Rotation

I. Kinematics
- Angular analogs

II. Dynamics
- Torque and Rotational Inertia

III. Work and Energy

IV. Angular Momentum
- Bodies and particles

V. Elliptical Orbits

	The student will be able to:	HW:
1	Define angular position, angular displacement, angular velocity, angular acceleration and solve related problems in fixed axis kinematics.	1 – 5
2	Define torque, lever arm (moment arm), and solve related problems.	6 – 8
3	Define rotational inertia (moment of inertia), use provided formulas of such to solve related problems.	9 – 11
4	Solve rotational dynamics problems using relation between torque, rotational inertia, and angular acceleration for fixed axis.	12 – 18
5	Define rotational kinetic energy and work and solve related problems.	19 – 22
6	Define angular momentum and angular impulse and solve related problems.	23 – 26
7	State and apply conservation of angular momentum to solve related problems.	
8	Analyze orbital motion, including elliptical orbits, using conservation of angular momentum and energy.	27 – 30

Work, Energy, Power for Rotation

- The definitions and units for work, energy, and power do not change for rotational motion!
- Key difference is the relation of work to torque and the relation of kinetic energy to angular speed.
- The equations are just as expected using the analogous rotational quantities:

$$W = \tau\theta$$

$$K = \frac{1}{2}I\omega^2$$

Work, Energy, Power for Rotation

- The work energy theorem and conservation of energy are exactly the same!

$$\Sigma W = \Delta K$$

$$\Sigma W_{NC} + U_1 + K_1 = U_2 + K_2$$

$$W = \tau\theta$$

$$K = \frac{1}{2} I\omega^2$$

Rotation

I. Kinematics
- Angular analogs

II. Dynamics
- Torque and Rotational Inertia

III. Work and Energy

IV. Angular Momentum
- Bodies and particles

V. Elliptical Orbits

	The student will be able to:	HW:
1	Define angular position, angular displacement, angular velocity, angular acceleration and solve related problems in fixed axis kinematics.	1 – 5
2	Define torque, lever arm (moment arm), and solve related problems.	6 – 8
3	Define rotational inertia (moment of inertia), use provided formulas of such to solve related problems.	9 – 11
4	Solve rotational dynamics problems using relation between torque, rotational inertia, and angular acceleration for fixed axis.	12 – 18
5	Define rotational kinetic energy and work and solve related problems.	19 – 22
6	Define angular momentum and angular impulse and solve related problems.	23 – 26
7	State and apply conservation of angular momentum to solve related problems.	
8	Analyze orbital motion, including elliptical orbits, using conservation of angular momentum and energy.	27 – 30

Momentum and Impulse for Rotation

- Recall that linear momentum is the product of mass and velocity and that impulse is the product of force and time – apply the analogous quantities of rotation to get the rotational equivalents.
- Recall that net force equals rate of change in linear momentum – a similar statement can be made for torque and angular momentum.
- Recall that linear momentum is conserved when net external force is zero – essentially the same can be said for angular momentum.

Angular Momentum is the product of rotational inertia and angular velocity.

$$\vec{L} = I\vec{\omega}$$

Angular momentum is a vector in the same direction as angular velocity.

Its magnitude has SI units of: kg m^2/s.

Torque, Angular Momentum, Angular Impulse

Starting with Newton's 2nd Law for rotation it can be shown that net torque equals rate of change in angular momentum…

$$\vec{\tau}_{net} = \frac{\Delta \vec{L}}{\Delta t}$$

$$\vec{\tau}_{net}\left(\Delta t\right) = \Delta \vec{L}$$

…which leads to a formula that equates net angular impulse with change in angular momentum.

Conservation of Angular Momentum

The total angular momentum of an isolated system of objects will remain constant over time.

For two objects that interact with one another:

$$\vec{L}_1 + \vec{L}_2 = \vec{L}_1' + \vec{L}_2'$$

$$\underbrace{I_1\vec{\omega}_1 + I_2\vec{\omega}_2}_{\text{total angular momentum } \textit{before} \text{ an interaction}} = \underbrace{I_1\vec{\omega}_1' + I_2\vec{\omega}_2'}_{\text{total angular momentum } \textit{after} \text{ the interaction}}$$

Internal and External Torques

In order for the total angular momentum of a particular system to remain constant there must be **no net external torque** on the system.

When an object *outside* the system interacts with an object *inside* the system this is called **external torque**.

When objects within a system interact with one another this is called **internal torque**. Internal torques *have no effect on total angular momentum*!

Rotation

I. Kinematics
- Angular analogs

II. Dynamics
- Torque and Rotational Inertia

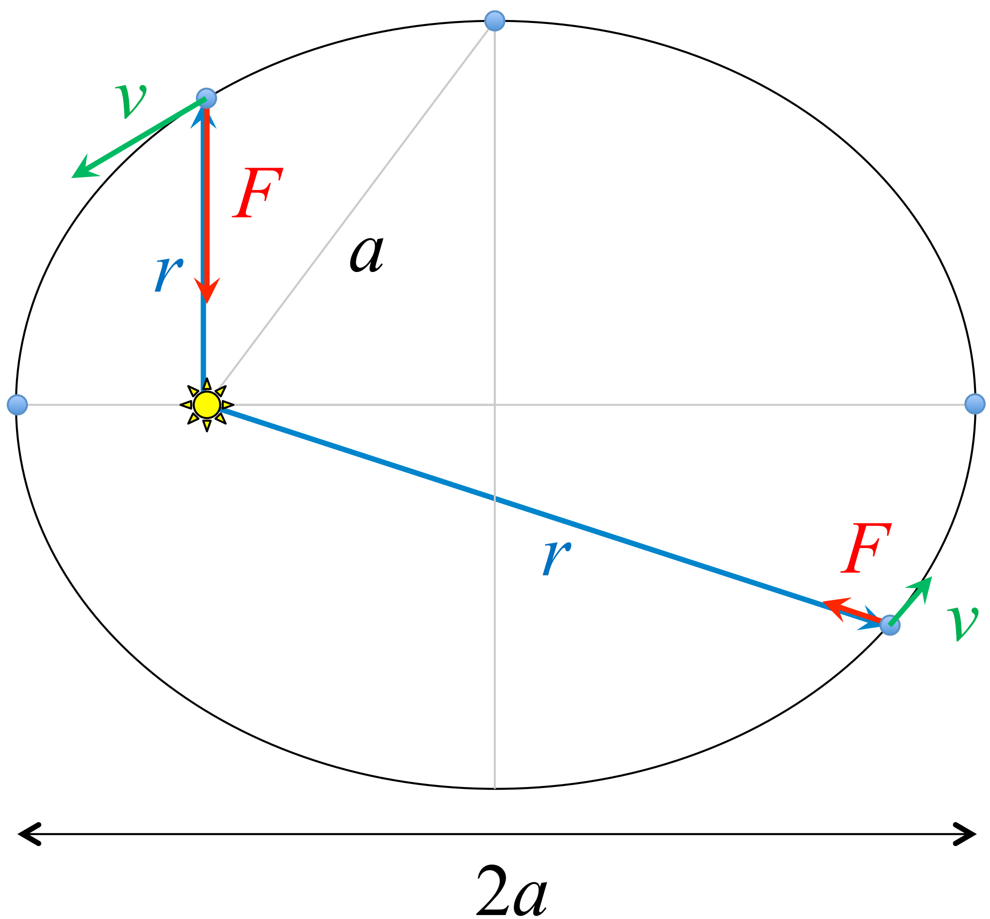
III. Work and Energy

IV. Angular Momentum
- Bodies and particles

V. Elliptical Orbits

	The student will be able to:	HW:
1	Define angular position, angular displacement, angular velocity, angular acceleration and solve related problems in fixed axis kinematics.	1 – 5
2	Define torque, lever arm (moment arm), and solve related problems.	6 – 8
3	Define rotational inertia (moment of inertia), use provided formulas of such to solve related problems.	9 – 11
4	Solve rotational dynamics problems using relation between torque, rotational inertia, and angular acceleration for fixed axis.	12 – 18
5	Define rotational kinetic energy and work and solve related problems.	19 – 22
6	Define angular momentum and angular impulse and solve related problems.	23 – 26
7	State and apply conservation of angular momentum to solve related problems.	
8	Analyze orbital motion, including elliptical orbits, using conservation of angular momentum and energy.	27 – 30

Special Focus: Elliptical Orbits



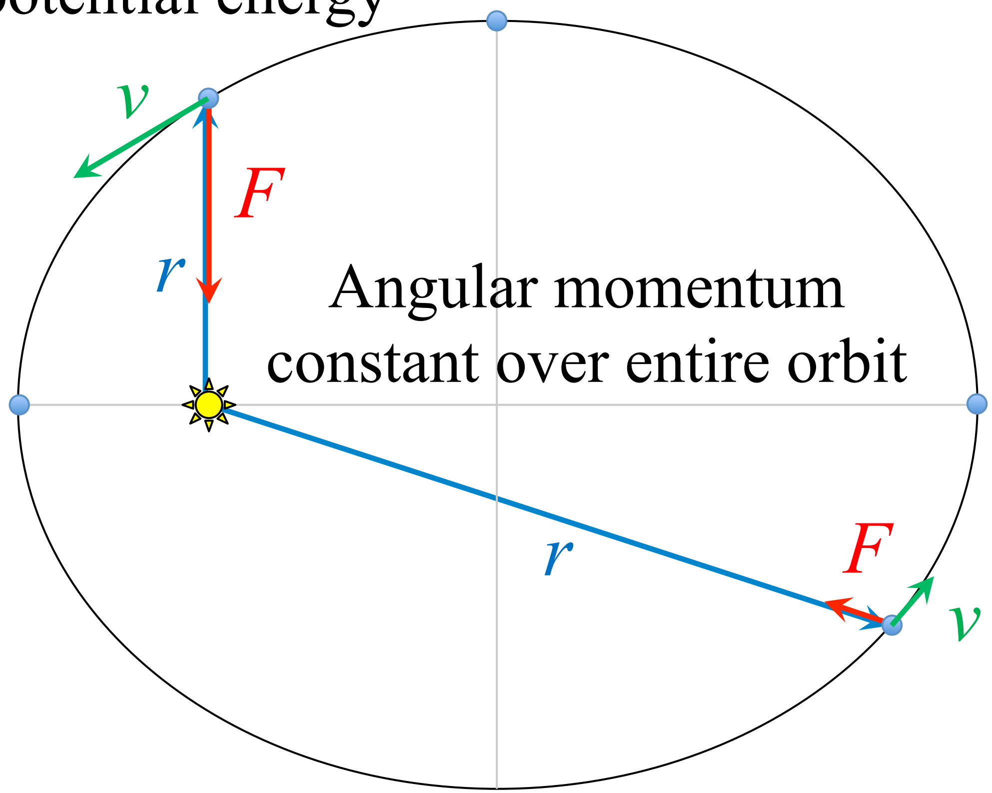
The position, *r*, velocity, *v*, and gravitational force, *F*, all vary in magnitude and direction as an object follows an elliptical orbit.

The diagram illustrates the major axis, 2*a*, and the semi-major axis, *a*, of the ellipse. The value of *a* is usually referred to as the average distance between the two objects. (the symbol "*a*" does not represent acceleration in this diagram!)

This type of motion can be analyzed by using conservation of angular momentum and conservation of energy…

Elliptical Orbits and Conservation Laws

Greater kinetic energy lower potential energy



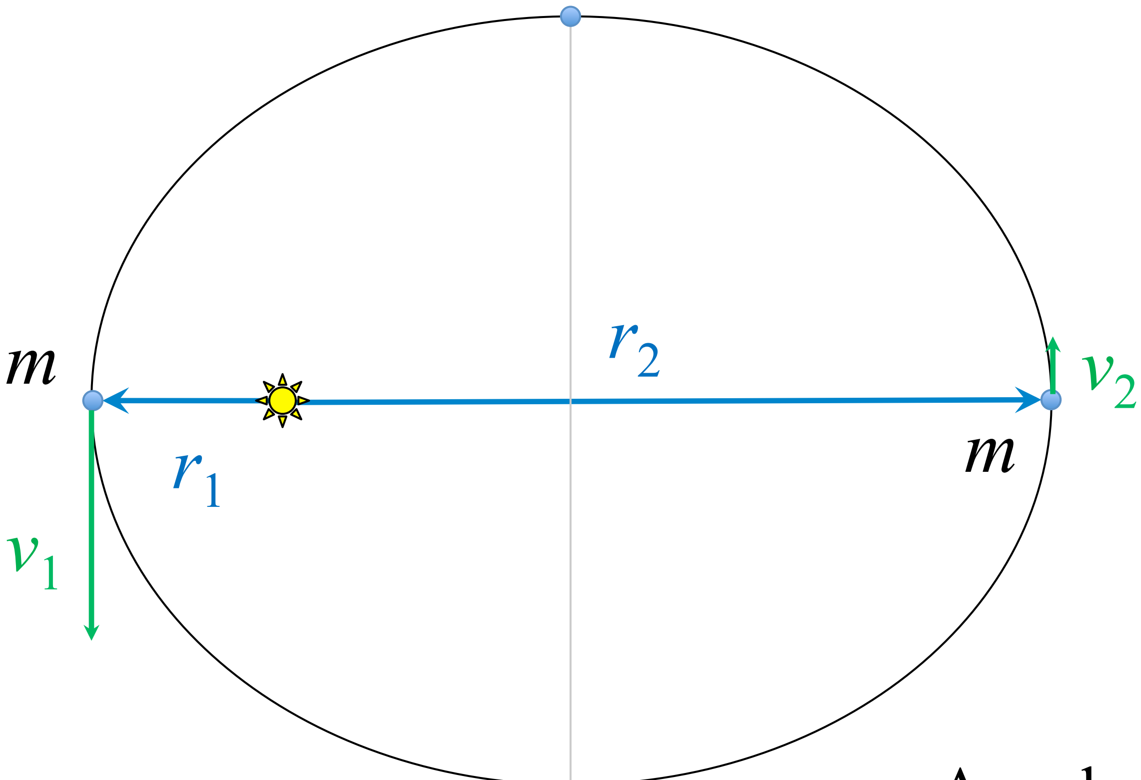
Lower kinetic energy greater potential energy

As shown in the figure the force of gravity acting on the satellite is always directed toward the central body. Taking the central body to be the axis of revolution there is zero torque due to gravity. Because there is no torque the **angular momentum of the system remains constant**.

And because gravity is a conservative force and no other forces are signifcant the **total mechanical energy of the system remains constant**. As the satellite nears the central body it is like falling down and potential energy is converted to kinetic energy.

Elliptical Orbits and Angular Momentum

The angular momentum should remain constant over the course of the entire orbit. Angular momentum is easiest to determine at the nearest and farthest points in the orbit.



$$L = I\omega = \left(mr^2\right)\left(\frac{v}{r}\right)$$

$$\boxed{L = rmv}$$

Angular momentum of satellite ($r \perp v$)

$$L = L'$$

$$\boxed{r_1 m v_1 = r_2 m v_2}$$

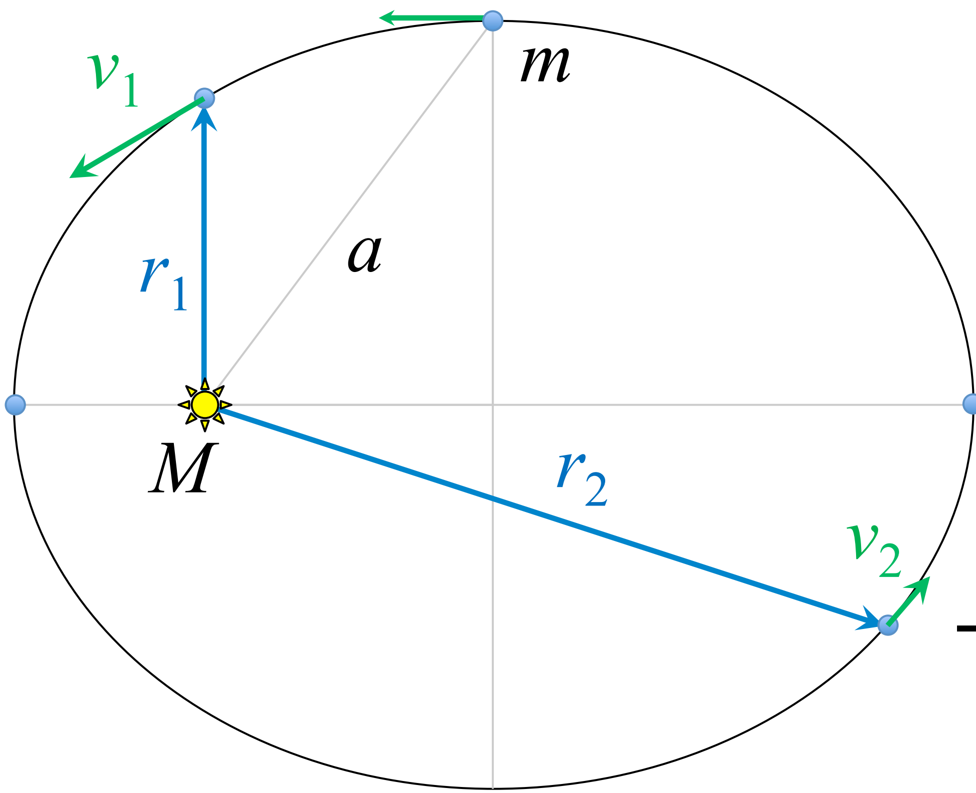
Angular momentum nearest the central body

Angular momentum farthest from central body

Note: finding angular momentum at any other point in the orbit is complicated by the fact that r and v are not perpendicular.

Elliptical Orbits and Angular Momentum

The total mechanical energy of the system remains constant because only the conservative force of gravity acts. There is a special formula for this potential energy.



$$U_G = -G\frac{Mm}{r}$$

Potential energy of the system

$$E = U_1 + K_1 = U_2' + K_2'$$

$$-G\frac{Mm}{r_1} + \frac{1}{2}mv_1^2 = -G\frac{Mm}{r_2} + \frac{1}{2}mv_2^2$$

$$E = -G\frac{Mm}{2a}$$

Total energy of the system

Note: the angles of *r* and *v* are do not affect energy and so this calculation can be done for any and every point in the orbit.