The Sun's Wobble

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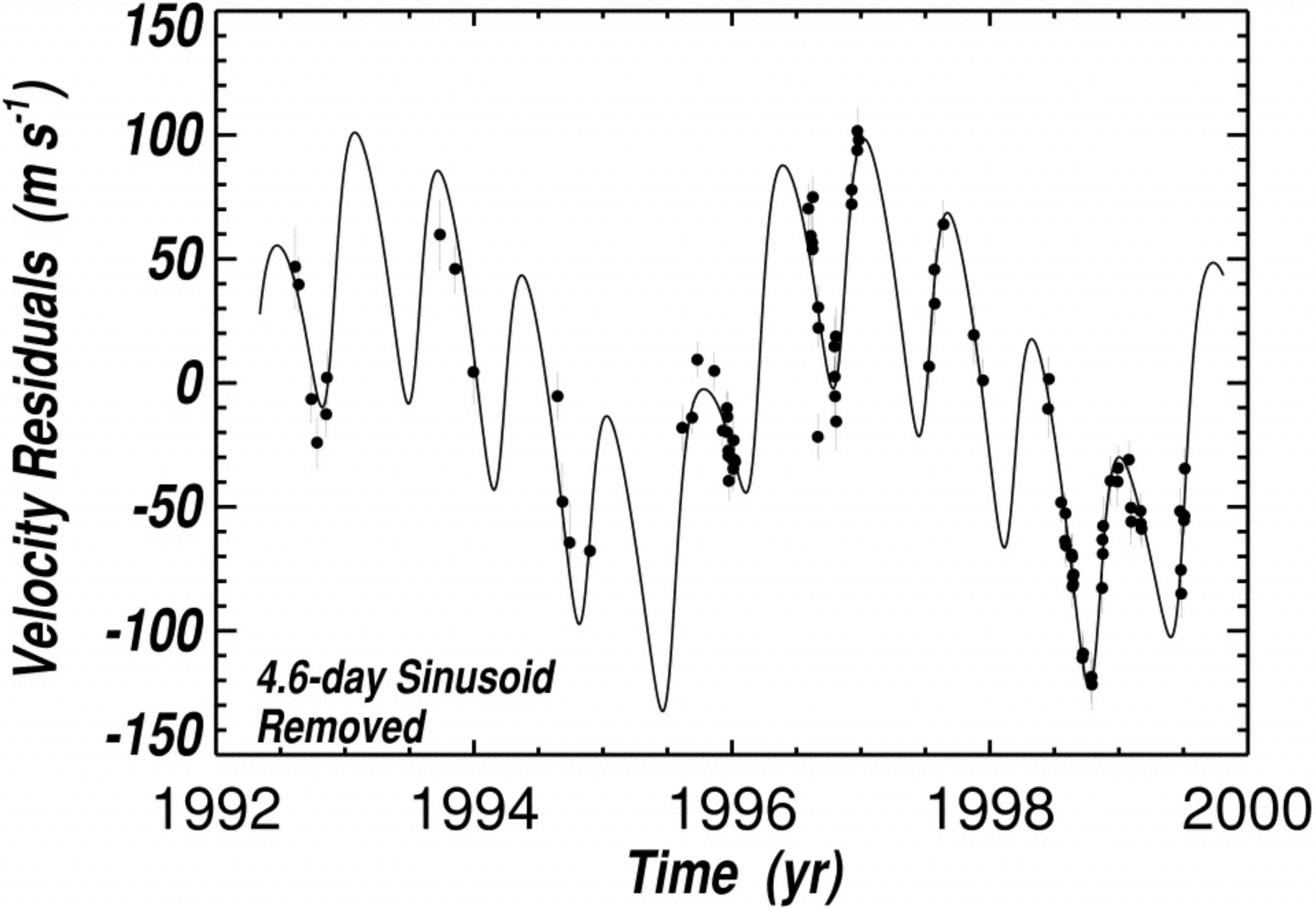
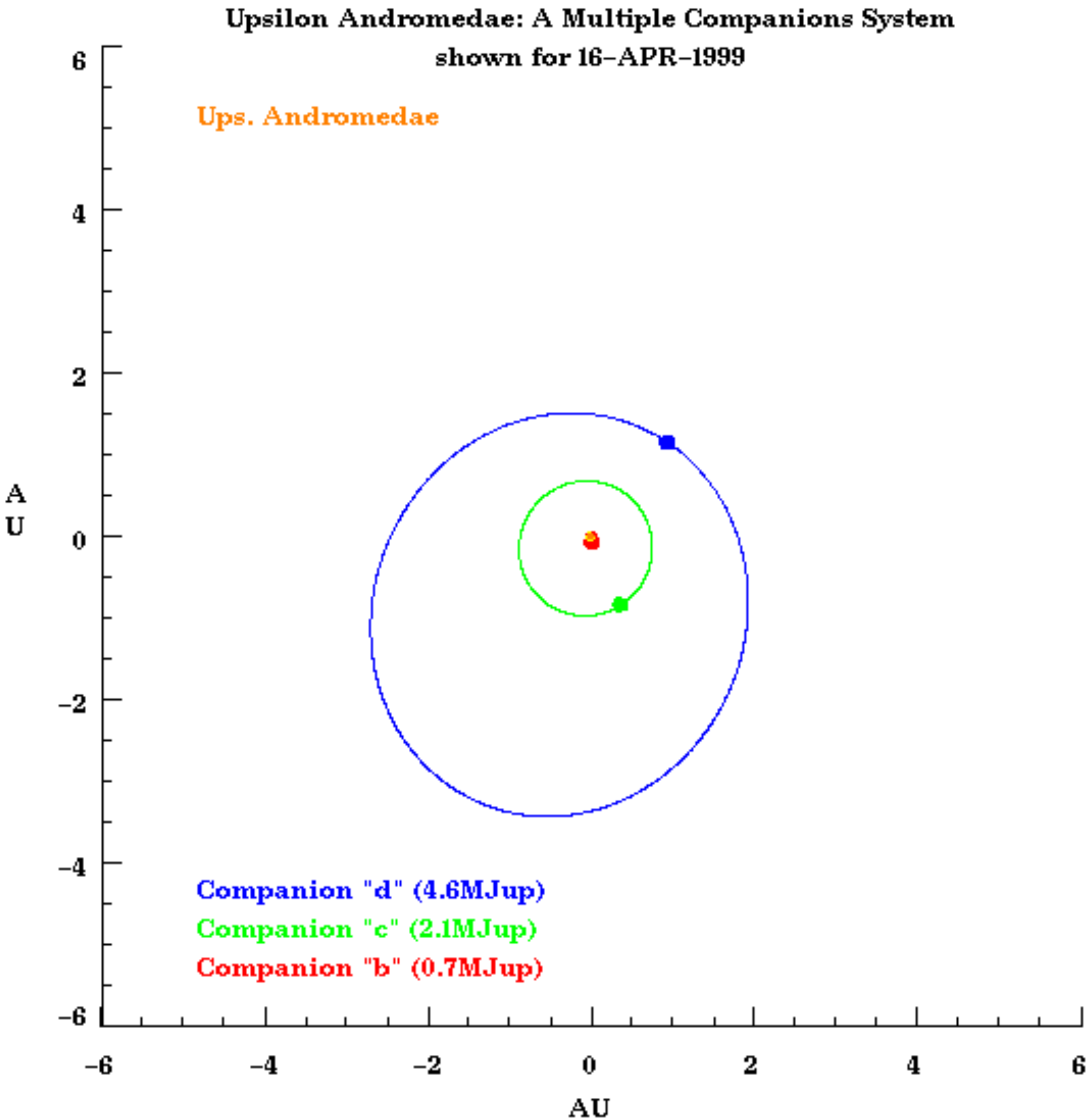
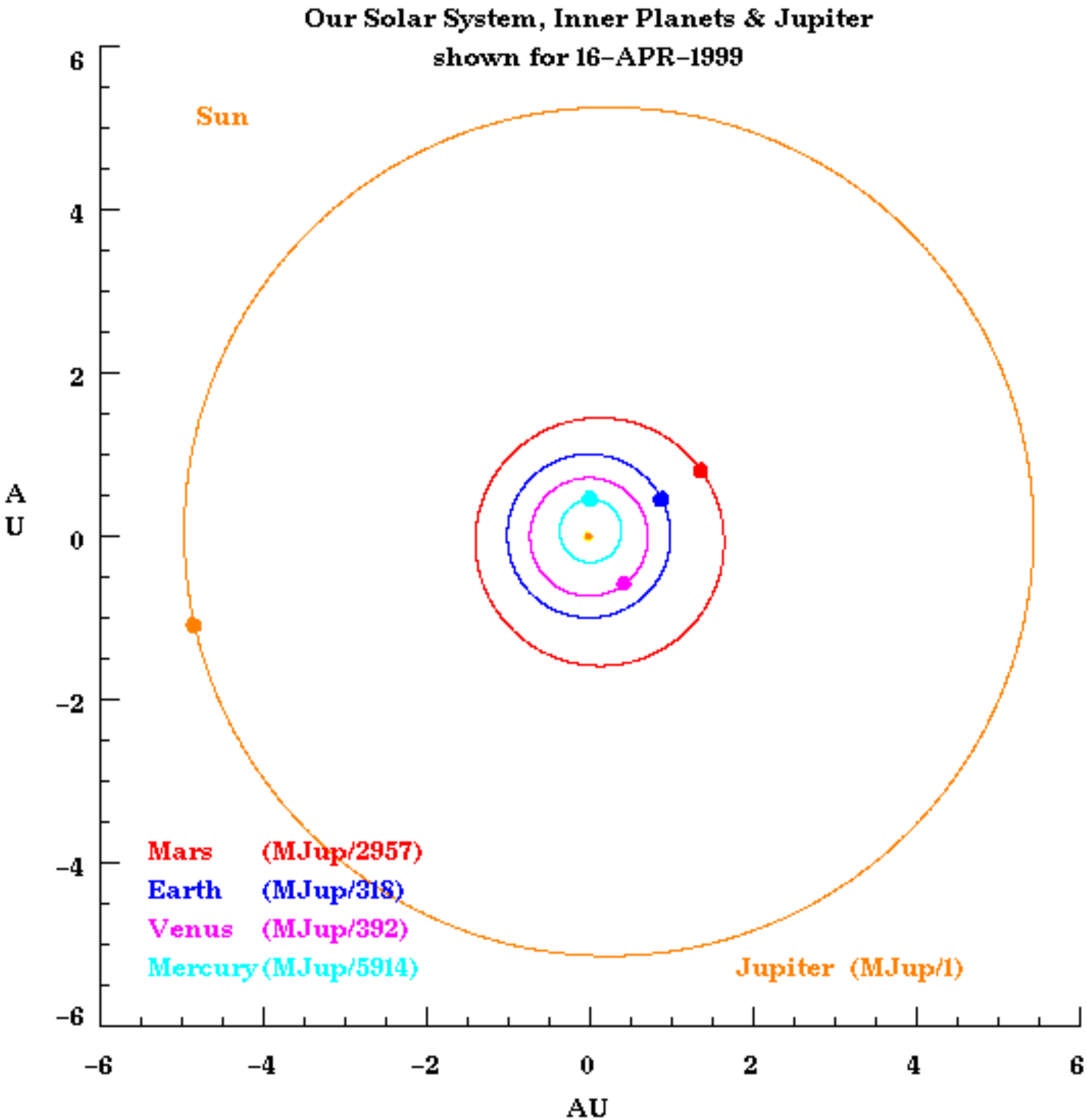


Velocity Residuals (m s-1)
150
100
50
0
-50
-100
-150
4.6-day Sinusoid
Removed
1992
1994
1996
1998
2000
Time (yr)

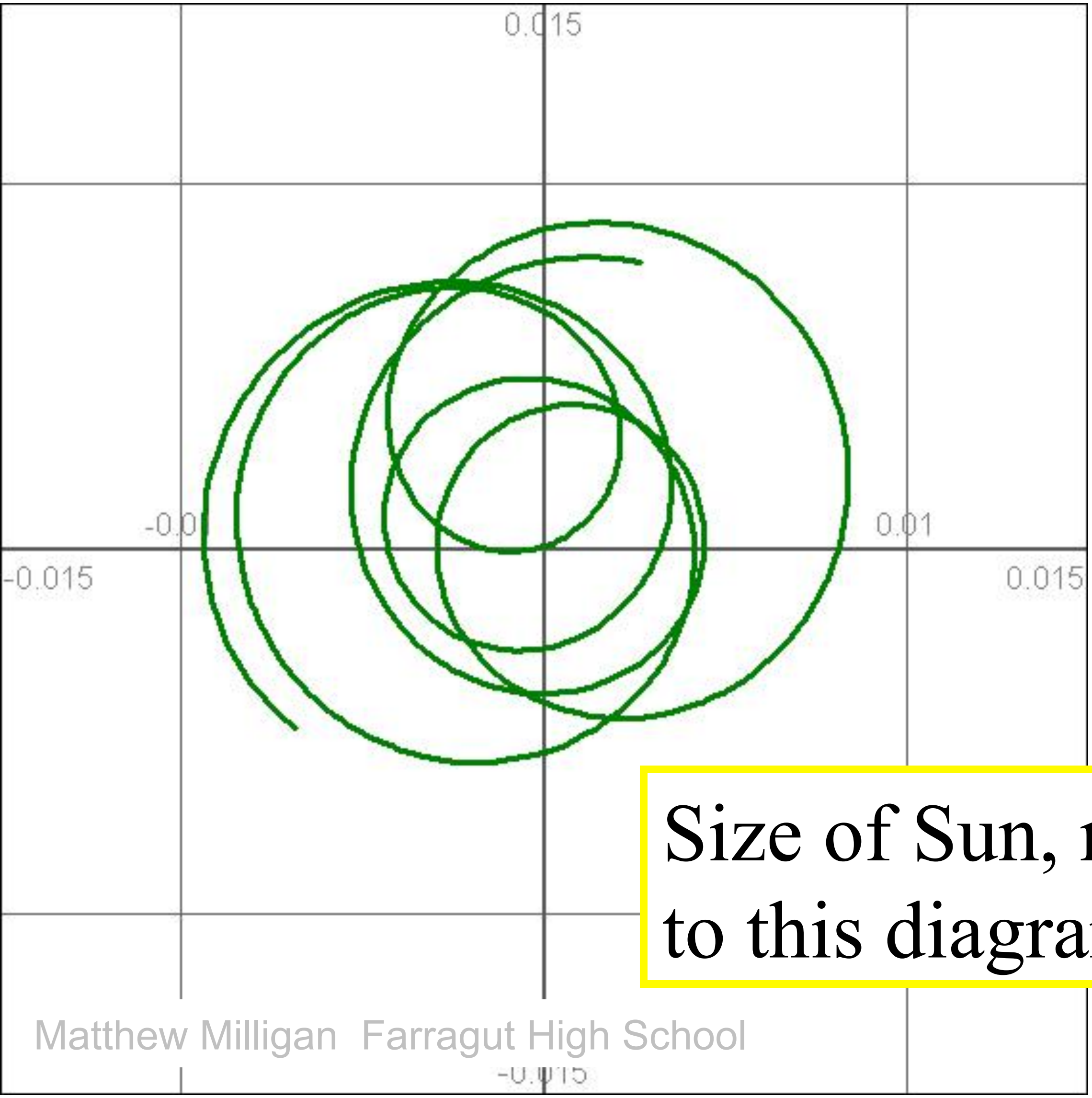

Upsilon Andromedae: A Multiple Companions System
shown for 16–APR–1999
Ups. Andromedae
Companion "d" (4.6MJup)
Companion "c" (2.1MJup)
Companion "b" (0.7MJup)
AU
AU
6
4
2
0
–2
–4
–6

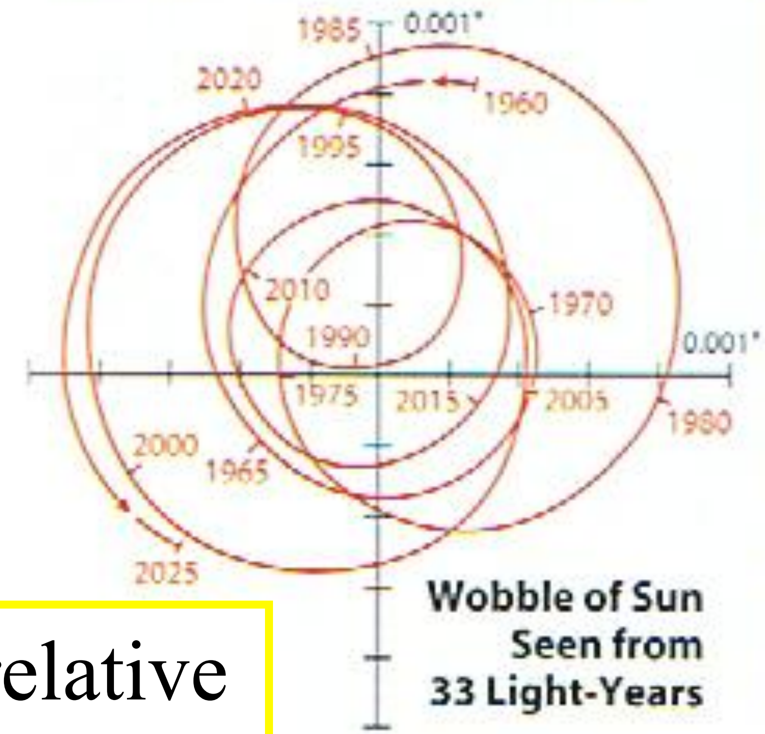

Our Solar System, Inner Planets & Jupiter
shown for 16–APR–1999
Sun
A
U
6
4
2
0
−2
−4
−6
AU
−6
−4
−2
0
2
4
6
Mars (MJup/2957)
Earth (MJup/318)
Venus (MJup/392)
Mercury (MJup/5914)
Jupiter (MJup/1)

As shown in the May 2003 issue of Sky and Telescope magazine this diagram shows how the Sun "wobbles" due to the combined effect of the gravity of each of the nine planets.

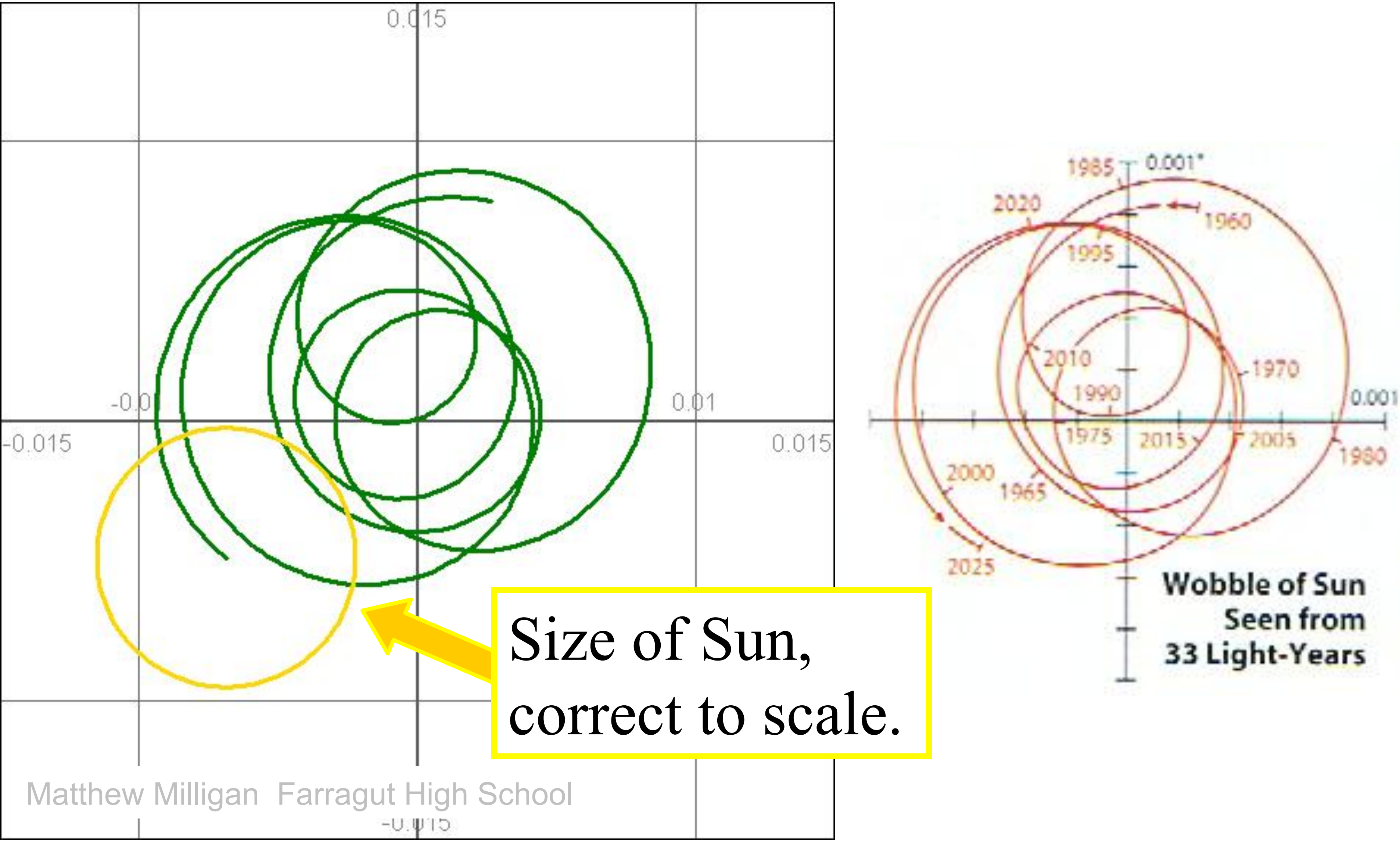
NASA's Space Interferometry Mission (SIM), scheduled for launch in 2009, will perform astrometric measurements of stars to reveal minuscule sideways wobbles caused by planets as small as several Earth masses. *Below:* The wobble of our Sun due to its orbiting planets (mostly Jupiter and Saturn) as would be seen from 33 light-years away. SIM should have a precision approaching 1 microarcsecond, a thousandth the length of each axis here.

Challenge:
Use AP level physics
to produce this graph!





Size of Sun, relative to this diagram = ?


0.015
-0.01
0.01
-0.015
0.015
-0.015
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Size of Sun, correct to scale.
1985
0.001"
2020
1960
1995
2010
1970
1990
0.001"
1975
2015
2005
1980
2000
1965
2025
Wobble of Sun Seen from 33 Light-Years

Which four planets have the greatest *gravitational pull* on the Sun?
(in order…)

1. Jupiter

2. Venus

3. Saturn

4. Earth

Which four planets have the greatest *effect on the position* of the Sun?
(in order…)

1. Jupiter

2. Saturn

3. Neptune

4. Uranus

Mass and Orbital Radius (relative to Earth)

	m	r	$m \cdot r$	m/r^2
Mercury	0.0553	0.387	0.0214	0.369
Venus	0.815	0.723	0.59	1.56
Earth	1.00	1.00	1.00	1.00
Mars	0.107	1.52	0.163	0.0461
Jupiter	318	5.20	1650	11.8
Saturn	95.1	9.54	907	1.04
Uranus	14.5	19.2	279	0.0395
Neptune	17.1	30.1	513	0.0189
Pluto	0.0021	39.5	0.0829	0.000001

	$r \cdot m$	m/r^2
Mercury	0.0214	0.369
Venus	0.59	1.56
Earth	1.00	1.00
Mars	0.163	0.0461
Jupiter	1650	11.8
Saturn	907	1.04
Uranus	279	0.0395
Neptune	513	0.0189
Pluto	0.0829	0.000001

In order of gravitational force on Sun…

	$r \cdot m$	m/r^2
Jupiter	1650	11.8
Venus	0.59	1.56
Saturn	907	1.04
Earth	1.00	1.00
Mercury	0.0214	0.369
Mars	0.163	0.0461
Uranus	279	0.0395
Neptune	513	0.0189
Pluto	0.0829	0.000001

In order of influence on wobble of Sun…

	$r \cdot m$	m/r^2
Jupiter	1650	11.8
Saturn	907	1.04
Neptune	513	0.0189
Uranus	279	0.0395
Earth	1.00	1.00
Venus	0.59	1.56
Mars	0.163	0.0461
Pluto	0.0829	0.000001
Mercury	0.0214	0.369

But why does the value of *rm* matter?

Modeling the Sun's Wobble:

- Assume the solar system to be an isolated system with no external forces.
- The center of mass of the system should not accelerate relative to distant stars based on this assumption.
- The internal forces of gravitation have no effect on the center of mass.
- At all points in time the Sun and planets must be positioned such that the center of mass maintains a "constant" position.

$$\vec{r}_{CM} = \frac{\Sigma m\vec{r}}{\Sigma m}$$

$$0 = \frac{m_{sun}\vec{r}_{sun} + \left(\Sigma m\vec{r}\right)_{planets}}{\Sigma m}$$

$$0 = m_{sun}\vec{r}_{sun} + \left(\Sigma m\vec{r}\right)_{planets}$$

$$m_{sun}\vec{r}_{sun} = -\left(\Sigma m\vec{r}\right)_{planets}$$

Graphing the Sun's Wobble:

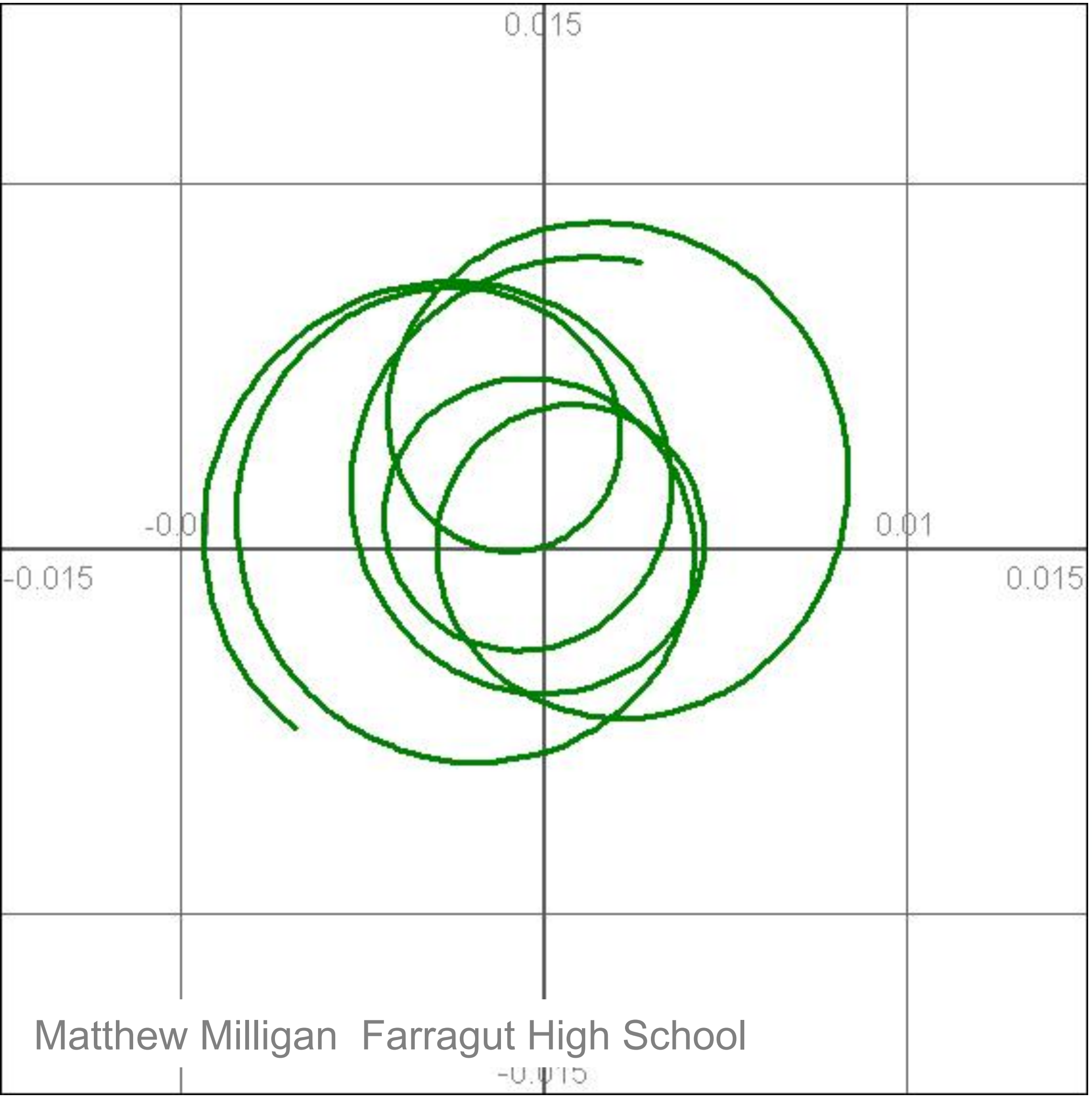
- Let the center of mass be the origin of a coordinate system
- Assume the planets move in perfect circles about this origin (not true)
- Model the motion of each planet by parametric equations of the form:
  $x = r\cos(\omega t + \delta)$ and $y = r\sin(\omega t + \delta)$
- Set $x_{cm} = 0$ and $y_{cm} = 0$ and then solve for the x and y coordinates of the Sun's position

$$m_{sun}\vec{r}_{sun} = -\left(\sum m\vec{r}\right)_{planets}$$

$$\vec{r}_{sun} = \frac{-\left(\sum m\vec{r}\right)_{planets}}{m_{sun}}$$

$$x_{sun} = -\frac{m_1 r_1 \cos\left(\omega_1 t + \delta_1\right) + m_2 r_2 \cos\left(\omega_2 t + \delta_2\right) + \ldots}{m_{sun}}$$

$$y_{sun} = -\frac{m_1 r_1 \sin\left(\omega_1 t + \delta_1\right) + m_2 r_2 \sin\left(\omega_2 t + \delta_2\right) + \ldots}{m_{sun}}$$


0.015
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0.01
-0.015
0.015
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1985
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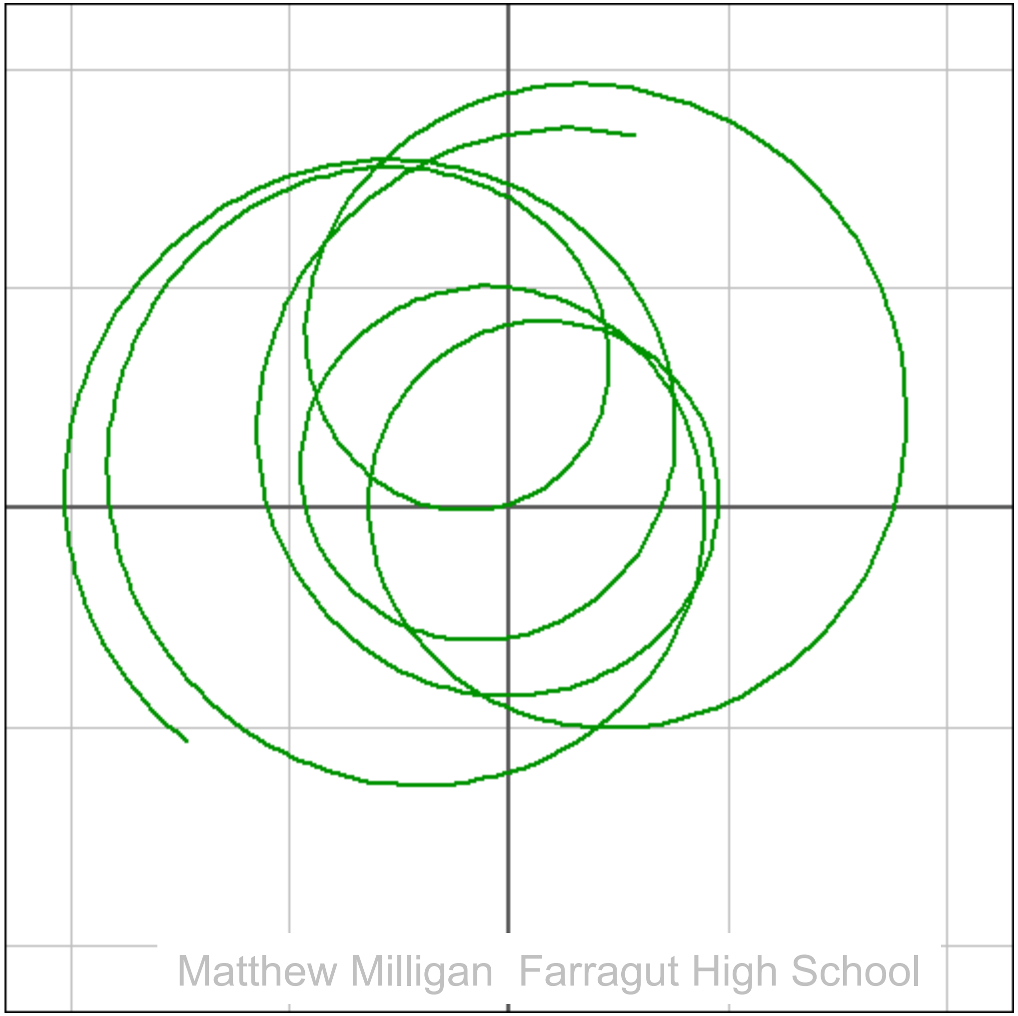
Wobble of Sun
Seen from
33 Light-Years

Sun's Path Including the Effect of:



Venus

Earth

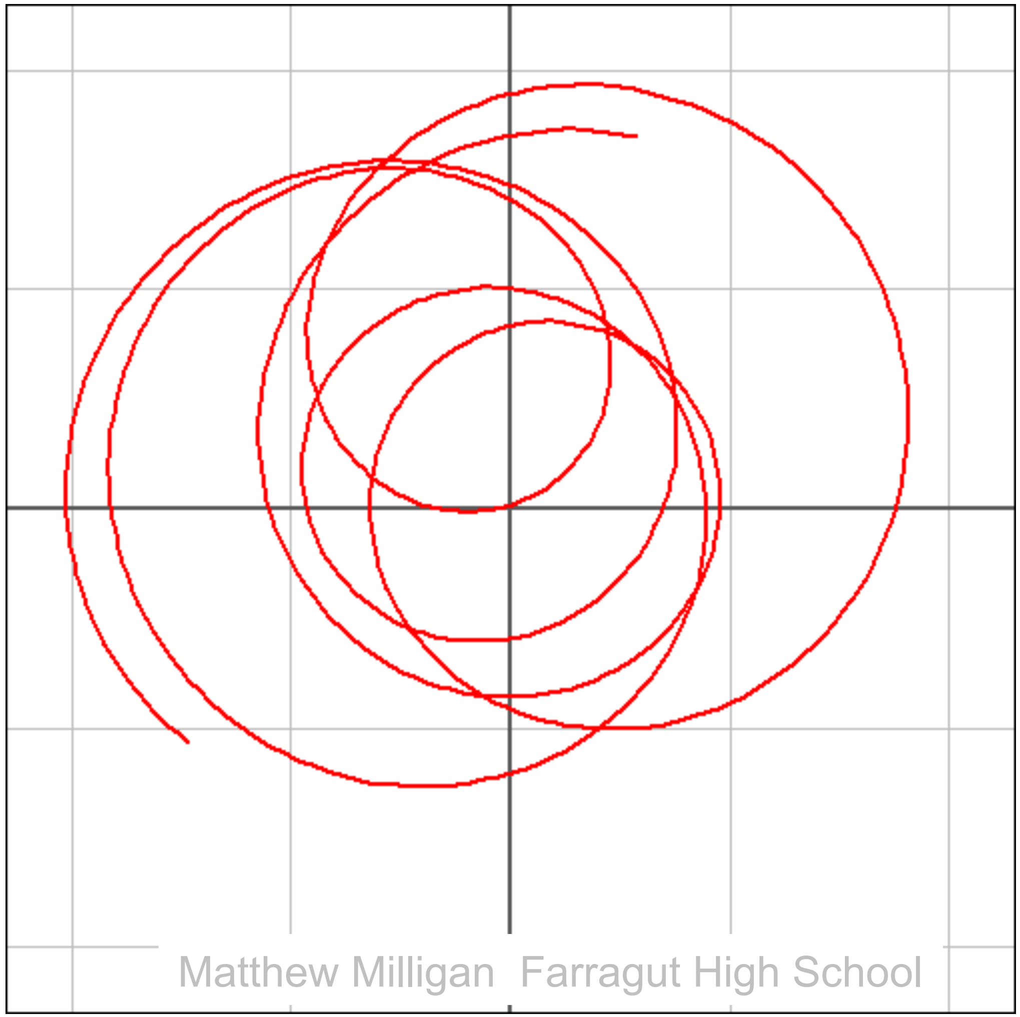
Jupiter

Saturn

Uranus

Neptune

Sun's Path Including the Effect of:



Earth

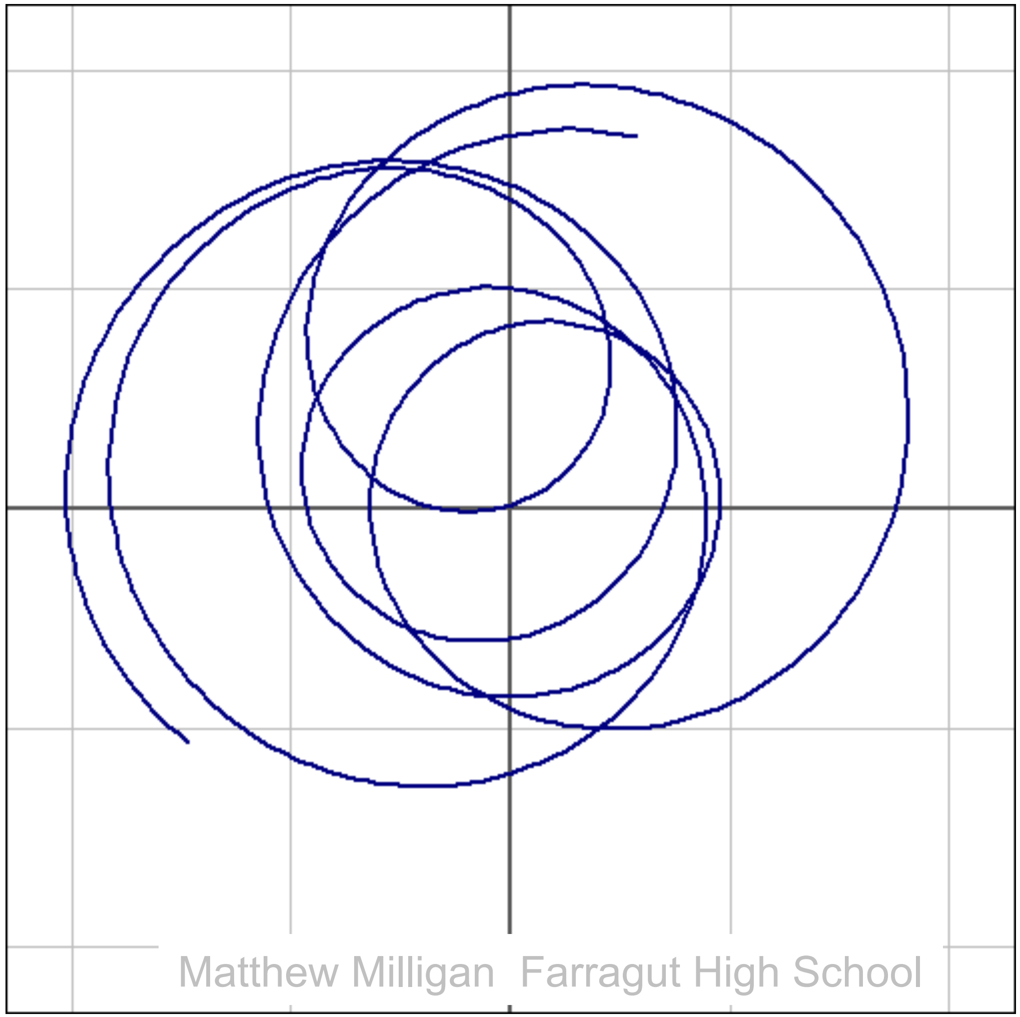
Jupiter

Saturn

Uranus

Neptune

Sun's Path Including the Effect of:



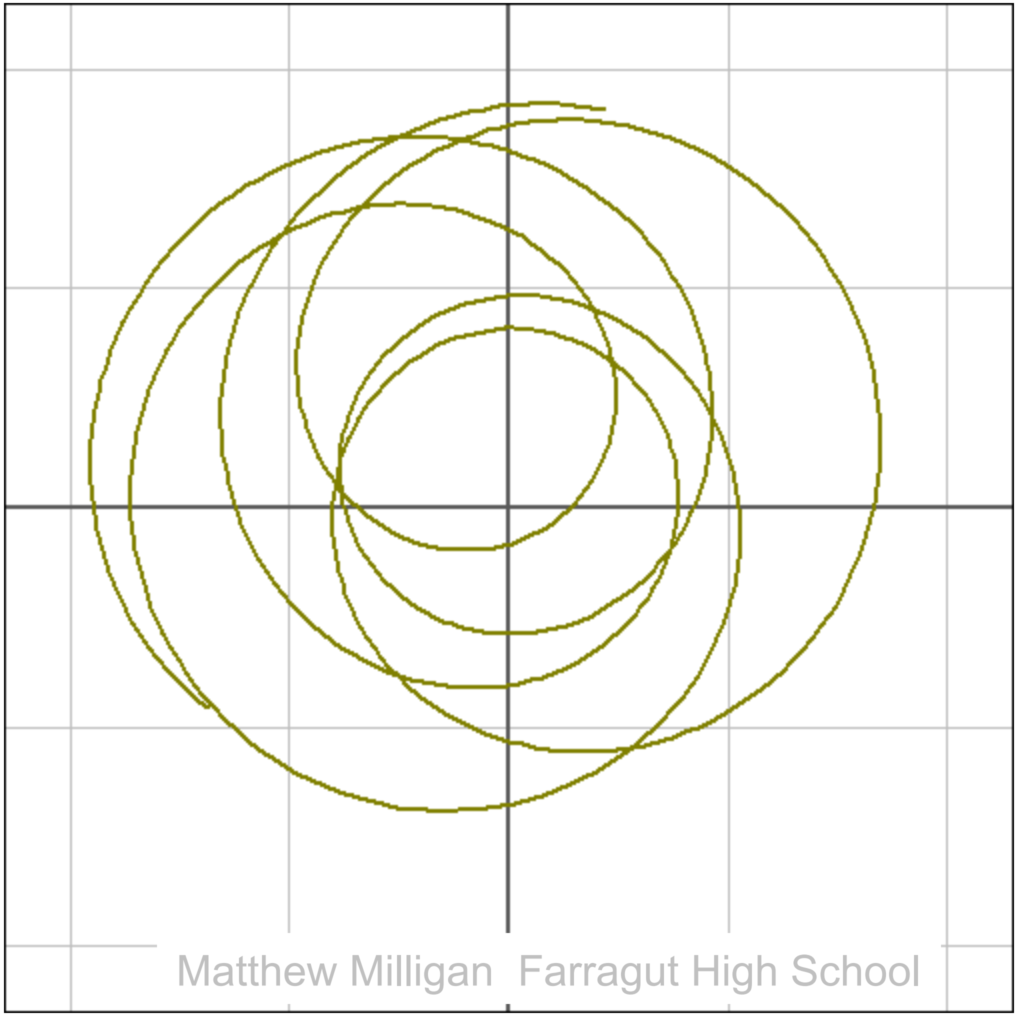
Jupiter

Saturn

Uranus

Neptune

Sun's Path Including the Effect of:

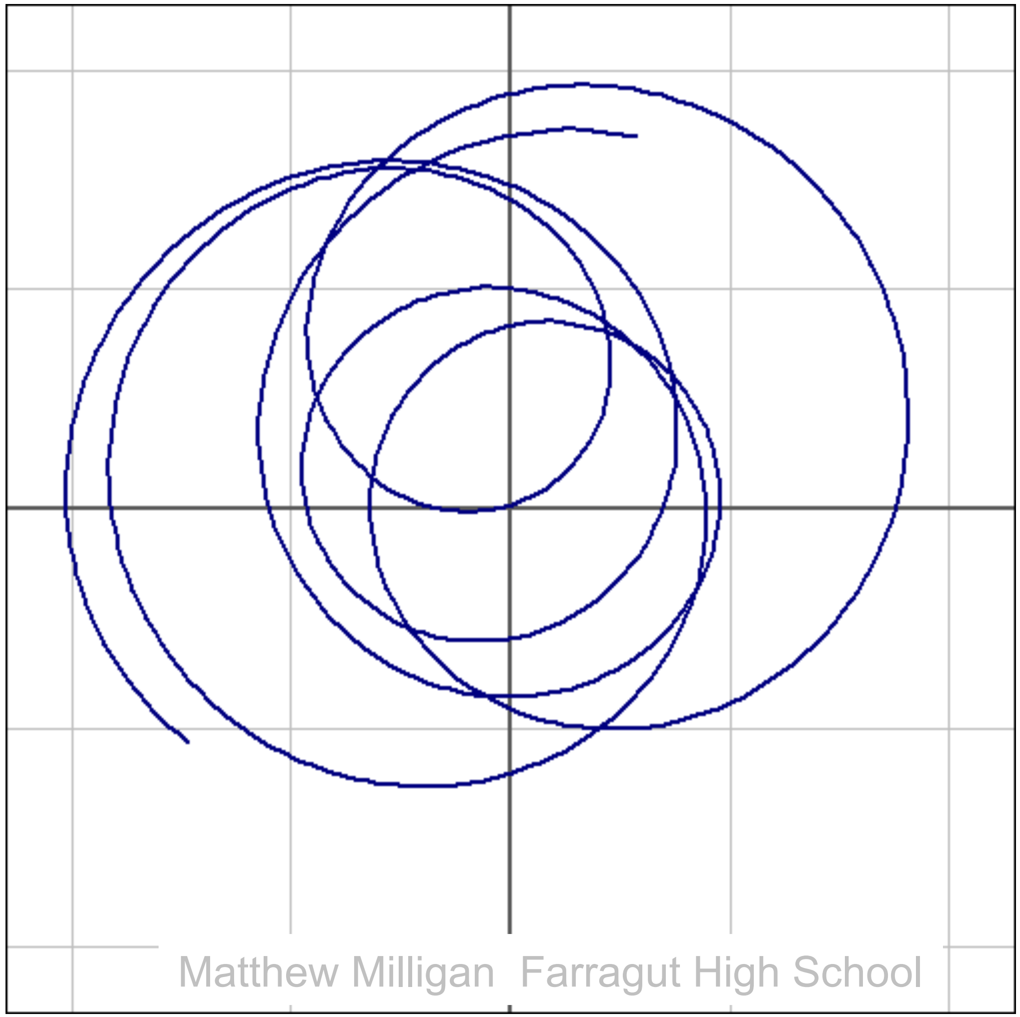


Jupiter

Saturn

Neptune

Sun's Path Including the Effect of:



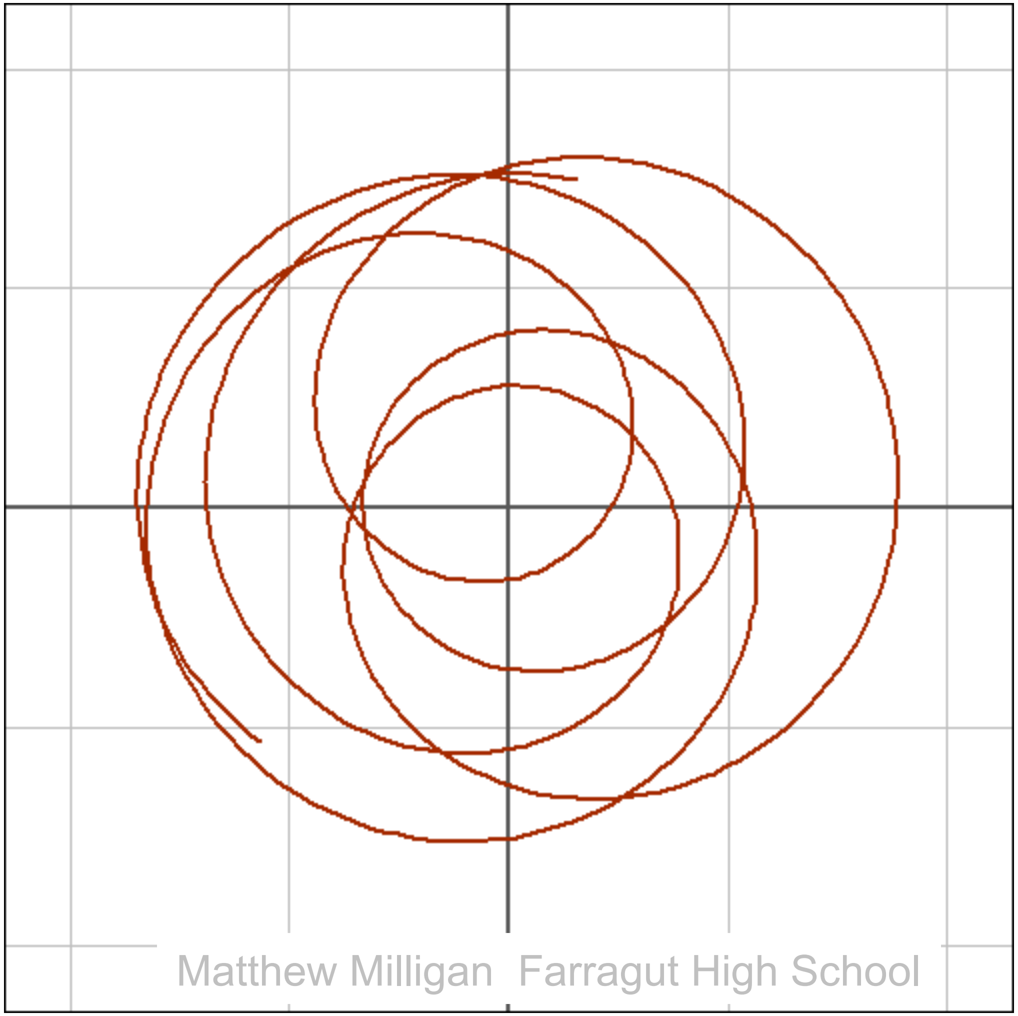
Jupiter

Saturn

Uranus

Neptune

Sun's Path Including the Effect of:

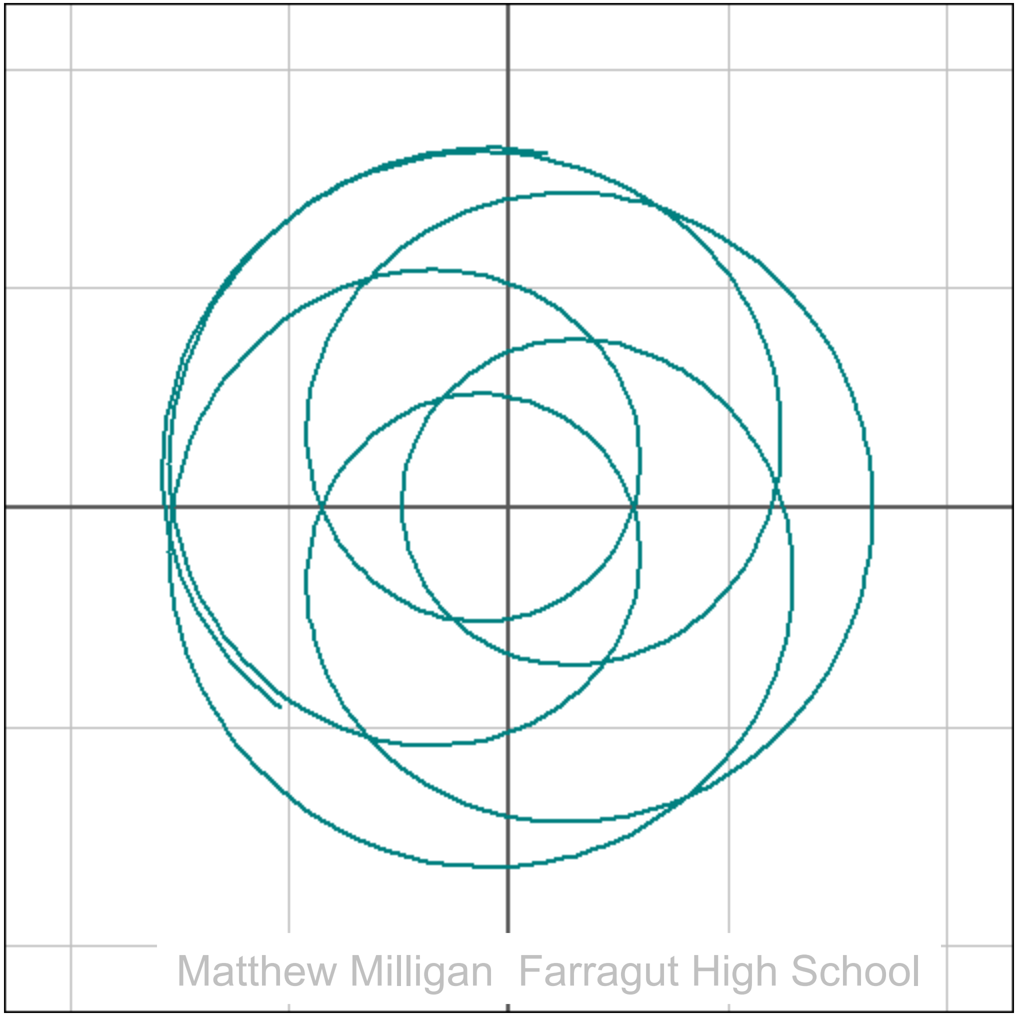


Jupiter

Saturn

Uranus

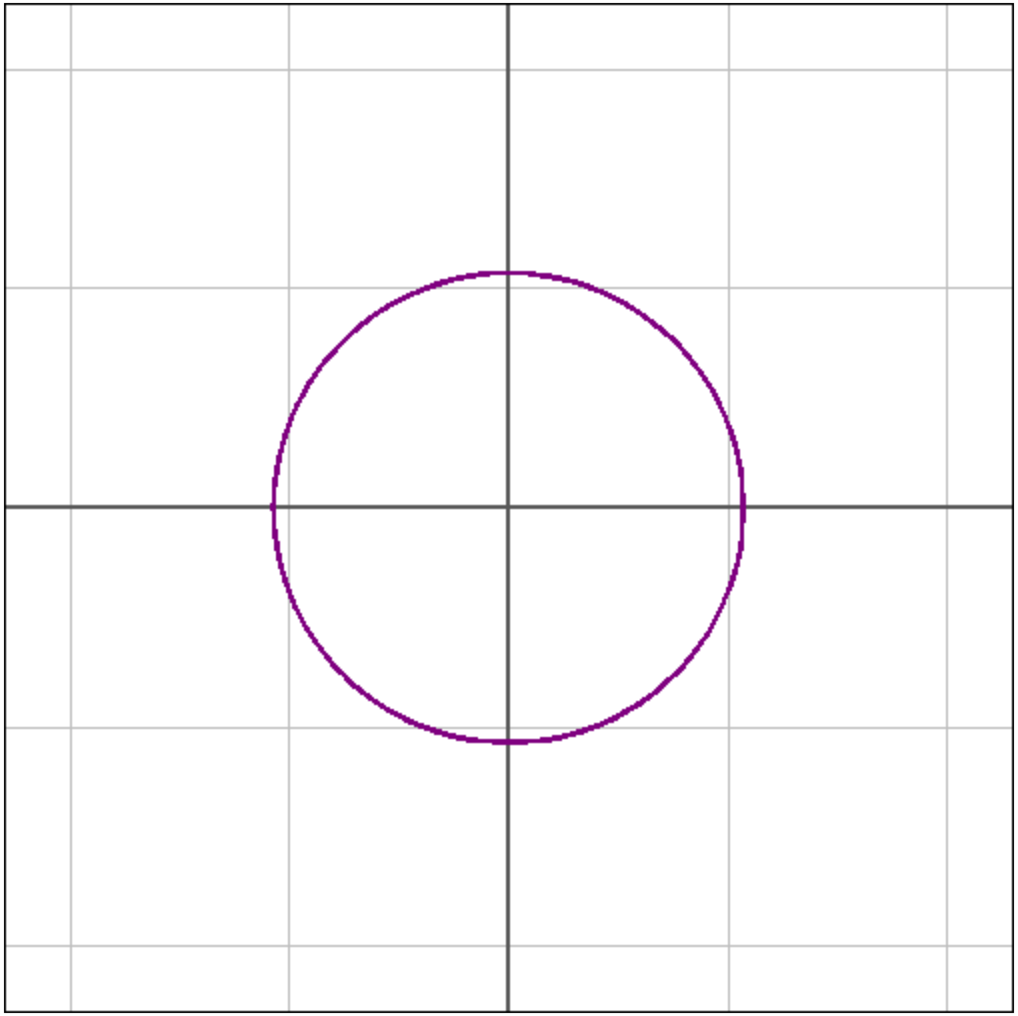
Sun's Path Including the Effect of:



Jupiter

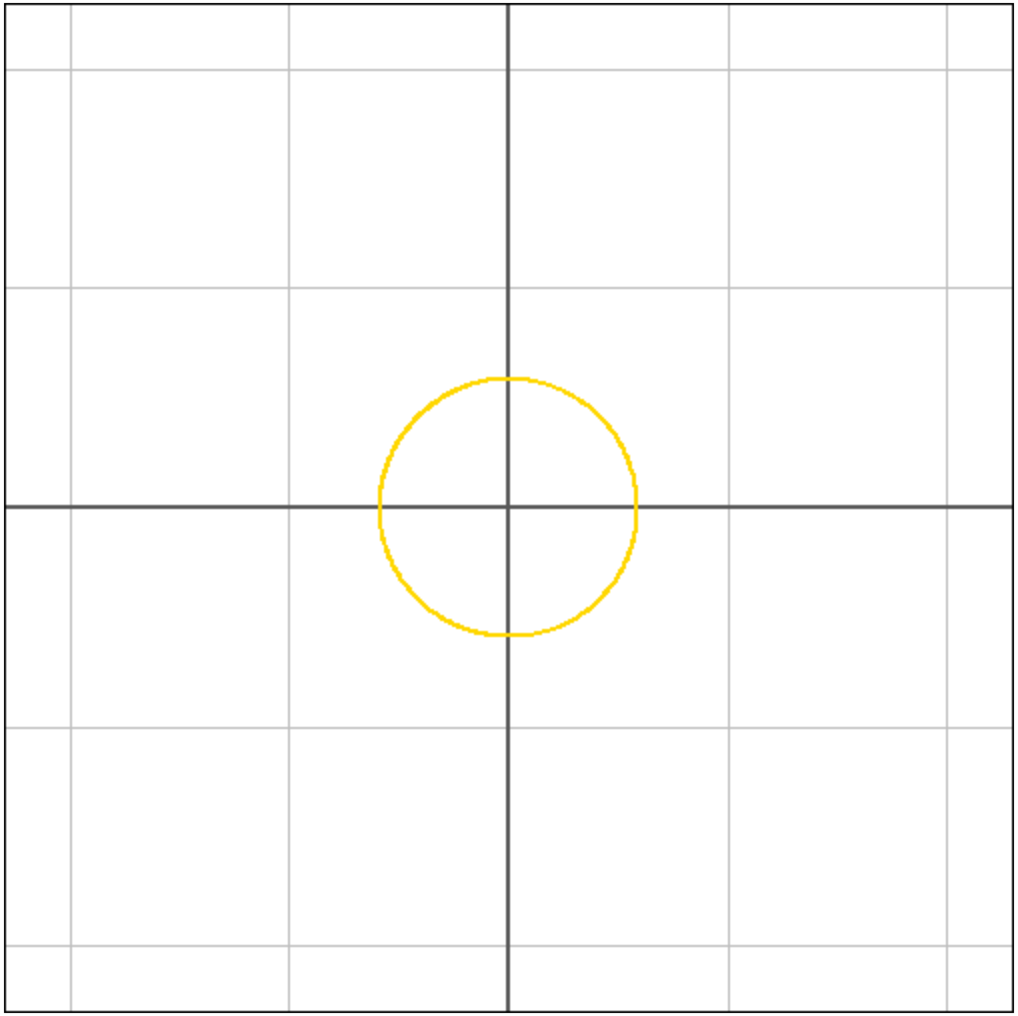
Saturn

Sun's Path Including the Effect of:



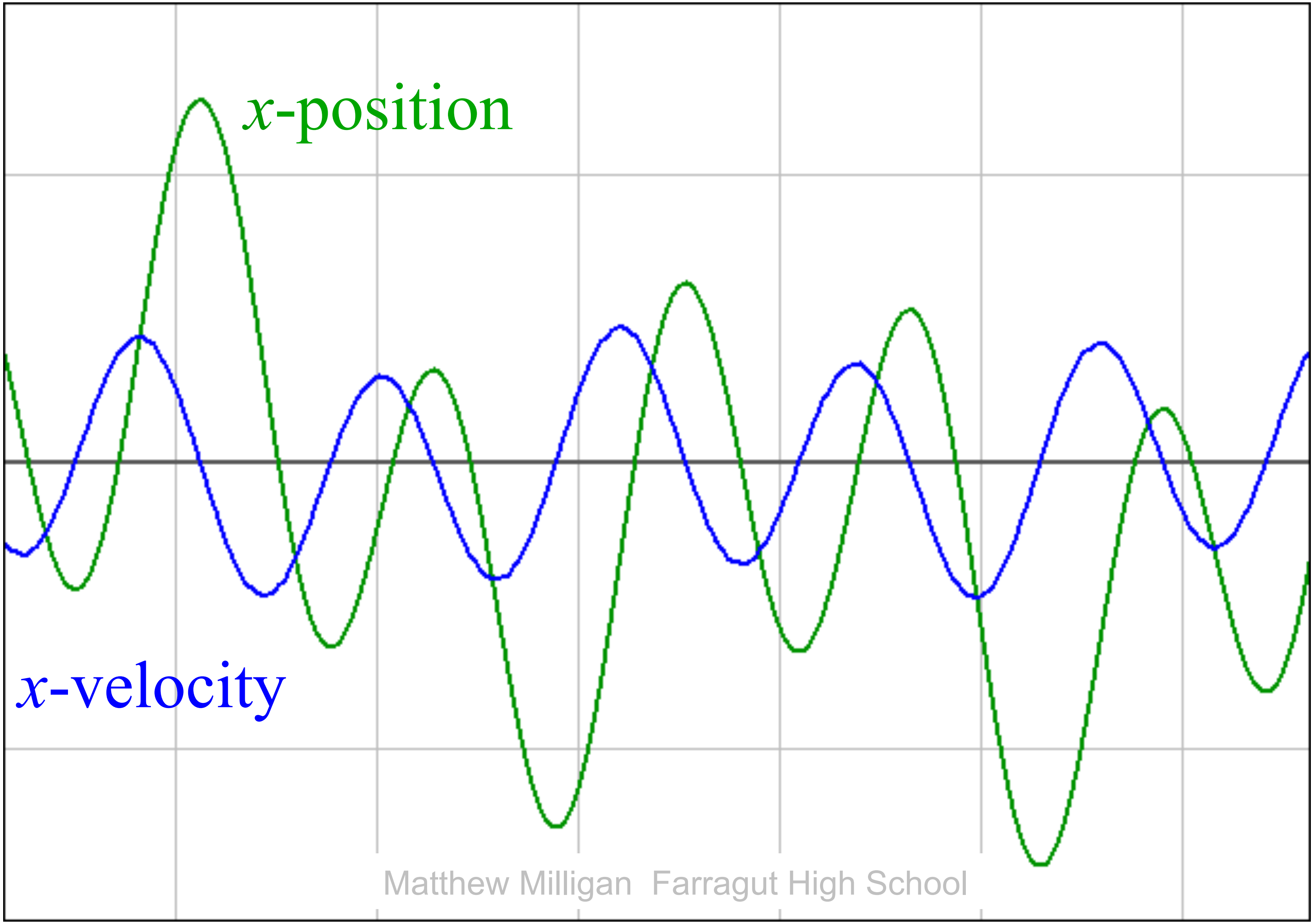
Jupiter

Sun’s Path Including the Effect of:

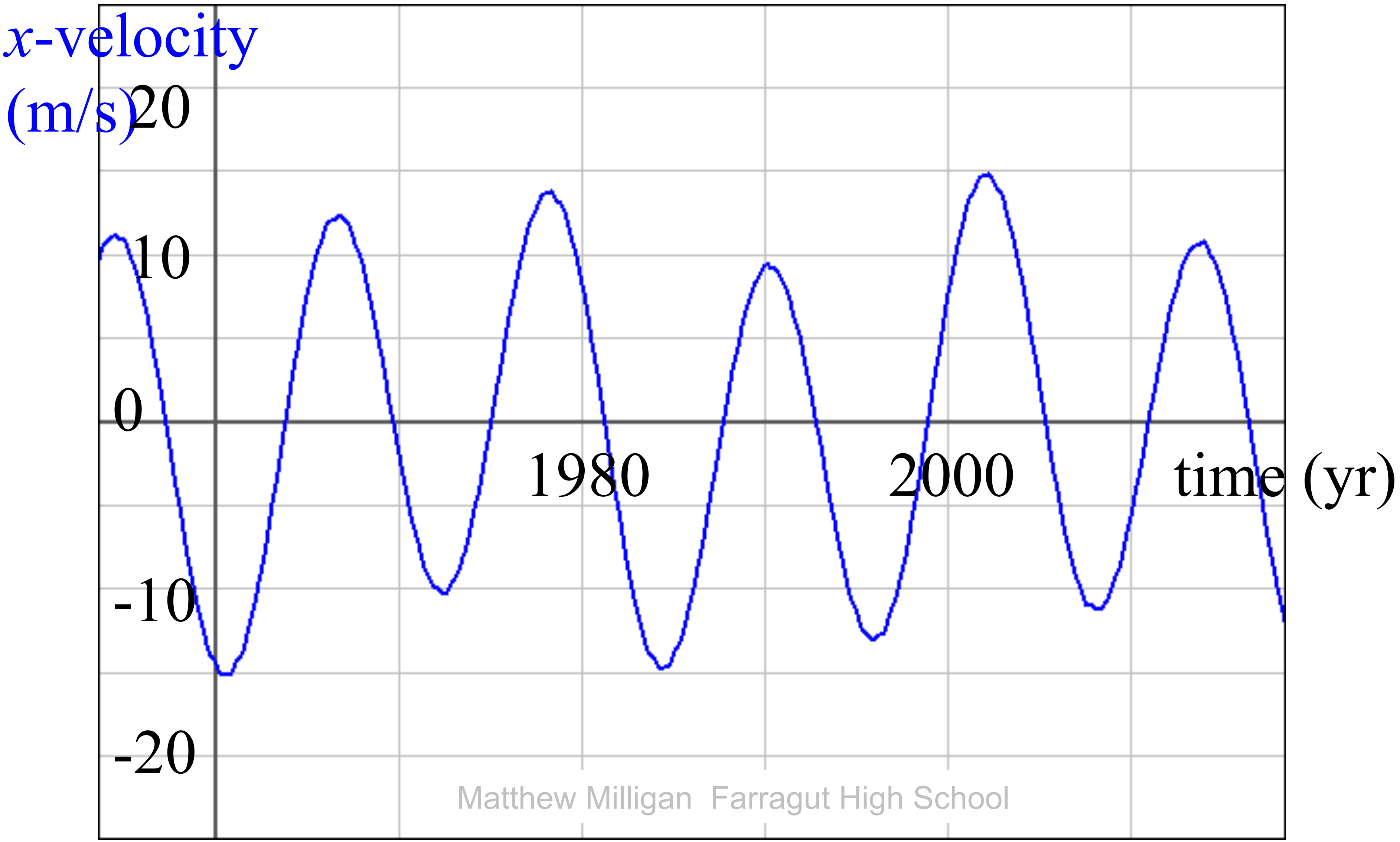


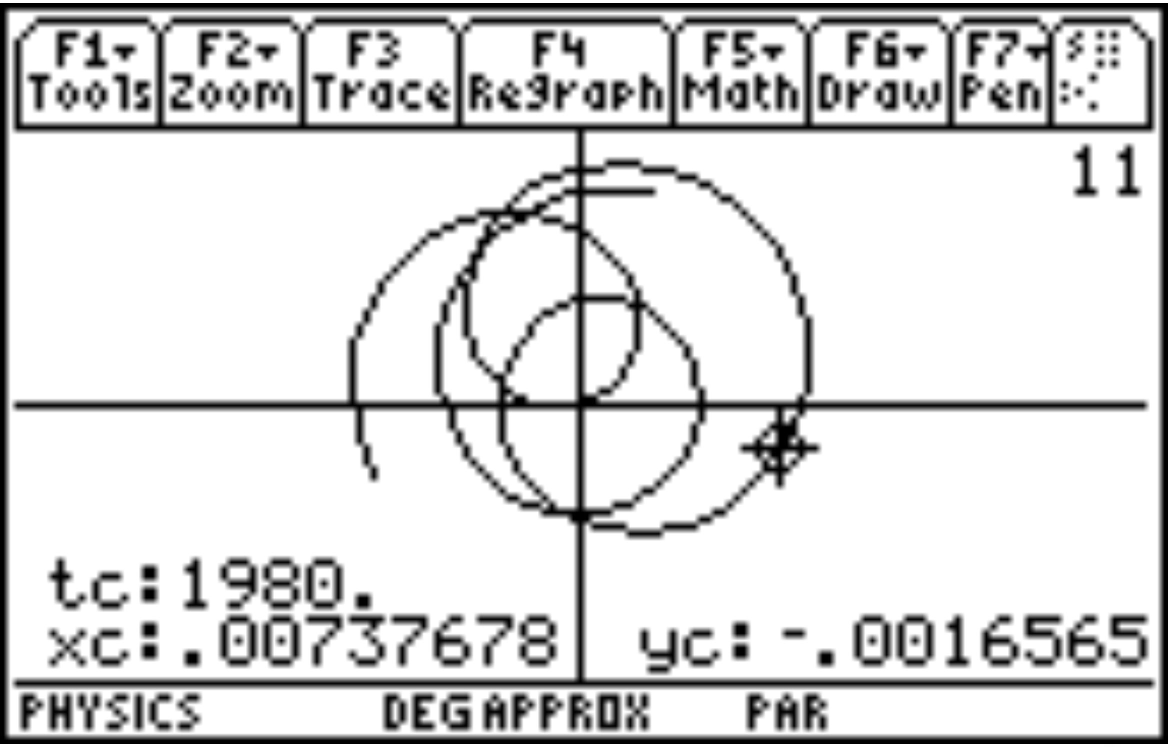
Saturn

x-components of the Sun's Wobble



x-components of the Sun's Wobble

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F3 Trace
F4 Regraph
F5 Math
F6 Draw
F7 Pen
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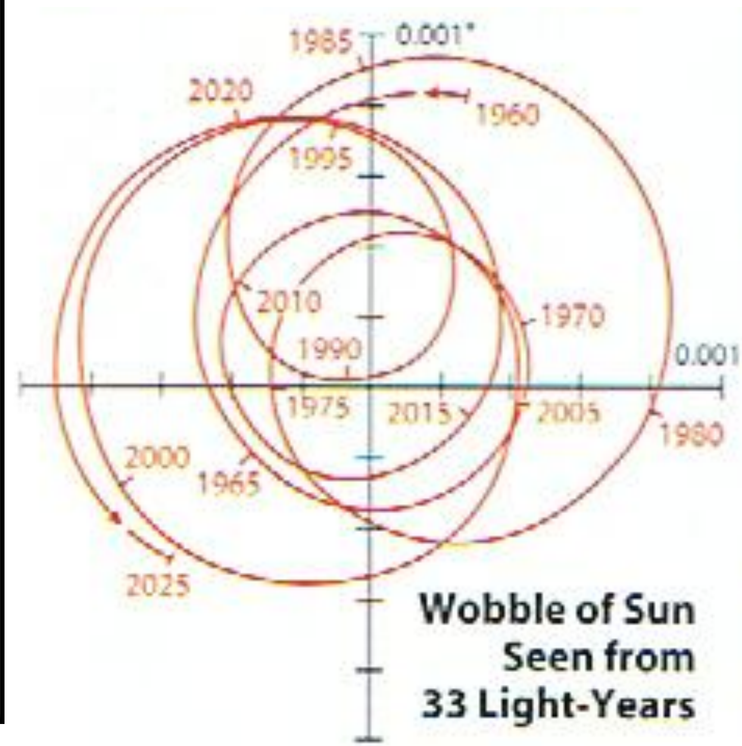
xc:.00737678
yc:-.0016565
PHYSICS
DEG APPROX
PAR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Wobble of Sun
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So, how does one actually create this graphs using a graphing calculator?

Positions of Planets Jan. 1, 1960

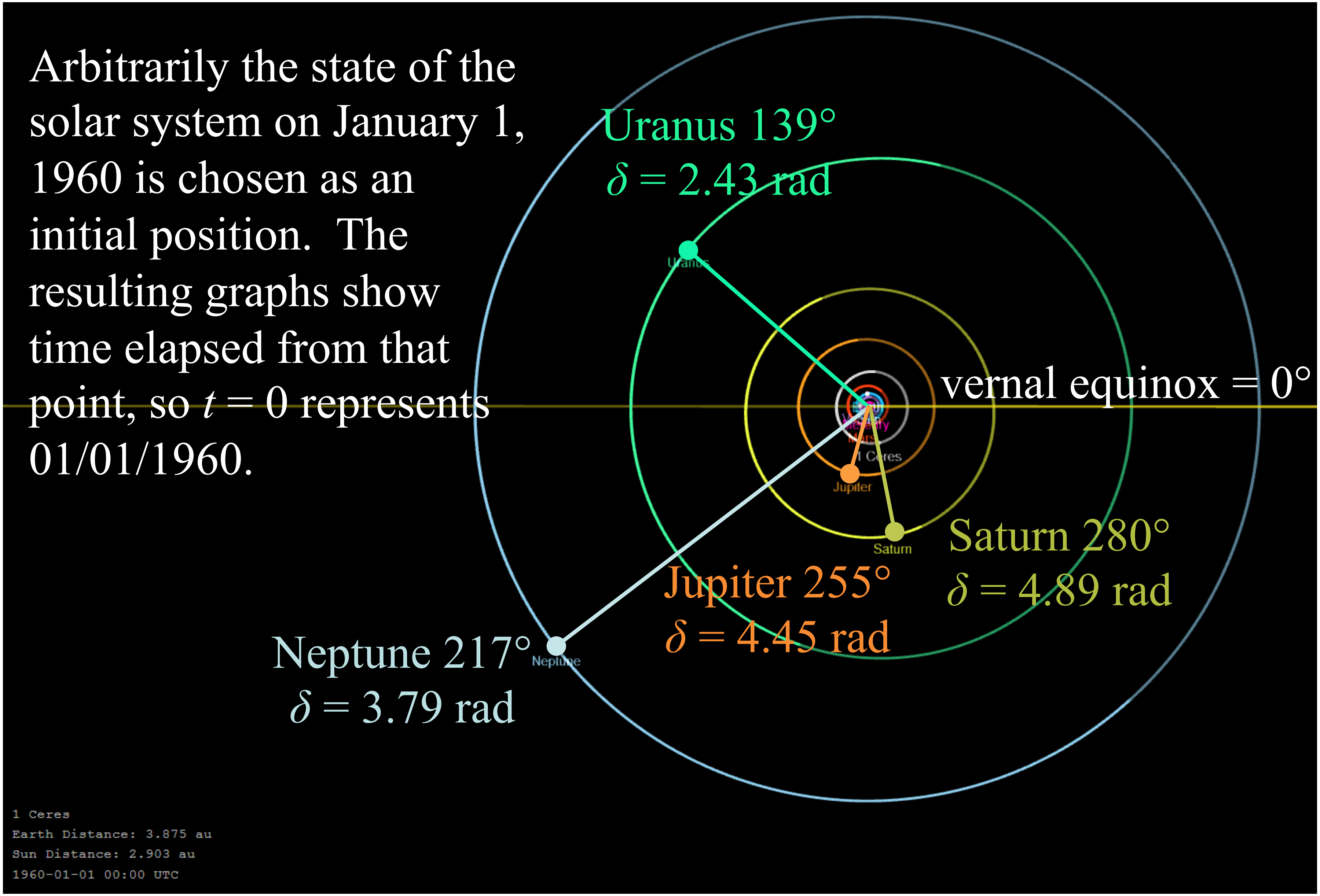
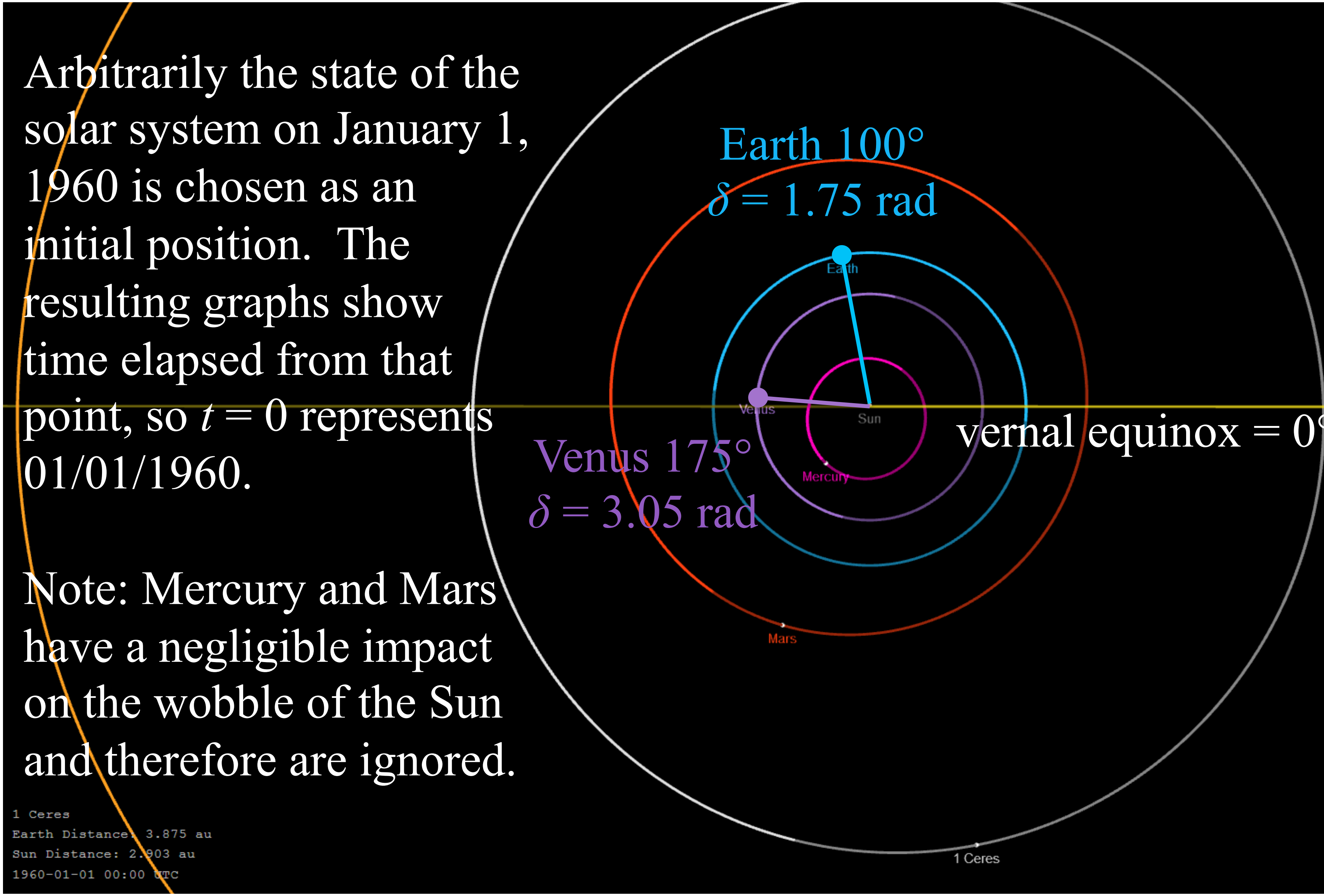


image credit: JPL Solar System Dynamics

Positions of Planets Jan. 1, 1960



Arbitrarily the state of the solar system on January 1, 1960 is chosen as an initial position. The resulting graphs show time elapsed from that point, so $t = 0$ represents 01/01/1960.

Note: Mercury and Mars have a negligible impact on the wobble of the Sun and therefore are ignored.

image credit: JPL Solar System Dynamics

System Data Relative to Earth

	m	r	T	δ
Venus	0.815	0.723	0.615	3.054
Earth	1.00	1.00	1.00	1.745
Jupiter	318	5.20	11.86	4.450
Saturn	95.1	9.54	29.42	4.887
Uranus	14.5	19.2	83.75	2.426
Neptune	17.1	30.1	163.7	3.787

Using TI-Calculator

- Save data into lists of numbers.
  TI-83/84: {0.815,1,318,95.1,14.5,17.1}$\rightarrow L_1$
  TI-89: {0.815,1,318,95.1,14.5,17.1}$\rightarrow m$
- Set mode to Parametric and Radian.
- Enter $x(t)$ and $y(t)$.
  TI-83/84: $x(t) = -\text{sum}(L_1 * L_2 * \cos(2 * \pi * T/L_3 + L_4))$
  TI-89: $x(t) = -\text{sum}(m * r * \cos(2 * \pi * t/T + \delta))$
- Adjust Window parameters: $0 < t < 40$, steps of 0.5 and $-3000 < x$ and $y < 3000$. Then Zoom Square.

	m or L_1	r or L_2	T or L_3	δ or L_4
Venus	0.815	0.723	0.615	3.047
Earth	1.00	1.00	1.00	1.748
Jupiter	318	5.20	11.86	4.460
Saturn	95.1	9.54	29.42	4.887
Uranus	14.5	19.2	83.75	2.428
Neptune	17.1	30.1	163.7	3.798

TI-83/84: $x(t) = -\text{sum}(L_1 * L_2 * \cos(2 * \pi * T / L_3 + L_4))$

TI-89: $x(t) = -\text{sum}(m * r * \cos(2 * \pi * t / T + \delta))$

Adjust Window parameters: $0 < t < 40$, steps of 0.5 and $-3000 < x$ and $y < 3000$. Then Zoom Square.

Note: divide by 1550 to get position in Sun radii.