# Advanced Kinematics

I. **Vector addition/subtraction**

II. **Components**

III. Relative Velocity

IV. Projectile Motion

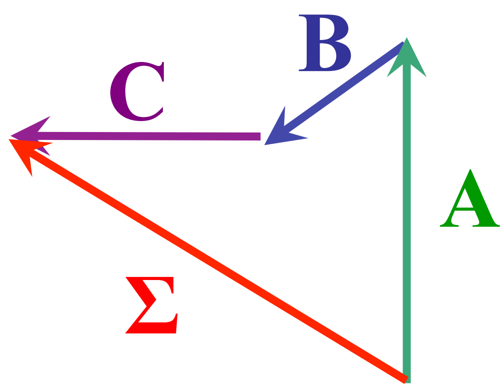
V. Use of Calculus
(nonuniform acceleration)

VI. Parametric Equations

| | The student will be able to: | HW: |
|---|---|---|
| 1 | Calculate the components of a vector given its magnitude and direction. | 1 – 2 |
| 2 | Calculate the magnitude and direction of a vector given its components. | 3 – 4 |
| 3 | Use vector components as a means of analyzing/solving 2-D motion problems. | 5 – 6 |
| 4 | Add or subtract vectors analytically (using trigonometric calculations). | 7 – 9 |
| 5 | Use vector addition or subtraction as a means of solving relative motion problems. | 10 – 15 |
| 6 | State the horizontal and vertical relations for projectile motion and use the same to solve projectile problems. | 16 – 24 |
| 7 | Use derivatives to determine speed, velocity, or acceleration and solve for extrema and/or zeros. | 25 – 27 |
| 8 | Use integrals to determine distance, displacement, change in speed or velocity and solve for functions thereof given initial conditions. | 28 – 31 |
| 9 | Solve problems involving parametric equations that describe motion components | 32 – 34 |

# Rule for Vector Addition

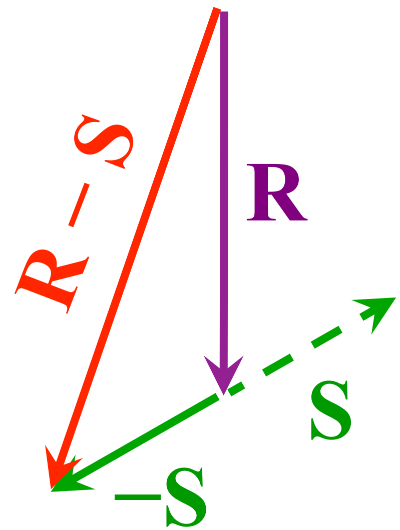
To add vectors, place the vectors head-to-tail. The resultant sum is the vector that extends from the tail of the first to the head of the last.




$$\vec{A} + \vec{B} + \vec{C} = \vec{\Sigma}$$

# Vector Subtraction

$\mathbf{R}$ = 20.0 m, 270.0°
$\mathbf{S}$ = 10.0 m, 30.0°
<u>$-\mathbf{S}$ = 10.0 m, 210.0°</u>
$\mathbf{R} - \mathbf{S} = \mathbf{R} + (-\mathbf{S})$
$\mathbf{R} - \mathbf{S}$ = 26.5 m, 109.1°



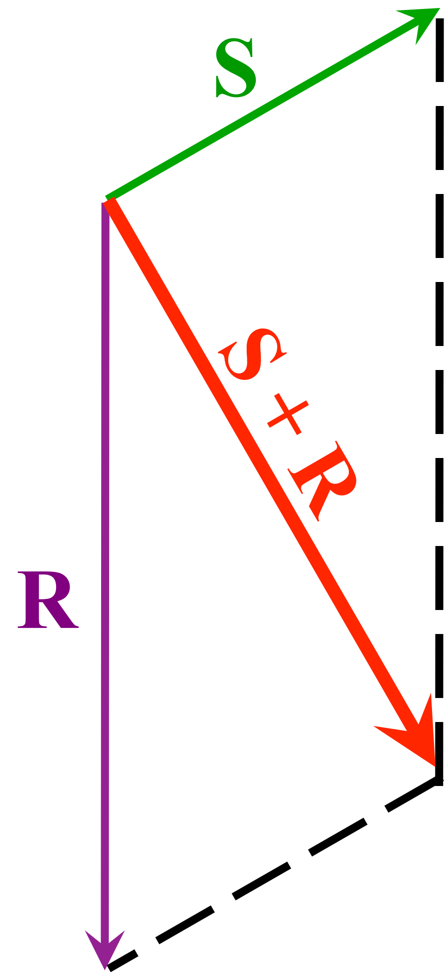

To subtract a vector, add its opposite.

A vector's opposite has the same magnitude but opposite direction (differs by 180°).

# Parallelogram Rule

Vector addition and subtraction may also be visualized by the parallelogram formed by placing tail-to-tail…

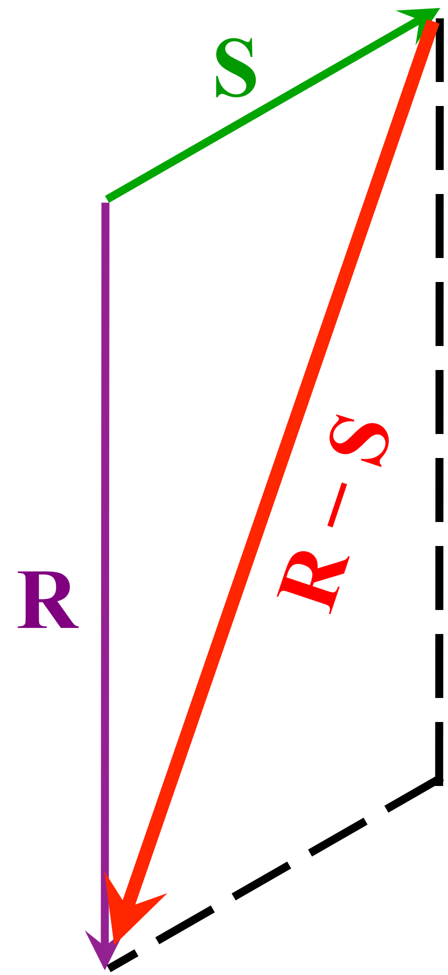
…the sum extends along a diagonal outward from the tails.



# Parallelogram Rule

Vector addition and subtraction may also be visualized by the parallelogram formed by placing tail-to-tail…

…the difference is along a diagonal from head to head.



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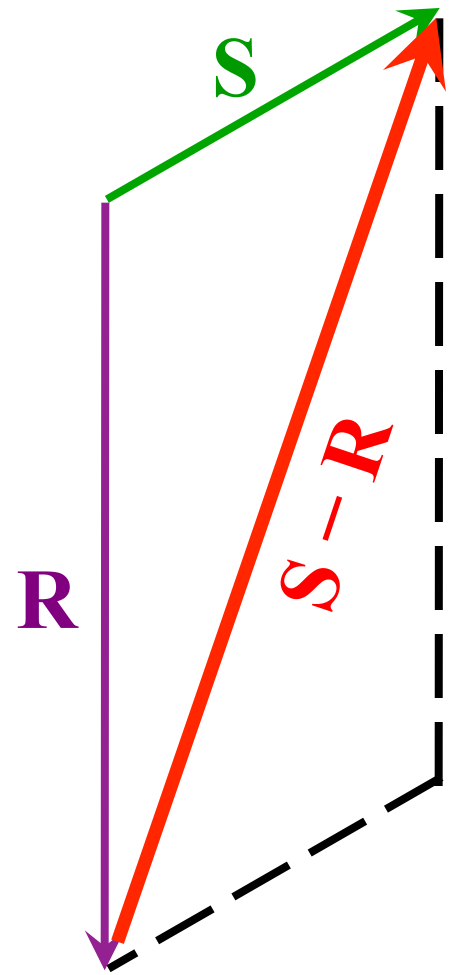
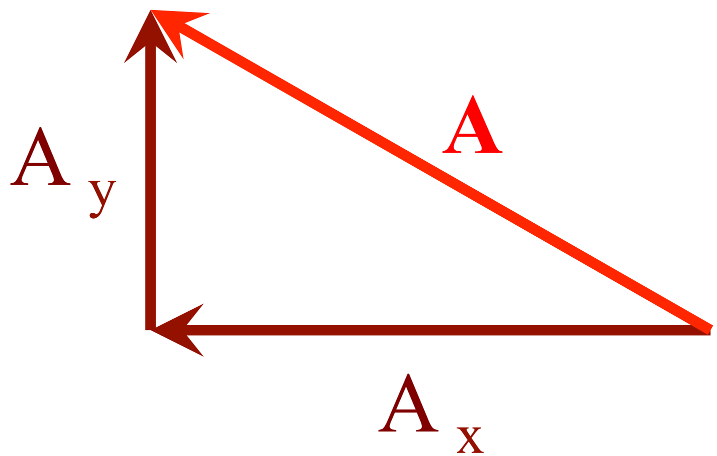


# Components

- Components are “parts that make up a whole”.
- A vector’s components indicate the partial amounts extending in perpendicular directions.
- Components indicate *how much up or down* and *how much left or right* a vector points.
- Any given vector is equal to the sum of its components by the head-to-tail rule or parallelogram rule.

Example of correct notation and terminology:




**A** = 10.0 m, 150.0°

This is "the vector".

$A_x = -8.66$ m
$A_y = 5.00$ m

These are "the components" of the vector.

Vector **A** points 5.00 m up and 8.66 m to the left.

# Unit Vectors

- A “unit vector” is a convenient alternate notation for indicating components and vector directions.
- By definition a unit vector always has a magnitude of exactly 1 and a particularly defined direction.
- Unit vectors for a given coordinate system are always perpendicular to one another.

# Unit Vectors

Rectangular Coordinates:

$\hat{i}$ = 1 in the positive $x$-direction

$\hat{j}$ = 1 in the positive $y$-direction

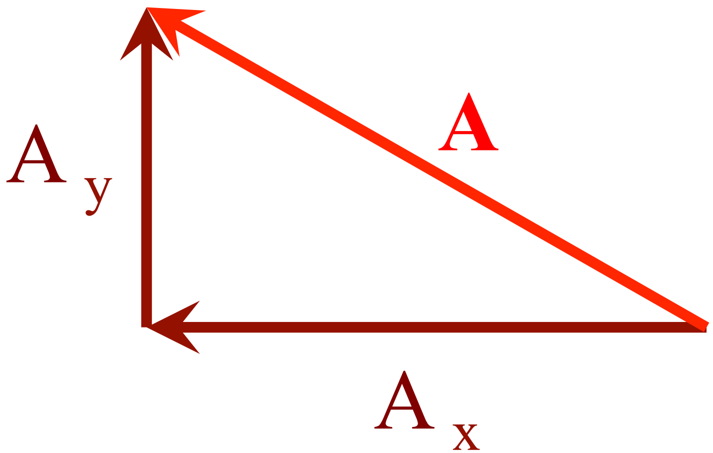
Polar Coordinates:

$\hat{r}$ = 1 in the positive radial direction

$\hat{\theta}$ = 1 in a direction perpendicular to $\hat{r}$ and in a counterclockwise sense

Example of alternate notation:

**A** = 10.0 m, 150.0°




$$\vec{A} = \left(-8.66\,\hat{i} + 5.00\,\hat{j}\right)\text{m}$$

Vector **A** points 5.00 m up and 8.66 m to the left.