Momentum and Impulse

I. Momentum and Impulse
- concepts and definition
- relation to force

II. Conservation of Momentum
- internal and external force
- elasticity

	The student will be able to:	HW:
1	Define and calculate momentum using appropriate SI units. ✓	1
2	Define and calculate impulse and solve problems relating impulse, momentum, and force. ✓	2 – 6
3	State and apply the law of conservation of momentum with proper consideration to internal and external forces. Define and analyze center of mass position and velocity.	7 – 10
4	Use conservation of momentum to solve related problems.	11 – 21
5	Define elastic and inelastic collisions and use the definitions to solve related problems.	22 – 27

Conservation of Momentum

The total momentum of an isolated system of objects will remain constant over time.

For two objects that interact with one another:

$$\vec{p}_1 + \vec{p}_2 = \vec{p}_1' + \vec{p}_2'$$

$$\underbrace{m_1\vec{v}_1 + m_2\vec{v}_2}_{\text{total momentum } \textit{before} \text{ an interaction}} = \underbrace{m_1\vec{v}_1' + m_2\vec{v}_2'}_{\text{total momentum } \textit{after} \text{ the interaction}}$$

Conservation of Momentum

The total momentum of an isolated system of objects will remain constant over time.

The same reasoning may be extended to interactions of three or more objects…

$$\vec{p}_1 + \vec{p}_2 + \vec{p}_3 = \vec{p}_1' + \vec{p}_2' + \vec{p}_3'$$

Internal and External Forces

In order for the total momentum of a particular system to remain constant there must be **no net external force** on the system.

When an object *outside* the system interacts with an object *inside* the system this is called **external force**.

When objects within a system interact with one another this is called **internal force**. Internal forces *have no effect on the total momentum*!

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Center of Mass

The center of mass of a system is located at distances inversely proportional to the masses of the system. It is closest to the greatest mass.

The velocity of the center of mass is unaffected by internal forces and can only be changed by external forces.

The total mass of a system multiplied by the velocity of the center of mass is equal to the total momentum of the system.

Center of Mass

The position of the center of mass of a system is a "balancing point" that represents the location of the system as a whole:

$$\vec{x}_{CM} = \frac{m_1\vec{x}_1 + m_2\vec{x}_2 + \cdots}{m_1 + m_2 + \cdots}$$

where m = mass, x = position

The velocity of the center of mass represents the the system's overall motion:

$$\vec{v}_{CM} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + \cdots}{m_1 + m_2 + \cdots}$$

where m = mass, v = velocity

Center of Mass

The position of the center of mass of a system is a "balancing point" that represents the location of the system as a whole:

$$\vec{x}_{CM} = \frac{\Sigma m_i \vec{x}_i}{\Sigma m_i}$$

where m = mass, x = position

The velocity of the center of mass represents the the system's overall motion:

$$\vec{v}_{CM} = \frac{\Sigma m_i \vec{v}_i}{\Sigma m_i}$$

where m = mass, v = velocity

Center of Mass

The total momentum of a system is equal to the total mass times the velocity of the center of mass.

$$\vec{v}_{CM} = \frac{\Sigma m_i \vec{v}_i}{\Sigma m_i} = \frac{\Sigma \vec{p}_i}{\Sigma m_i}$$

$$(\Sigma m_i)\vec{v}_{CM} = \Sigma \vec{p}_i$$

$$\vec{p}_{sys} = (\Sigma m_i)\vec{v}_{CM}$$

A system's momentum and velocity can be changed only by external forces!

Elasticity

Characterizing Collisions

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II. Conservation of Momentum
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- **elasticity**

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Variety in Collisions

- <u>All</u> collisions will illustrate conservation of momentum and conservation of energy.
- However, depending on the nature of the objects involved, only a certain amount of <u>kinetic</u> energy will remain after the collision.
- In certain situations there may be conservation of *kinetic* energy.

Elasticity

In a perfectly **elastic** collision the total kinetic energy of the system remains *constant*.

The total kinetic energy of the system will be *reduced* in an **inelastic** collision.

In a "perfectly inelastic" collision the objects stick together and the reduction in kinetic energy is maximized.

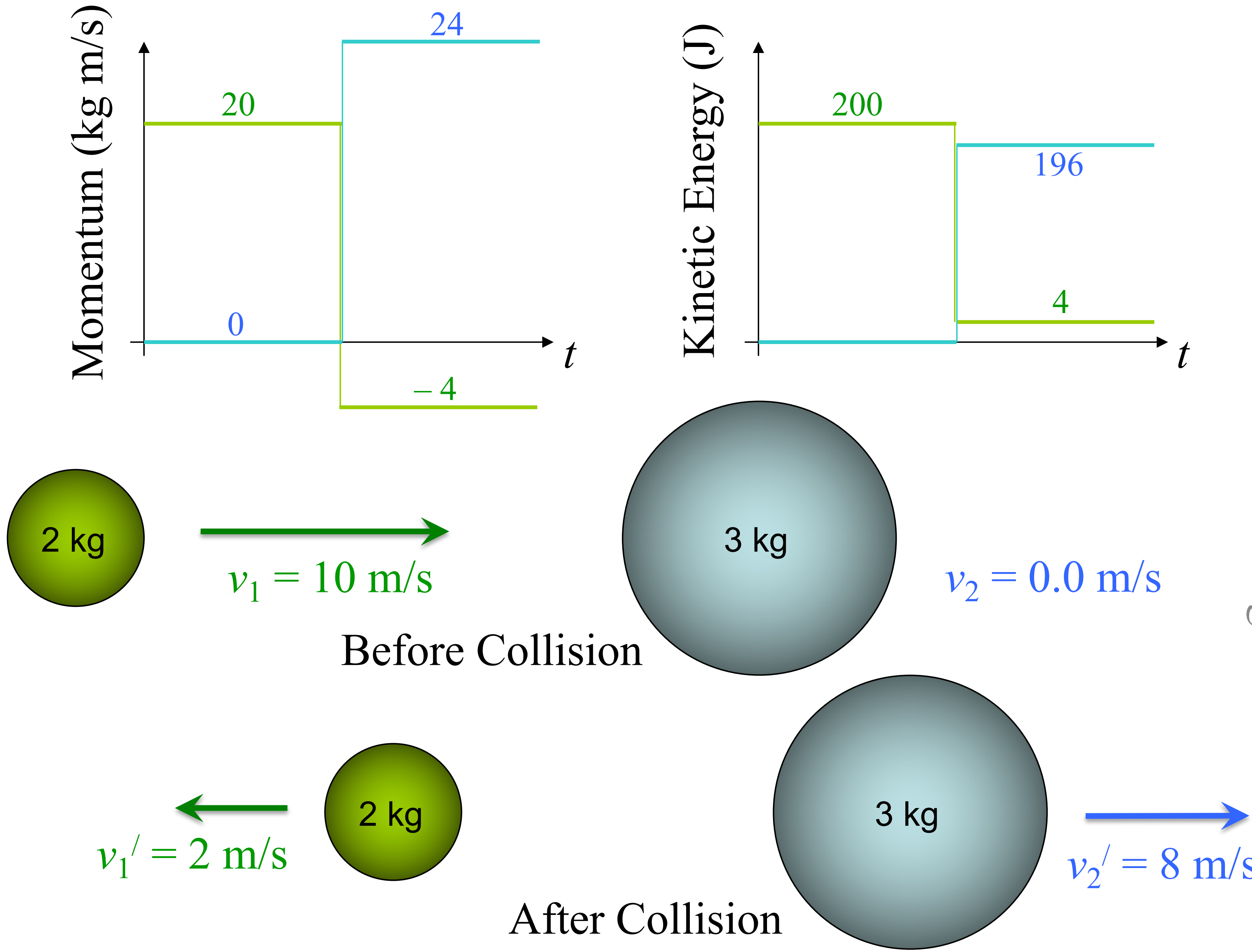
Quantifying Elasticity

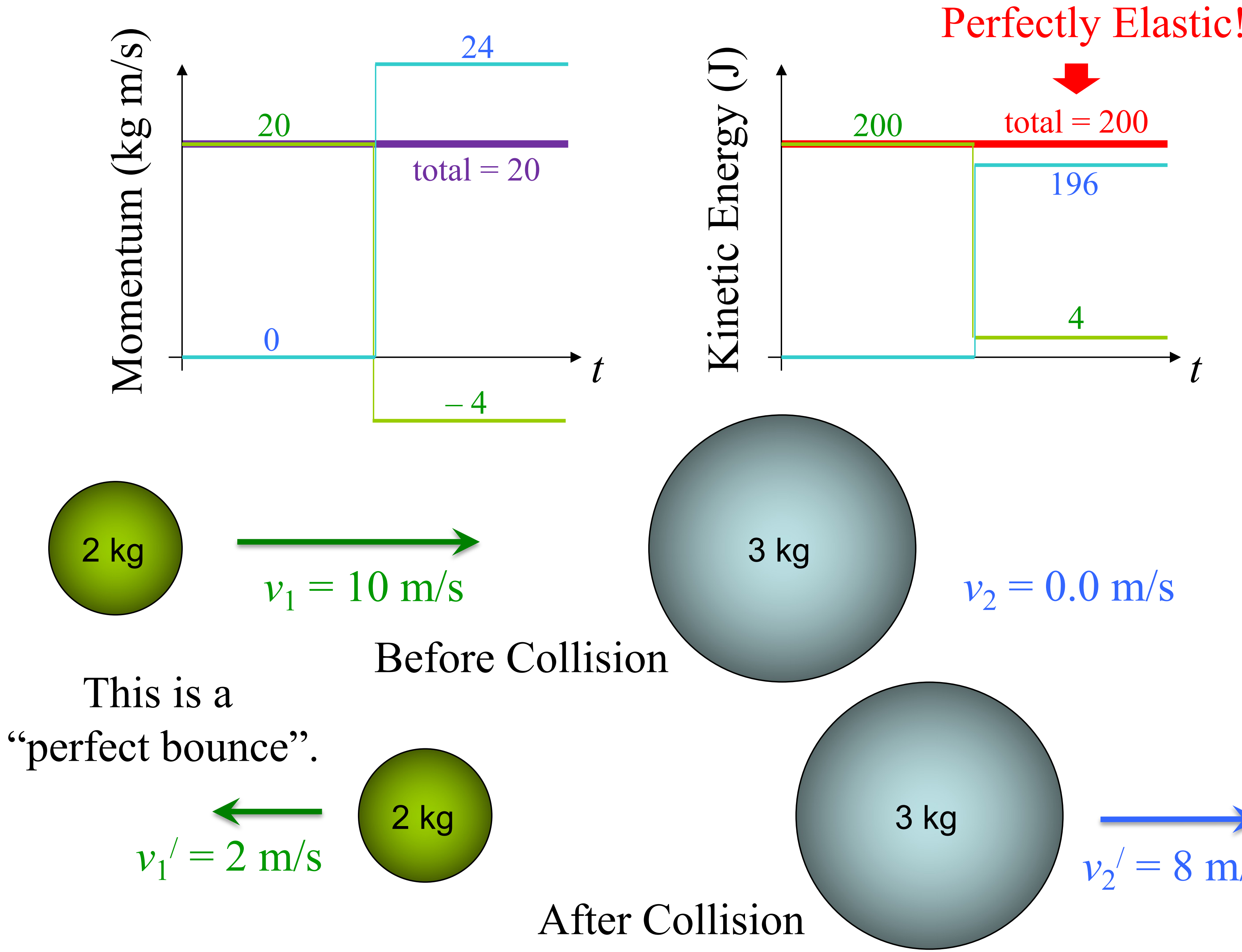
For two objects in a perfectly **elastic** collision:

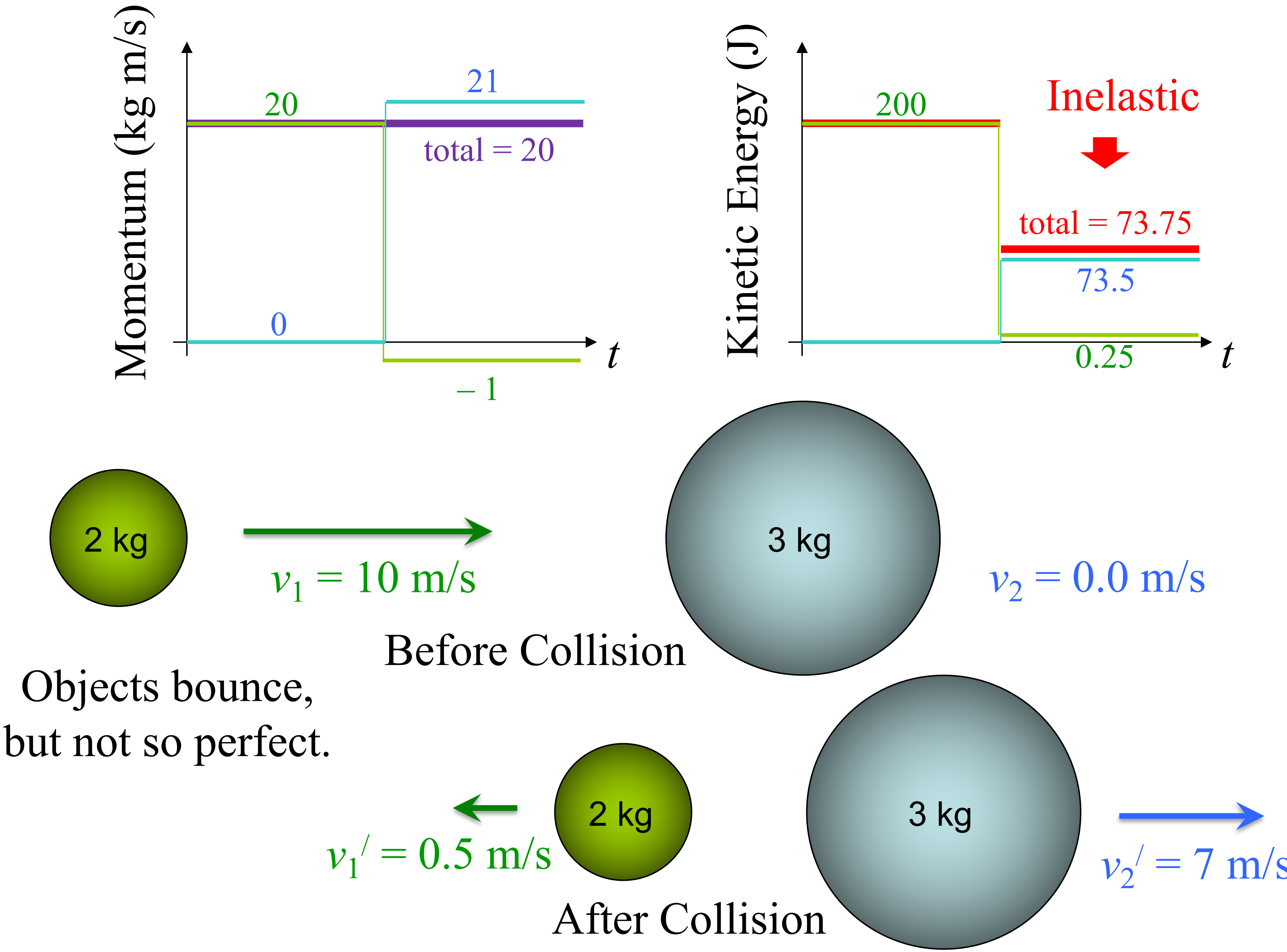
$$KE_1 + KE_2 = KE_1' + KE_2'$$

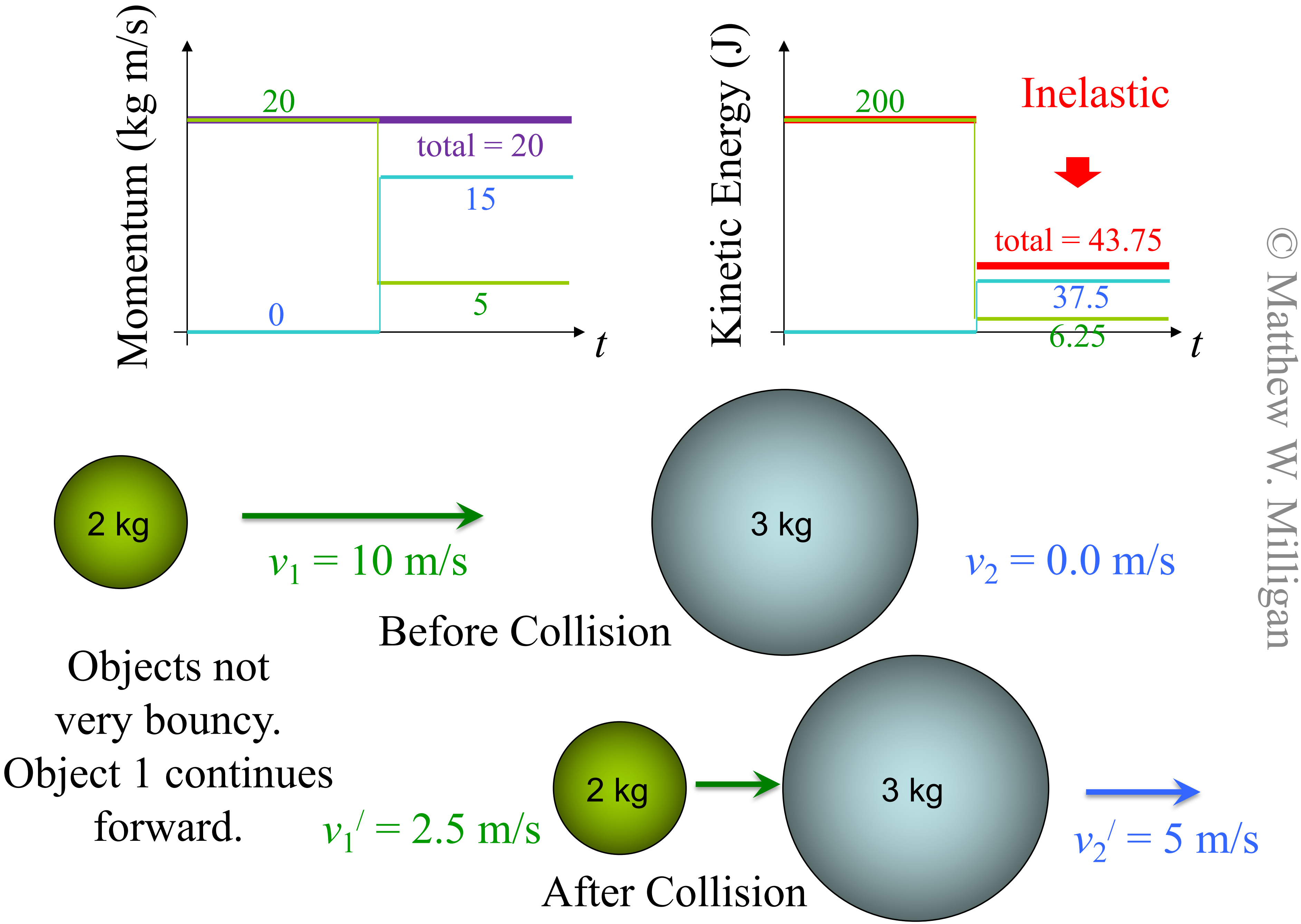
For two objects in an **inelastic** collision:

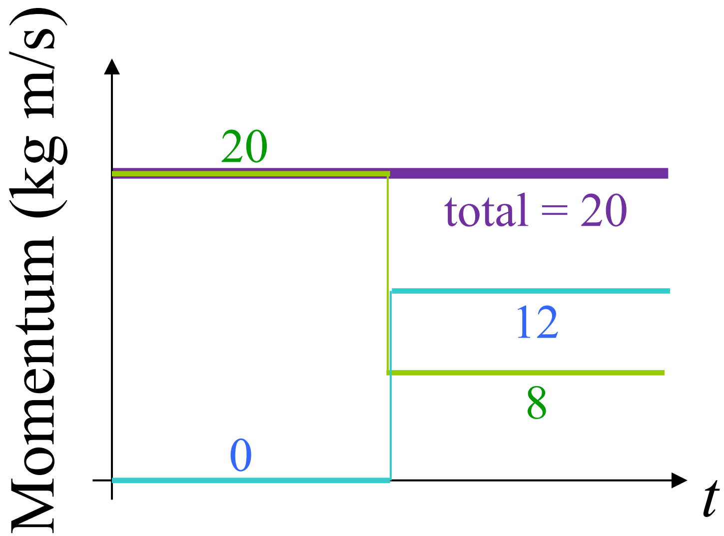
$$KE_1 + KE_2 > KE_1' + KE_2'$$

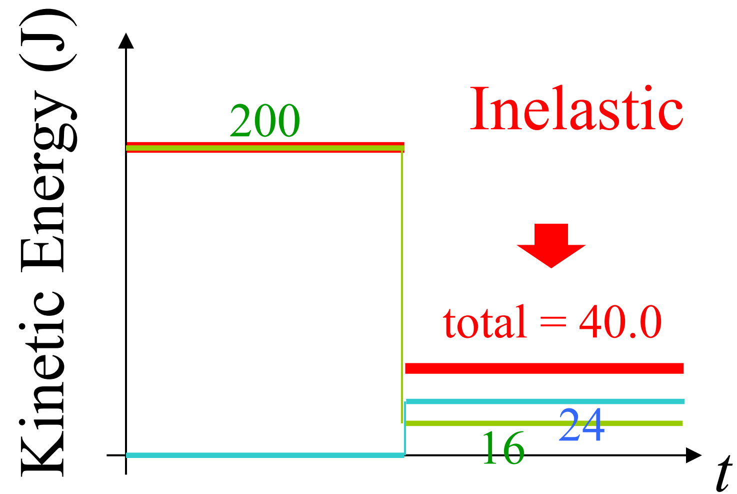

Momentum (kg m/s)
24
20
0
– 4
t
Kinetic Energy (J)
200
196
4
t
2 kg
v1 = 10 m/s
3 kg
v2 = 0.0 m/s
Before Collision
2 kg
v1/ = 2 m/s
3 kg
v2/ = 8 m/s
After Collision

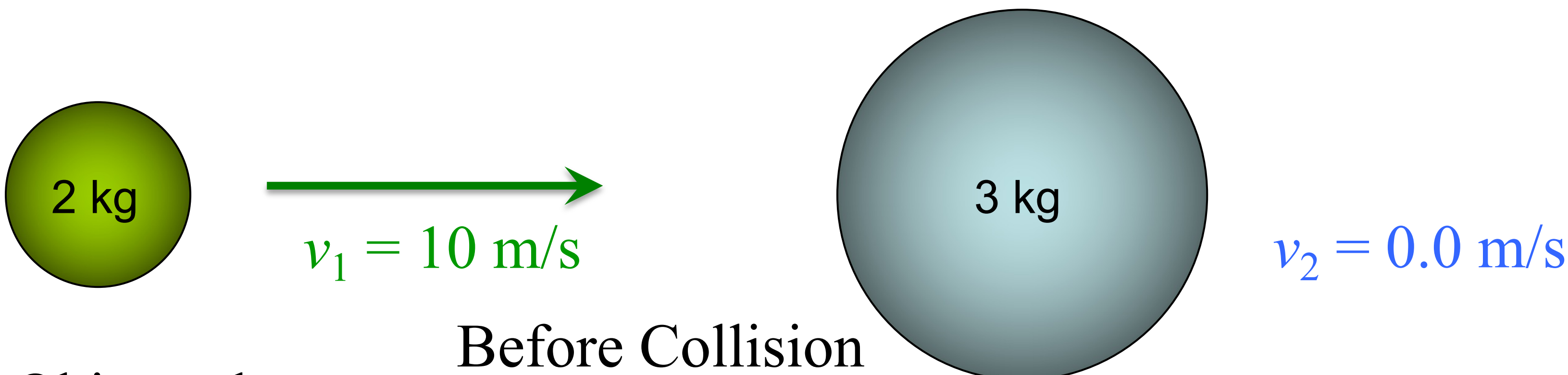

Perfectly Elastic!
Momentum (kg m/s)
20
24
total = 20
0
– 4
t
Kinetic Energy (J)
200
total = 200
196
4
t
2 kg
$v_1 = 10$ m/s
3 kg
$v_2 = 0.0$ m/s
Before Collision
This is a
"perfect bounce".
2 kg
$v_1' = 2$ m/s
3 kg
$v_2' = 8$ m/s
After Collision


Momentum (kg m/s)
20
21
total = 20
0
– 1
t
Kinetic Energy (J)
200
Inelastic
total = 73.75
73.5
0.25
t
2 kg
$v_1 = 10$ m/s
3 kg
$v_2 = 0.0$ m/s
Before Collision
Objects bounce,
but not so perfect.
2 kg
$v_1' = 0.5$ m/s
3 kg
$v_2' = 7$ m/s
After Collision


Momentum (kg m/s)
20
total = 20
15
0
5
t
Kinetic Energy (J)
200
Inelastic
total = 43.75
37.5
6.25
t
2 kg
$v_1 = 10$ m/s
3 kg
$v_2 = 0.0$ m/s
Before Collision
Objects not very bouncy. Object 1 continues forward.
$v_1' = 2.5$ m/s
2 kg
3 kg
$v_2' = 5$ m/s
After Collision
© Matthew W. Milligan







Objects do not bounce at all, but rather stick together. This is said to be "perfectly inelastic".