Measurement & Calculation

I. SI Units, Prefixes, Orders of Magnitude

II. Rates

III. Skinny Triangles

IV. Circles, Arcs

V. Spherical Coordinates

	The student will be able to:	HW:
1	Utilize and convert SI units and other appropriate units in order to solve problems.	1
2	Utilize the concept of orders of magnitude to compare amounts or sizes.	2 – 3
3	Solve problems involving rate, amount, and time.	4 – 7
4	Solve problems involving "skinny triangles."	8 – 13
5	Solve problems relating the radius of a circle to diameter, circumference, arc length, and area.	14 – 15
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SI Units

The Power of Ten!

SI Units

- The SI or International System of units provides the basis for all scientific measurements.
- There are seven base units; all other units are "built" from these seven.
- Three of the most basic units, the meter, the second, and the kilogram, are based upon the planet Earth.

the meter

- 1793: 1 meter = one ten millionth the distance from the north pole to the equator, passing through Lyons.
- 1889: 1 meter = the length of a certain metal bar kept near Paris.
- 1960: 1 meter = 1650763.73 wavelengths of orange-red light of krypton-86.
- 1983: 1 meter = distance traveled by light in a vacuum in 1/299792458th of a second.

the meter – fun trivia

- Based on the original definition the circumference of the Earth would be exactly 40 000 000 m, but the circumference based on the current definition is 40 007 863 m.
- This means that the prototype meter bar created in 1889 was “too short” by 0.0002 m or 0.2 mm.

the second

- originally: 1 second = 1/86400th of the mean solar day.
- 1967: 1 second = 9192631770 vibrations of cesium-133 atom placed under certain conditions

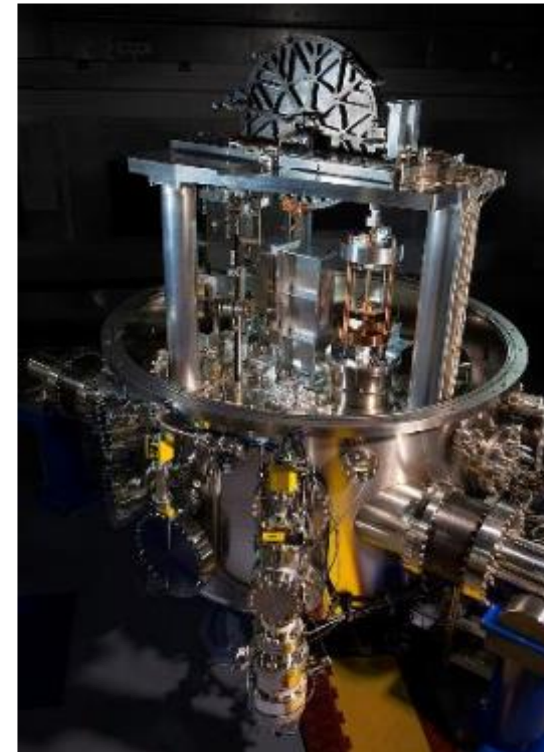
the kilogram

- originally: 1 kilogram = mass of one liter (1000 cm^3) of water
- 1900: 1 kilogram = mass of a certain chunk of metal kept near Paris.
- 2019: 1 kilogram is the mass such that Planck's constant is set at exactly:
  $h = 6.62607015 \times 10^{-34}$ Js
  (or 1 kilogram is a rest mass with energy equal to a collection of photons whose frequencies sum to $1.356392489652 \times 10^{50}$ Hz)

the original kilogram

the kilogram – fun trivia

- Comparisons of the international prototype kilogram to national prototypes had shown drifts of up to 2×10^{-8} kg per year! (Say it isn't so!!)



- A "watt balance" (or "Kibble balance") allows a mass to be precisely related to Planck's constant and vice versa. This is one way to implement the new definition.

Previous (1983 - 2018) | Current (implemented May 2019!)

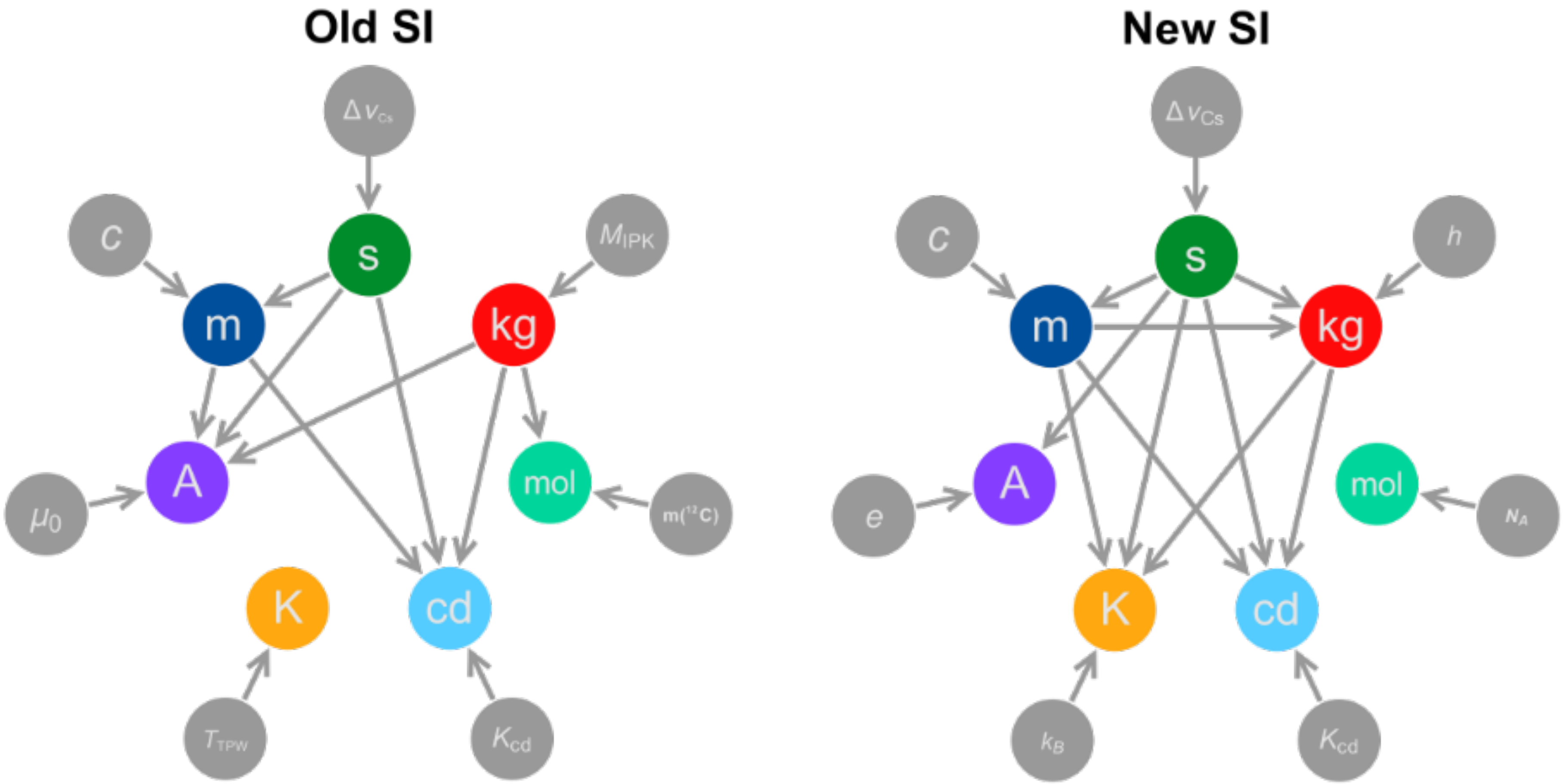


image credit: Emilio Pisanty

Metric Prefixes

factor	name	symbol	factor	name	symbol
10^{24}	yotta	Y	10^{-1}	deci	d
10^{21}	zetta	Z	10^{-2}	centi	c
10^{18}	exa	E	10^{-3}	milli	m
10^{15}	peta	P	10^{-6}	micro	μ
10^{12}	tera	T	10^{-9}	nano	n
10^{9}	giga	G	10^{-12}	pico	p
10^{6}	mega	M	10^{-15}	femto	f
10^{3}	kilo	k	10^{-18}	atto	a
10^{2}	hecto	h	10^{-21}	zepto	x
10^{1}	deka	da	10^{-24}	yocto	y

What You Need to Know!

- Memorize values of the most commonly used prefixes:
  nano-, micro-, milli-, centi-,
  kilo-, mega-, and giga-
- Memorize the value of a liter:
  $1 \text{ L} = 1000 \text{ cm}^3$ or $1 \text{ mL} = 1 \text{ cm}^3$
- Be able to do any unit conversion involving any of the above!

2.5 dozen eggs = ? eggs

42 days = ? weeks

$$2.5\ \cancel{\text{dozen eggs}} \times \frac{12\ \text{eggs}}{1\ \cancel{\text{dozen eggs}}} = 30\ \text{eggs}$$

$$42\ \cancel{\text{days}} \times \frac{1\ \text{week}}{7\ \cancel{\text{days}}} = 6\ \text{weeks}$$

$$2.5\ \cancel{\text{kilometers}} \times \frac{1 \times 10^3\ \text{m}}{1\ \cancel{\text{kilometer}}} = 2500\ \text{m}$$

$$42\ \cancel{\text{min.}} \times \frac{1\ \text{hr.}}{60\ \cancel{\text{min.}}} = 0.70\ \text{h}$$

Factor Label Method

- A conversion factor is an equivalence of two values.
- Multiply or divide by the factor(s) and label the units.
- Check that unwanted units cancel and desired units remain.

Minimum wage of $7.25 per hour is equal to how many cents per second?

7.25 dollars/h = ? cents/s

Minimum wage of \$7.25 per hour is equal to how many cents per second?

$$\frac{7.25\ \cancel{\text{dollar}}}{1\ \cancel{\text{hour}}} \cdot \frac{100\ \text{cents}}{1\ \cancel{\text{dollar}}} \cdot \frac{1\ \cancel{\text{hr}}}{60\ \cancel{\text{min}}} \cdot \frac{1\ \cancel{\text{min}}}{60\ \text{s}} = 0.20\ ¢/\text{s}$$

$$\frac{7.25\ \cancel{\text{meter}}}{1\ \cancel{\text{hour}}} \cdot \frac{100\ \text{cm}}{1\ \cancel{\text{meter}}} \cdot \frac{1\ \cancel{\text{hr}}}{60\ \cancel{\text{min}}} \cdot \frac{1\ \cancel{\text{min}}}{60\ \text{s}} = 0.20\ \text{cm/s}$$

Converting a speed of 7.25 meters per hour into centimeters per second is very similar!

Units Saved* My Life!

- What is the maximum safe radon exposure?
  CRC Handbook: 3×10^{-8} μCi/cm^3
  Random dude in cafeteria: 20 pCi/L
- Convert both values to Ci/m^3 and compare!

*(sort of)

Notice that because 1 meter = 100 cm it follows that 1 cubic meter = 100^3 cubic centimeters:

$$\frac{3\times10^{-8}\ \mu Ci}{cm^3}\cdot\frac{1\ Ci}{10^6\ \mu Ci}\cdot\frac{100^3\ cm^3}{1^3\ m^3} = 3\times10^{-8}\ \mu Ci$$

$$\frac{20\ pCi}{L}\cdot\frac{10^{-12}\ Ci}{1\ pCi}\cdot\frac{1\ L}{1000\ cm^3}\cdot\frac{100^3\ cm^3}{1^3\ m^3} = 2\times10^{-8}\ \mu Ci$$

Notice that the liter is defined as 1000 cubic centimeters.

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Order of Magnitude

An order of magnitude is an approximate factor of ten.

How fast is light?

- The speed of light is 299792458 m/s. The speed of sound in air is 343 m/s. The speed of a car is about 25 m/s. How many orders of magnitude greater is the speed of light?

$299792458/343 = 874030 \approx 10^6$ – light is 6 orders of magnitude faster than sound
$299792458/25 = 1.199 \times 10^8 \approx 10^8$ – light is 8 orders of magnitude faster than a car

How far is it to Mars?

- The distance from Earth to Mars using a Hohmann transfer is 300 Gm. The distance from Earth to Moon 380 Mm. The distance around the Earth is 40000 km. How many orders of magnitude greater is the trip to Mars?

$G = 10^9$, $M = 10^6$, $9 – 6 = 3$, so Mars is 3 orders of magnitude farther than the Moon
$G = 10^9$, $k = 10^3$, $9 – 3 = 6$, so Mars is 6 orders of magnitude farther than circumnavigating the Earth

How far is it to Alpha Centauri?

- The distance to Alpha Centauri is about 4 light-years. The distance to the Sun is about 8 light-minutes. How many orders of magnitude greater is Alpha Centauri?

4 years = 2103780 minutes, 2103780/8 = $2.63 \times 10^5 \approx 10^5$
The nearest star similar to the Sun is 5 orders of magnitude farther away than the Sun.