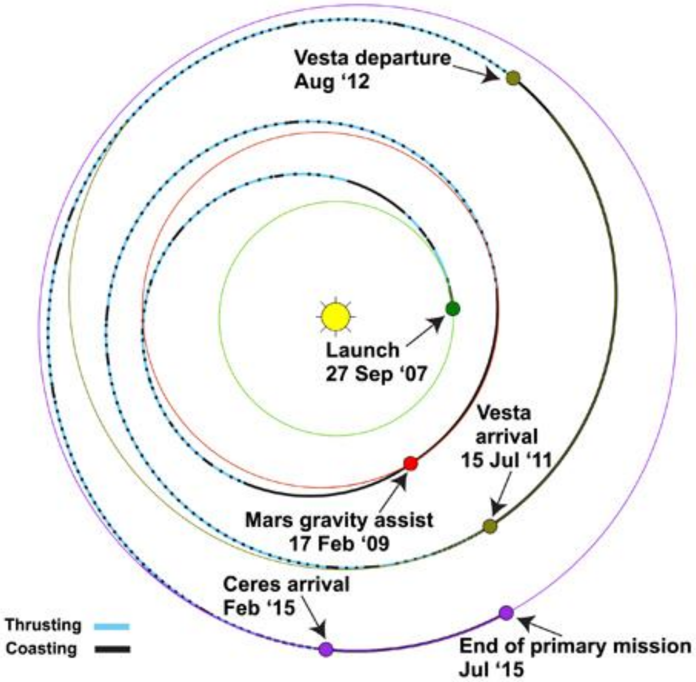
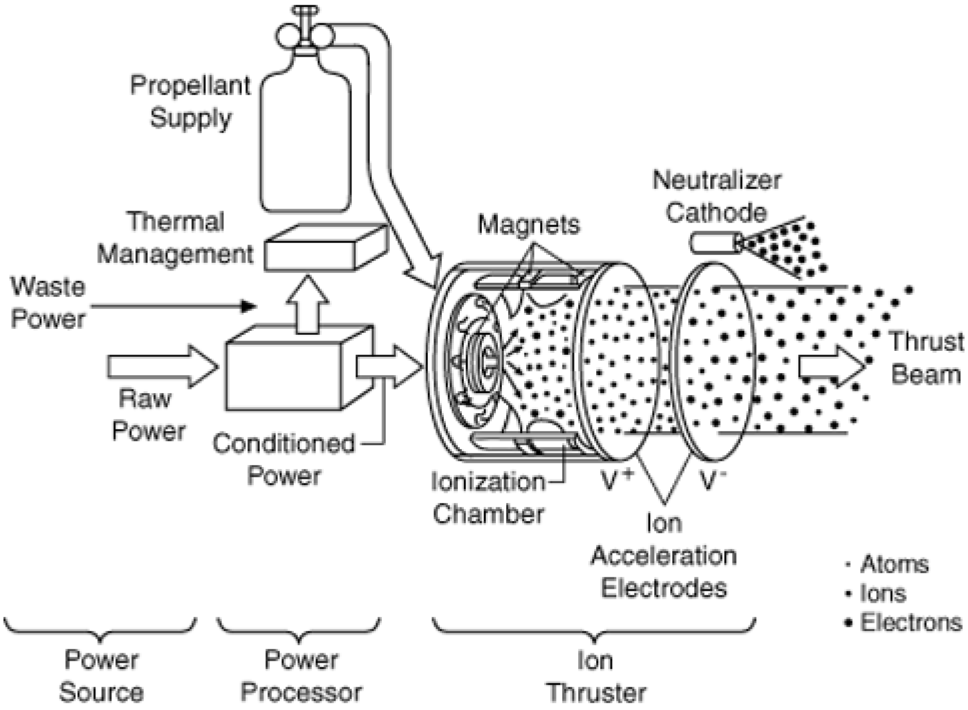
The Physics of DAWN – Ion Propulsion to the Asteroid Belt



1. The DAWN spacecraft had mass 1240 kg at the beginning of its mission. Of this total mass, 430 kg was Xenon "fuel" to be expended and expelled into space by the ion propulsion system over the course of DAWN's mission. (a) The thrust was initially at its maximum 92 mN – what was the initial acceleration rate, disregarding gravity? (b) Toward the end of the mission the thrust will decrease because the engines use solar power and the spacecraft is farther from the Sun – what will be the acceleration rate at minimum thrust 19 mN if the Xenon fuel is nearly expended?
2. The Xenon fuel exits the nozzle of the ion propulsion system at a speed ranging from 20 km/s to 31 km/s. Assume the entire 430 kg of Xenon is ejected into space at an average speed of 25 km/s – by what amount could this change the speed of the spacecraft? Ignore gravity. (Use conservation of momentum – this can be viewed as essentially a "recoil" problem! Note: this is an oversimplification, but still gives a realistic "ballpark figure".)
3. DAWN traveled a distance of 2.8 billion km going from Earth to the asteroid Vesta. Assuming an initial speed of about 33 km/s and a constant acceleration equal to the numerical average of the values found in the previous problem, calculate the amount of time it would take. Compare to the time shown in the chart above. Why the discrepancy?
4. Using the same values as the previous problem what would have been the speed of DAWN arriving at Vesta? Seeing as how Vesta moves slower in its orbit than does Earth, why would this have been problematic if it had been the true speed?
5. In reality DAWN arrived at Vesta moving at about the same speed relative to the Sun as the asteroid moves in its orbit – 20.5 km/s. (The work done by the ion engine did not increase the *speed* of the spacecraft.) (a) Use an average thrust of 56 mN and the distance 2.8 billion km to calculate work done by the ion engine, assuming its thrust was aligned with the direction of travel. (b) Using a mass of 1000 kg for the spacecraft and the initial speed 33 km/s, by how much did DAWN's gravitational potential energy increase as it moved away from the Sun? (This is the "primary task" of the ion engines – to move away from the Sun.)

6. After it arrived, DAWN entered an orbit with radius 2700 km around Vesta (mass 2.59×10^{20} kg) (a) What speed (relative to the asteroid) was required to enter this orbit? (b) Find the orbital period.
7. After spending time in the previous orbit, the ion propulsion system was used to lower the spacecraft to an orbit with a radius of 950 km. (a) Find the speed required for the lower orbit. (b) Find the average work done on DAWN by the gravity of Vesta as it moved from one orbit to the other, assuming its mass was 970 kg. (This is equivalent to finding the *reduction* in potential energy *mgh*, using an average value for *g*). (c) Use the work-energy theorem to determine the work done by the ion engine. (d) Assuming a thrust of 64 mN is opposing the forward motion of the spacecraft, what distance is traveled by DAWN during this transition? (e) Using an average value for the speed determine the number of days required to make the transition.



8. Each Xenon atom (mass 2.18×10^{-25} kg) is singly ionized (loses one electron) and is then accelerated to a speed of 31 km/s over a distance of 1.0 mm between oppositely charged electrode grids. (a) Determine the voltage difference between the two oppositely charged grids. (b) Find the strength of the electric field between the grids.
9. (a) Using the maximum thrust of 92 mN and exhaust speed 31 km/s, determine the mass of Xenon fuel “burned” per second. (b) Repeat for minimum thrust values 19 mN and 20 km/s.
10. Use results from the previous two problems: (a) Determine the current between the two grids at maximum thrust. (b) What is the electrical power associated with this? (This electrical energy comes from photovoltaic solar panels – *i.e.* DAWN is a solar-powered, electric engine spaceship!)

Images from NASA and JPL

Answers:

1. a. 7.4×10^{-5} m/s^2
   b. 2.3×10^{-5} m/s^2
2. 13 km/s
3. 8.0×10^{7} s or 930 days (actual = 1390 days) The discrepancy is mainly due to the effect of the Sun’s gravity. Also this result is based on constant acceleration, which is not the case.
4. 37 km/s This would be problematic because the spacecraft would be going a lot faster around the Sun than the asteroid and would “fly right by”! The small thrust of the ion propulsion would not be nearly sufficient to rapidly slow the spacecraft enough to go into orbit – instead its inertia would carry it past the intended destination.
5. a. 160 GJ
   b. 490 GJ
6. a. $8\underline{0}$ m/s
   b. 59 h
7. a. 130 m/s
   b. 18 MJ
   c. –13 MJ
   d. 2.0×10^{8} m
   e. 21 days
8. a. 650 V
   b. 650 kN/C
9. a. 3.0×10^{-6} kg
   b. 9.5×10^{-7} kg
10. a. 2.2 A
    b. 1.4 kW