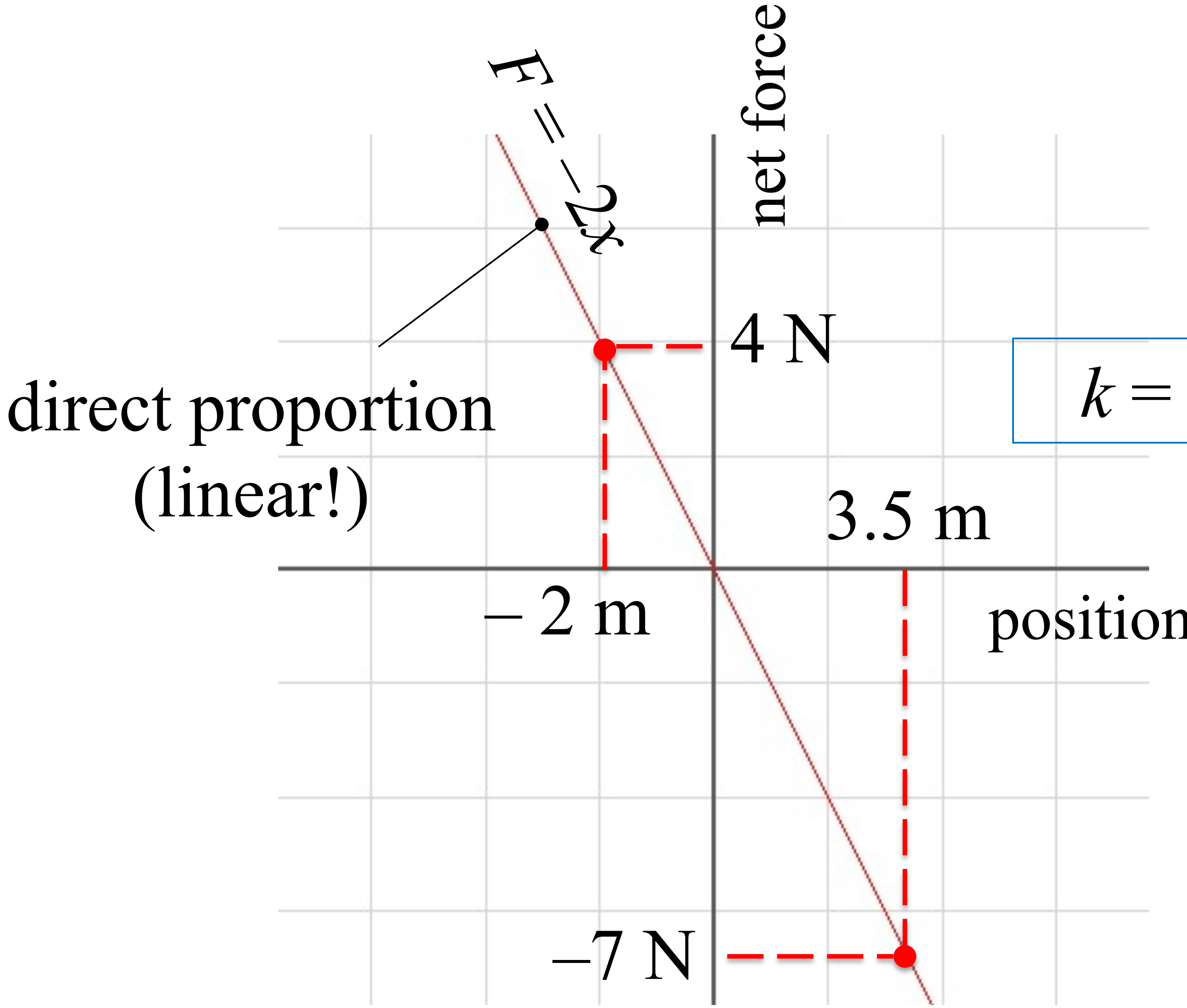
The red arrows indicate the net force.

What would be the value of k for this example?



$$k = \left|\frac{F}{x}\right| = \frac{4}{2} = \frac{7}{3.5}$$

$k = 2$ N/m



net force
F = –2x
direct proportion
(linear!)
4 N
3.5 m
– 2 m
position
–7 N

$k = |\text{ slope }|$

Resulting Sinusoidal Motion

$$\Sigma\vec{F} = m\vec{a}$$

$$-kx = m\frac{d^2x}{dt^2}$$

If the net force meets the condition $F = -kx$, then the position function $x(t)$ must satisfy the differential equation of motion shown above. An example of a solution has the form:

$$x = A\sin(\omega t + \delta)$$

A = the amplitude of the oscillation
ω = angular frequency
δ = phase angle

$$\omega = \sqrt{\frac{k}{m}}$$

Resulting Motion

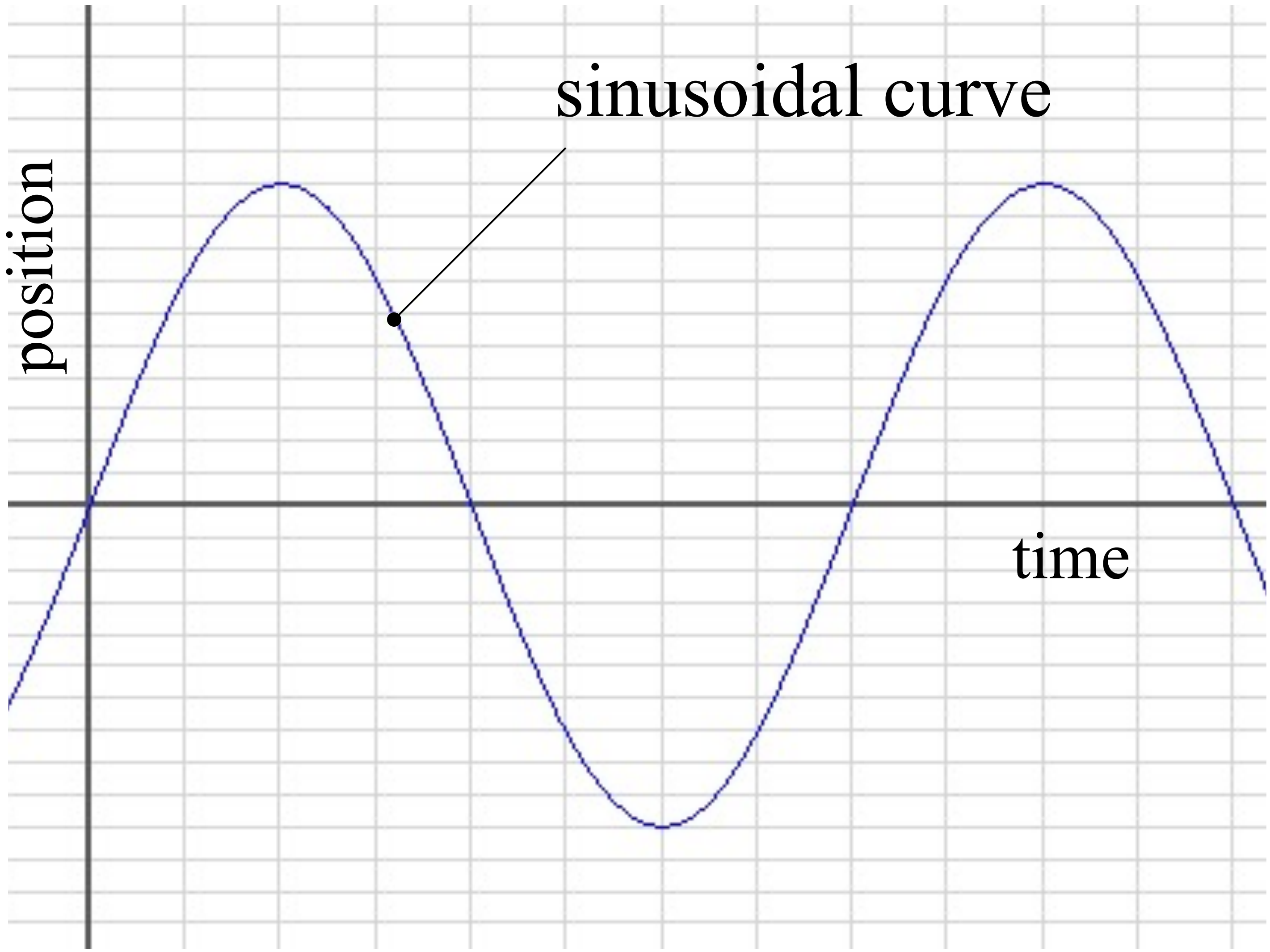
$$T = 2\pi\sqrt{\frac{m}{k}}$$

Where: T = period of oscillation

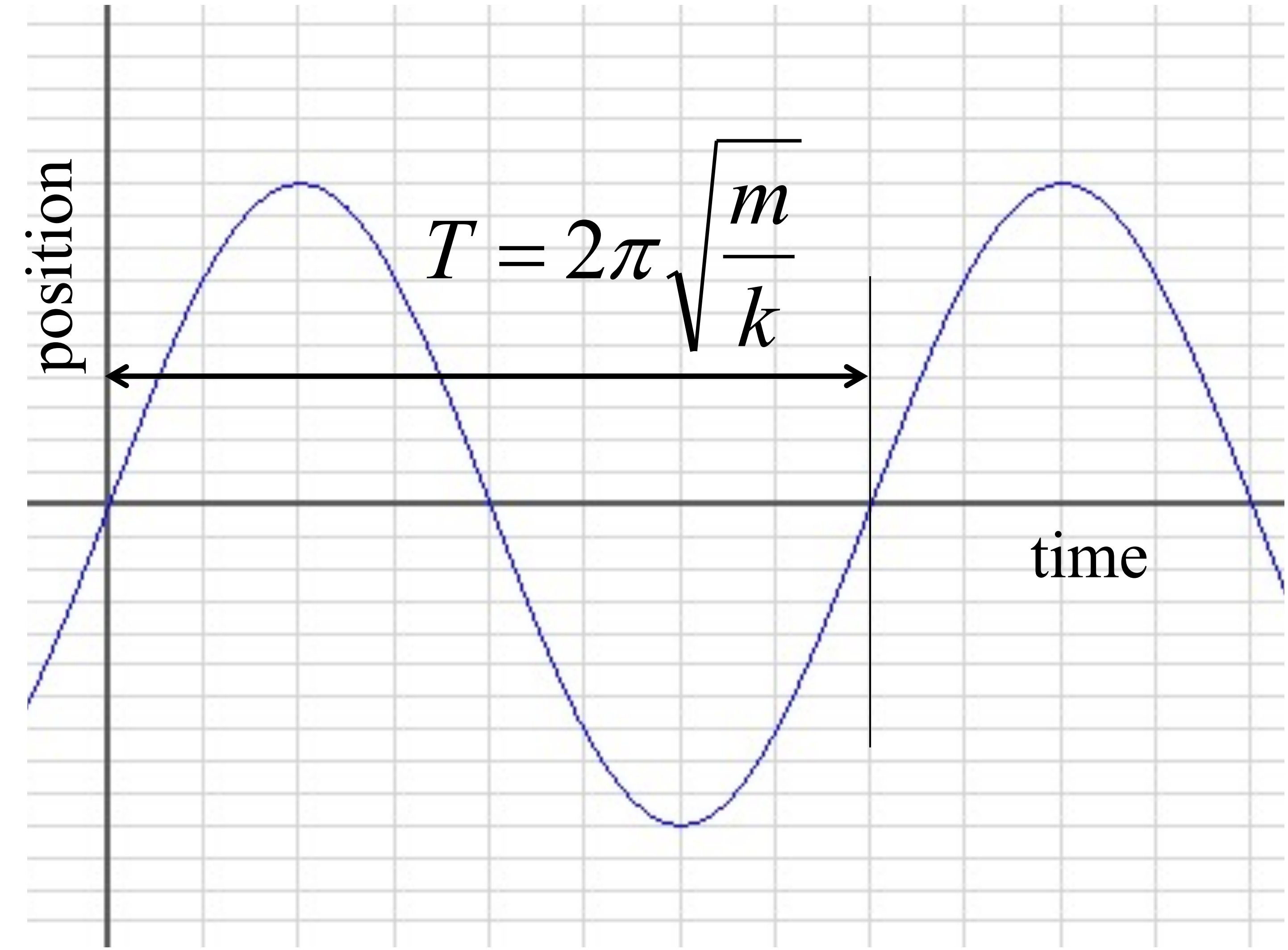
k = the constant from $\mathbf{F} = -k\mathbf{x}$

m = mass of object

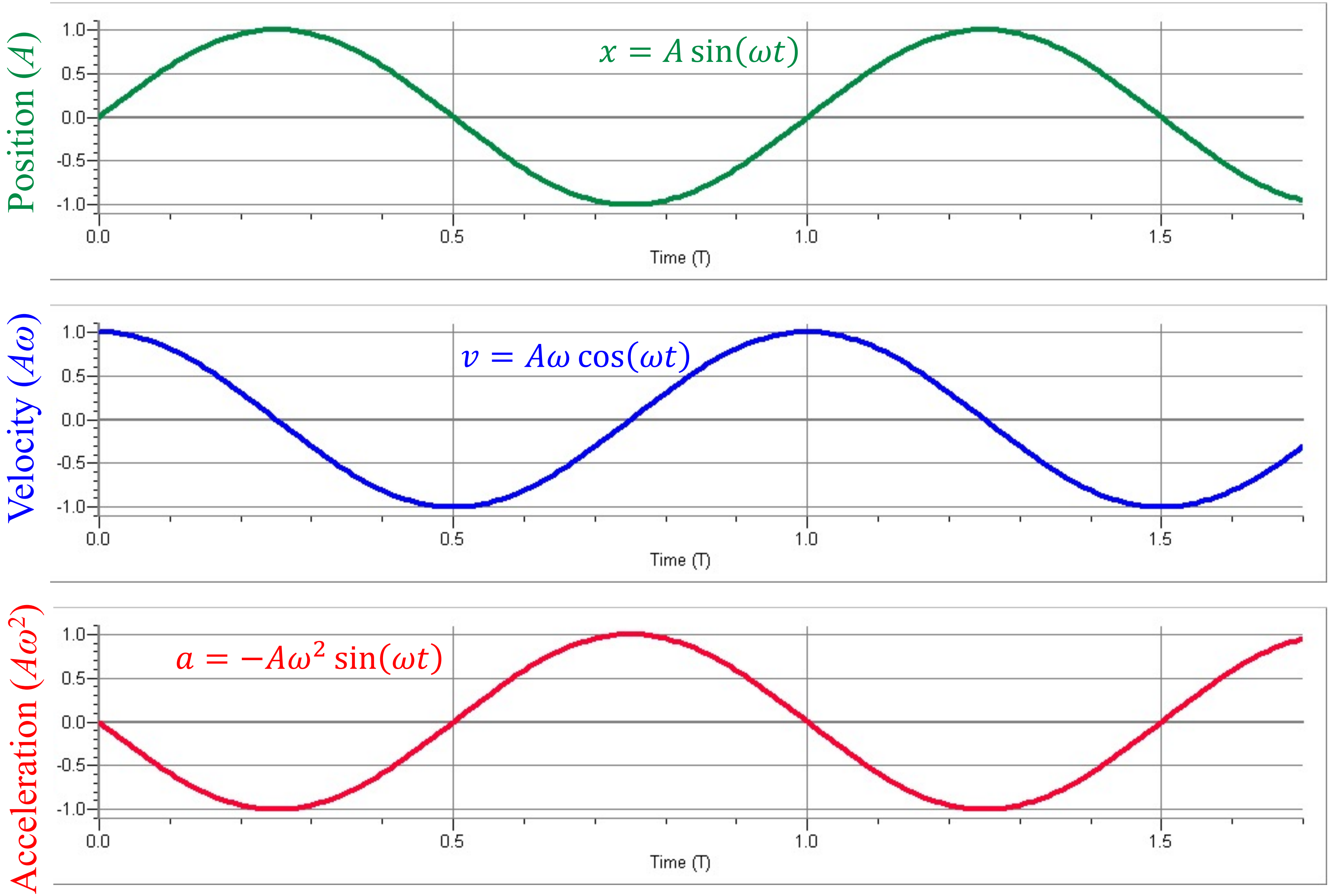
Position vs. Time



Position vs. Time



Kinematics of SHM

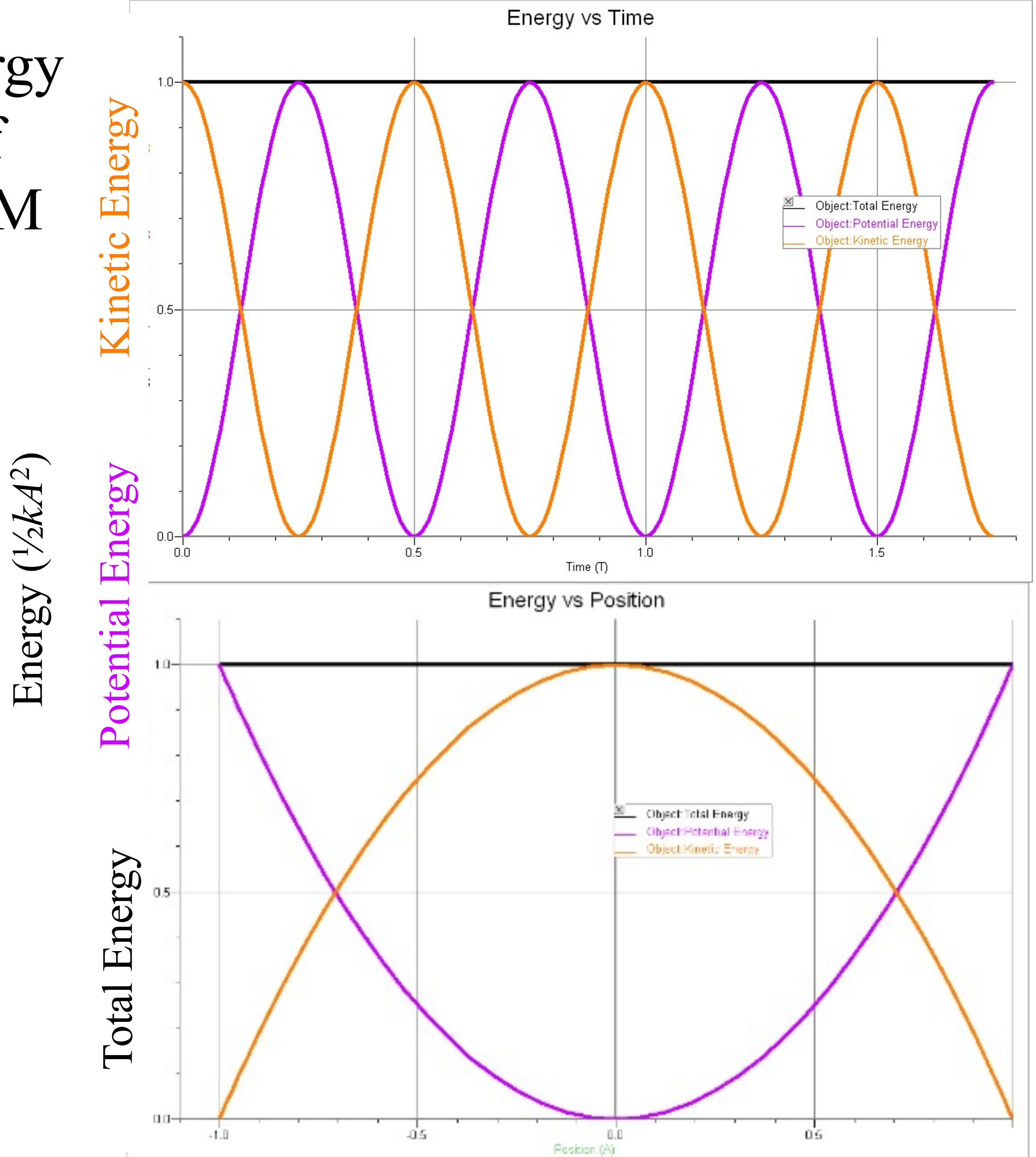


Notes on Period

$$T = 2\pi\sqrt{\frac{m}{k}}$$

It is remarkable what is *not* in this equation – the amplitude or size of the oscillation. In other words the period <u>does not</u> depend on the size of the oscillations!

Energy of SHM



$$K = \frac{mA^2\omega^2}{2}\cos^2(\omega t)$$

$$K = \frac{kA^2}{2}\cos^2(\omega t)$$

$$U = \frac{kA^2}{2}\sin^2(\omega t)$$

$$E = \tfrac{1}{2}kx^2 + \tfrac{1}{2}mv^2$$

$$E = \tfrac{1}{2}kA^2$$

Mass on a Spring

- According to **Hooke's Law** any common steel spring will apply a force that is proportional to its elongation or compression
- Every spring has a unique ratio of force to change designated as k, the "spring constant".
- Therefore a mass attached to a spring will undergo SHM.
- In this situation the spring constant is the same k as found in the condition for SHM.

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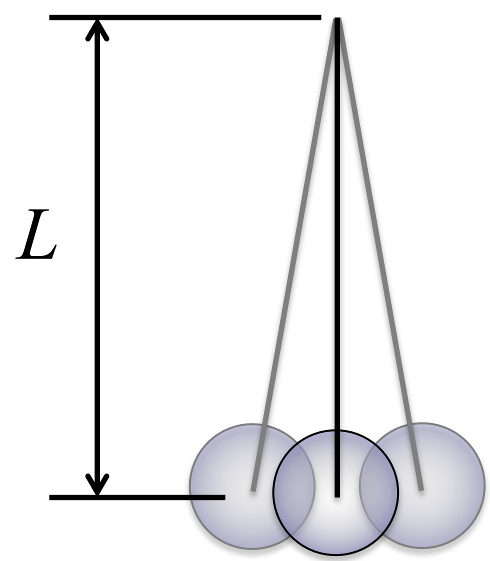
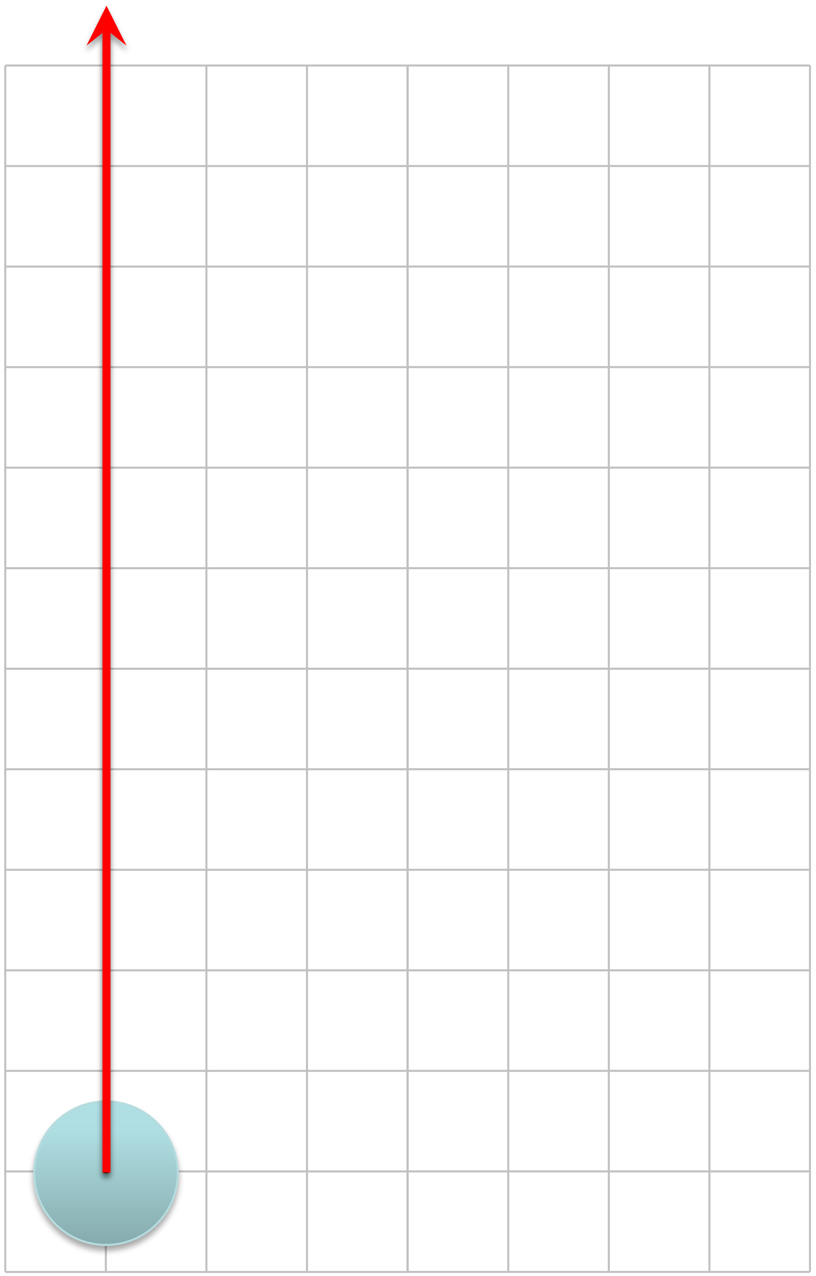
Pendulum

- A pendulum will exhibit SHM to a high degree of accuracy as long as the amplitude of its swing is less than 10° or so (from vertical).
- In this situation it can be shown that $k = mg/L$.
- Therefore the period of a pendulum is given by:



$$T = 2\pi\sqrt{\frac{L}{g}}$$

Simple Pendulum



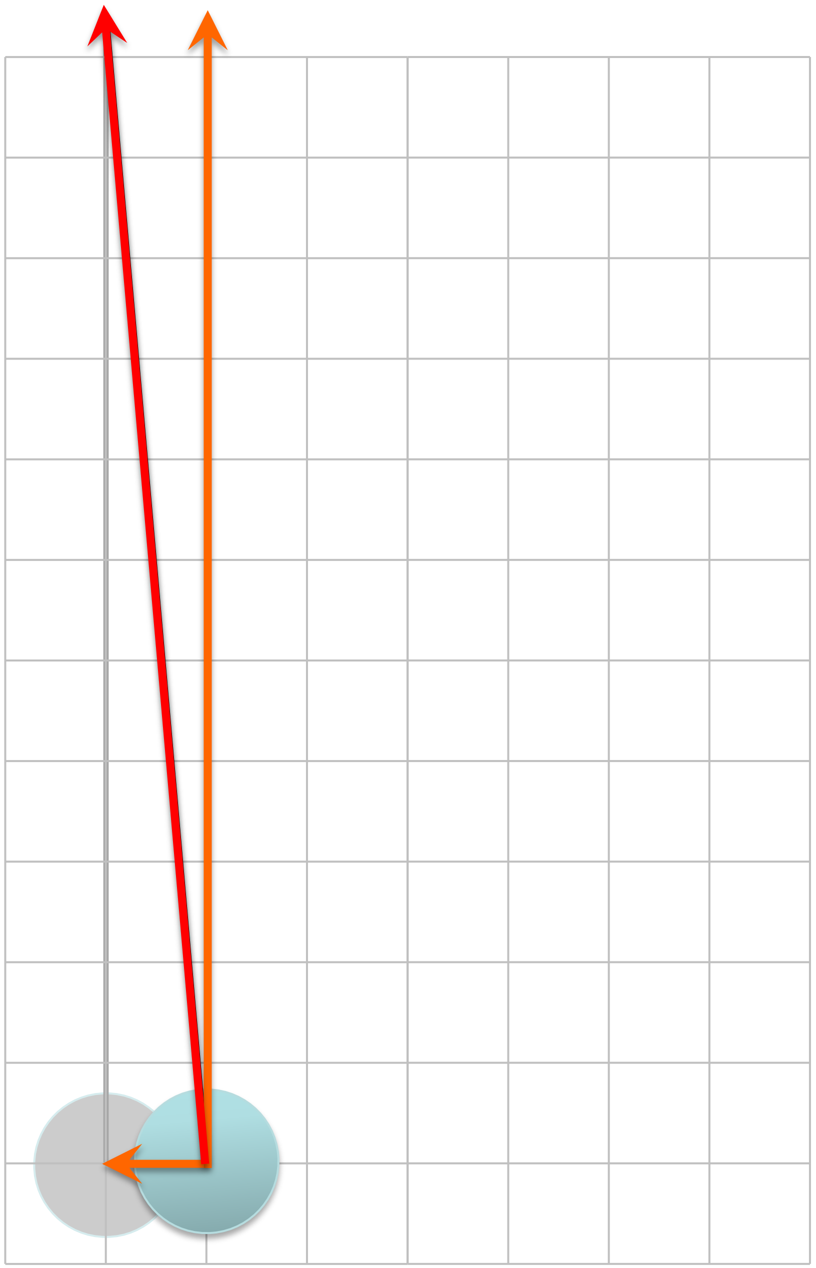
Simple Pendulum

The red arrow is tension in the string.

Orange arrows are the components of the tension.

On this and following pages the components T_x and T_y are shown correct to scale for an oscillating pendulum.

Go through the pages and pay attention to the relative sizes of the x and y components…

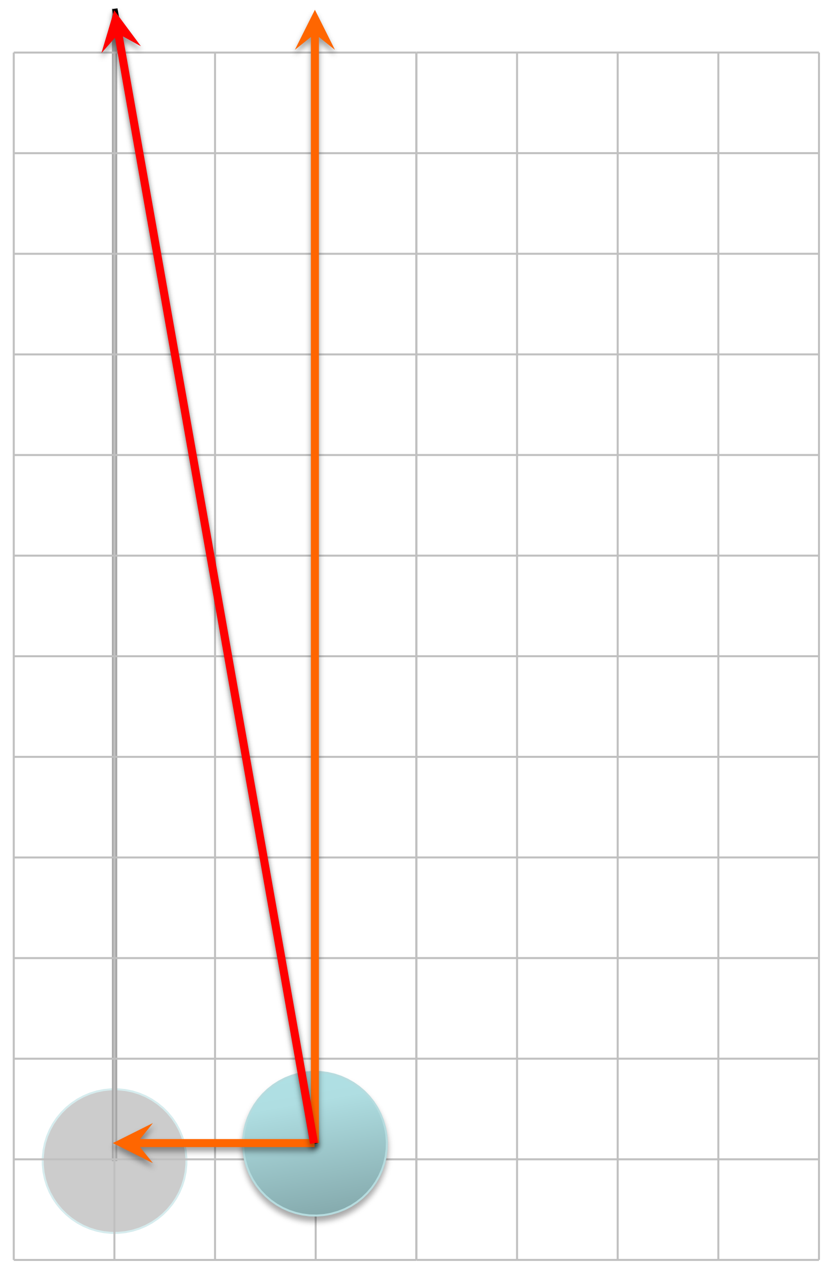


Simple Pendulum

The red arrow is tension in the string.

Orange arrows are the components of the tension.

Comparing this page to the previous one, note that the *x*-component of the tension force has essentially doubled as the *x*-coordinate of the object's position has doubled. The *y*-component of tension has changed very little.

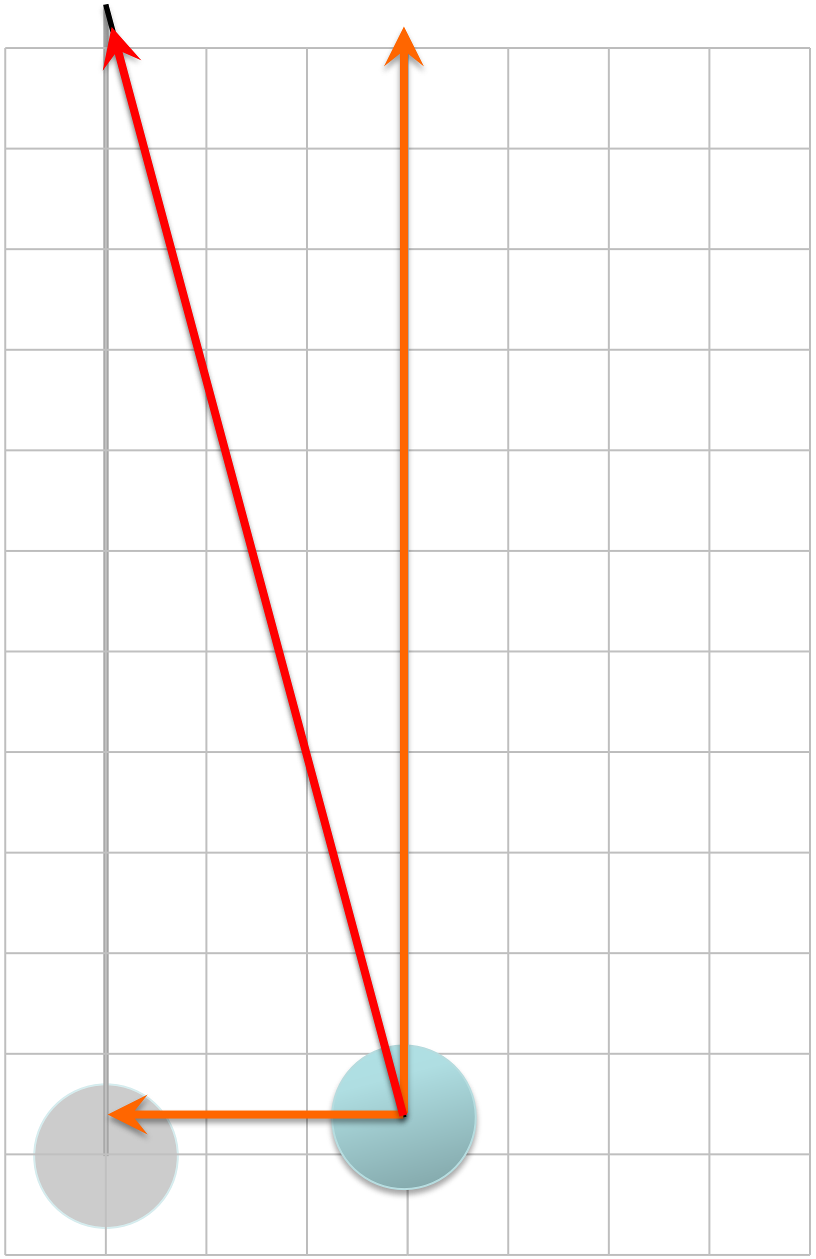


Simple Pendulum

The red arrow is tension in the string.

Orange arrows are the components of the tension.

Comparing this page to the previous two, note that the *x*-component of the tension force changes in proportion to the *x*-coordinate of the object's position. The *y*-component of tension has changed very little.

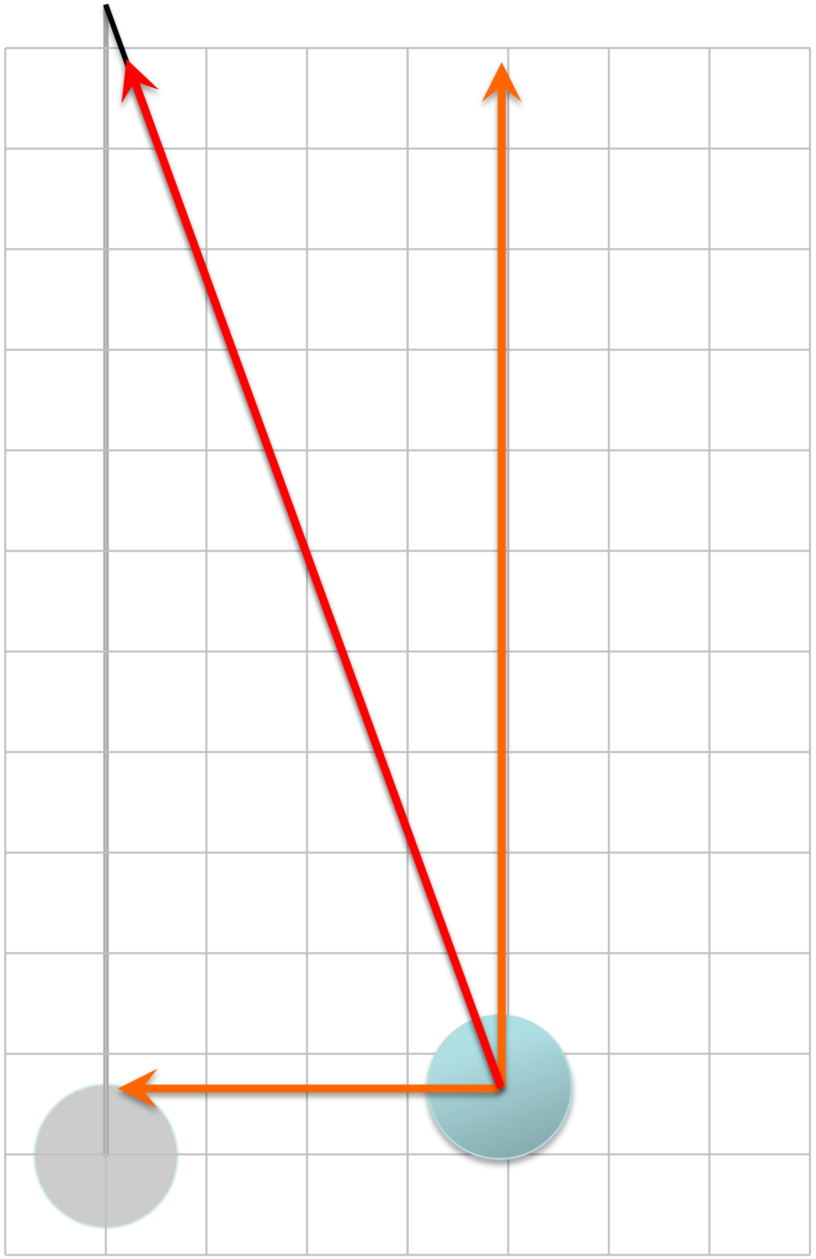


Simple Pendulum

The red arrow is tension in the string.

Orange arrows are the components of the tension.

Once the pendulum reaches an angle as great as this there is a barely noticeable deviance from the apparent direct proportion between T_x and x that was observed at smaller angles.

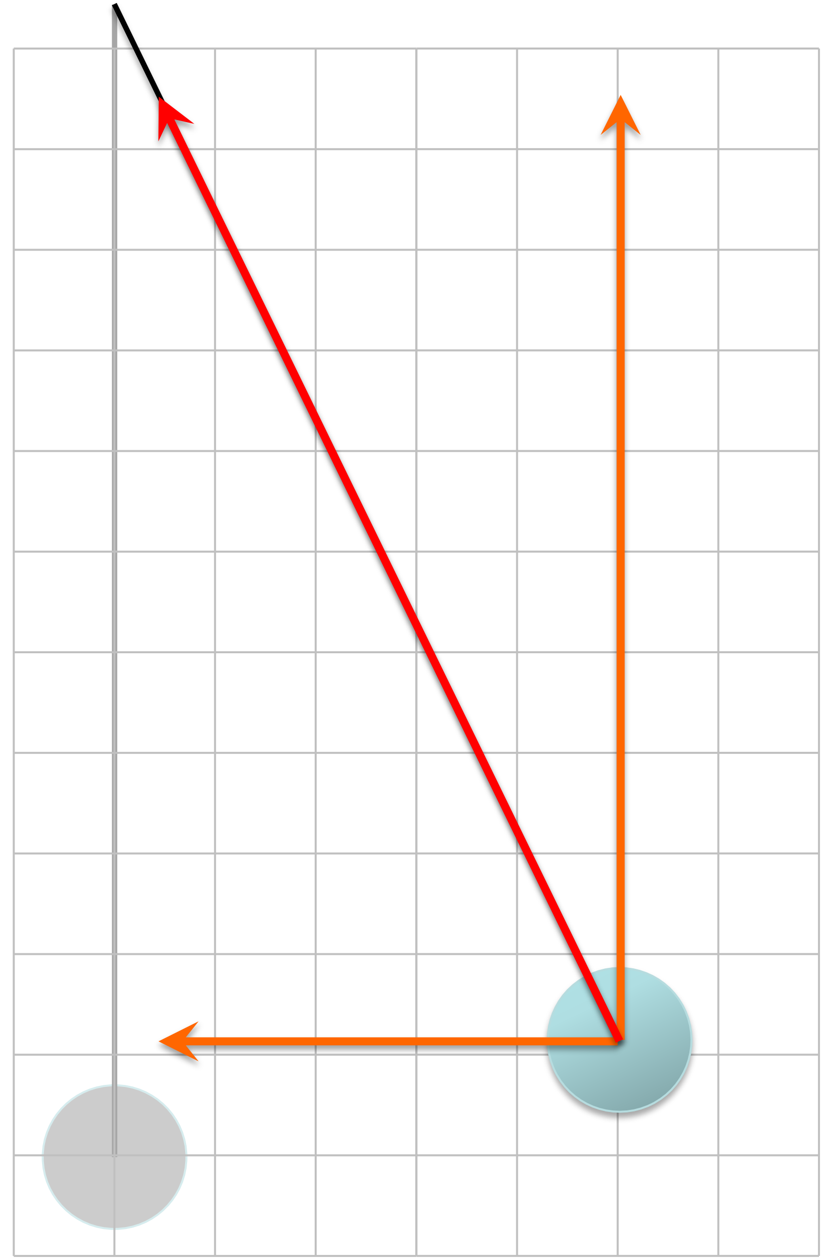


Simple Pendulum

The red arrow is tension in the string.

Orange arrows are the components of the tension.

The deviance from the apparent direct proportion observed at smaller angles is now quite obvious – the component T_x is clearly less than 5 times as great here, at x-coordinate = 5, than it was at x-coordinate = 1.

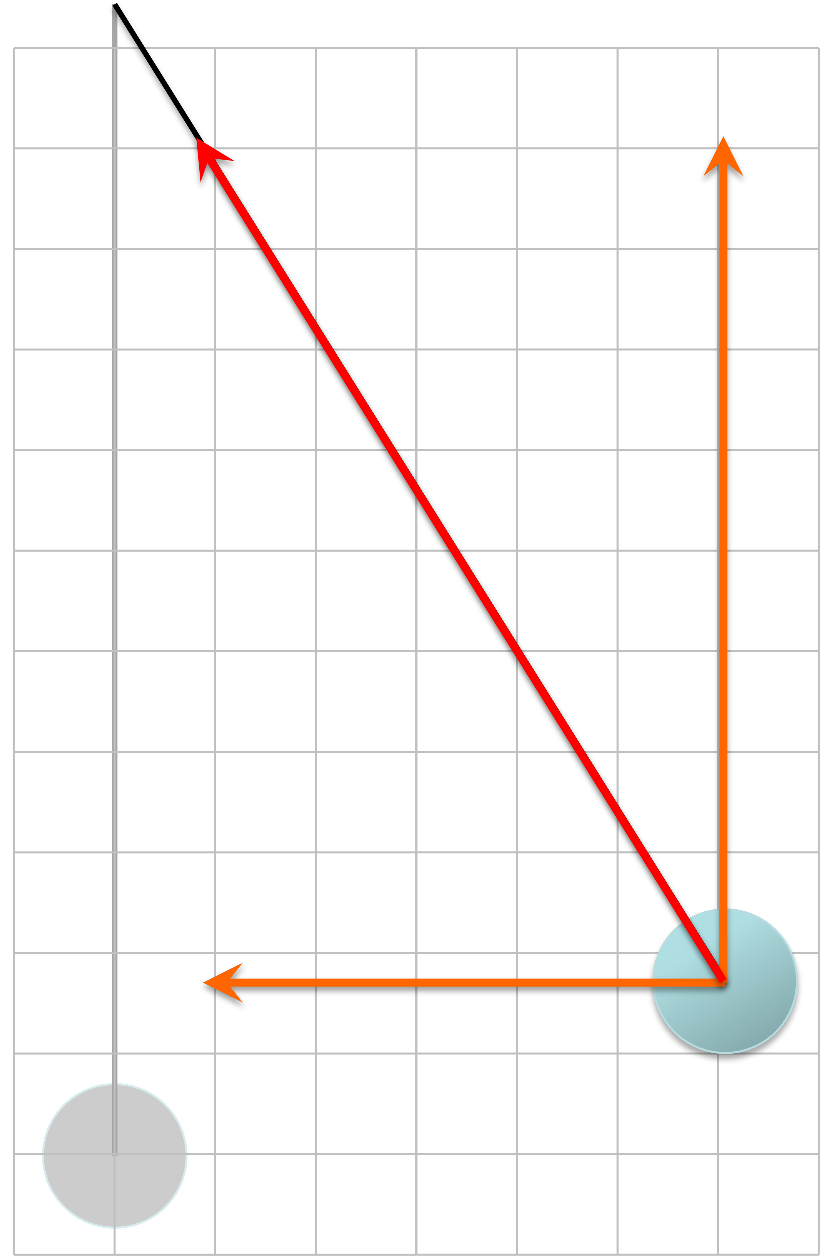


Simple Pendulum

The red arrow is tension in the string.

Orange arrows are the components of the tension.

The value of the component T_y continues to decrease as the angle increases. At no point is it equal to the weight of the object, because a pendulum is accelerating at all times.

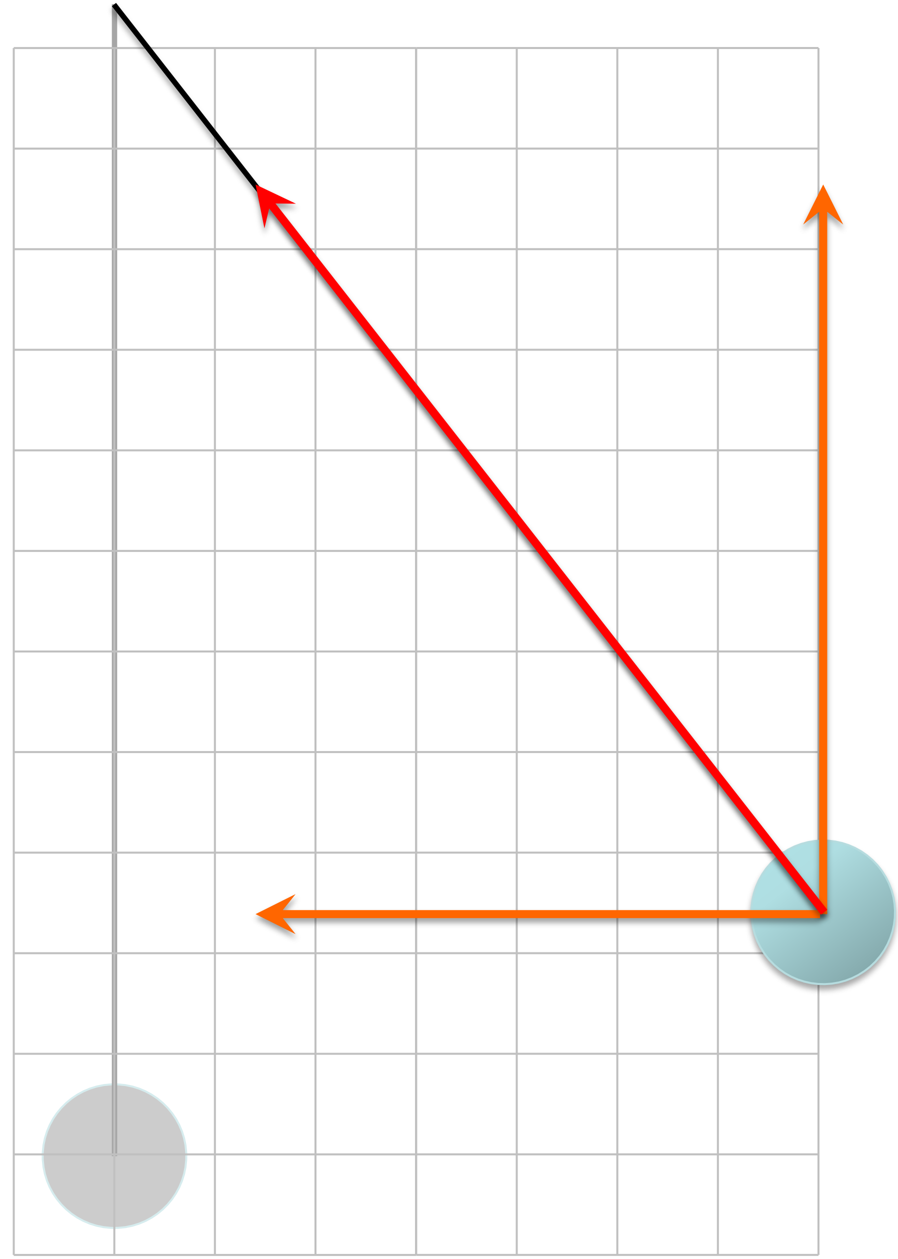


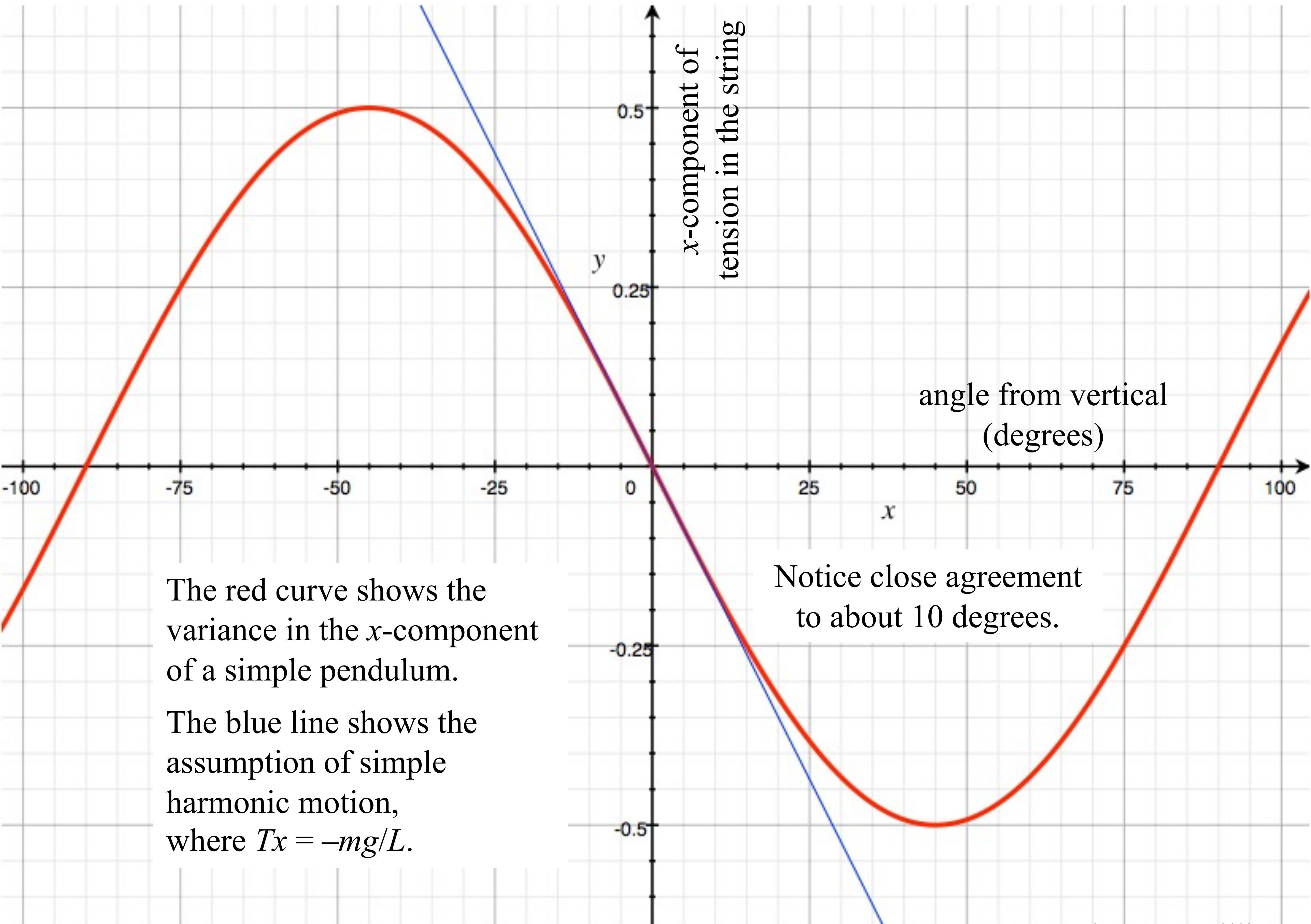
Simple Pendulum

The red arrow is
tension in the string.

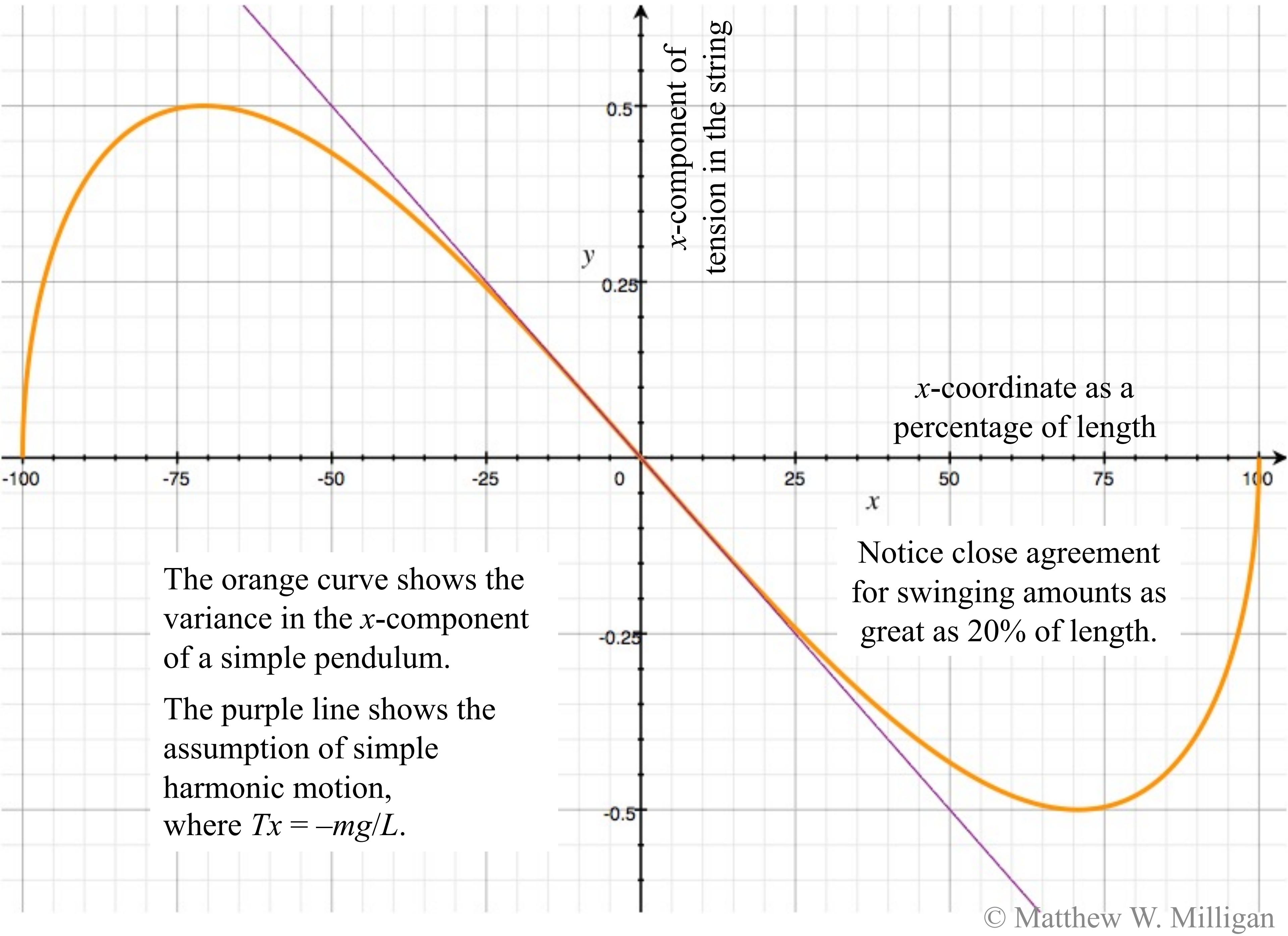
Orange arrows are the components of the tension.

Only if a pendulum swings a relatively small amount is it a good approximation to assume the vertical acceleration is zero and the net force is proportional to displacement from equilibrium.





0.5
0.25
-0.25
-0.5
y
x-component of
tension in the string
-100
-75
-50
-25
0
25
50
75
100
x
angle from vertical
(degrees)
The red curve shows the
variance in the x-component
of a simple pendulum.
The blue line shows the
assumption of simple
harmonic motion,
where Tx = –mg/L.
Notice close agreement
to about 10 degrees.


x-component of tension in the string
y
0.5
0.25
-0.25
-0.5
-100
-75
-50
-25
0
25
50
75
100
x
x-coordinate as a percentage of length
The orange curve shows the variance in the x-component of a simple pendulum.
The purple line shows the assumption of simple harmonic motion, where Tx = –mg/L.
Notice close agreement for swinging amounts as great as 20% of length.
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