Simple Harmonic Motion

Understanding Oscillations

Periodic Motion

I. Circular Motion
- kinematics & centripetal acceleration
- dynamics & centripetal force
- centrifugal force

II. Universal Gravitation
- Newton's "4th" Law
- force fields & orbits

III. Simple Harmonic Motion
- pendulums, springs, etc

	The student will be able to:	HW:
1	Define and calculate period and frequency.	
2	Apply the concepts of position, distance, displacement, speed, velocity, acceleration, and force to circular motion.	
3	State and correctly apply the relation between speed, radius, and period for uniform circular motion.	
4	State and correctly apply the relation between speed, radius, and centripetal acceleration for uniform circular motion.	1 – 14
5	Distinguish and explain the concepts of centripetal vs. centrifugal force.	15 –16
6	State and apply Newton's Law of Universal Gravitation.	17 – 28
7	Combine equations of circular motion and gravitation to solve problems involving orbital motion.	29 – 37
8	State and apply the relation between length, period, and *g* for a pendulum.	38 – 40
9	Solve problems involving application of Hooke's Law to the periodic motion of a mass attached to a spring. Also state and apply the relation between mass, period, and spring constant.	41 – 43

Basic Ideas:

- **Simple Harmonic Motion** (SHM) is a special type of oscillation that occurs under certain conditions.
- In order for SHM to occur, there must be a restoring force proportional to displacement from a position of equilibrium.
- The oscillation in SHM is sinusoidal (can be modeled by the sine function).

Condition for SHM

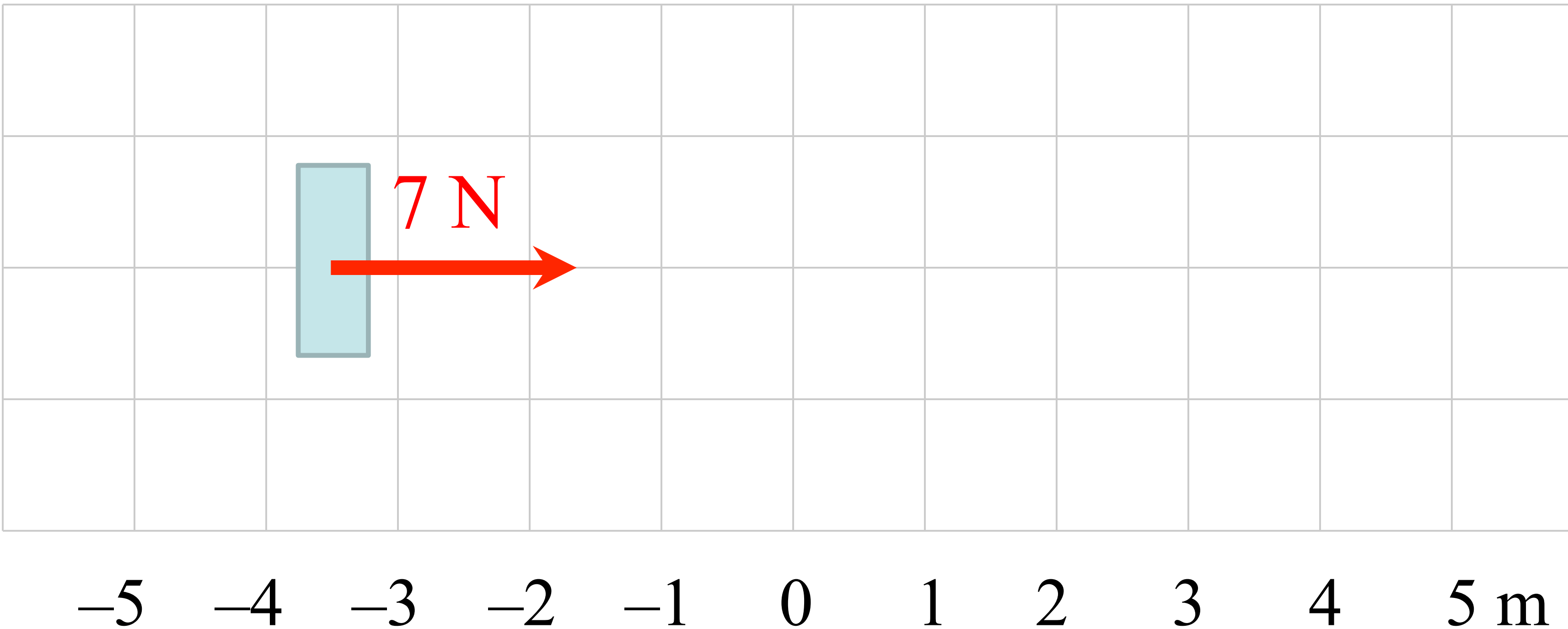
$$\vec{F} = -k\vec{x}$$

Where: **F** = net force acting on object

k = a positive constant

x = position relative to equilibrium

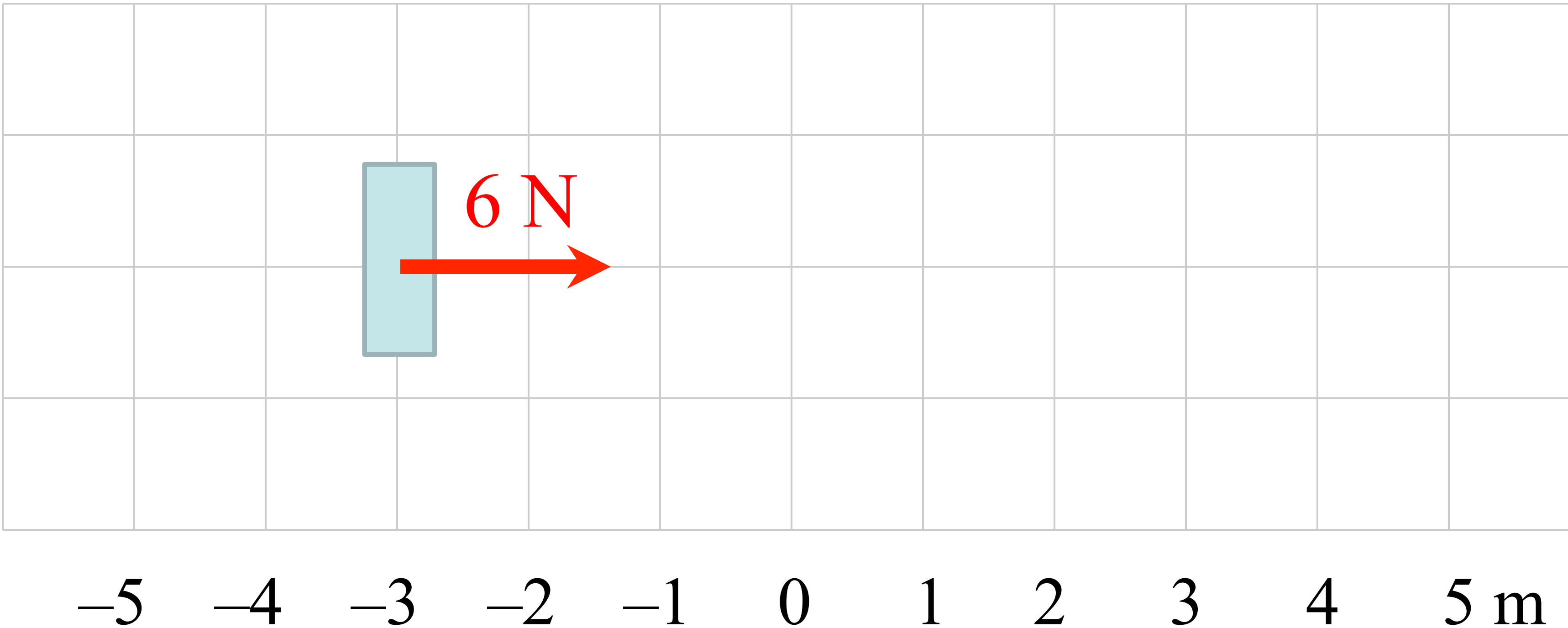
The red arrows indicate the net force:



Flip back and forth through seven pages to create a "stop action" animation of Simple Harmonic Motion!

page 1 of 7 $t = 0.0$ s

The red arrows indicate the net force:

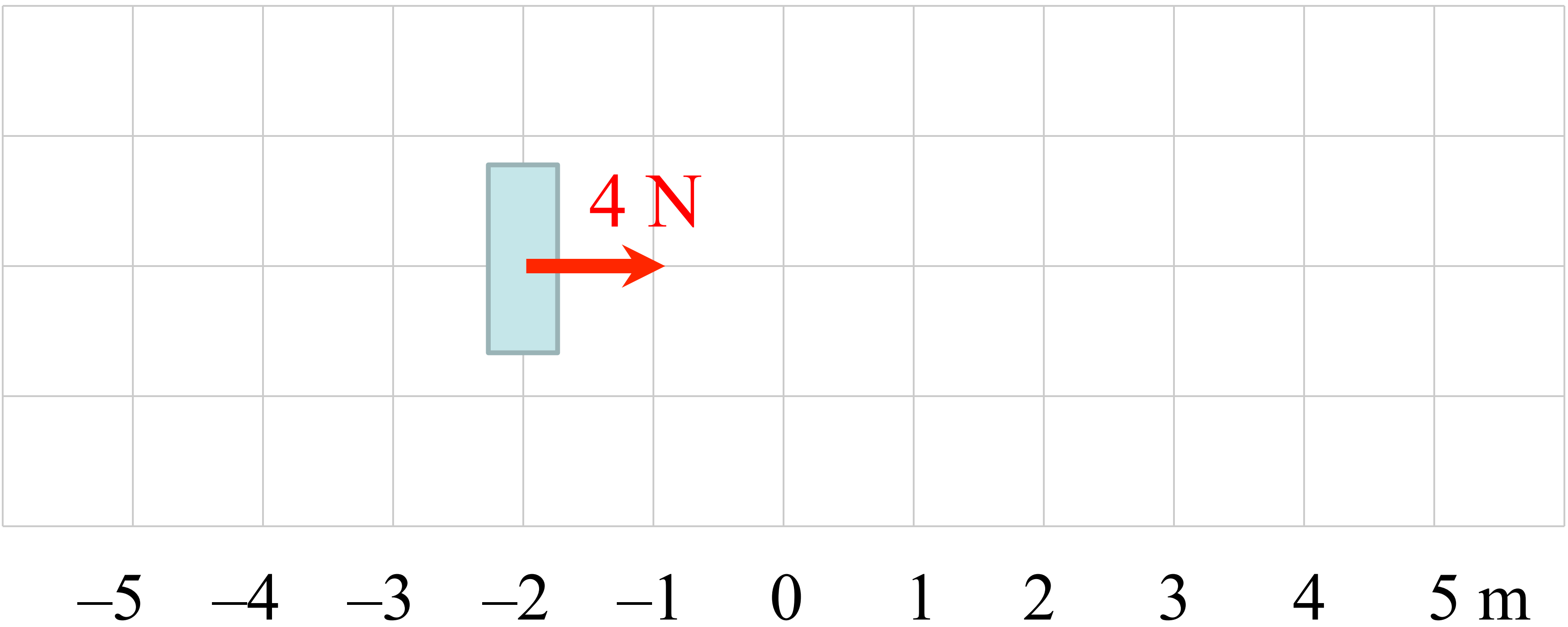


Flip back and forth through seven pages to create a "stop action" animation of Simple Harmonic Motion!

page 2 of 7

$t = 0.5$ s

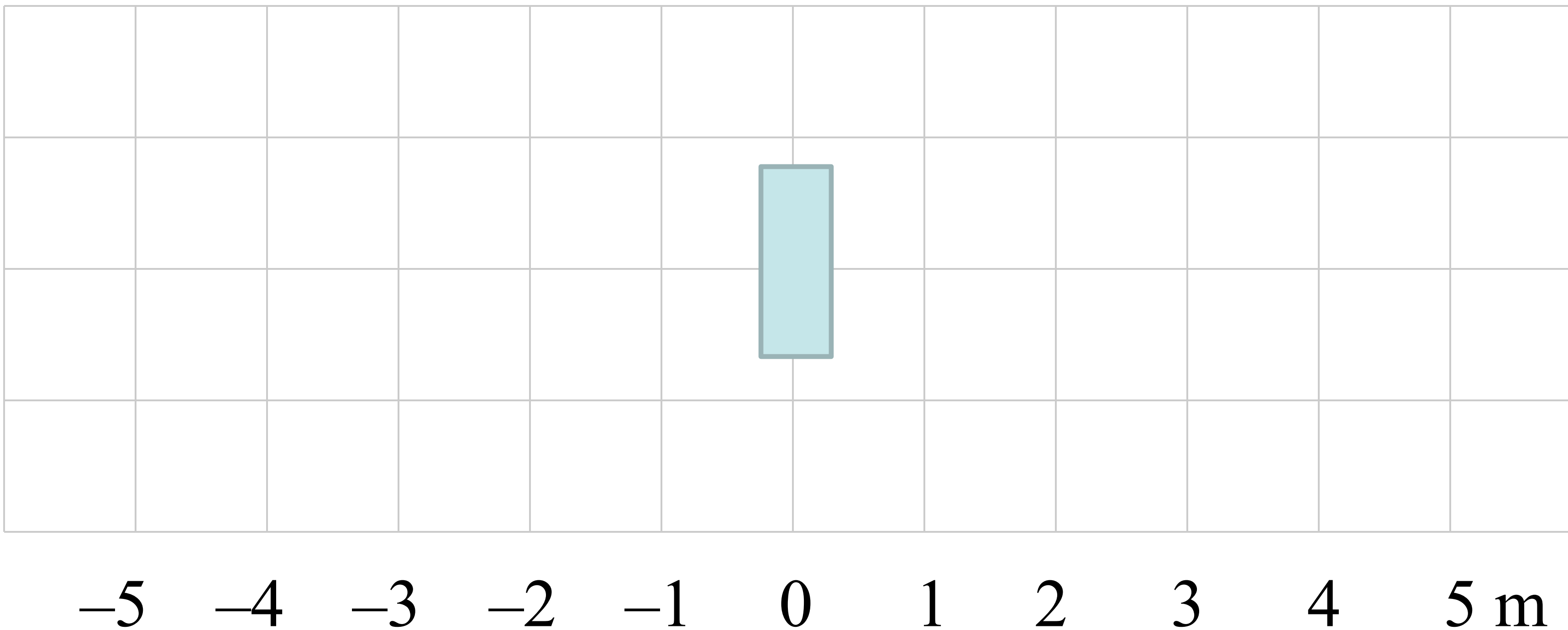
The red arrows indicate the net force:



Flip back and forth through seven pages to create a "stop action" animation of Simple Harmonic Motion!

 $t = 1.0$ s

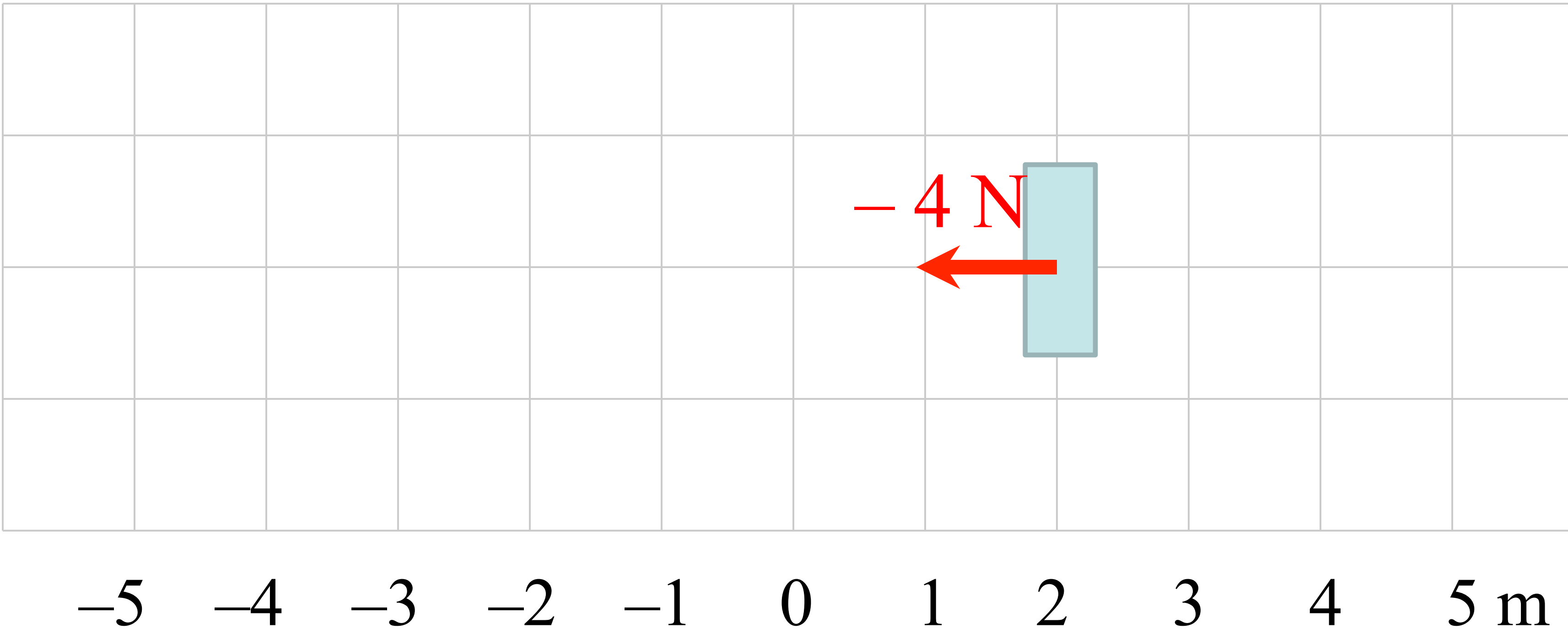
The red arrows indicate the net force:



Flip back and forth through seven pages to create a "stop action" animation of Simple Harmonic Motion!

page 4 of 7 $t = 1.5$ s

The red arrows indicate the net force:

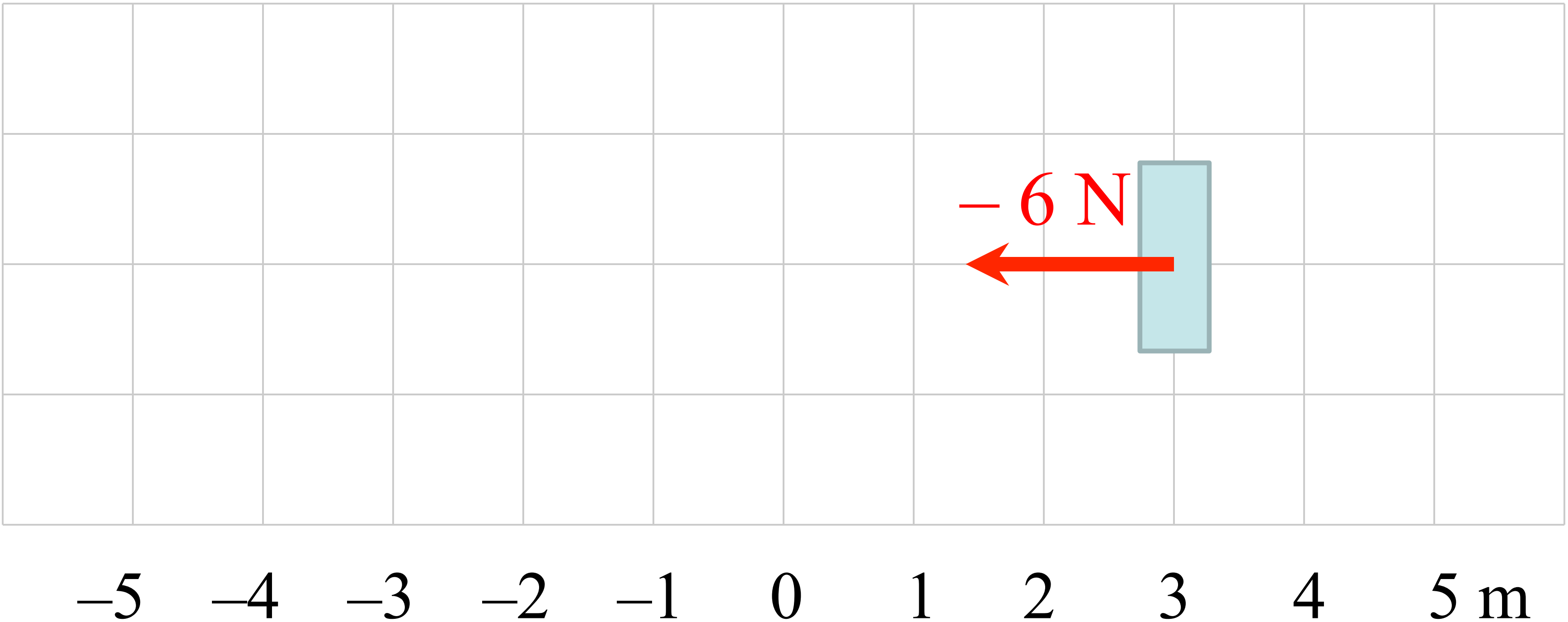


Flip back and forth through seven pages to create a "stop action" animation of Simple Harmonic Motion!

page 5 of 7

$t = 2.0$ s

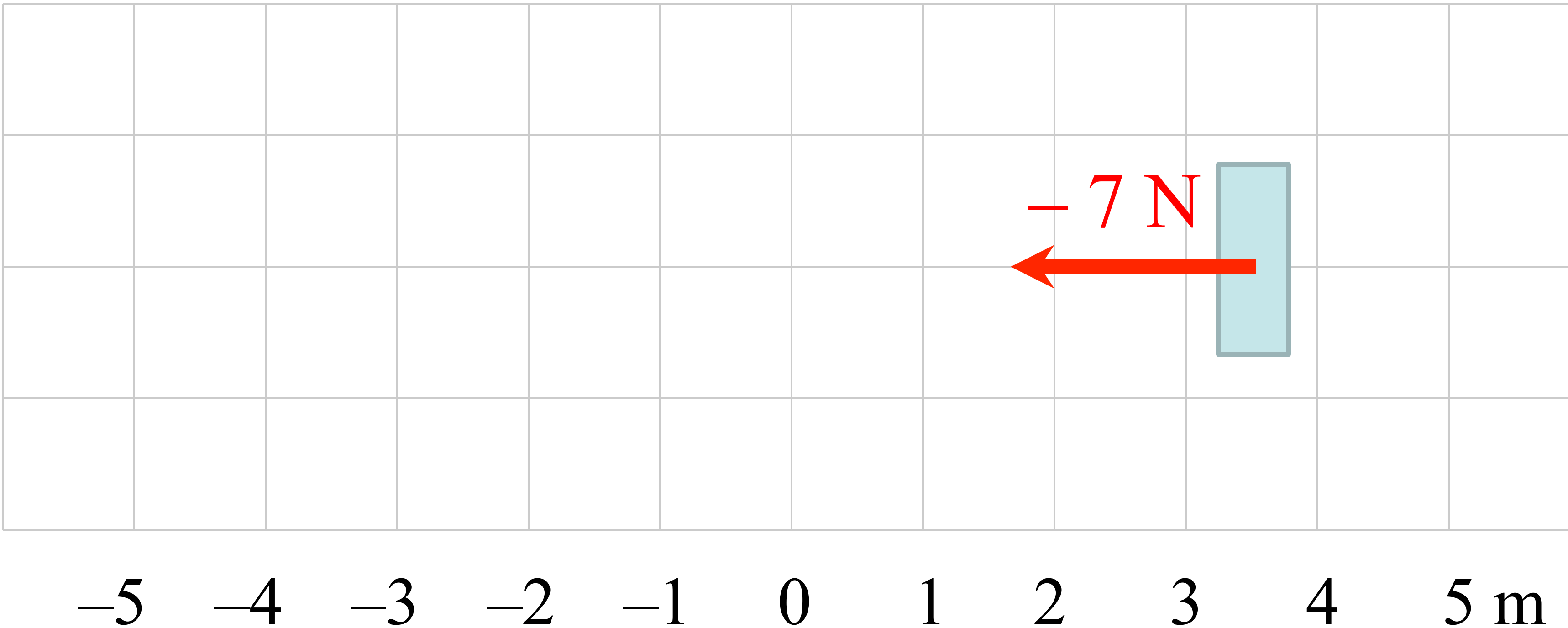
The red arrows indicate the net force:



Flip back and forth through seven pages to create a "stop action" animation of Simple Harmonic Motion!

page 6 of 7 $t = 2.5$ s

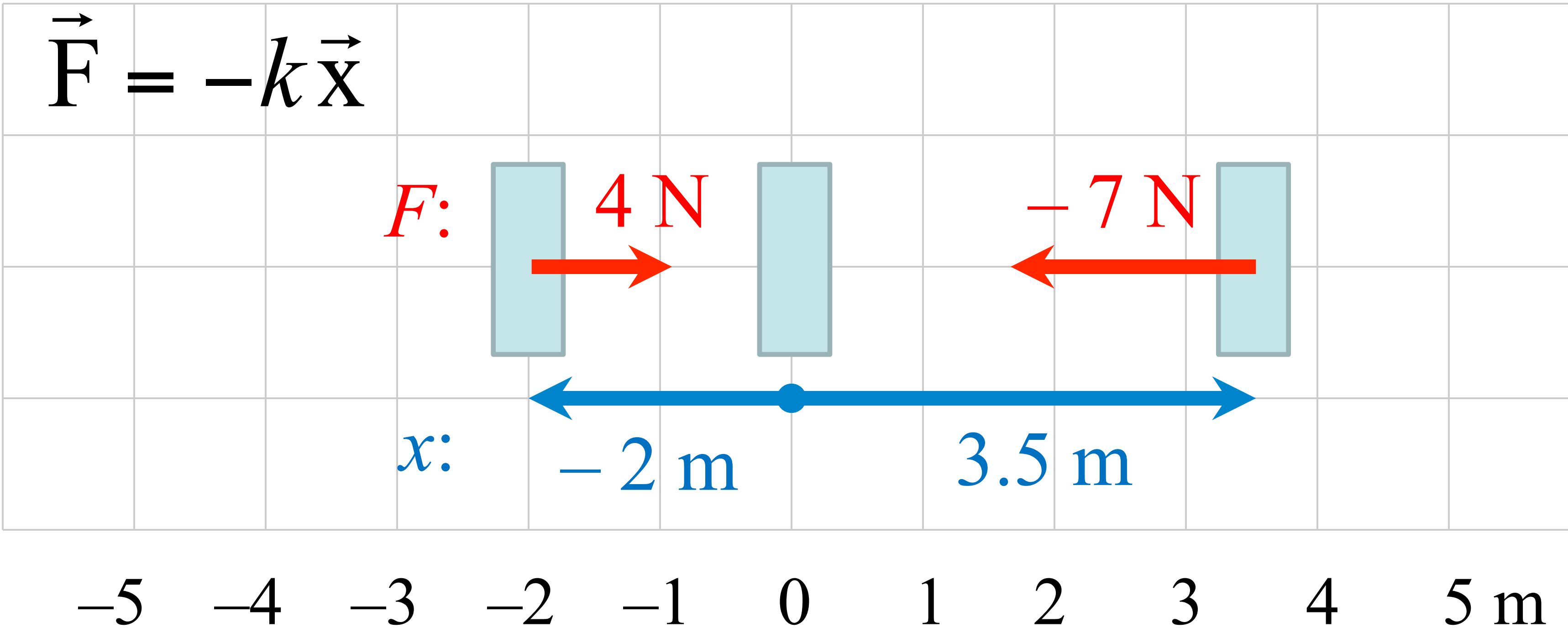
The red arrows indicate the net force:



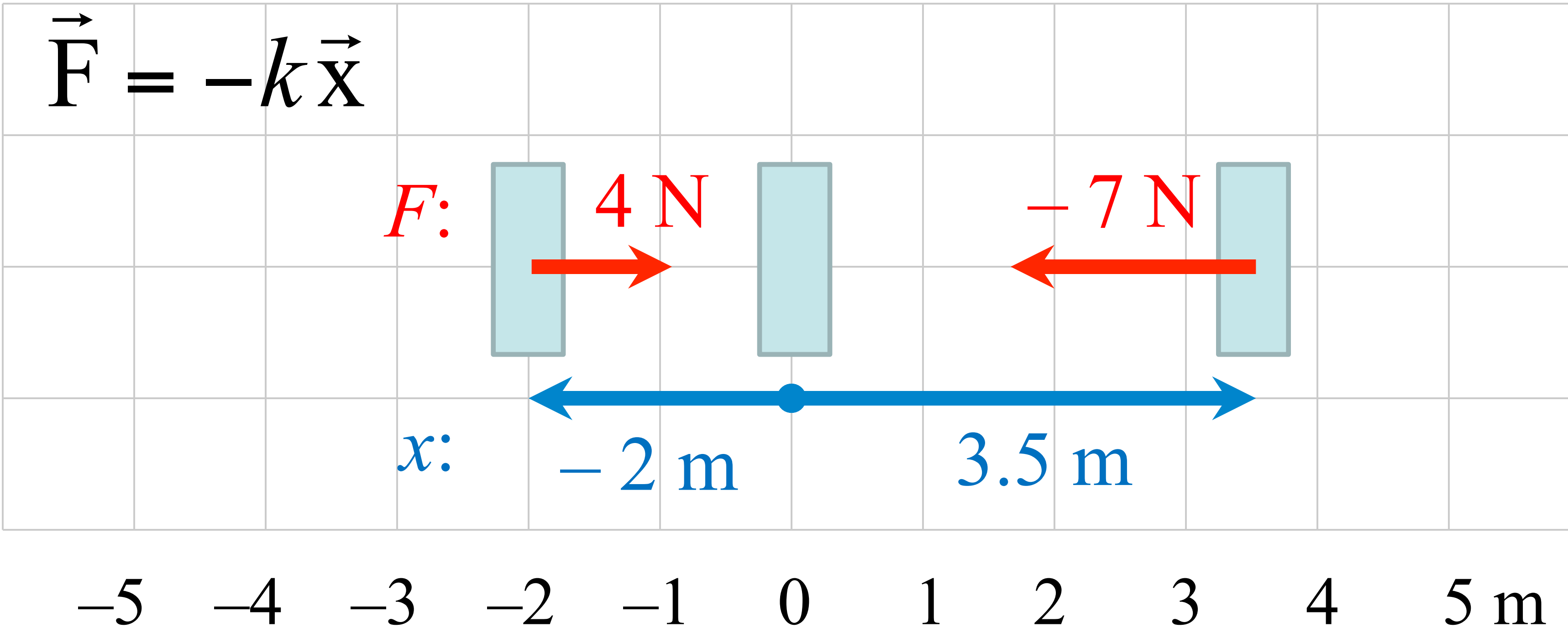
Flip back and forth through seven pages to create a "stop action" animation of Simple Harmonic Motion!

page 7 of 7 $t = 3.0$ s

What would be the value of k for this example?

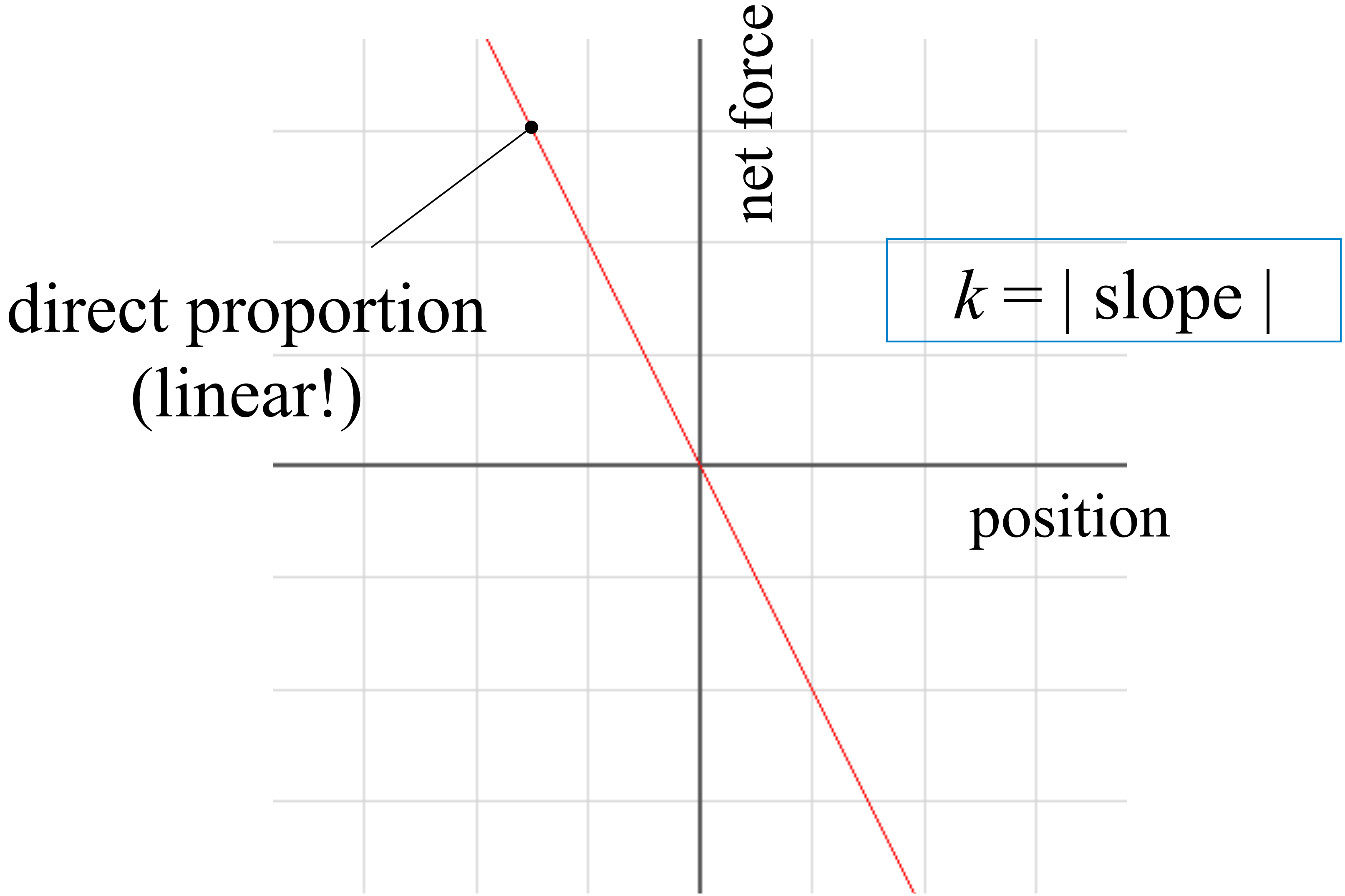


What would be the value of k for this example?



$$k = \left|\frac{F}{x}\right| = \frac{4}{2} = \frac{7}{3.5}$$

$k = 2$ N/m


net force
position
direct proportion
(linear!)
$k = |\text{ slope }|$

Resulting Motion

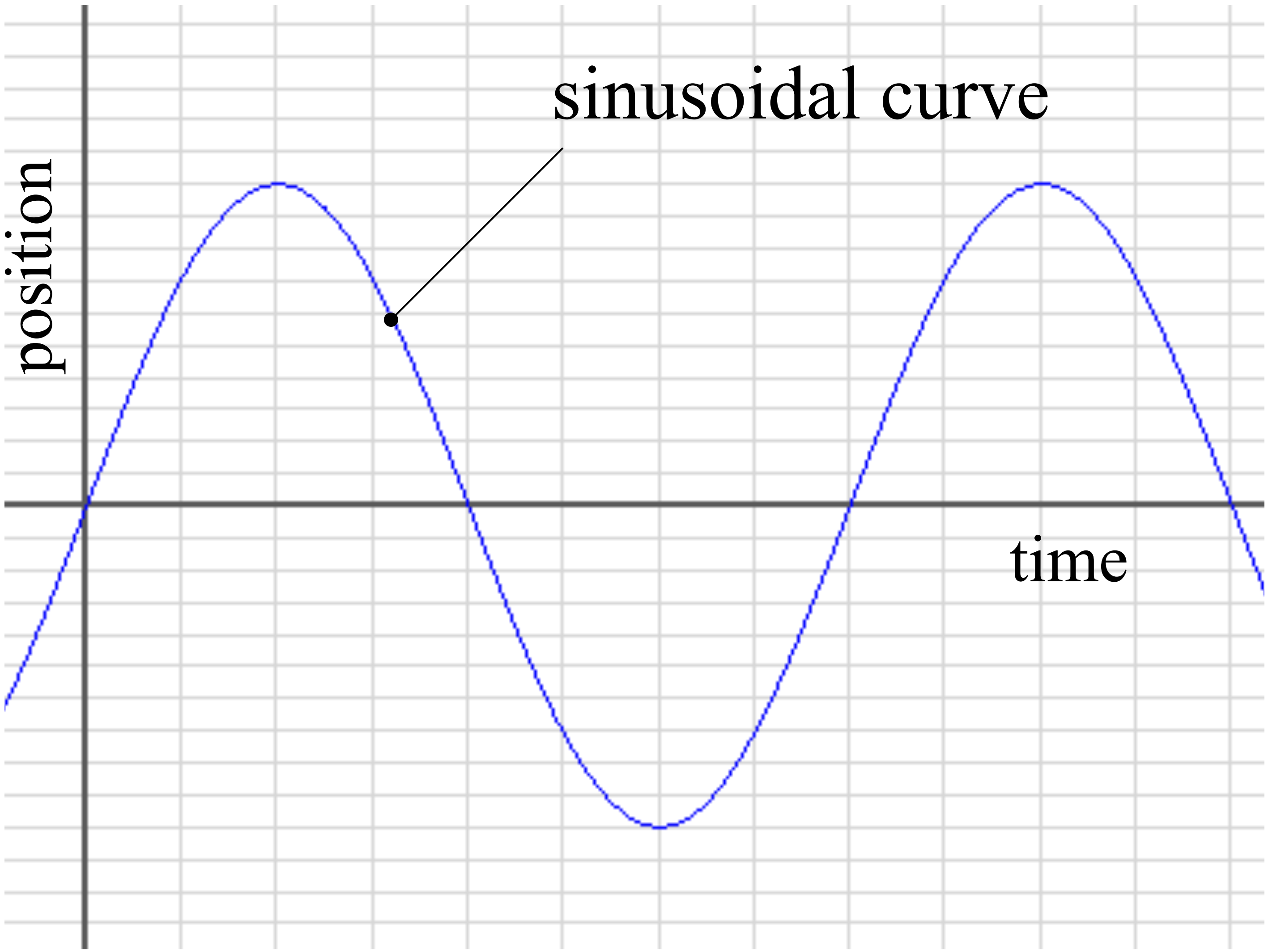
$$T = 2\pi\sqrt{\frac{m}{k}}$$

Where: T = period of oscillation

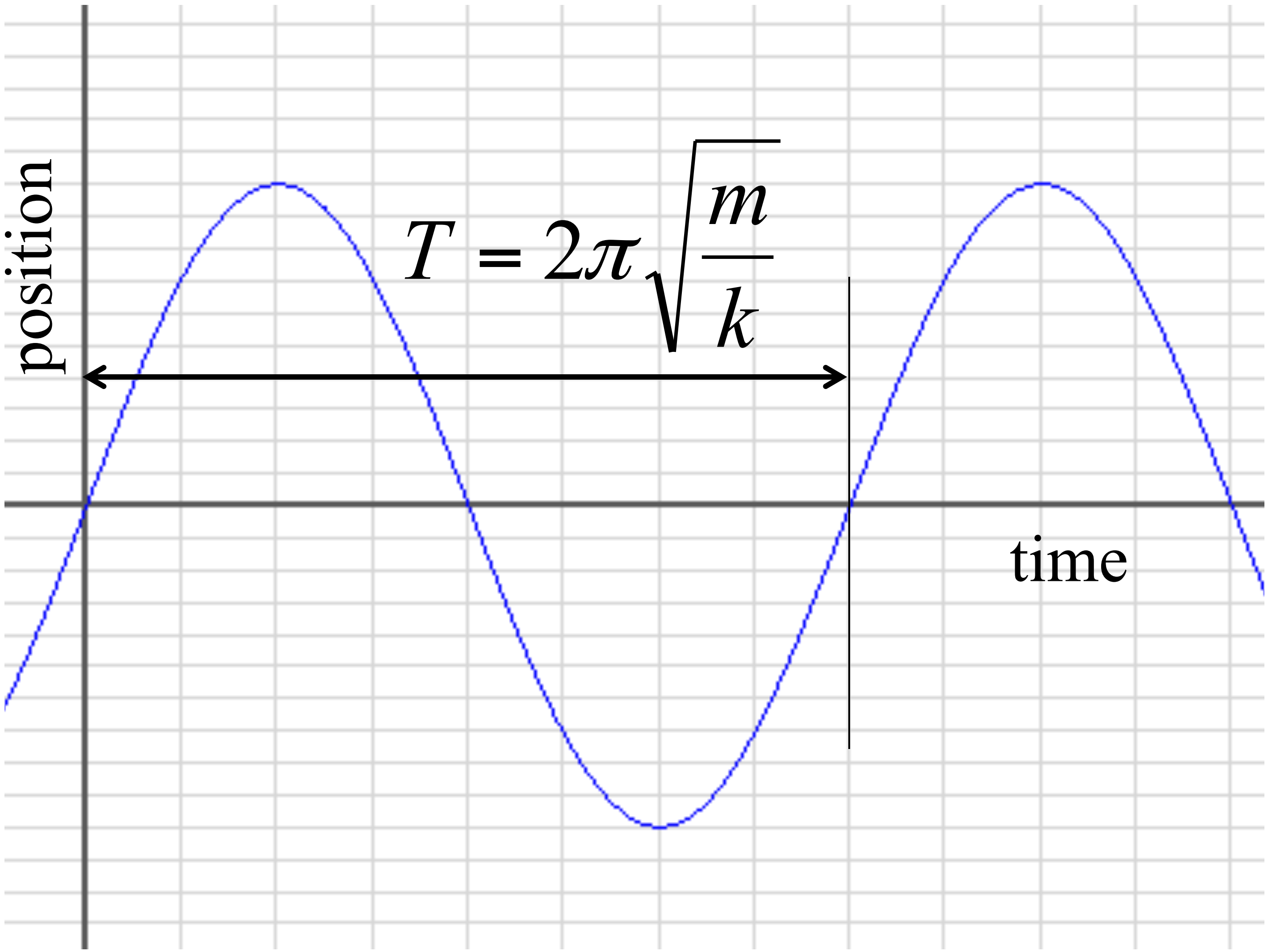
k = the constant from $\mathbf{F} = -k\mathbf{x}$

m = mass of object

Position vs. Time



Position vs. Time



Notes on Period

$$T = 2\pi\sqrt{\frac{m}{k}}$$

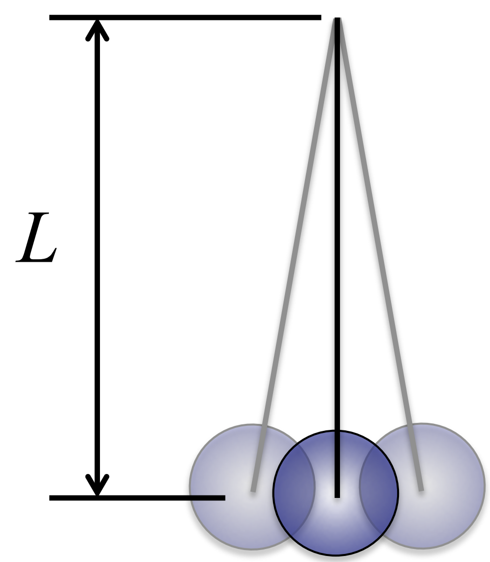
It is remarkable what is *not* in this equation – the amplitude or size of the oscillation. In other words the period <u>does not</u> depend on the size of the oscillations!

Mass on a Spring

- According to **Hooke's Law** any common steel spring will apply a force that is proportional to its elongation or compression
- Every spring has a unique ratio of force to change designated as k, the "spring constant".
- Therefore a mass attached to a spring will undergo SHM.
- In this situation the spring constant is the same k as found in the condition for SHM.

Pendulum

- A pendulum will exhibit SHM to a high degree of accuracy as long as the amplitude of its swing is less than 10° or so (from vertical).
- In this situation it can be shown that $k = mg/L$.
- Therefore the period of a pendulum is given by:



$$T = 2\pi\sqrt{\frac{L}{g}}$$