Cosmological Models

I. Planetary Motion
II. Aristotle and Ptolemy
III. Copernicus
IV. Galileo
V. Kepler's Laws
VI. Newton's Laws
VII. Einstein

The student will be able to:		HW:
1	Describe and illustrate the apparent motion of each of the eight planets as seen from Earth bringing special attention to the similarities and differences.	1 – 5
2	Define, illustrate, and apply the following concepts: direct or prograde motion, retrograde motion, conjunction, opposition, and elongation.	
3	Explain and illustrate aspects of ancient geocentric models of the universe including the concepts of deferents, epicycles, and the works of Ptolemy.	6 – 8
4	Explain and illustrate the heliocentric model of the universe proposed by Copernicus including its seven main points and its own inconsistencies.	9 – 11
5	Explain and illustrate how Galileo was able to provide evidence for the validity of the heliocentric model.	12
6	Desribe Tycho Brahe's contribution to the formation of Kepler's Laws.	13 – 14
7	Define and apply the characteristics of ellipses: focus, semi-major axis, semi-minor axis, and eccentricity.	15 – 16
8	Define, illustrate, and apply the concepts of aphelion and perihelion.	
9	Explain, illustrate, and apply Kepler's three laws of planetary motion and properties of ellipses to solve problems involving orbits.	17 – 21
10	Explain, illustrate, and apply methods for determining the absolute and relative scale of the solar system.	22 – 25
11	Explain, illustrate, and apply Newton's Laws of Motion and Universal Gravitation.	26 – 29
12	Compare and contrast Newton's Laws with Kepler's Laws.	30 – 32

Kepler
1571 – 1630 AD

Johannes Kepler was a German astronomer and mathematician. He worked with Tycho Brahe for a brief period, and eventually took over Tycho's position.

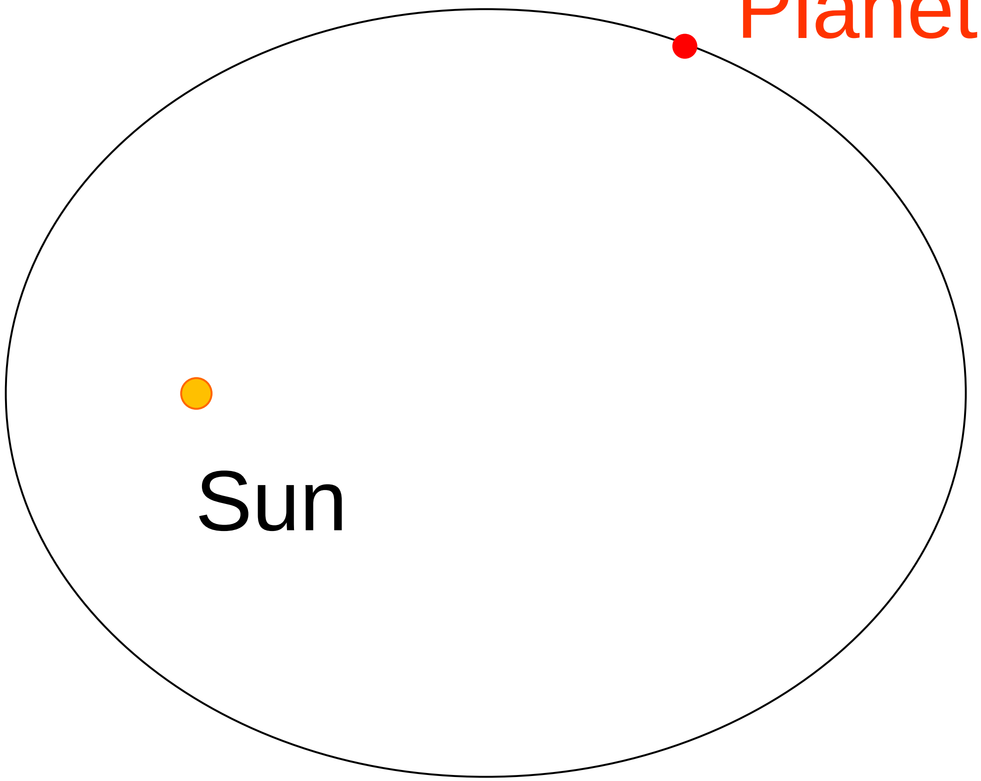
Using Tycho's detailed planetary data, Kepler was able to determine 3 Laws of Planetary Motion that defined a new cosmological model.

This model is *still in use*!

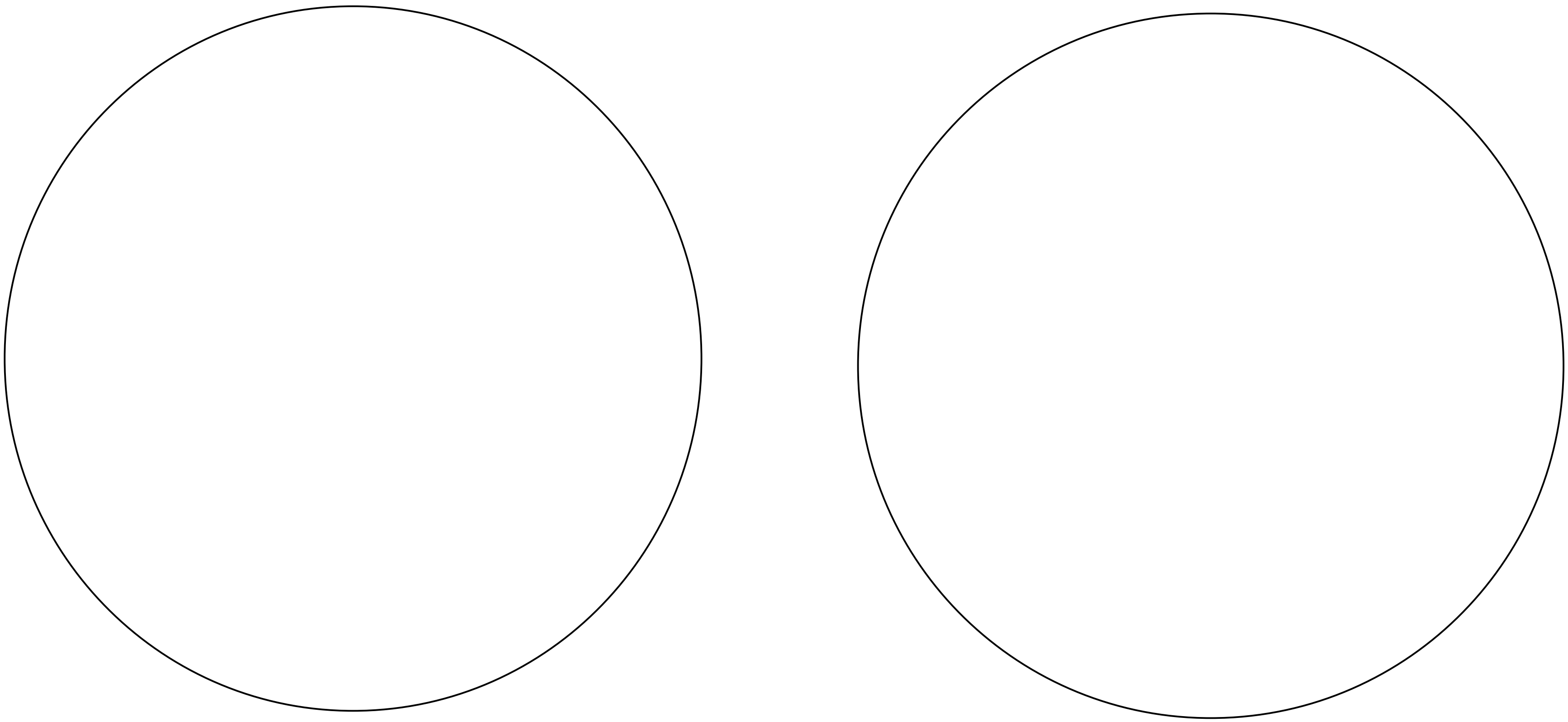
Kepler's Laws of Planetary Motion

1. The orbits of the planets are ellipses (not circles). The Sun is at a focus of each elliptical orbit.
2. An imaginary line connecting the Sun and any planet sweeps out equal areas of the ellipse in equal intervals of time.
3. The square of a planet's orbital period is proportional to the cube of the semi-major axis of its orbit.

1st Law – Orbits are Elliptical with Sun at Focus



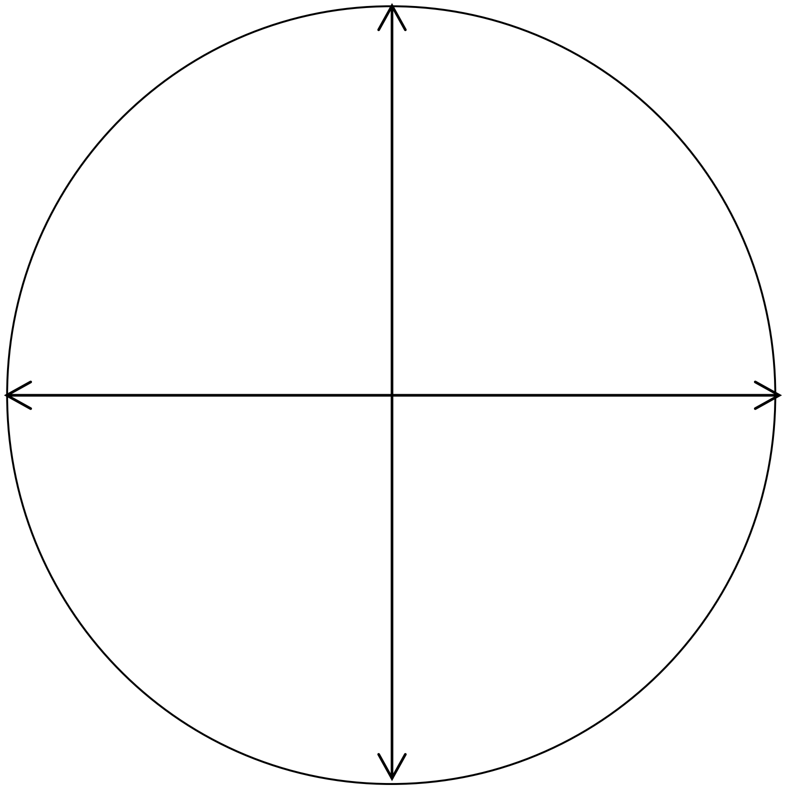
Gone is the Aristotlean idea of “perfect circles”!

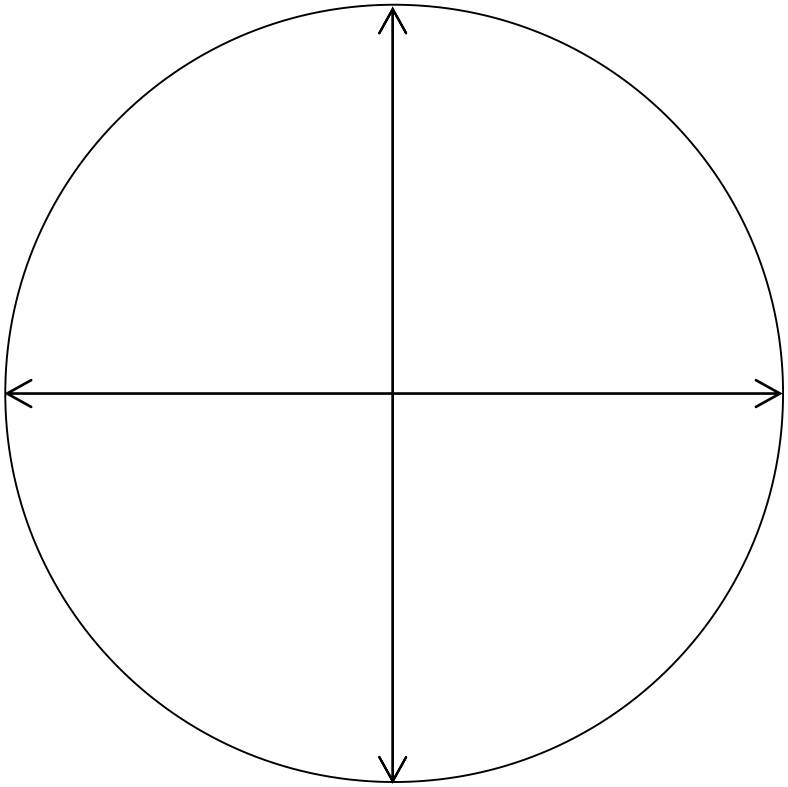


Which one is elliptical?

This one! (Barely!) Notice it is just a little taller than it is wide.

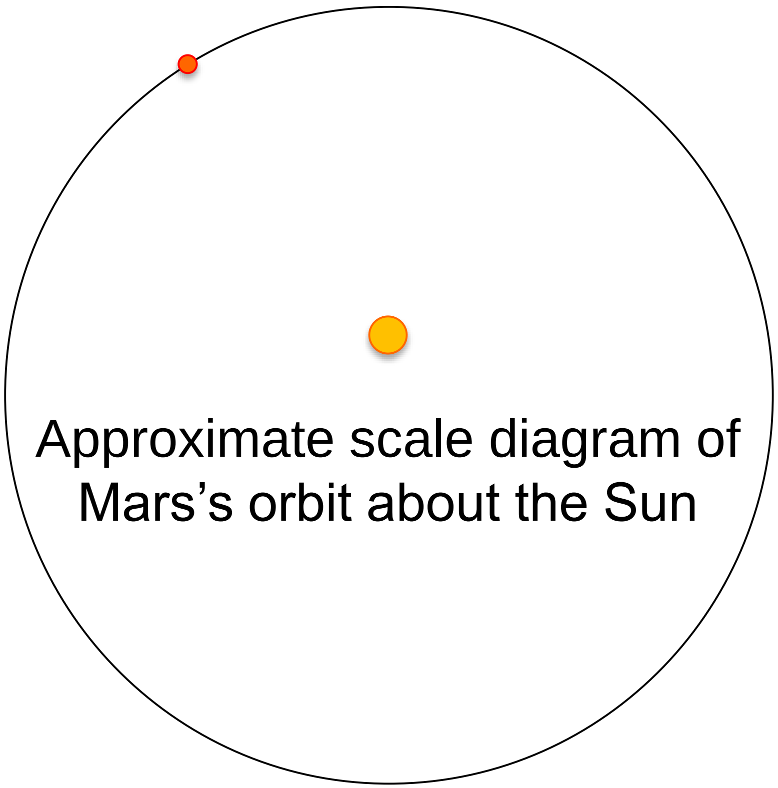
This is a perfect circle – not elliptical.





Amazingly Kepler was able to tell that orbits are not circular even though the discrepancy is about like what is shown here.

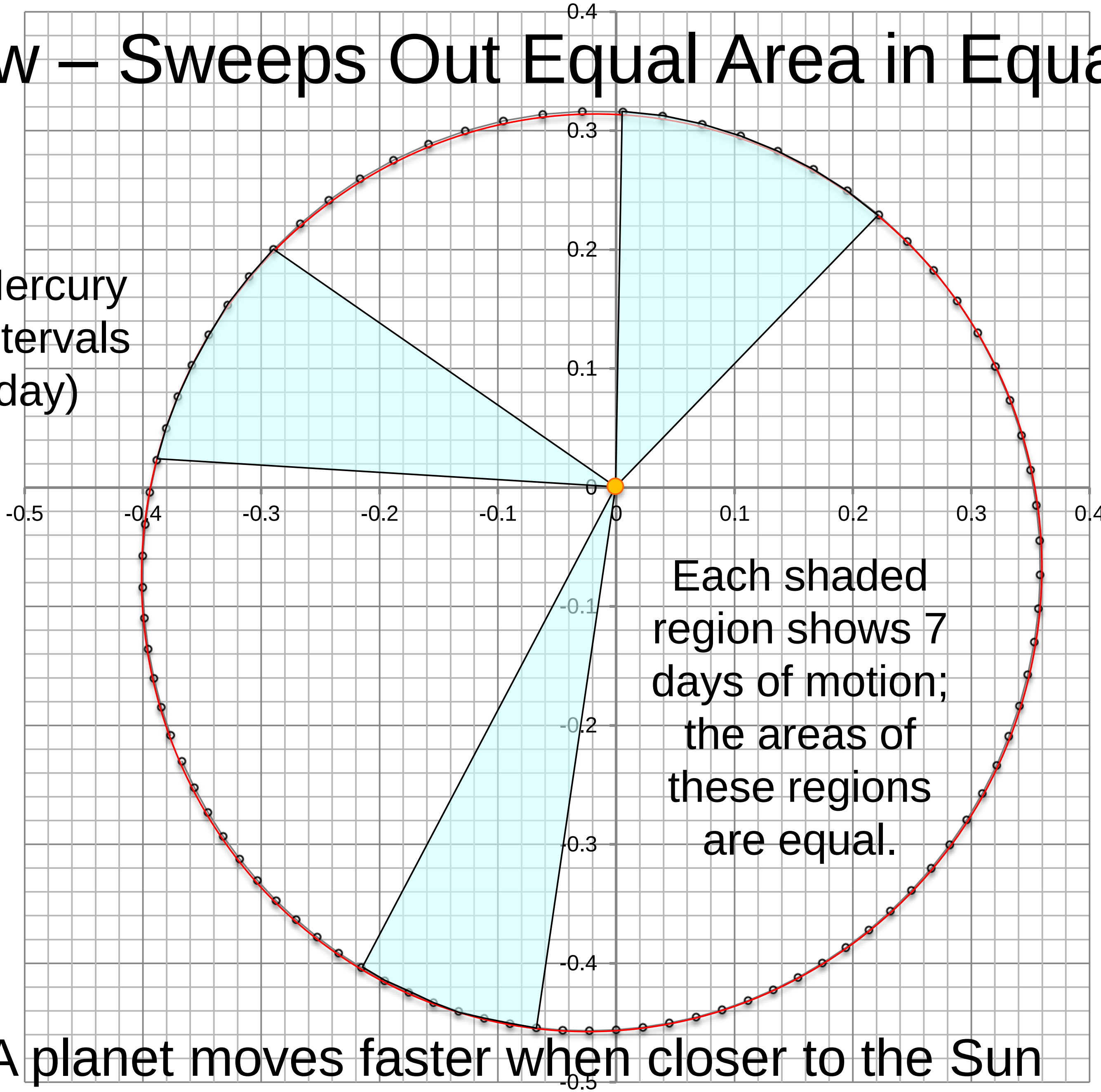
Kepler determined that Mars’s orbit is not circular but rather elliptical – in spite of the very small difference in shape!



The position of the Sun is noticeably off center – perhaps this aided Kepler’s determination.

2nd Law – Sweeps Out Equal Area in Equal Time

Orbit of Mercury (dots at intervals of one day)



Each shaded region shows 7 days of motion; the areas of these regions are equal.

A planet moves faster when closer to the Sun and slower when farther from the Sun.

3rd Law – p^2 is proportional to a^3

	a (AU)	p (yr)	a^3	p^2
Mercury	0.387	0.241		
Venus	0.723	0.615		
Earth	1.00	1.00	1.00	1.00
Mars	1.52	1.88		
Jupiter	5.20	11.86		
Saturn	9.54	29.42		

3^{rd} Law – p^2 is proportional to a^3

	a (AU)	p (yr)	a^3	p^2
Mercury	0.387	0.241	0.058	0.058
Venus	0.723	0.615	0.378	0.378
Earth	1.00	1.00	1.00	1.00
Mars	1.52	1.88	3.51	3.53
Jupiter	5.20	11.86	140.6	140.7
Saturn	9.54	29.42	868.3	865.5

3rd Law – p^2 is proportional to a^3

Note that this law is different than the first two because it deals with *all* of the planets, not the properties of an *individual* planet.

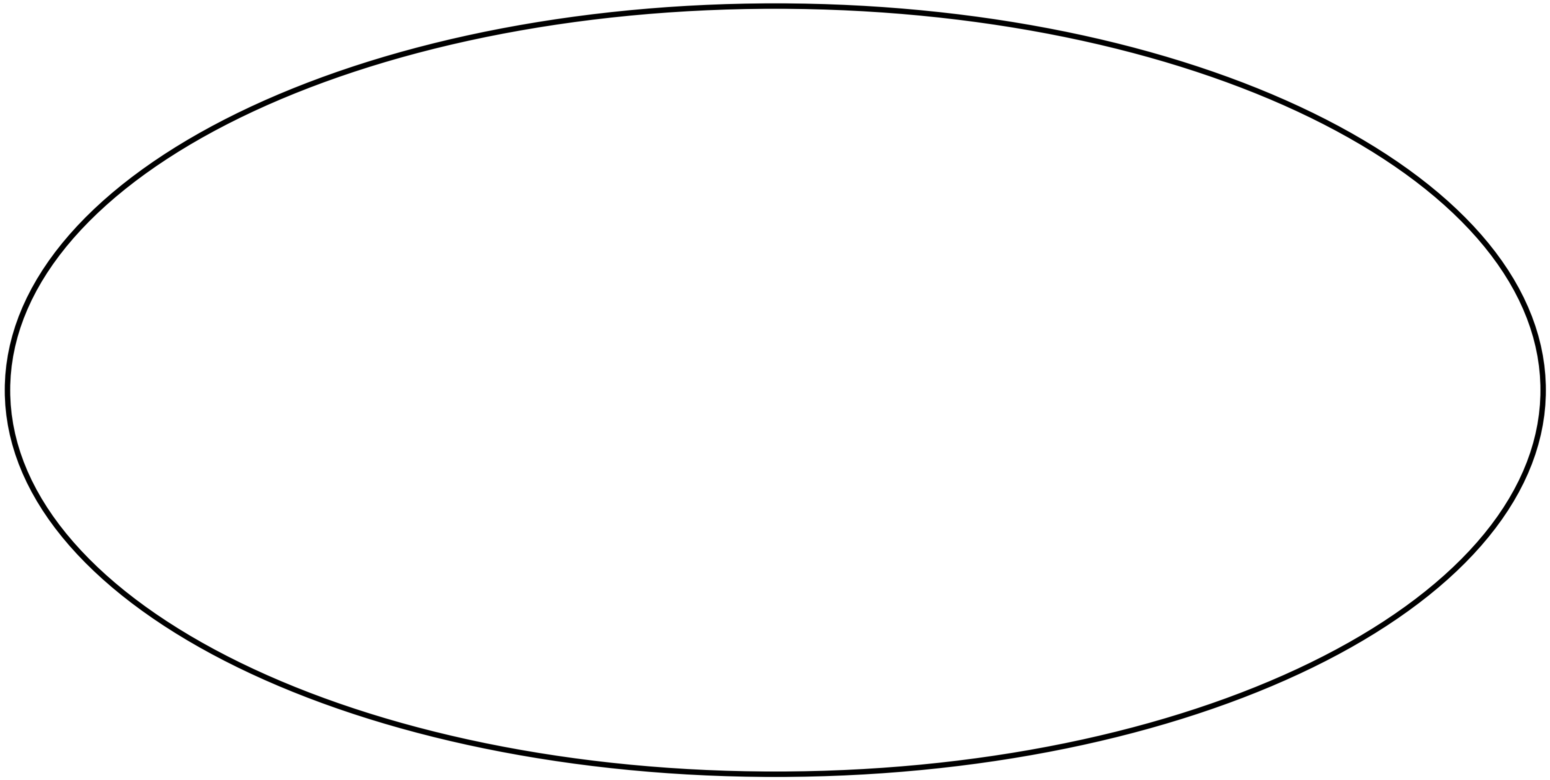
In simple terms, this law indicates that the farther from the Sun the more time it takes for an object to complete its orbit.

More on Kepler

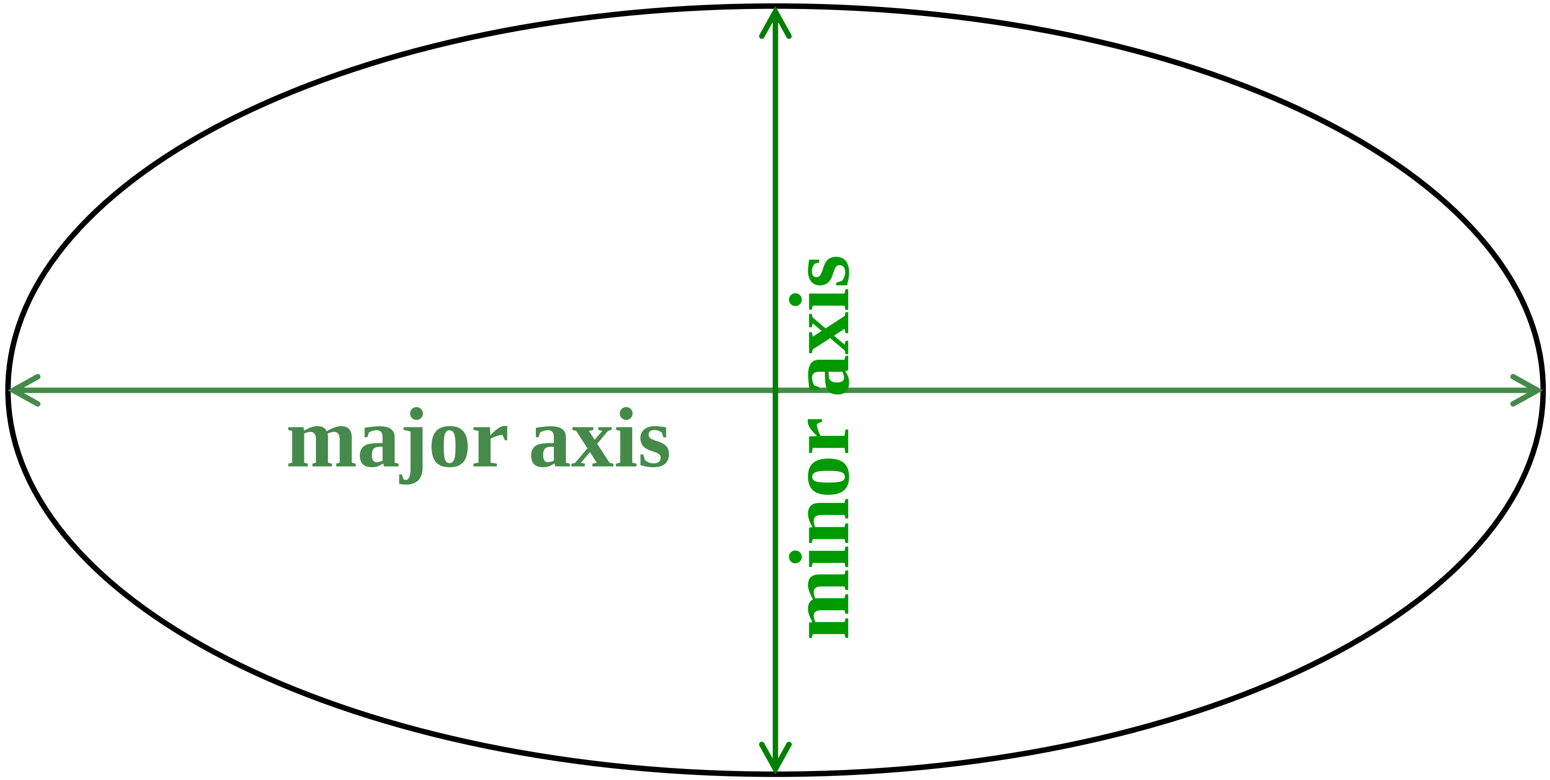
- Although Kepler worked out the *relative* sizes of the orbits, he was unable to determine the *absolute* sizes of the orbits.
- Taking the semi-major axis of Earth's orbit to be "one", he was able to determine other orbits as a multiple of this.
- This is the basis for the astronomical unit.
- The actual distance of 1 AU was first determined around 1761 by transit of Venus and then in 1964 by radar.

More on Kepler

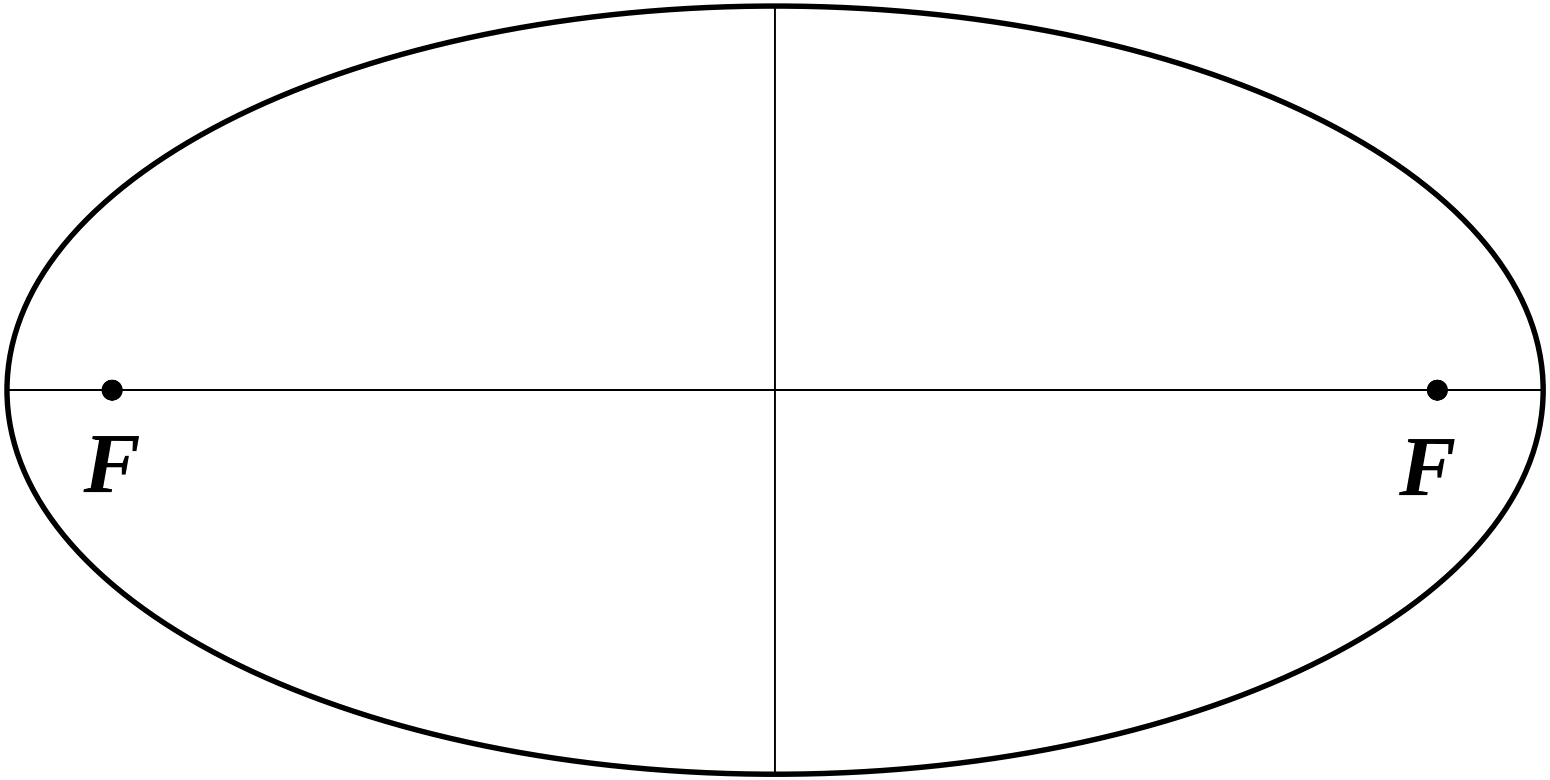
- As impressive and (still) useful as Kepler's Laws are, there is still something lacking...
- Kepler's Laws are *empirical*. This means that his laws are based purely on observation, with no underlying theory explaining *why*. (It is the *effect*, without the *cause*.)
- Kepler's Laws could be described as very technical and specific observations.



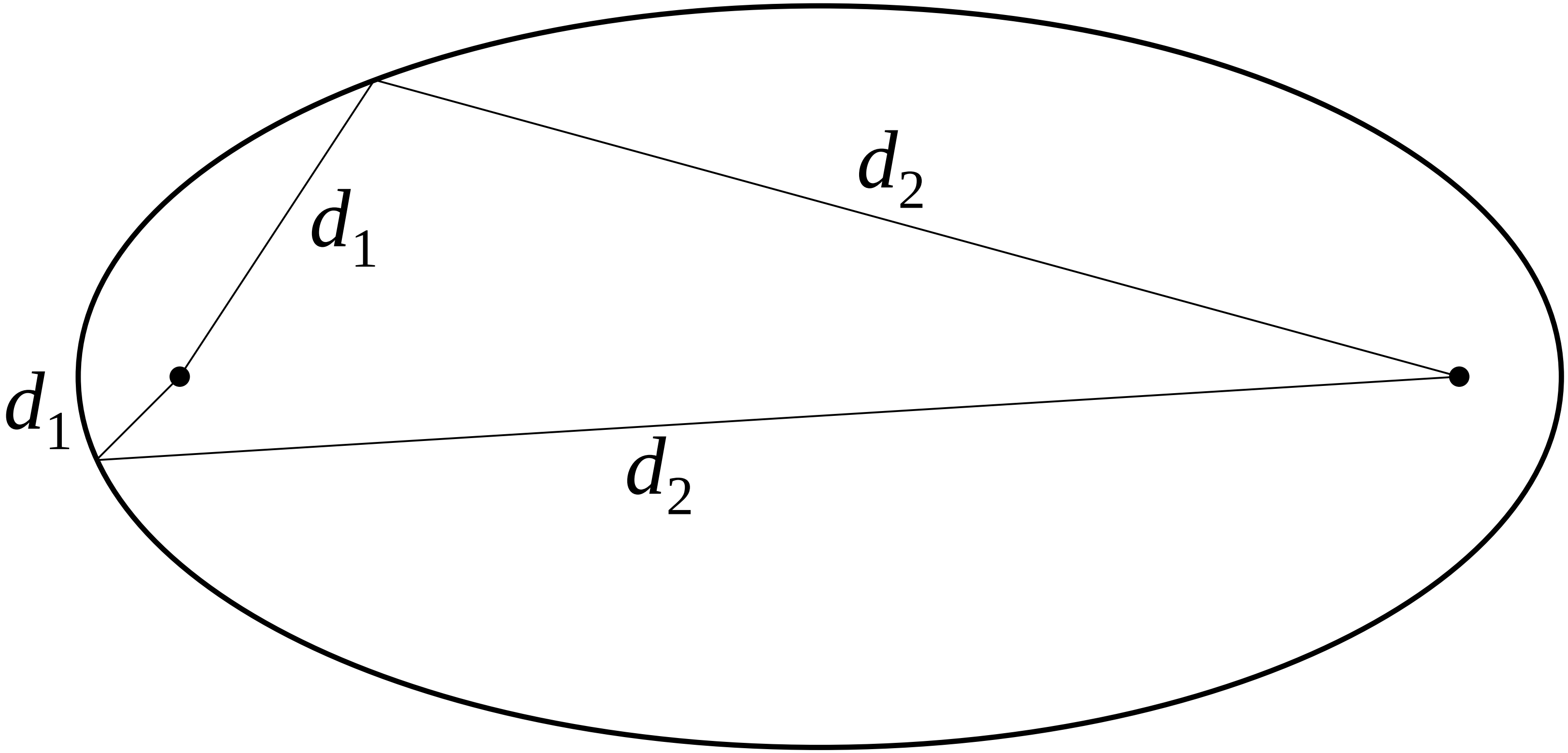
An ellipse is a unique oval shape
(not just *any* oval shape).



The "dimensions" of an ellipse are called the major axis and minor axis.

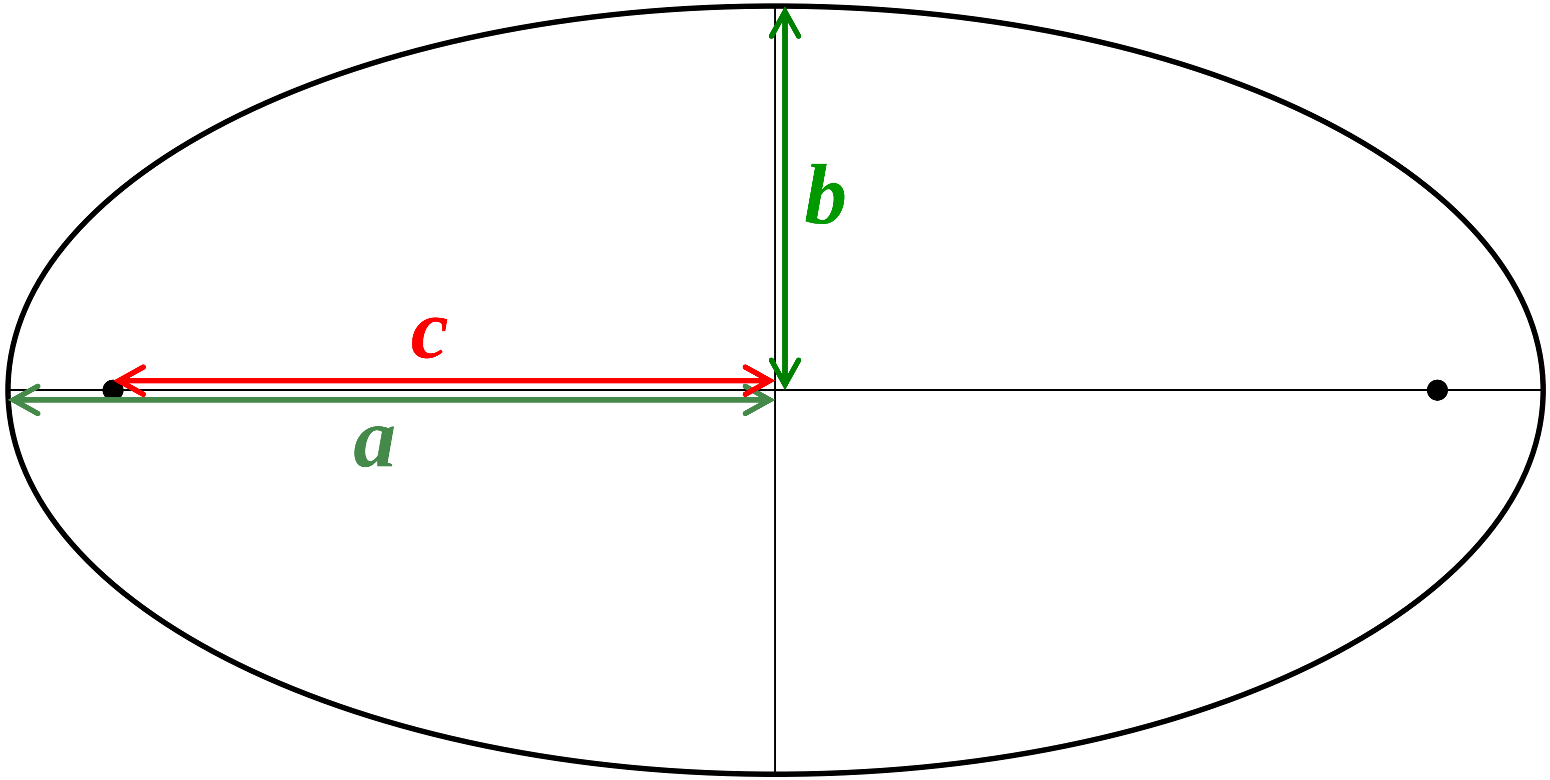


There are two unique points called foci.
Each focus is located on the major axis.



The sum of the distances to the foci is the same for any and every point on the ellipse.

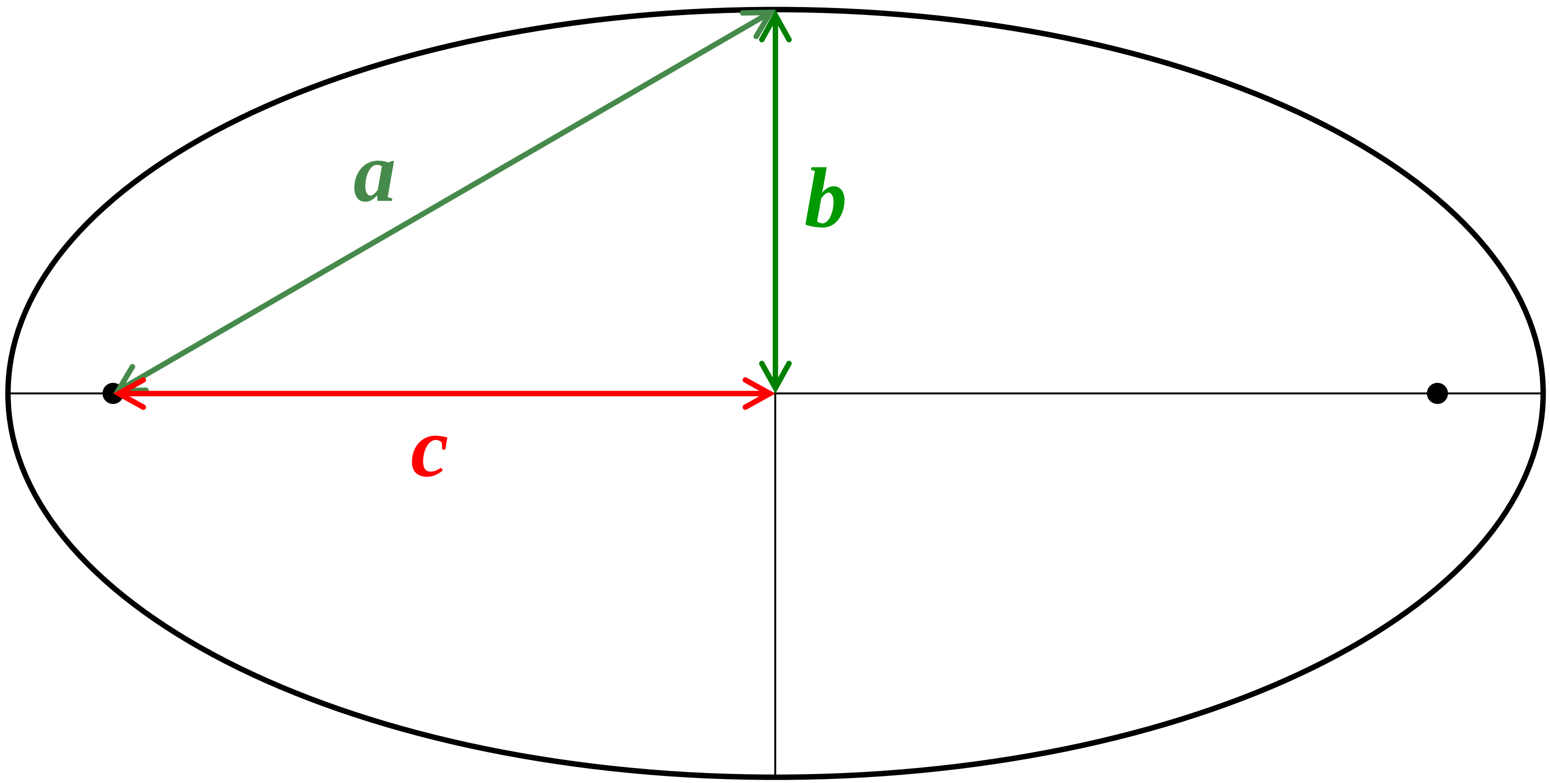
$$d_1 + d_2 = 2a = \text{constant}$$



a = semi-major axis

b = semi-minor axis

c = distance from center to focus

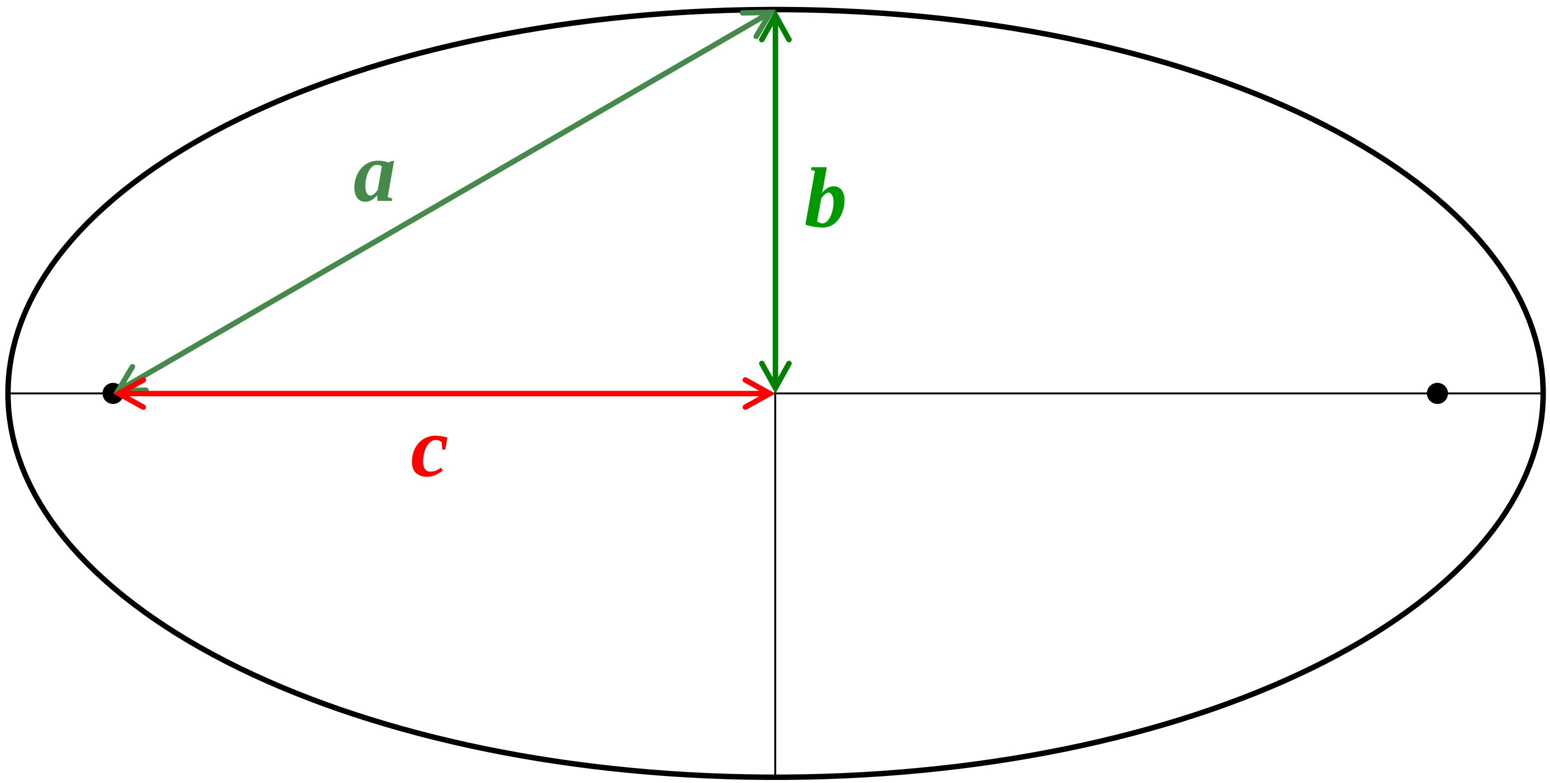


a = semi-major axis

b = semi-minor axis

c = distance from center to focus

note: $a^2 = b^2 + c^2$



The eccentricity, *e*, of the ellipse: $e = \dfrac{c}{a}$

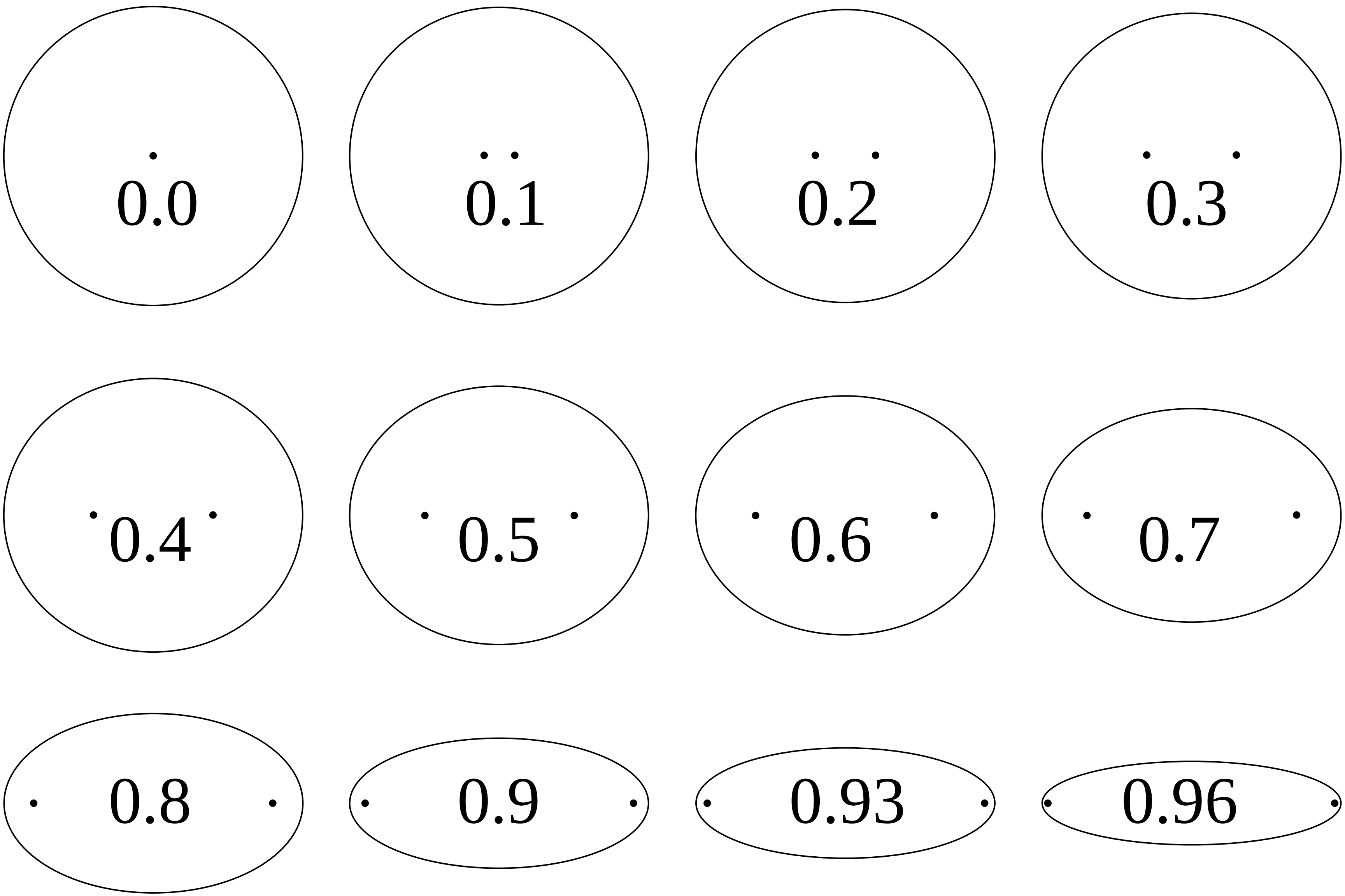
The value of *e* relates to how elongated the ellipse is and how far off-center the focii are located…

$0 < e < 1$

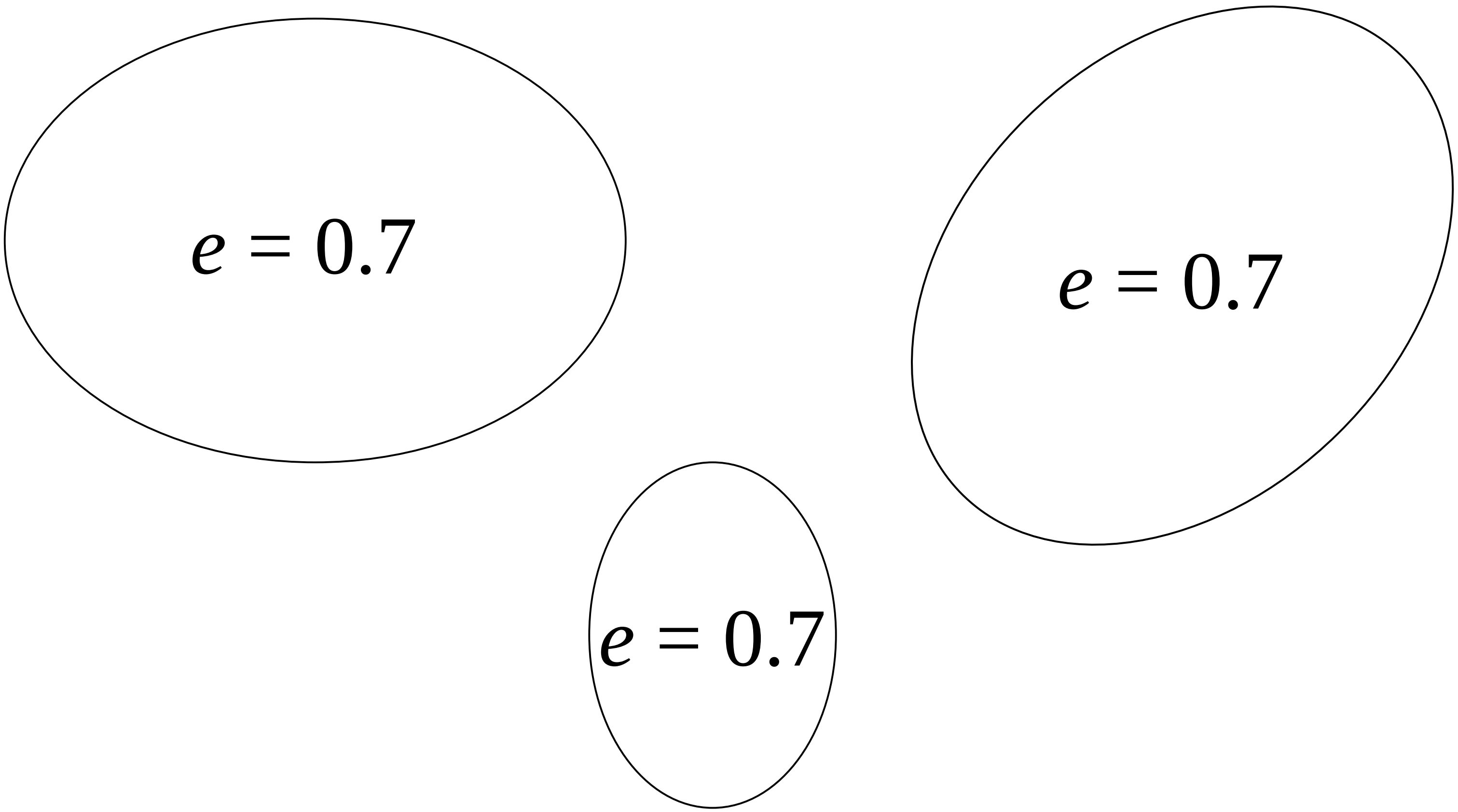
(closer to circular… …closer to parabolic)

Example Eccentricities

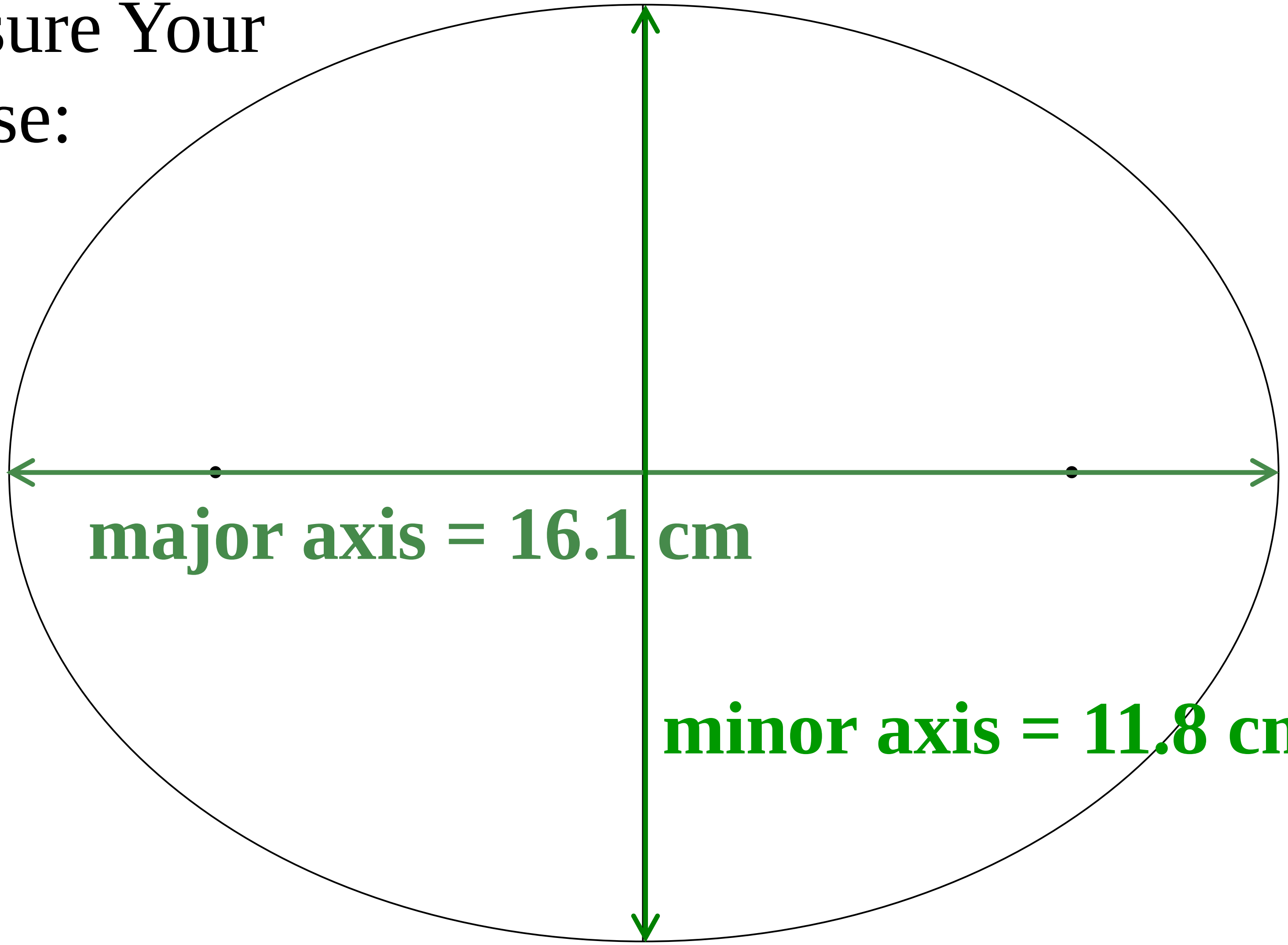
Example Eccentricities



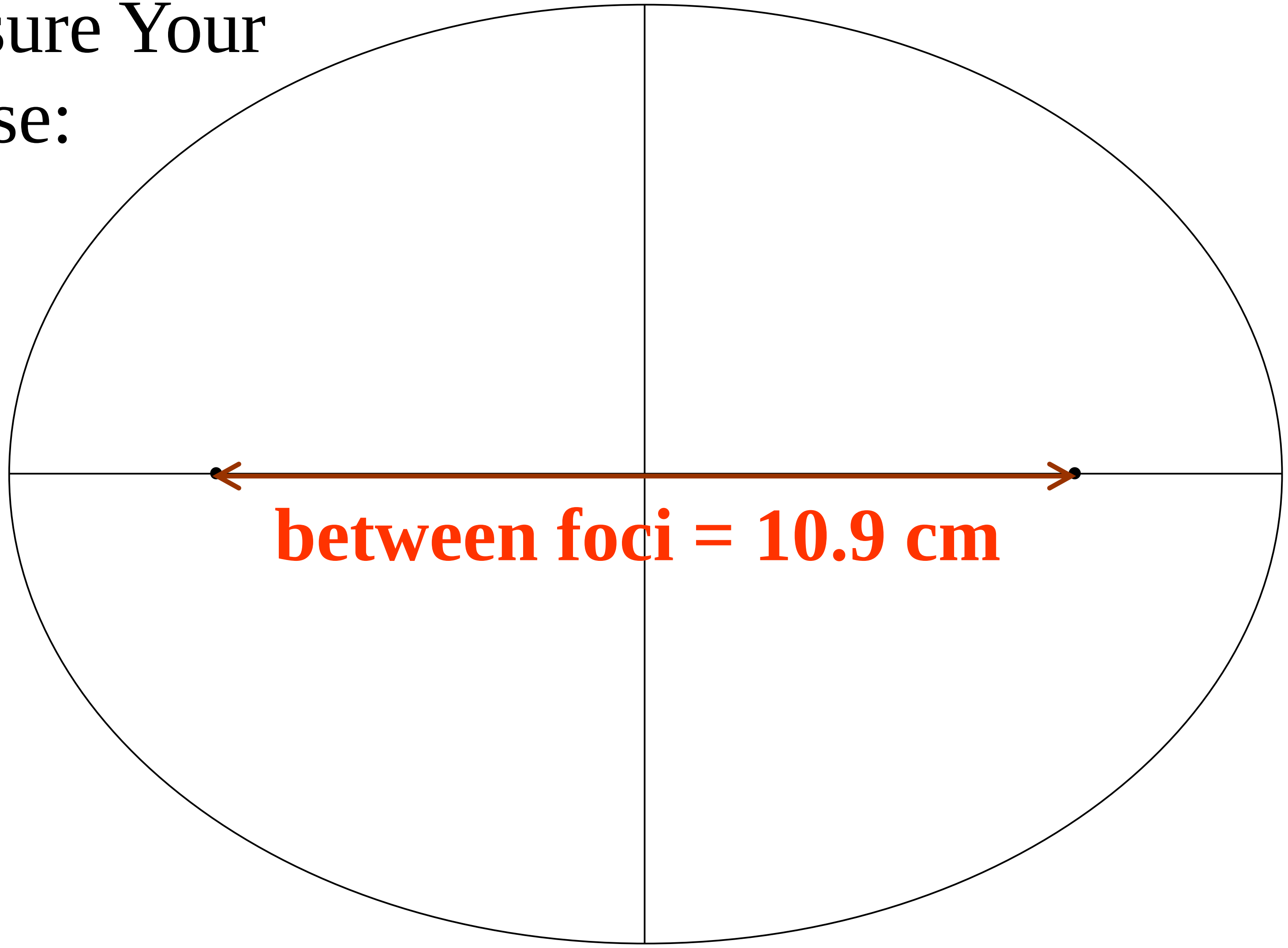
Eccentricity is <u>not</u> a measure of the *size* of an ellipse or its *orientation* – it depends only on the proportions of its dimensions.



Measure Your Ellipse:

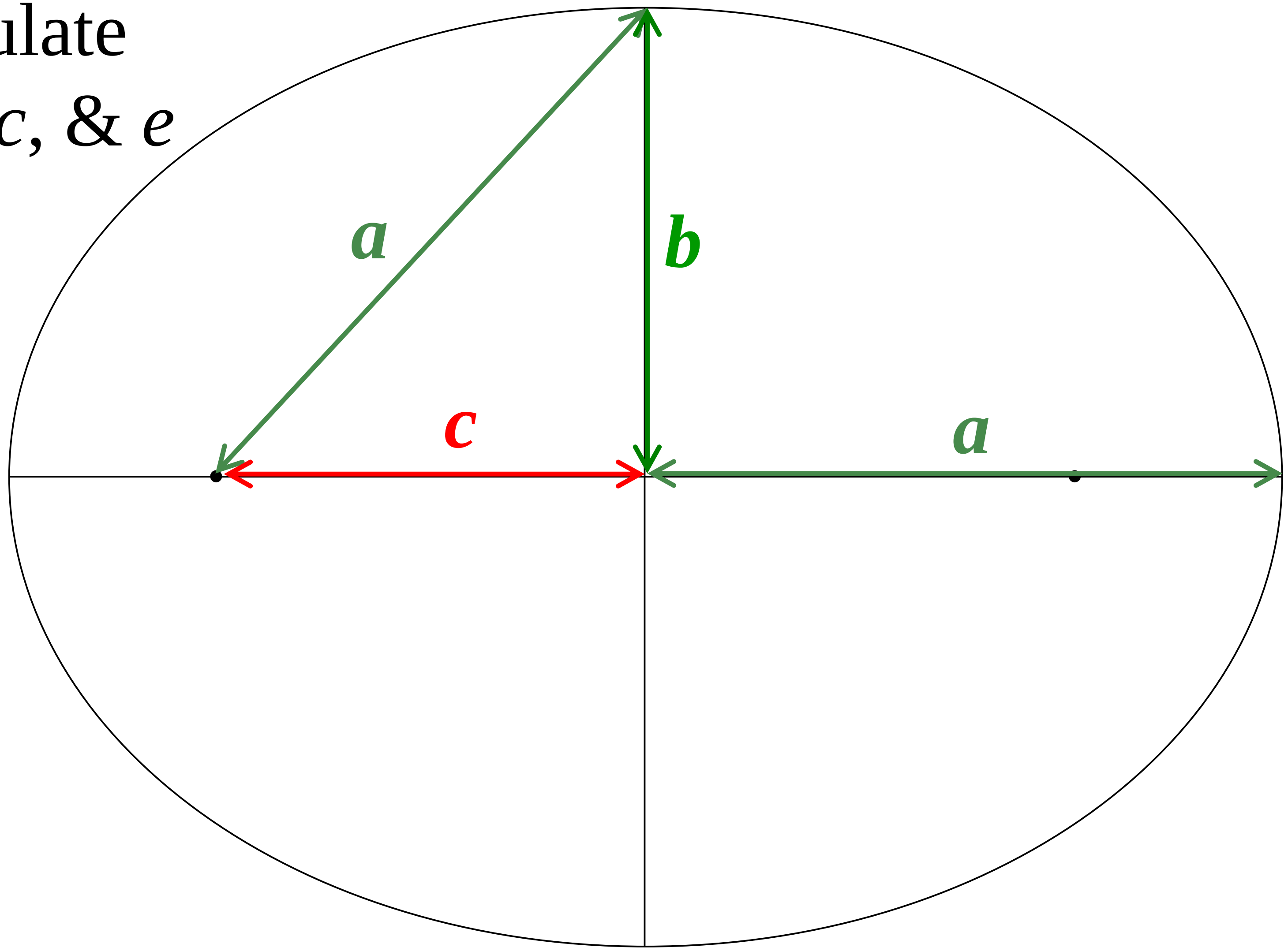


Measure Your Ellipse:



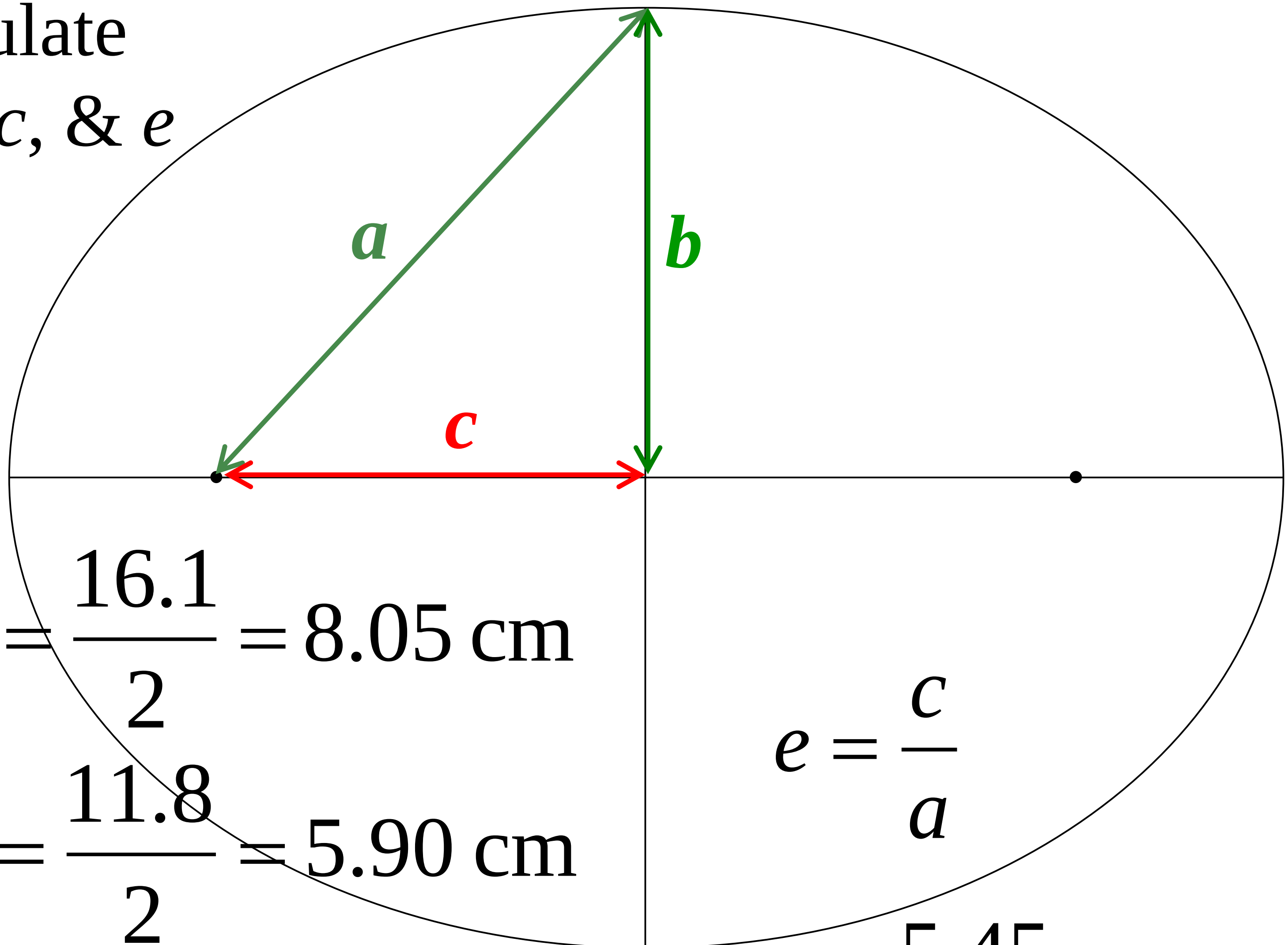
Calculate

a, b, c, & e



Calculate

a, b, c, & e



$$a = \frac{16.1}{2} = 8.05\ \text{cm}$$

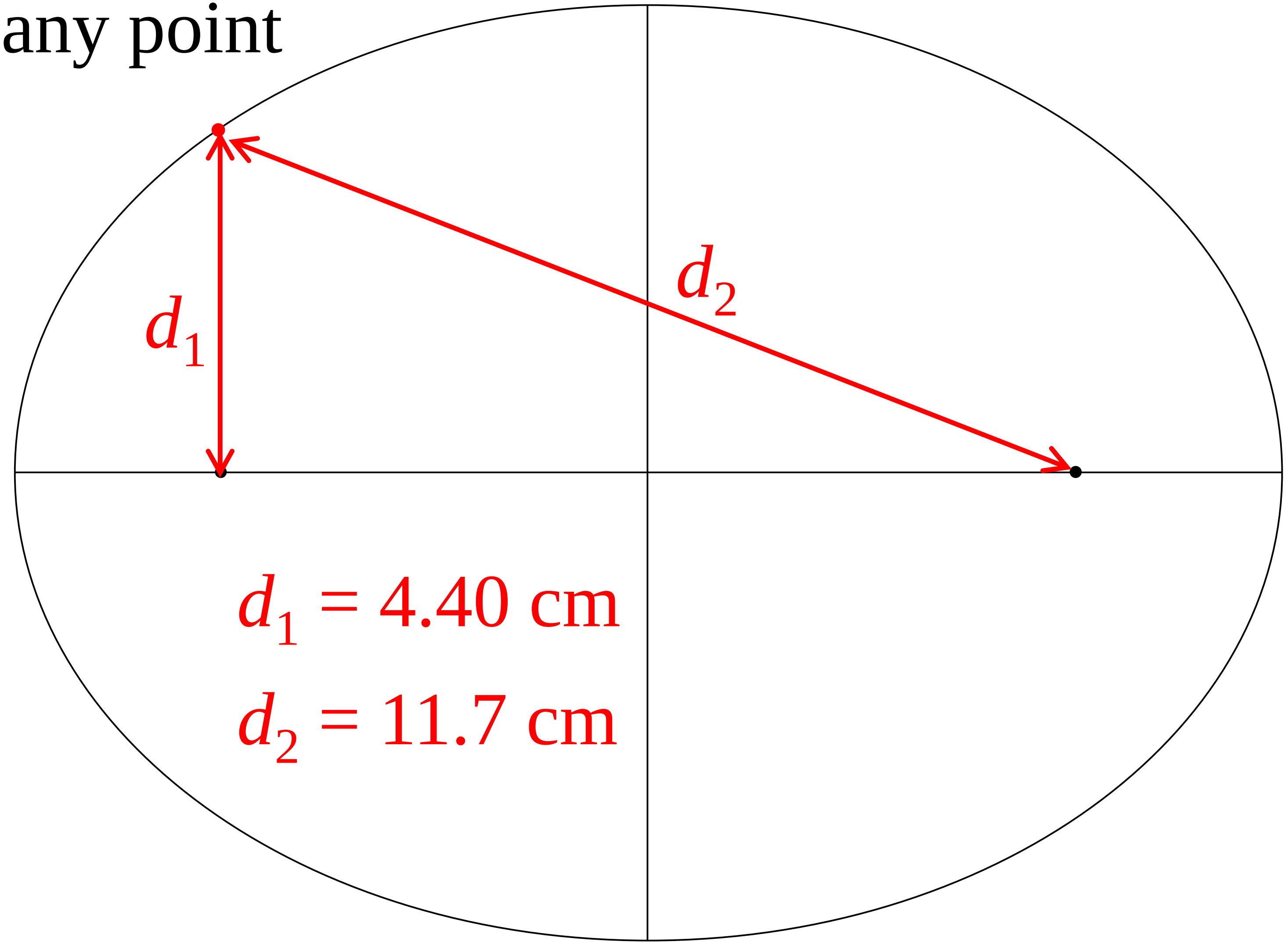
$$b = \frac{11.8}{2} = 5.90\ \text{cm}$$

$$c = \frac{10.9}{2} = 5.45\ \text{cm}$$

$$e = \frac{c}{a}$$

$$e = \frac{5.45}{8.05} = 0.677$$

Pick any point

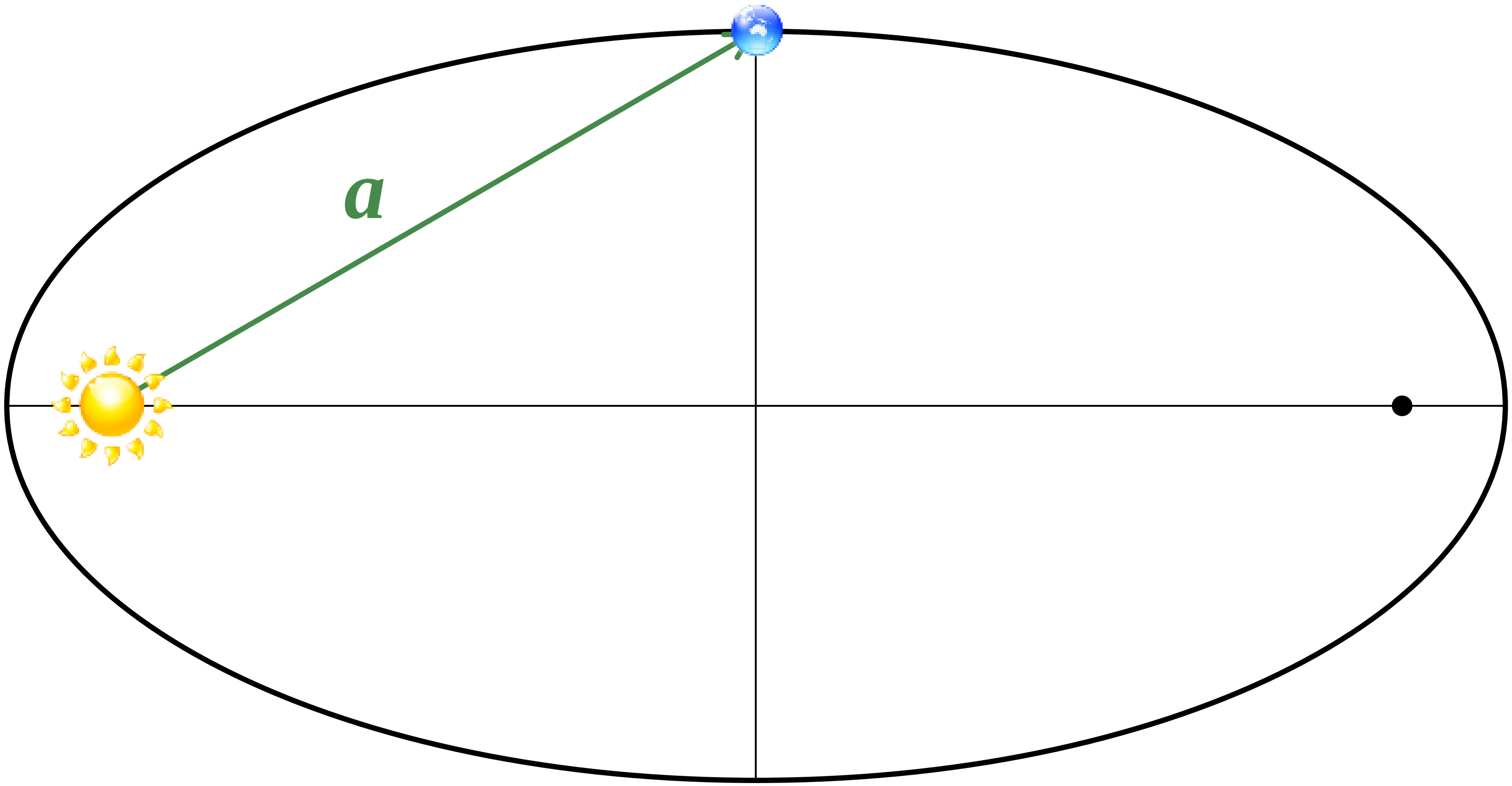


$d_1 = 4.40$ cm

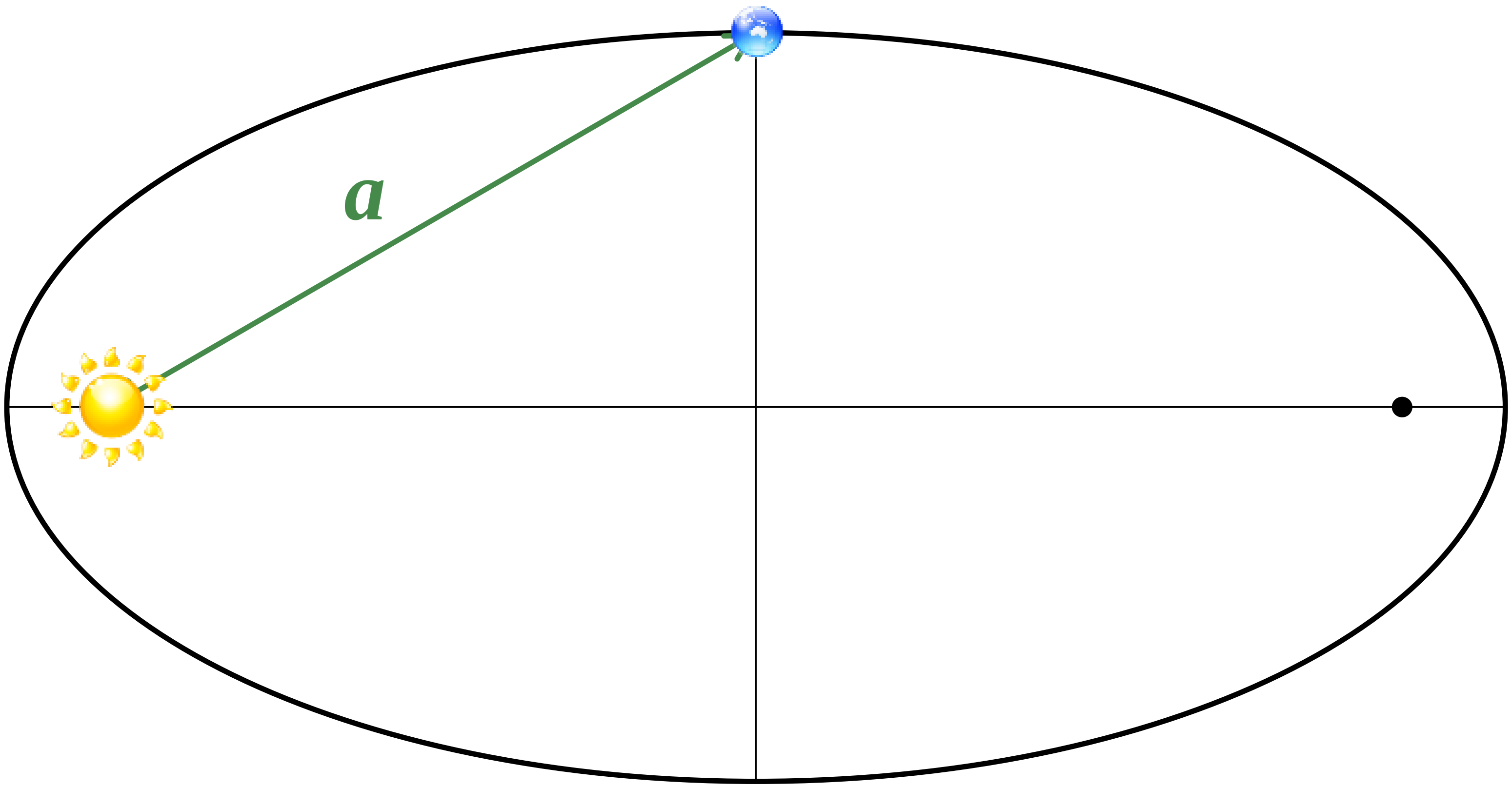
$d_2 = 11.7$ cm

$d_1 + d_2 = 11.7 + 4.40 = 16.1$ cm

$d_1 + d_2 = 2a$ for any point on ellipse!

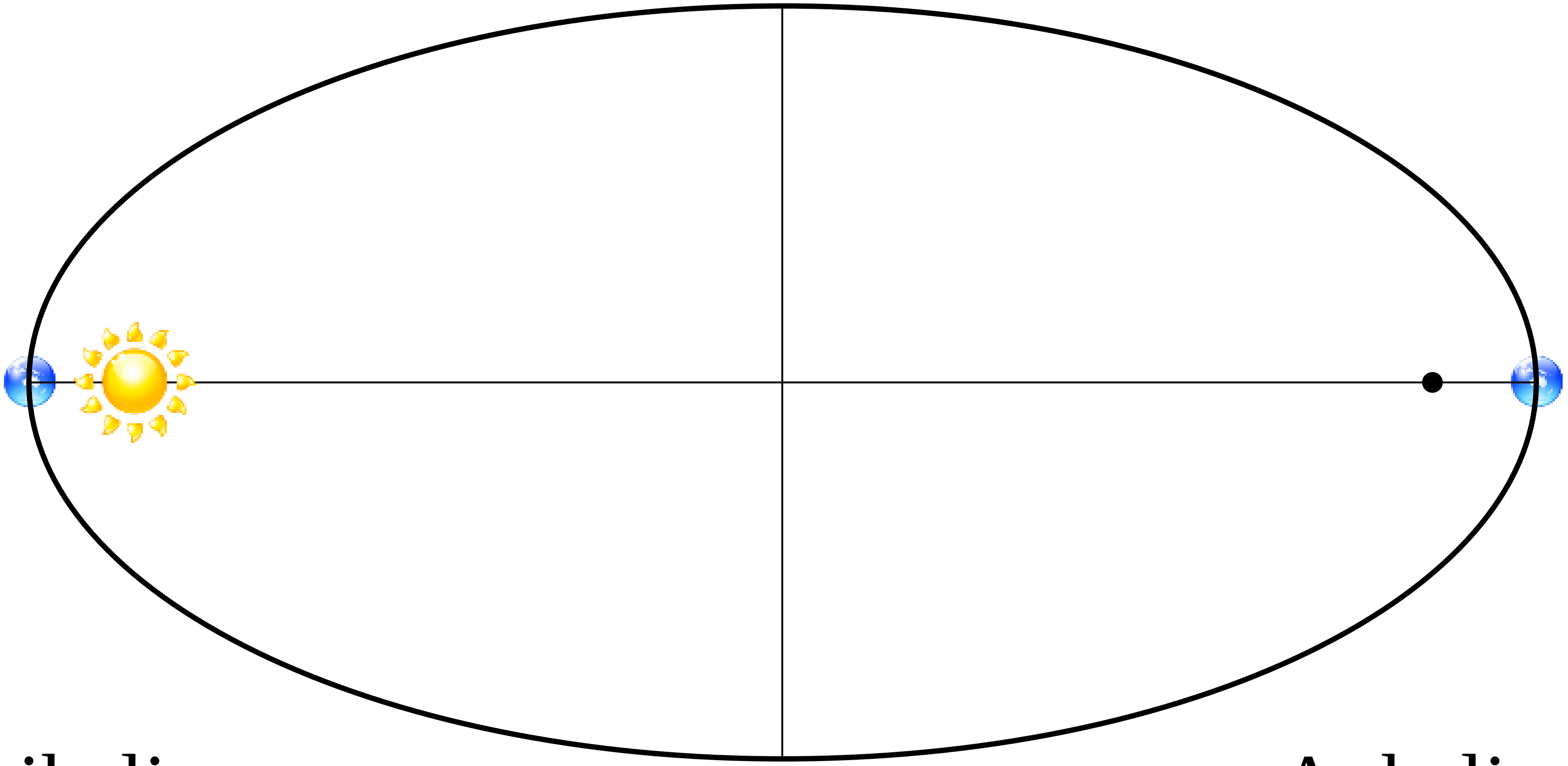


As applied to Kepler's Laws:
Sun is at one focus
Semi-major axis = average distance from Sun



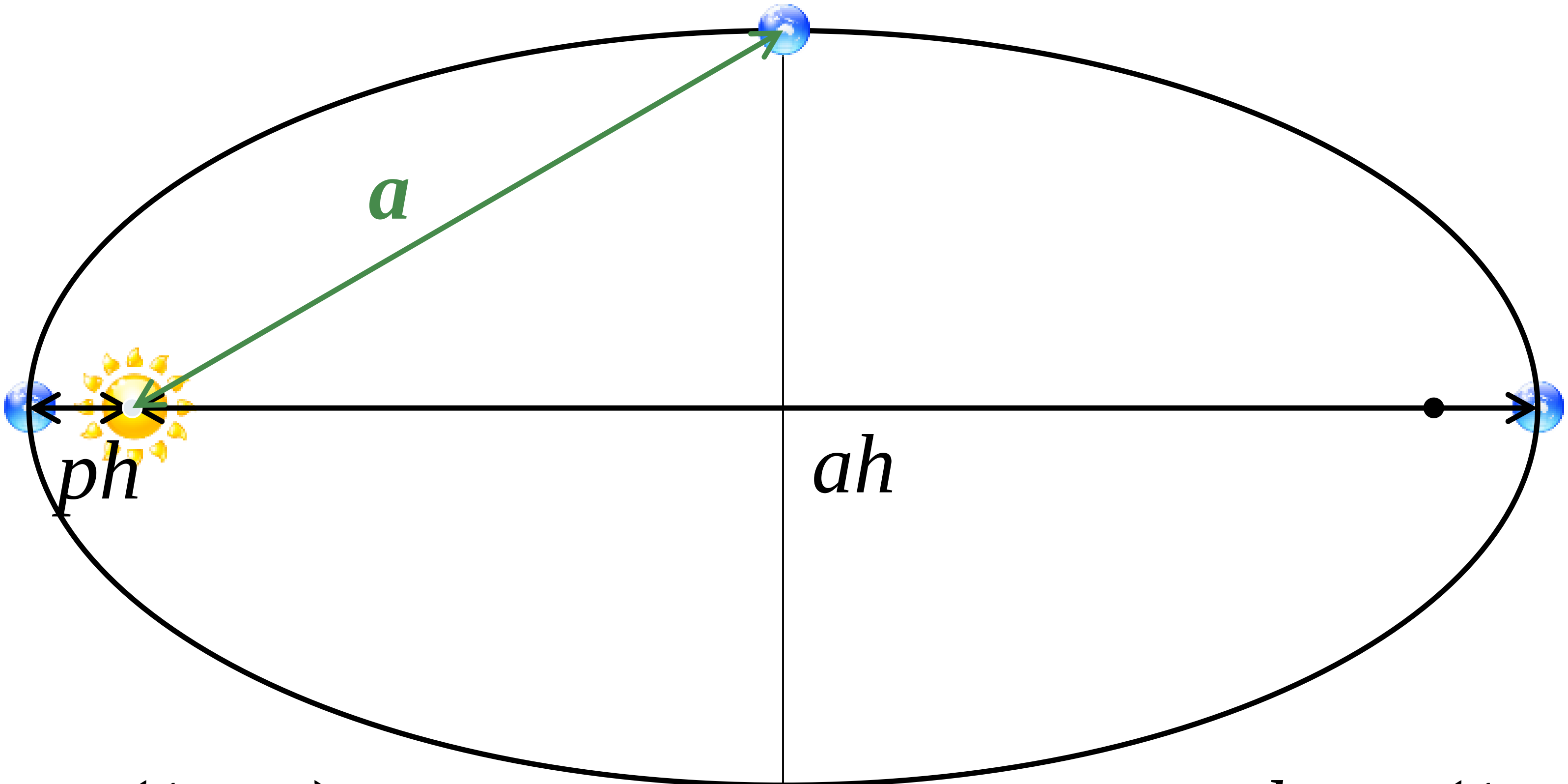
Note: the average distance from the Sun to the Earth is known as an "astronomical unit" or A.U.

For Earth, a = 1 A.U.



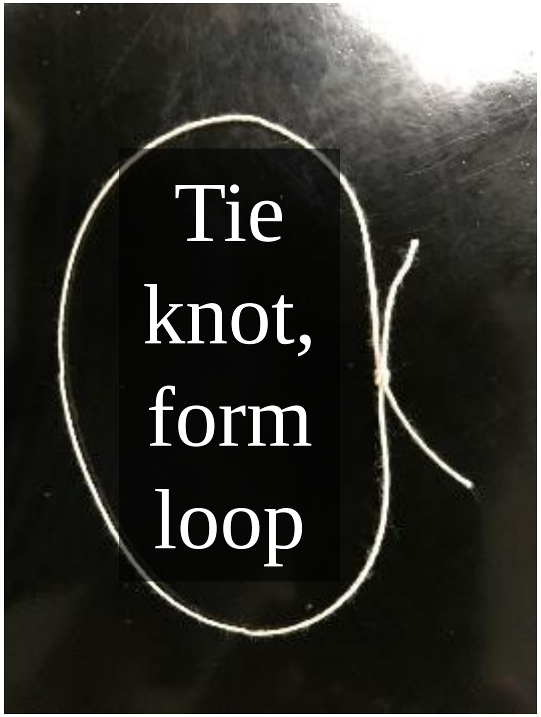
Perihelion =
point in orbit
closest to Sun

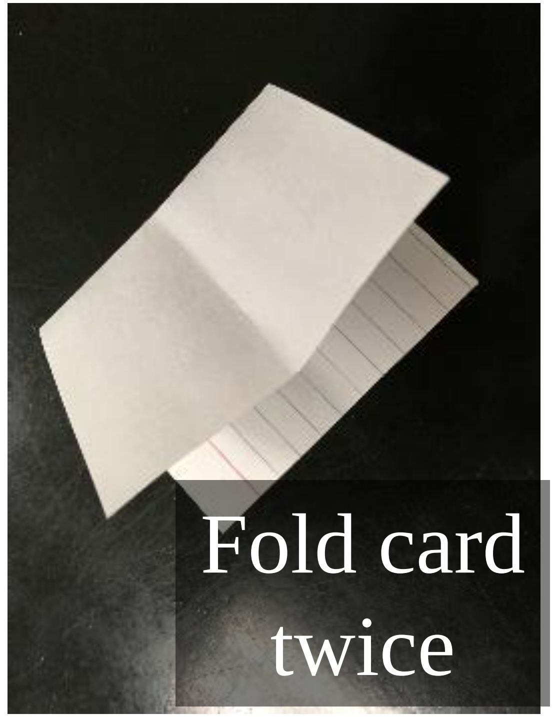
Aphelion =
point in orbit
farthest from Sun

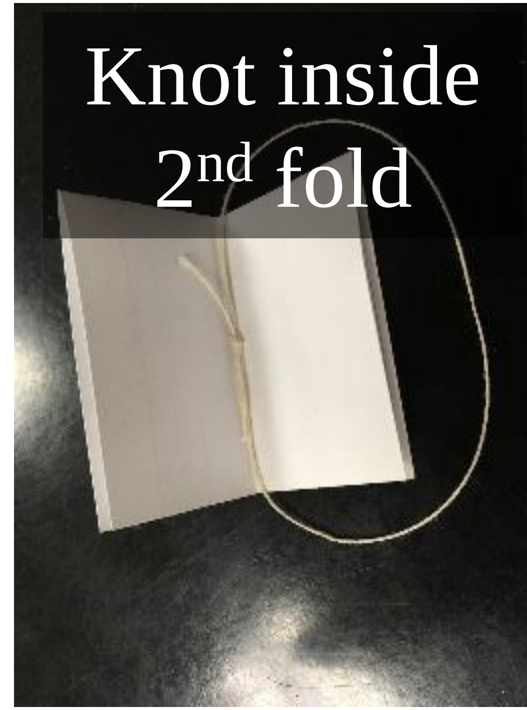


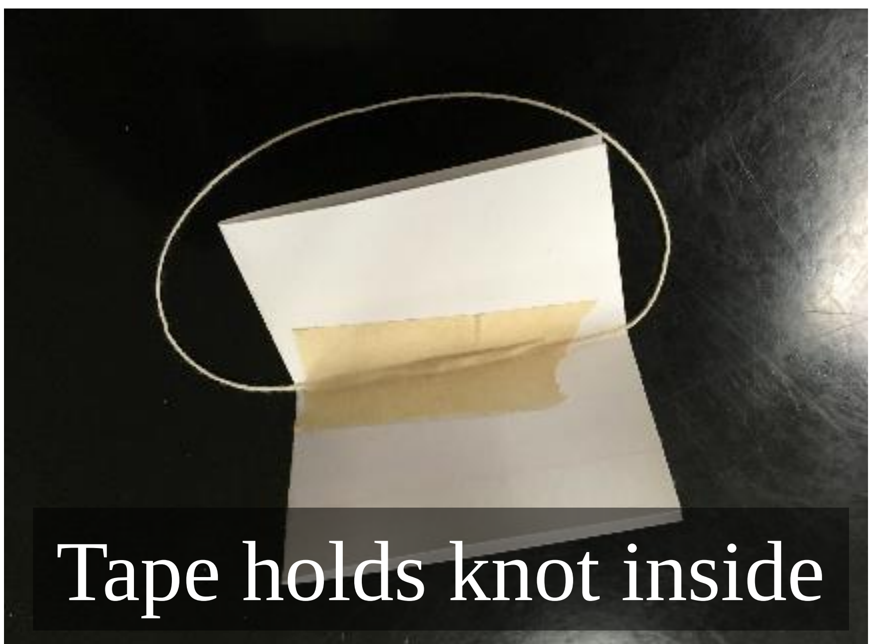
$ph = a(1 - e)$ $ah = a(1 + e)$

These formulas give the least and greatest distances from the Sun in terms of the average distance and the eccentricity.


Tie knot, form loop


Fold card twice

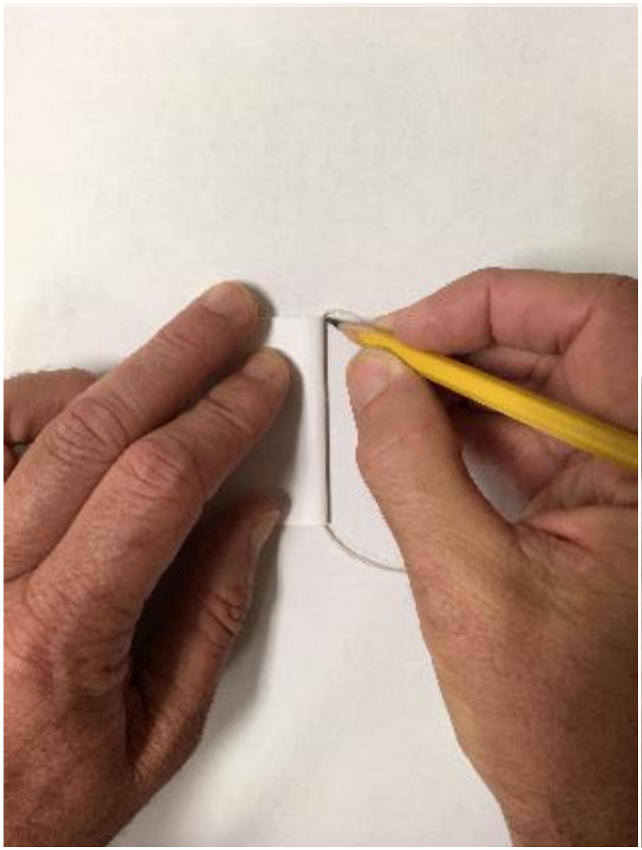

Knot inside 2nd fold

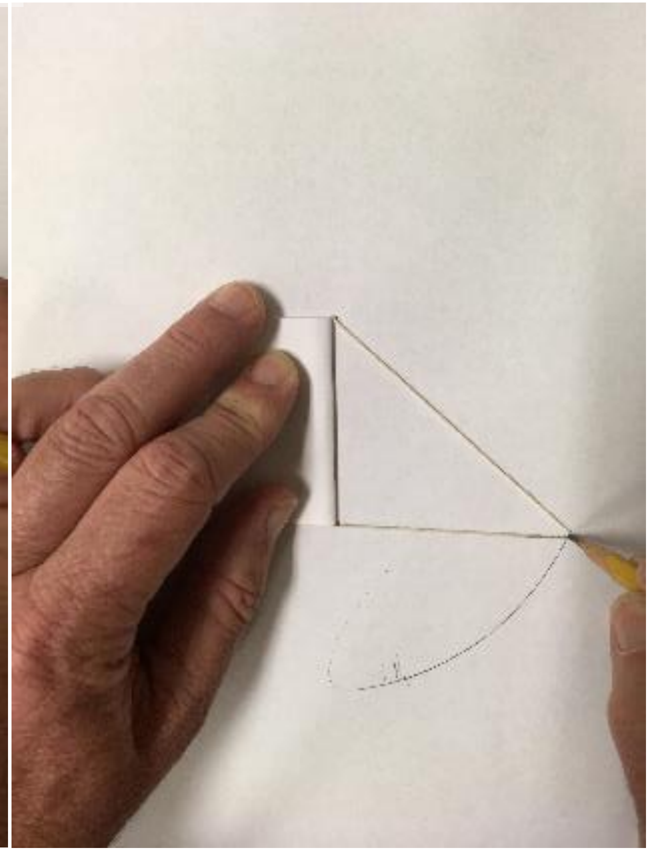

Tape holds knot inside

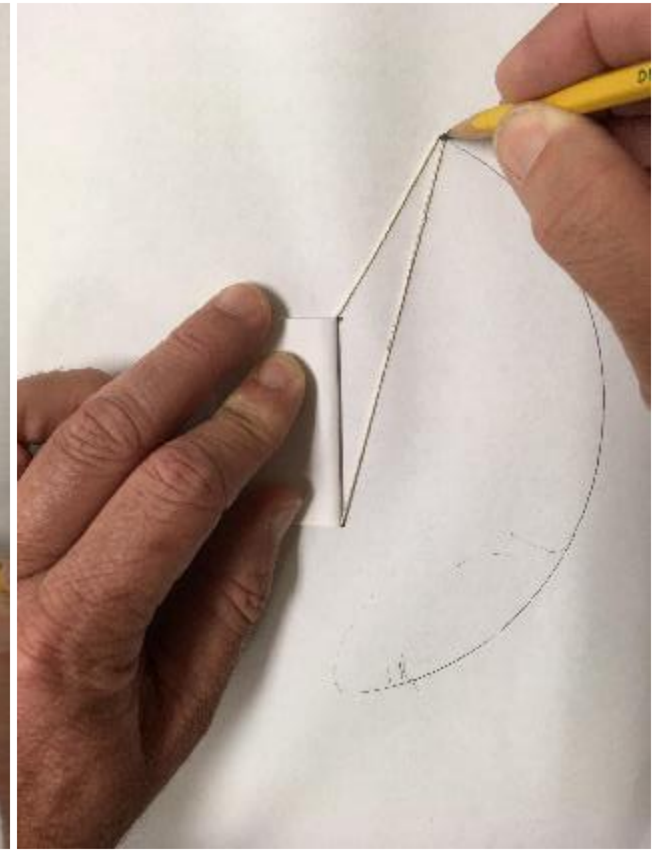

Finished!

Hold folded card, mark each end.

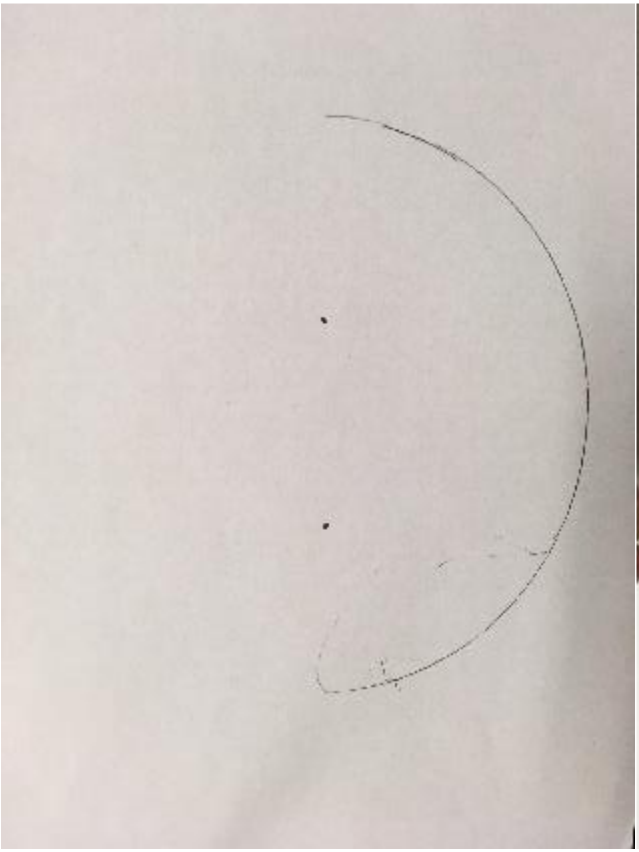
Pencil inside loop, draw half of ellipse.



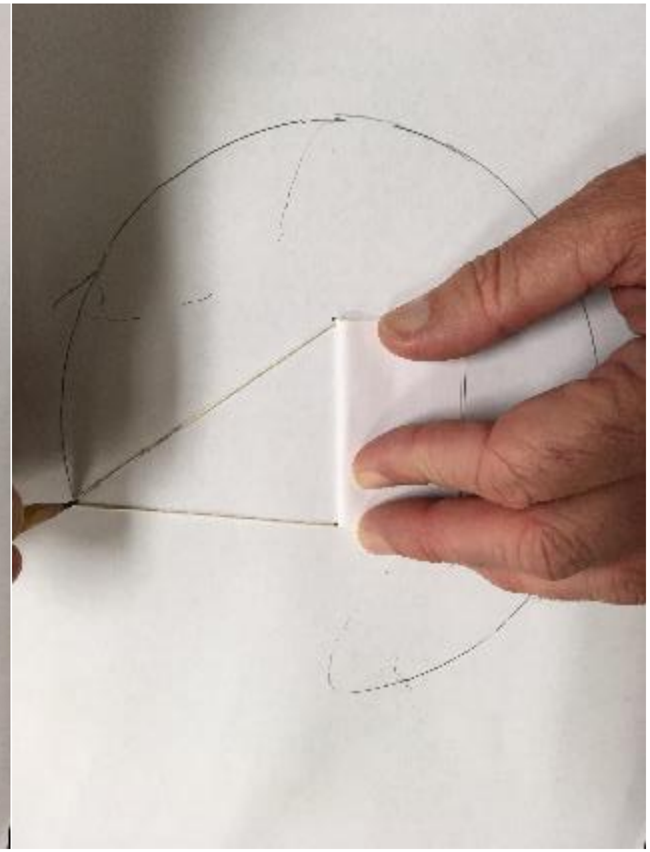


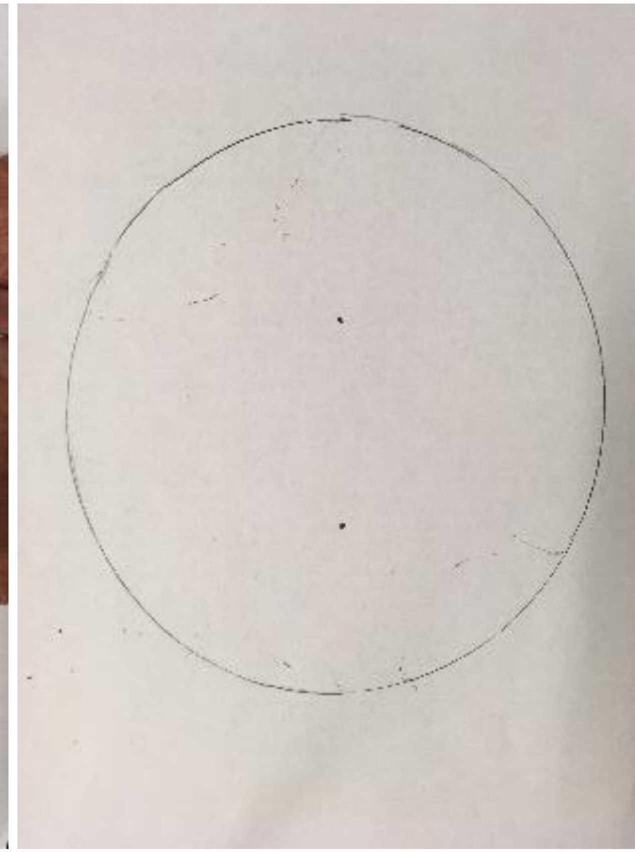


1st half done:



Reposition folded card against the two marks, draw 2nd half:





Finished!

1. Name planet, be creative!
2. Measure a, b, c.
3. Confirm:
   $d_1 + d_2 = 2a$
   $b^2 + c^2 = a^2$
4. Calculate the eccentricity:
   $e = c/a$
5. Use 1 cm = 1 AU calculate ph and ah distances.
6. Determine period.