Rotation

I. Kinematics
- Angular analogs

II. Dynamics
- Torque and Rotational Inertia

III. Work and Energy

IV. Angular Momentum
- Bodies and particles

V. Elliptical Orbits

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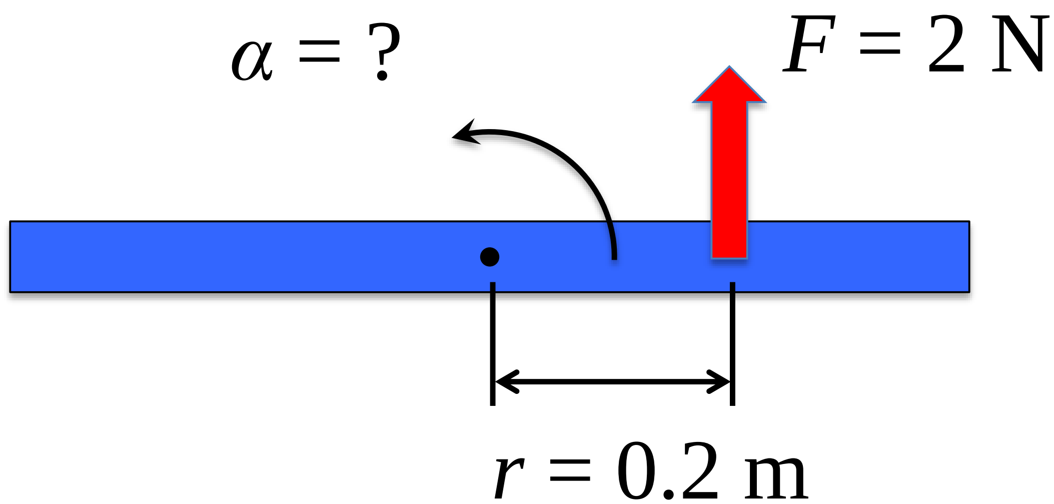
Force causes how much angular acceleration?

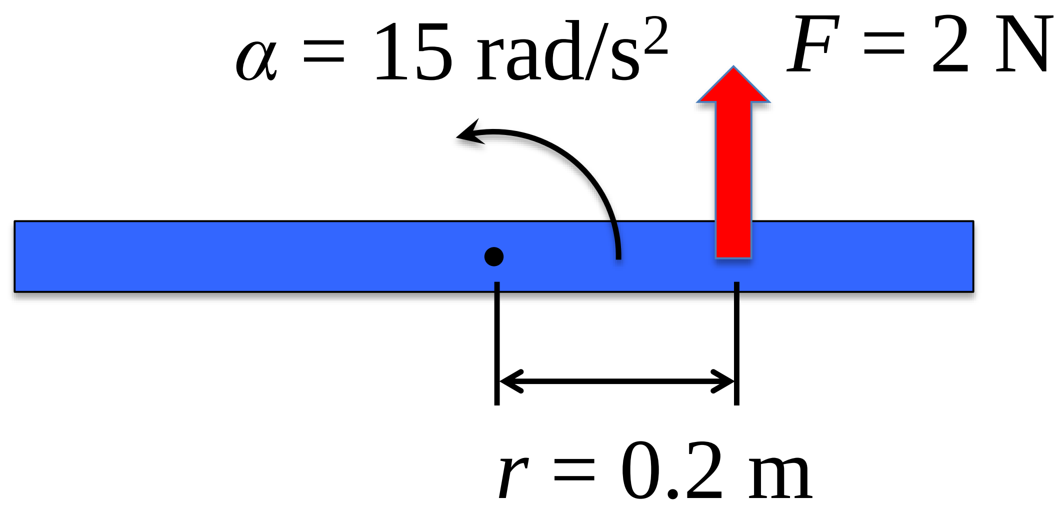
rod:
$m = 0.50$ kg
$L = 0.80$ m



For "regular motion" more force leads to more acceleration.
In fact, acceleration is *directly proportional* to force.
Can the same be said of rotational motion?

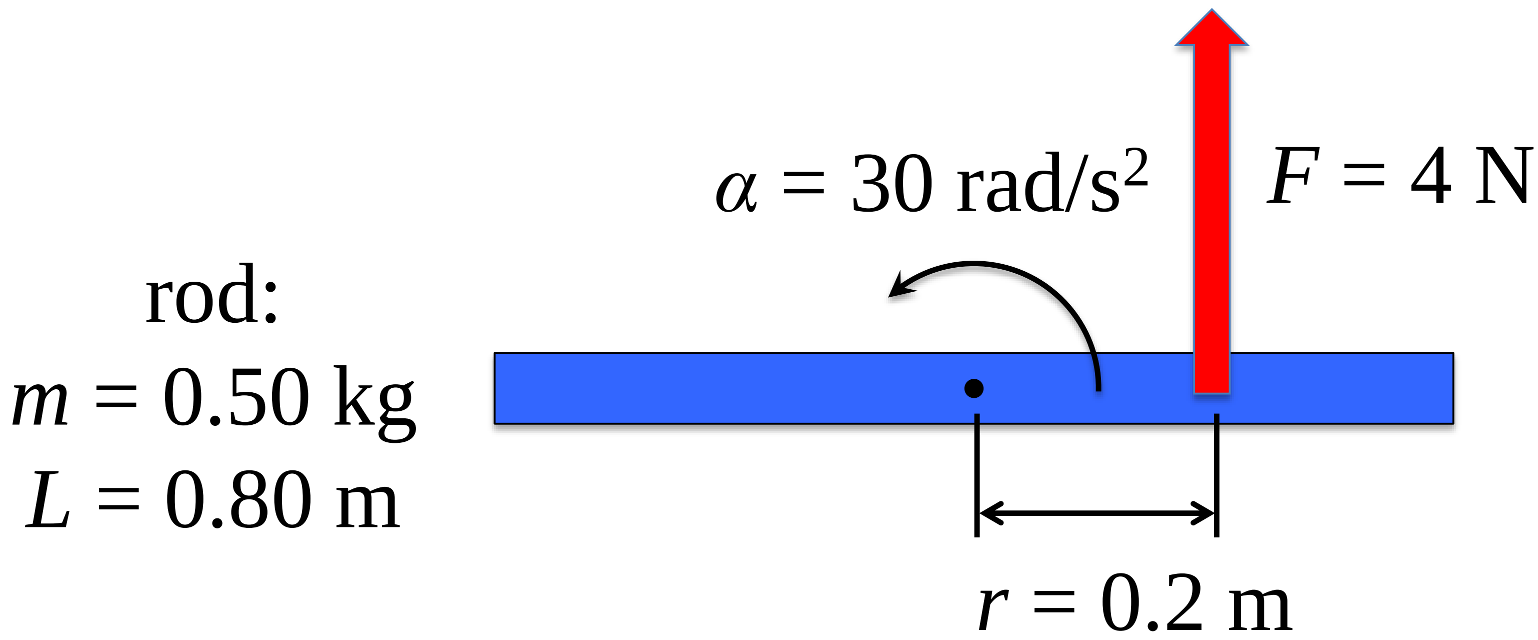
Force causes how much angular acceleration?

rod:
$m = 0.50$ kg
$L = 0.80$ m



In this example, a certain force applied to this particular rod would cause a certain amount of angular acceleration about a fixed axis passing through its center (disregarding friction).

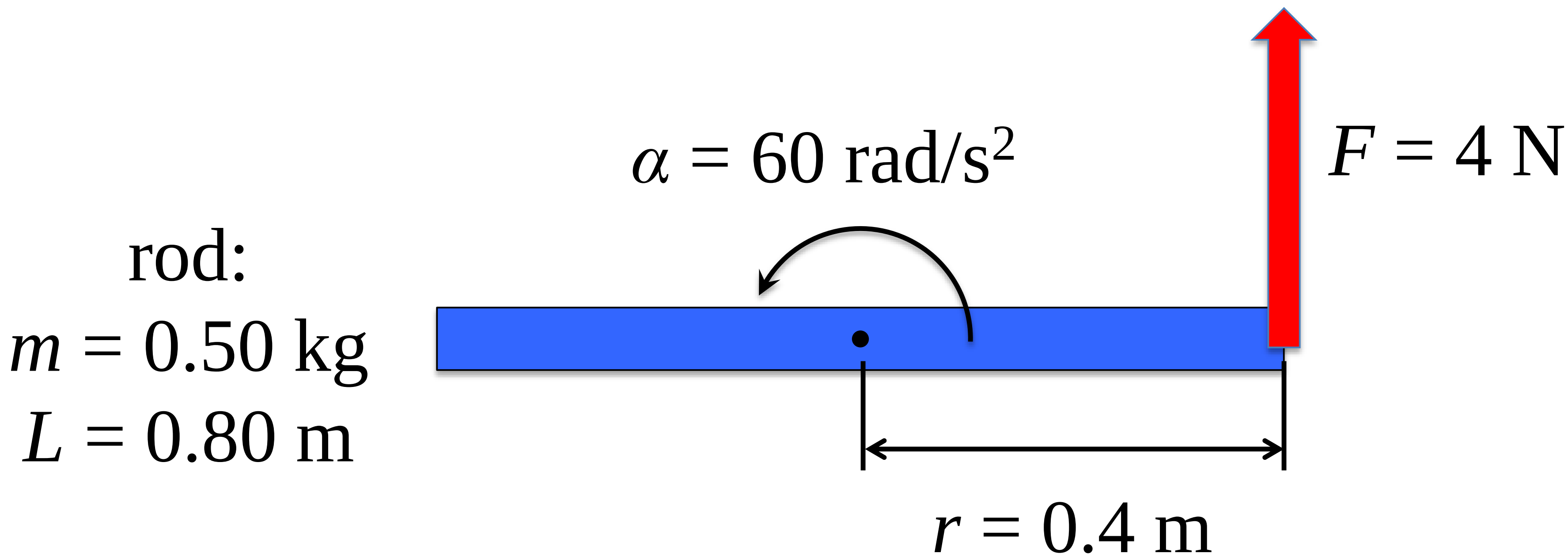
Force causes how much angular acceleration?



Twice as much force would cause twice the angular acceleration.

So, angular acceleration relates to force – but wait…

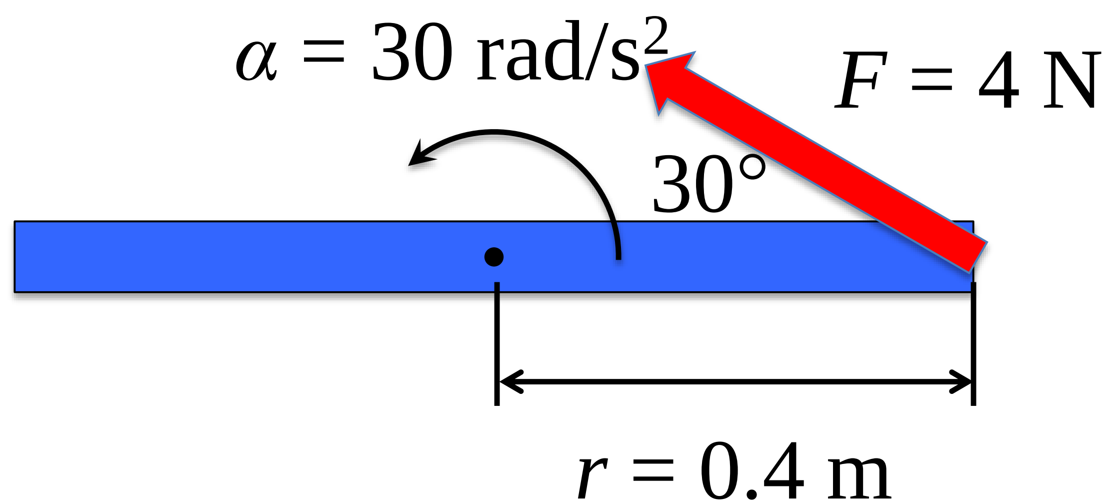
Force causes how much angular acceleration?



The *same* force applied at twice the radius also doubles angular acceleration. So angular acceleration not only relates to amount of force but also the *position at which the force acts*! But wait…

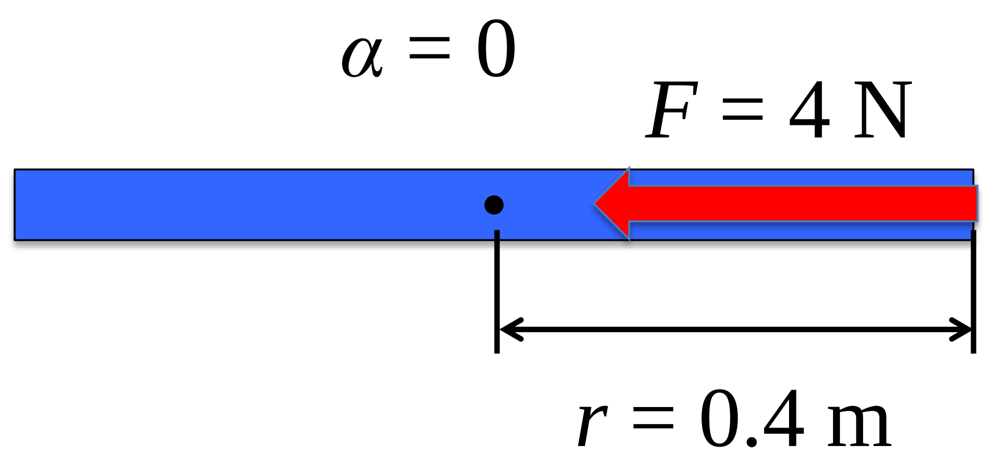
Force causes how much angular acceleration?

rod:
$m = 0.50$ kg
$L = 0.80$ m



The *same* force applied at the *same* radius but at an angle other than 90° causes *less* torque. So angular acceleration also relates to the *direction* in which the force acts! But wait…

Force causes how much angular acceleration?



rod:
$m = 0.50$ kg
$L = 0.80$ m

The *same* force applied at the *same* radius but pointing at the axle causes *zero* angular acceleration. So, it is possible for a force to act on the rod but cause no angular acceleration! This leads to…

Torque

- A torque is something that can cause angular acceleration.
- A force acting on an object can create torque on the object.
- The resulting torque depends not only on the force applied but also on the position at which the force is applied. Both the magnitude and the direction of the force and of the position are important!
- If a force is a “push or pull”, then a torque is a “twist” or a “torsion”.
- A torque is also sometimes referred to as a “moment” – especially in engineering.

Torque

$$\tau = r_{\perp} F$$

$$\tau = r F_{\perp}$$

$$\tau = rF \sin \theta$$

where: F = force

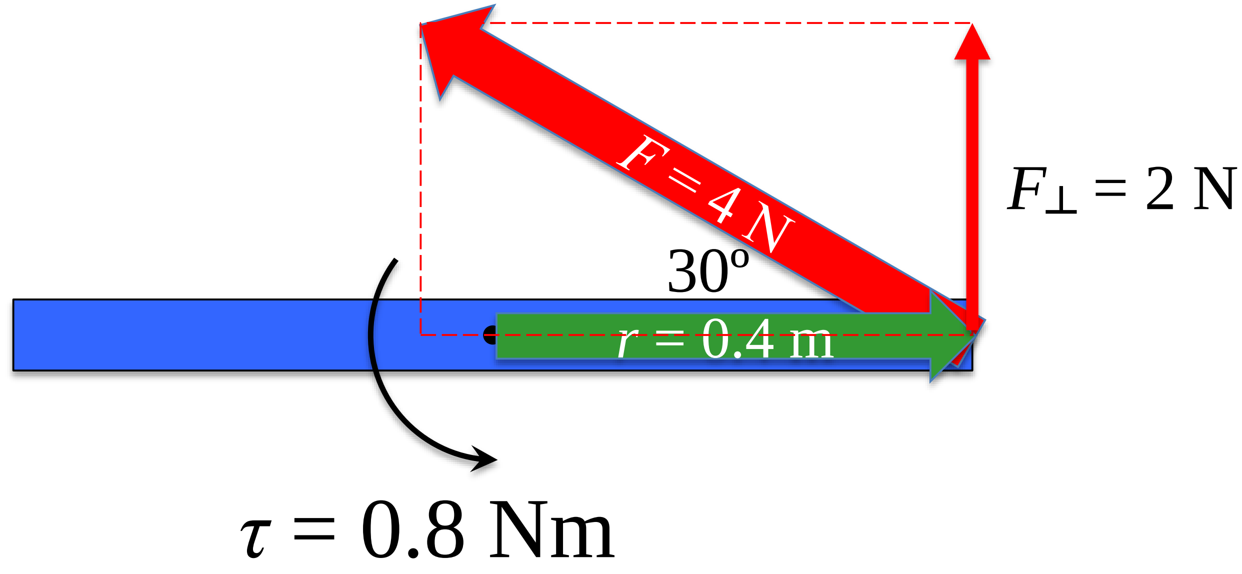
r = position at which force is applied relative to axis of rotation.

Torque

$$\tau = r_{\perp} F$$

$$\tau = r F_{\perp}$$

$$\tau = r F \sin\theta$$



example: $\tau = rF_{\perp}$

τ = (0.4 m)(4 N sin(30°))

τ = (0.4 m)(2 N)

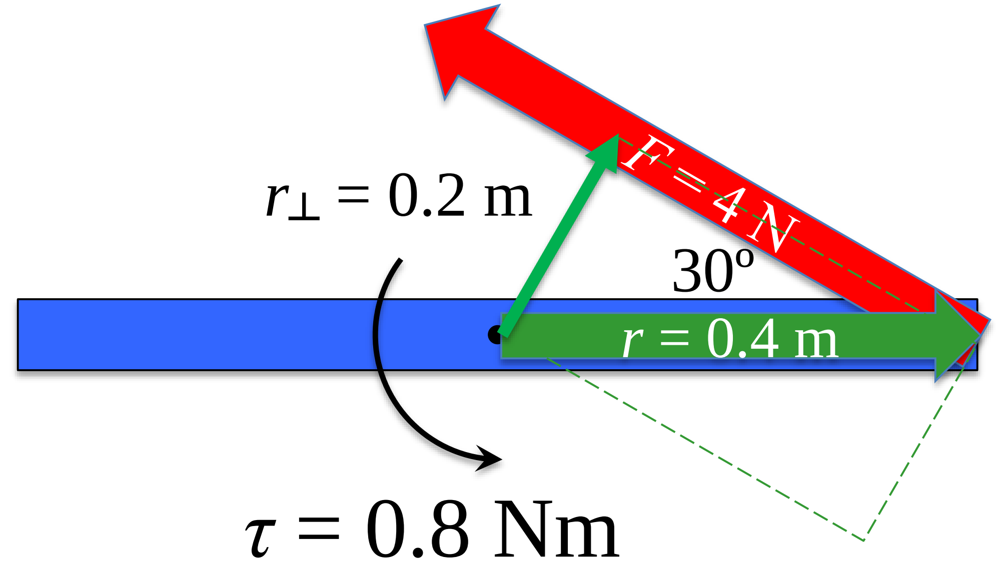
τ = 0.8 Nm counterclockwise

Torque

$$\tau = r_\perp F$$

$$\tau = rF_\perp$$

$$\tau = rF\sin\theta$$



example: $\tau = r_\perp F$

τ = (0.4 m sin(30°))(4 N)

τ = (0.2 m)(4 N)

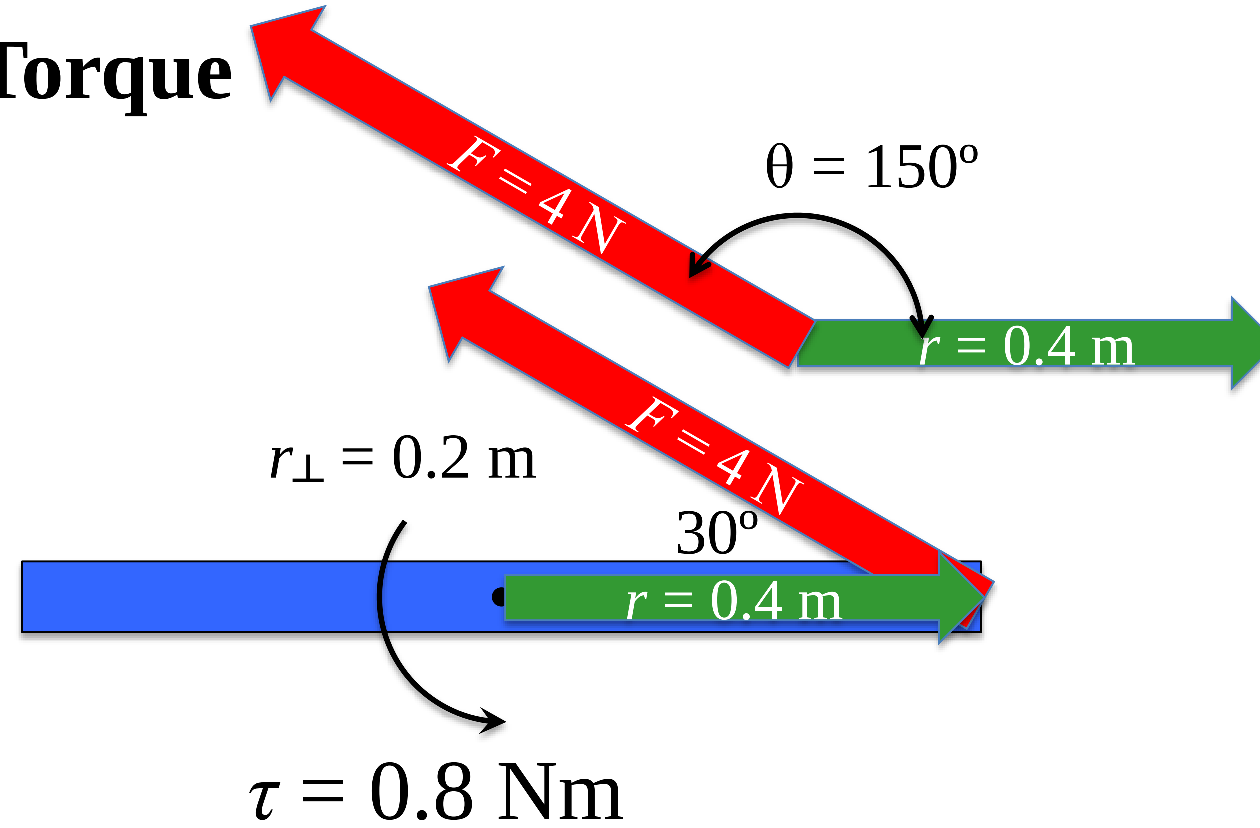
τ = 0.8 Nm counterclockwise

Torque

$$\tau = r_{\perp} F$$

$$\tau = r F_{\perp}$$

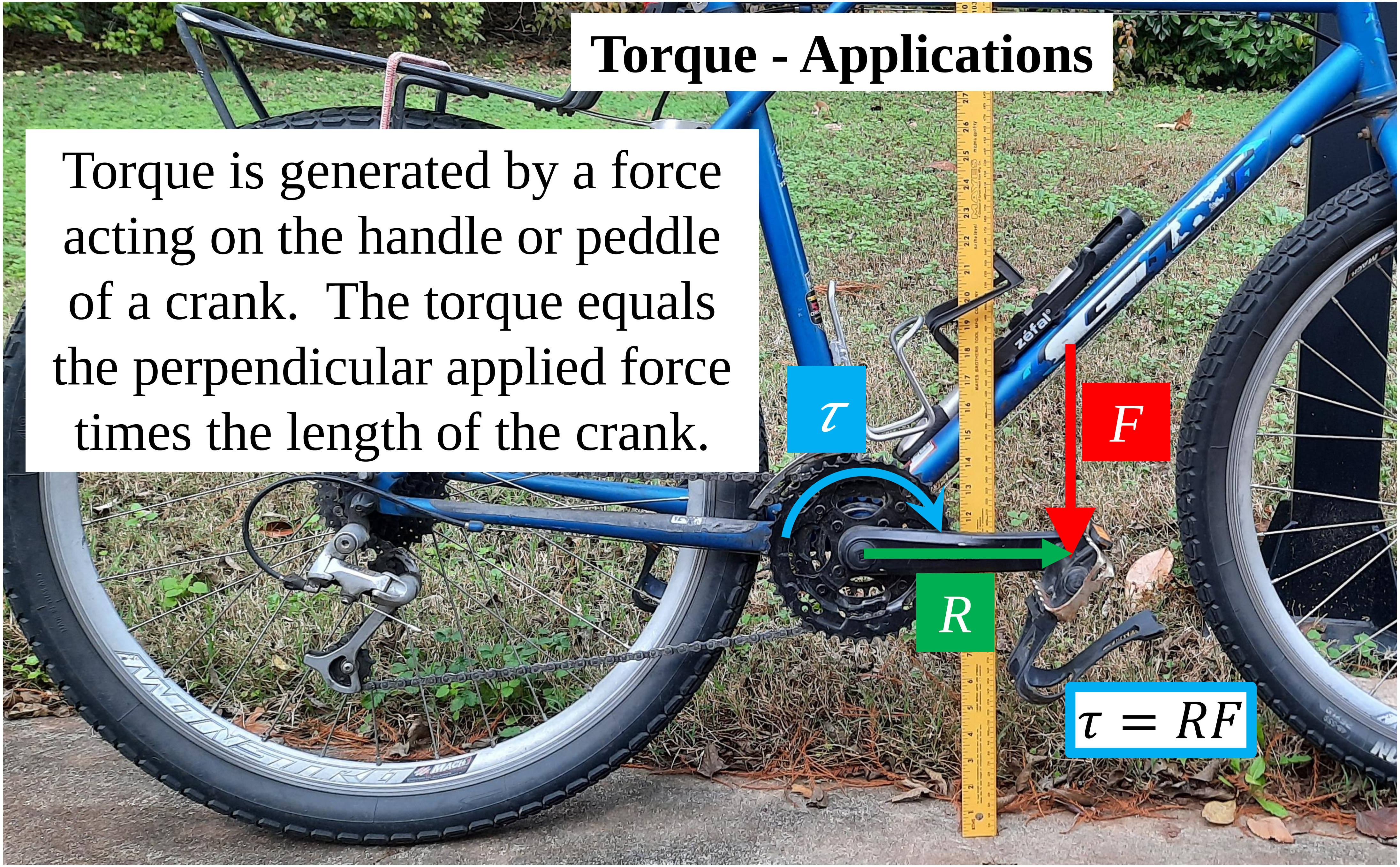
$$\tau = rF \sin\theta$$



example: $\tau = rF \sin\theta$

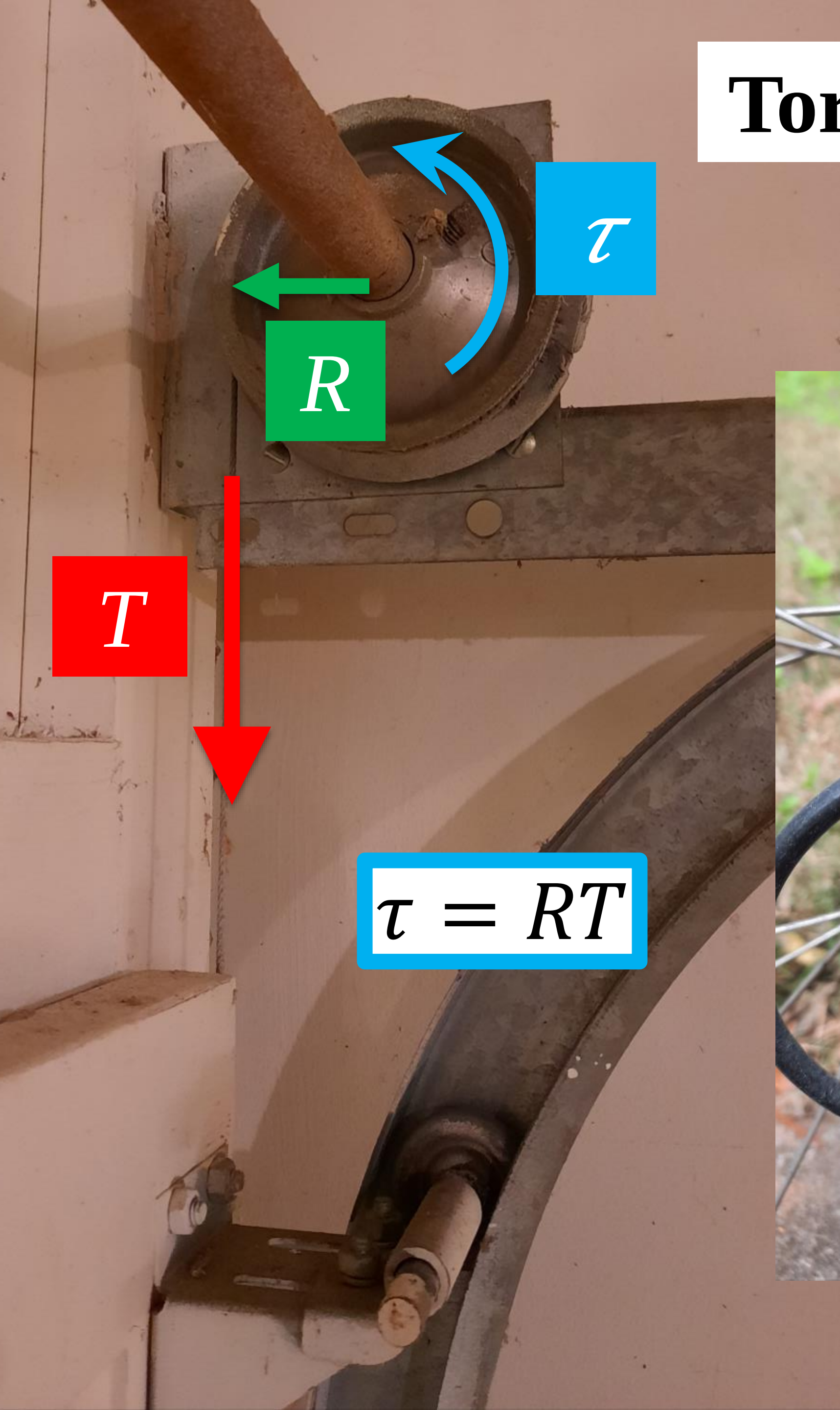
$\tau = (0.4\text{ m})(4\text{ N})\sin(150^{\circ})$

$\tau = 0.8$ Nm counterclockwise

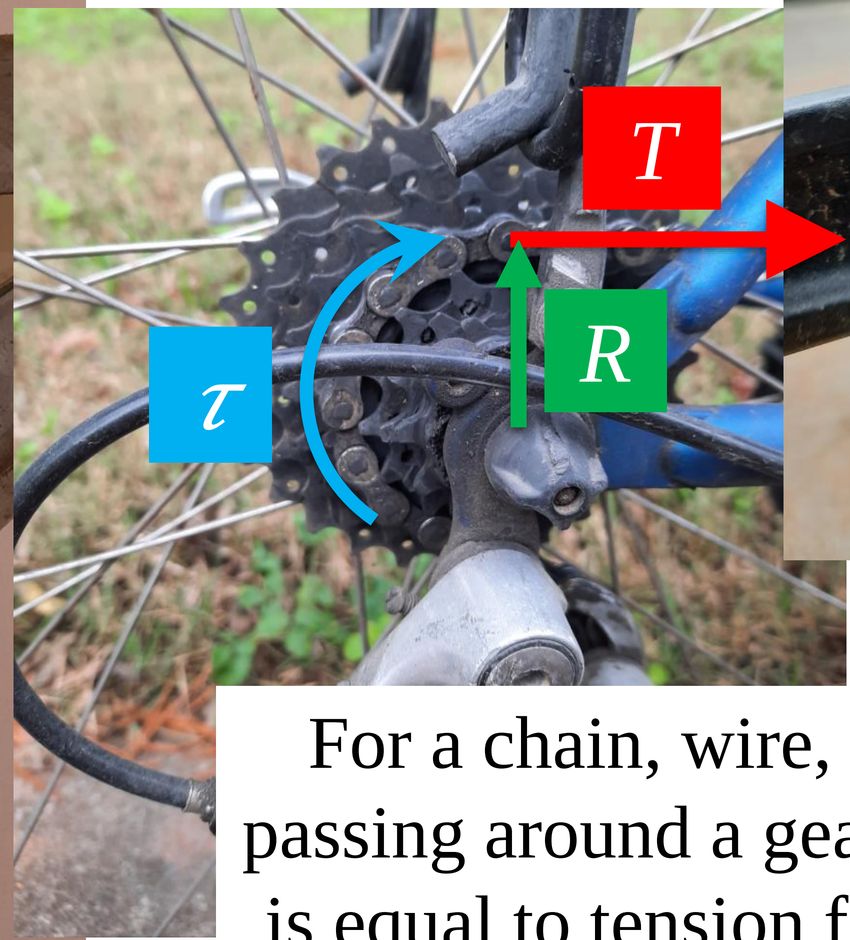


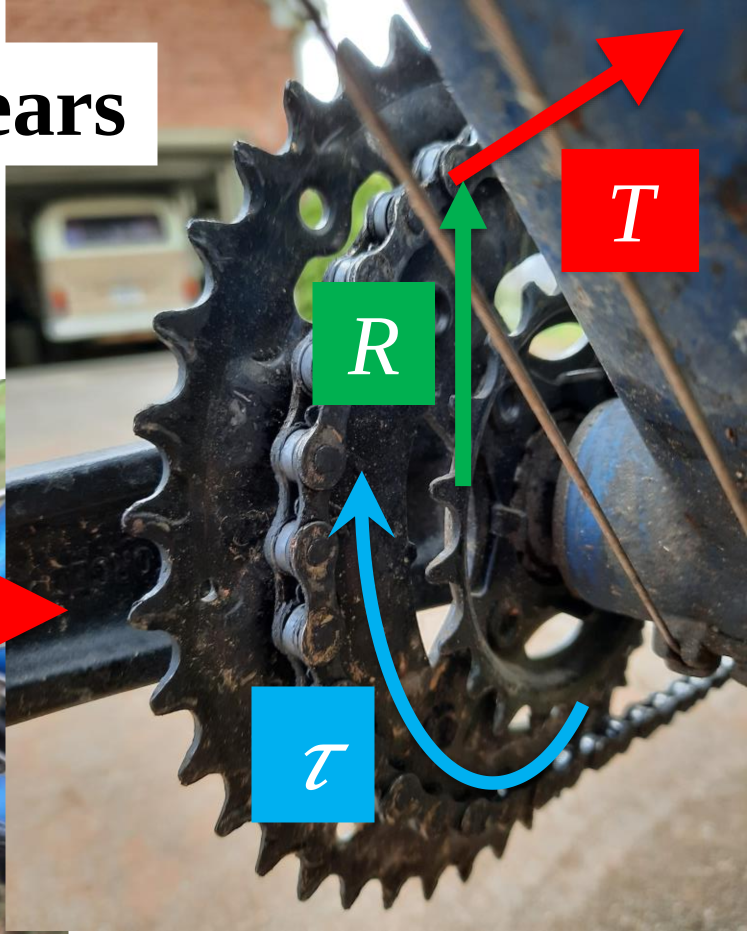
Torque - Applications
Torque is generated by a force acting on the handle or peddle of a crank. The torque equals the perpendicular applied force times the length of the crank.
τ
F
R
τ = RF

Torque – Pulleys & Gears



$$\tau = RT$$



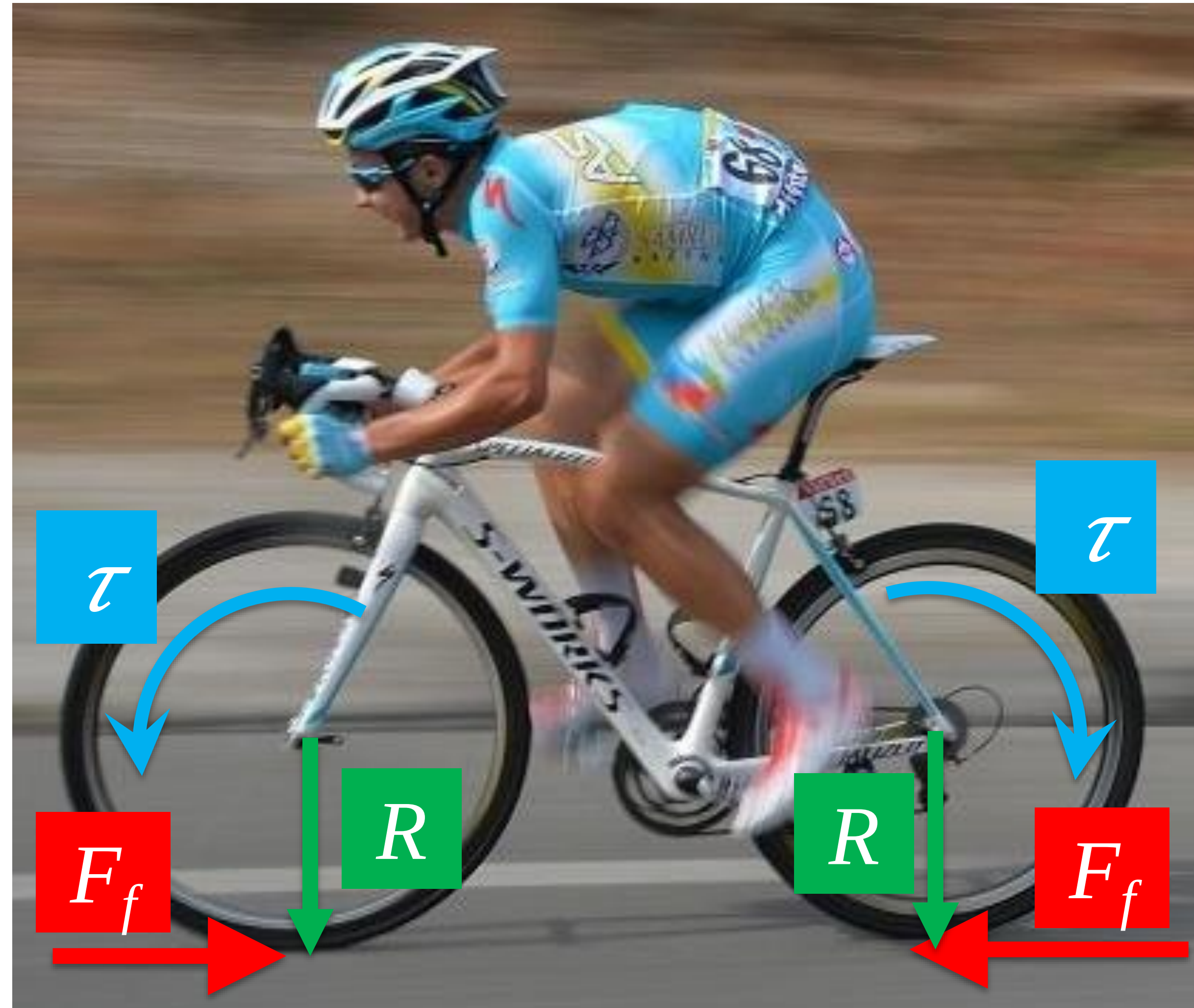


For a chain, wire, string, rope, etc. passing around a gear or pulley, torque is equal to tension force times radius.

Torque - Wheels

A common torque on a wheel results from friction. The amount of torque is equal to frictional force times radius of wheel.

(Note: There is also friction at the axle, with torque equal to frictional force times radius of axle – this torque is often ignored. Why might it this torque be negligible?)



$$\tau = RF_f$$

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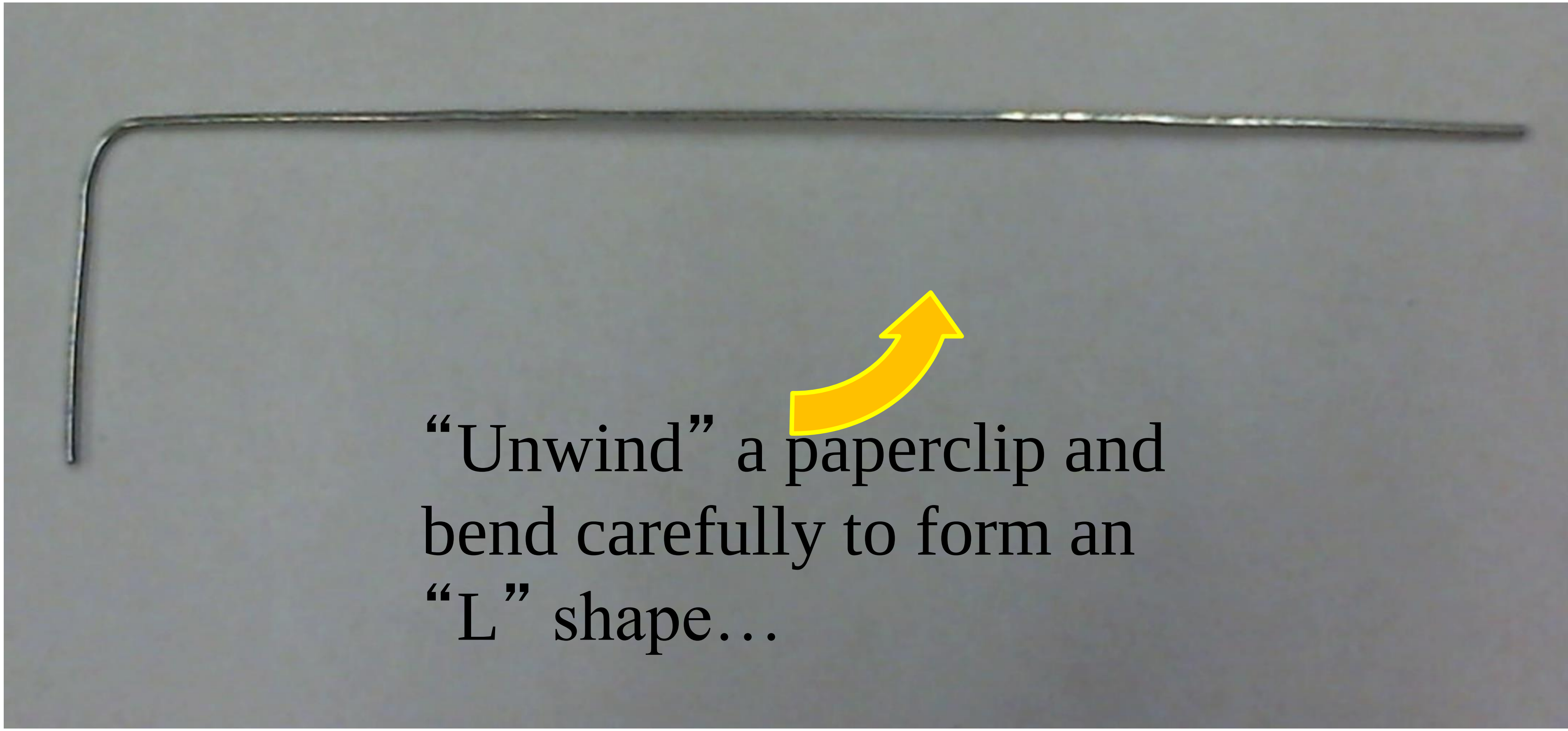
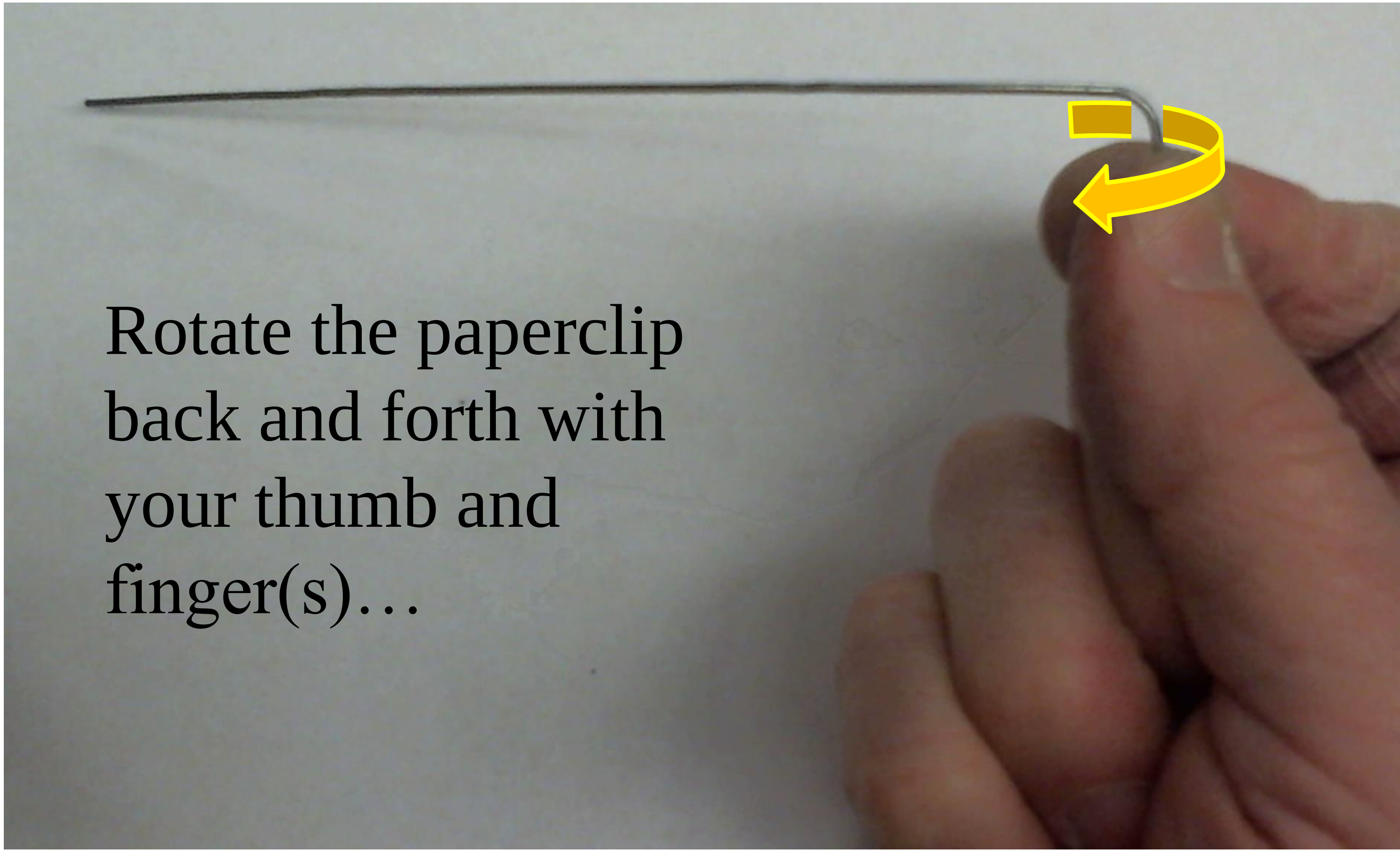
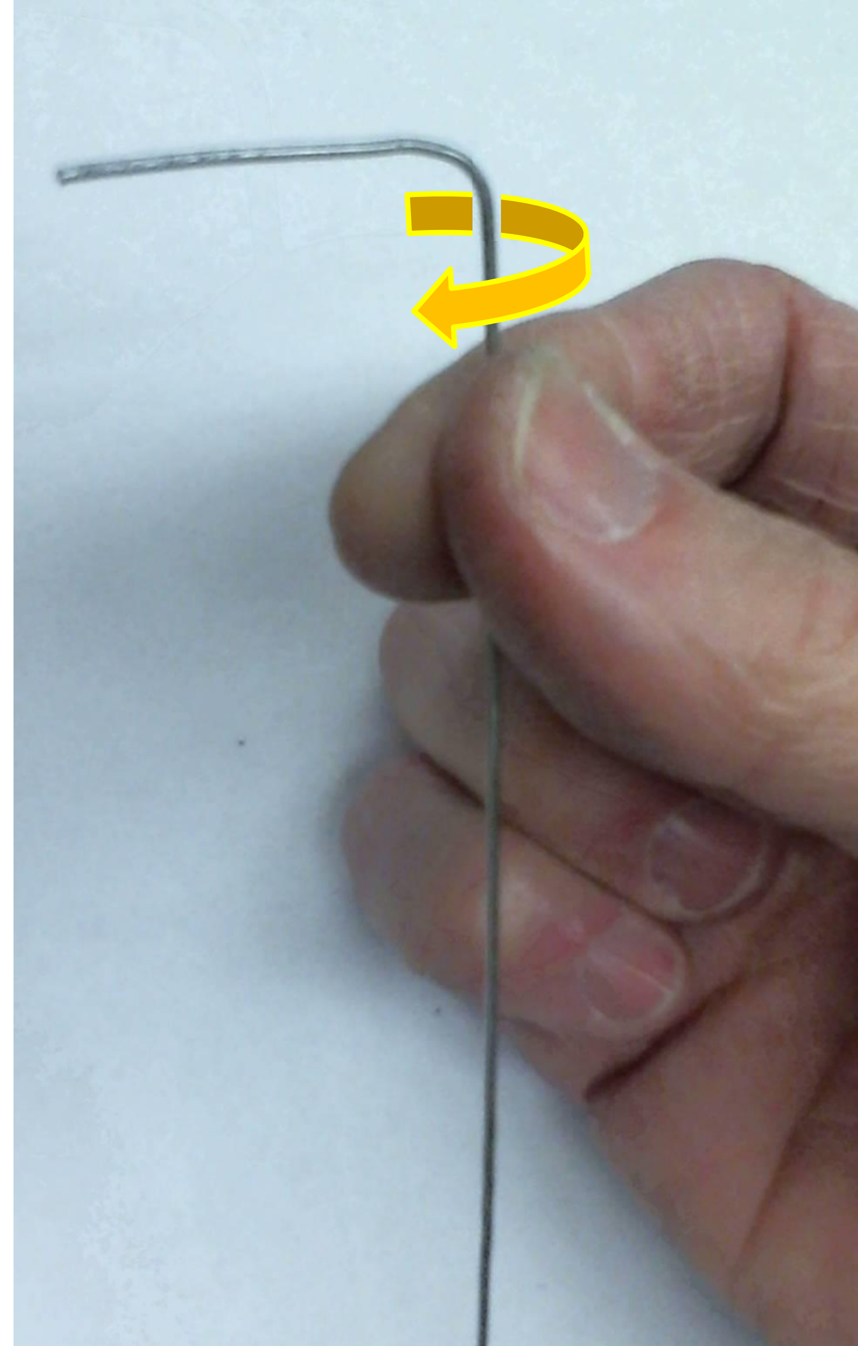
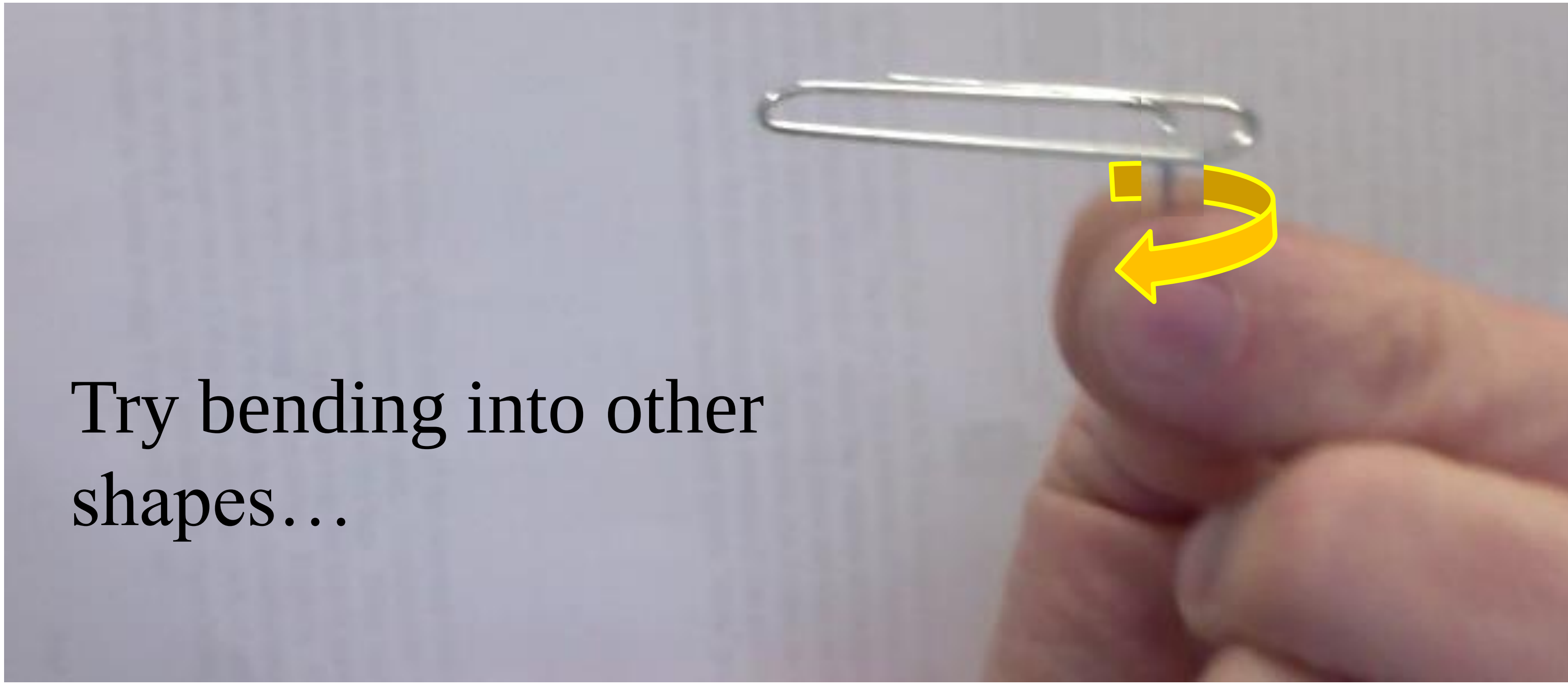
Rotational Inertia: Hands-On Experiment – Try it!




Rotate the paperclip back and forth with your thumb and finger(s)…

Repeat but now hold the *other* side of the “L”, so that the small leg of the “L” is swinging around instead of the long leg.





The torque on the paperclip results from your fingers acting on the "shaft", but the resulting angular acceleration is greatly affected by the *shape* of the paperclip. When more of its mass is located farther from the axis of rotation the paperclip is much "harder" to start moving or stop moving – it has greater "rotational inertia" – *i.e.* greater tendency to stay at rest and not rotate or to stay in motion and continue to rotate. This can be observed by the fact you must exert more force with your fingers when more of the mass is farther away from the axis of rotation – it "feels heavier"!

Newton's 2nd Law for Rotation

$$\vec{\tau}_{net} = I\vec{\alpha}$$

$$\Sigma\vec{\tau} = I\vec{\alpha}$$

where: τ = torque
I = rotational inertia
α = angular acceleration

But, what *exactly* is “rotational inertia”?!

$$\vec{\alpha} = \frac{\vec{\tau}_{net}}{I}$$

Whatever it is, the *more* of *it* the *less* the angular acceleration…

where: τ = torque

I = rotational inertia

α = angular acceleration

Mass and shape result in how much rotational inertia?

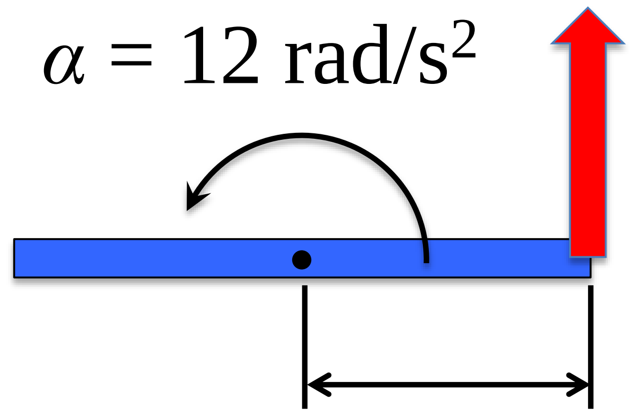
rod:

$m = 15$ kg

$L = 0.4$ m

$F = 12$ N

$\alpha = 12$ rad/s^2



$r = 0.2$ m

$\tau = 2.4$ Nm

$I = \tau/\alpha$

$I = 2.4/12$

$I = 0.2$ kg m^2

In this example, a certain torque on this particular rod would cause a certain angular acceleration about a fixed axis passing through its center (disregarding friction). Dividing torque by angular acceleration gives the rotational inertia, similar to dividing force by acceleration to find mass.

Mass and shape result in how much rotational inertia?

rod:

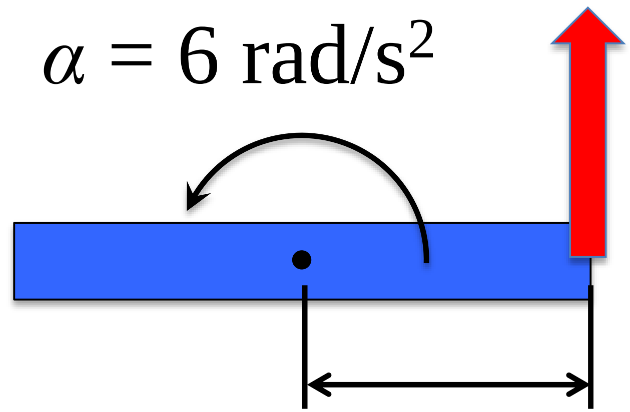
$m = 30$ kg

$L = 0.4$ m

$F = 12$ N

$\alpha = 6$ rad/s^2

$\tau = 2.4$ Nm



$r = 0.2$ m

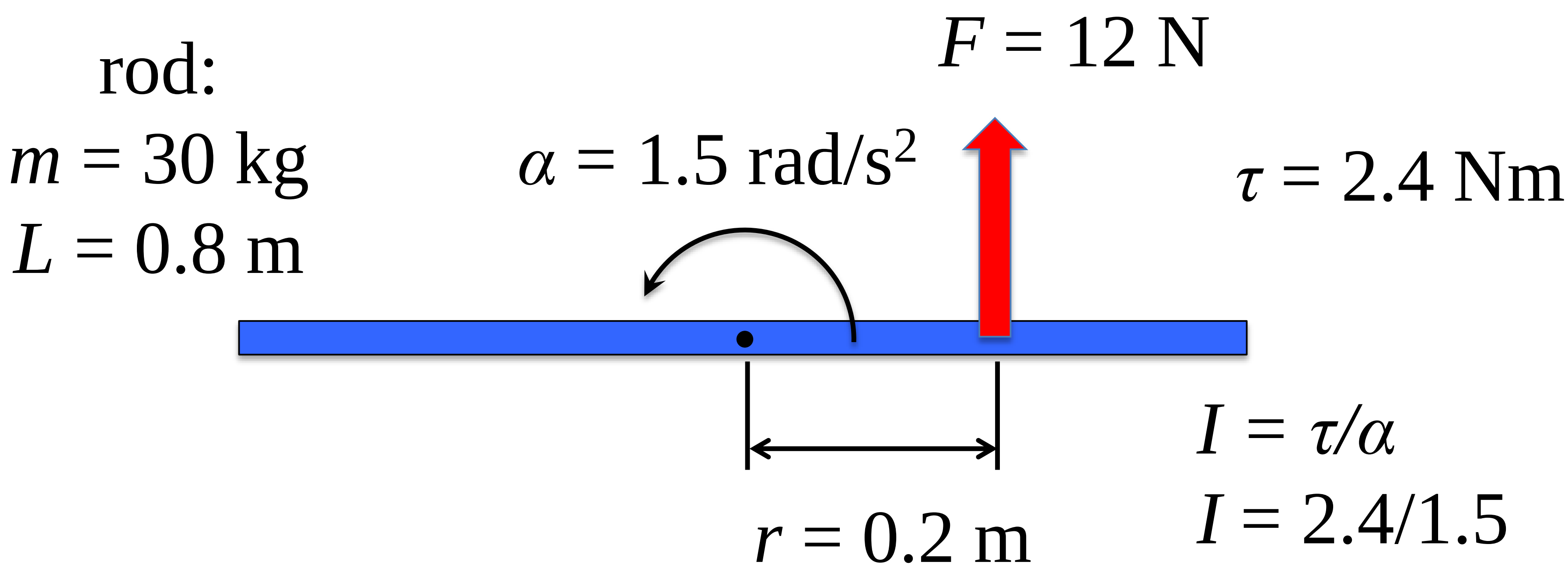
$I = \tau/\alpha$

$I = 2.4/6$

$I = 0.4$ kg m^2

If the bar were twice the mass but the same length as before, the same amount of torque as before would cause only half as much angular acceleration and so the rotational inertia would double. Therefore, not surprisingly, rotational inertia is proportional to mass. But wait…

Mass and shape result in how much rotational inertia?



$I = \tau/\alpha$

$I = 2.4/1.5$

$I = 1.6$ kg m^2

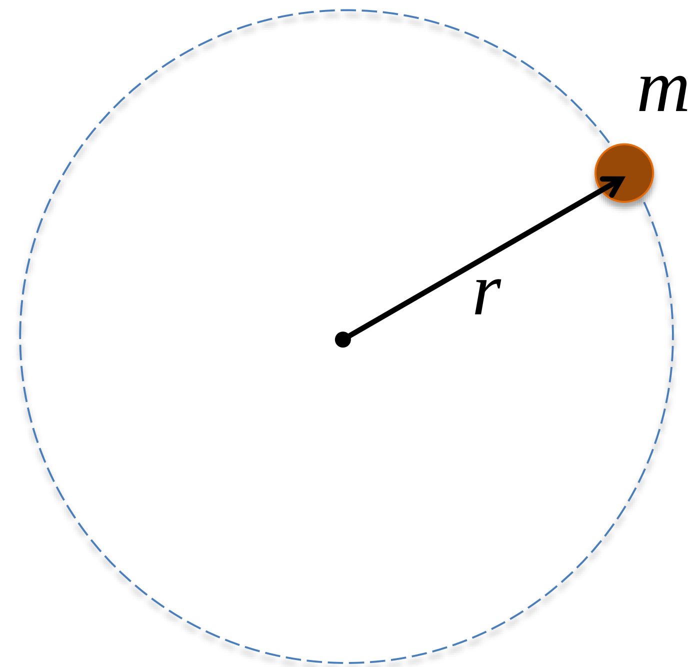
If the same mass bar as the previous example were made twice as long, the same torque as before would cause only fourth the angular acceleration as the previous example and so the rotational inertia would quadruple. This is because rotational inertia is proportional to radius squared.

Rotational Inertia

- As explained in Newton's Laws of Motion any object that has mass has inertia – tendency to maintain state of motion.
- However, for a rotating body this tendency also depends on the *arrangement* of mass relative to the axis.
- The quantity "rotational inertia" is defined to satisfy a rotational version of Newton's 2nd Law.
- Rotational inertia is proportional to mass.
- Rotational inertia is proportional to radius squared.

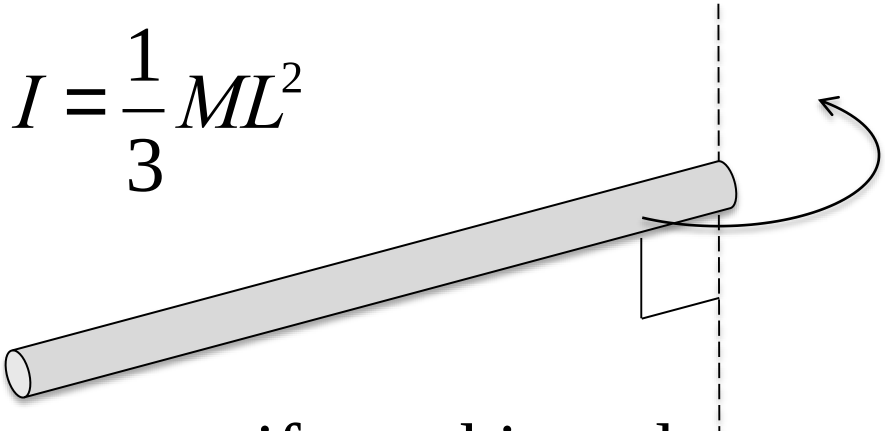
Rotational Inertia

$$I = mr^2$$



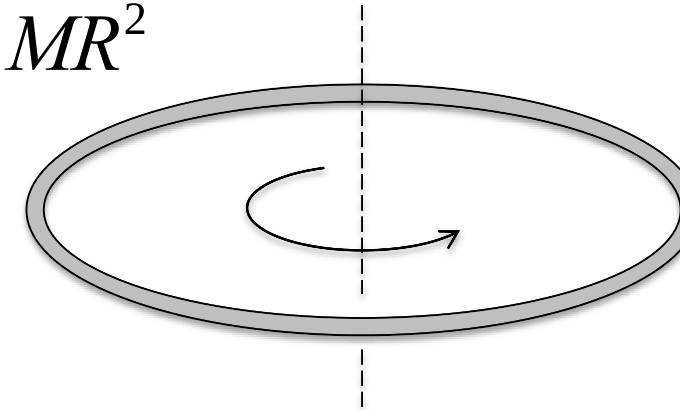
where: I = rotational inertia
m = point-like mass
r = radial distance from axis

$$I = \frac{1}{3}ML^2$$



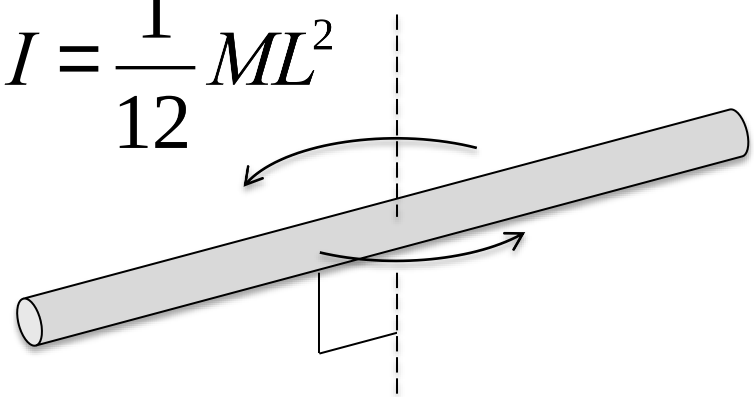
uniform thin rod,
perpendicular axis at end

$$I = MR^2$$



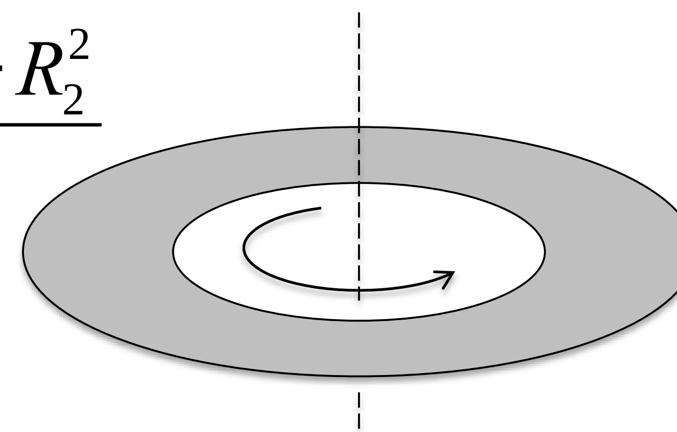
uniform thin ring,
perpendicular axis at center

$$I = \frac{1}{12}ML^2$$



uniform thin rod,
perpendicular axis at center

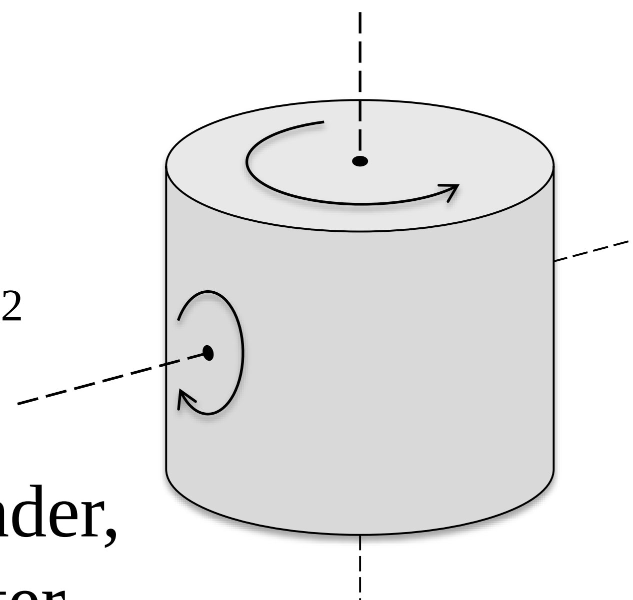
$$I = M\frac{R_1^2 + R_2^2}{2}$$



uniform thin “washer”,
perpendicular axis at center

uniform solid cylinder,
axis through center
parallel to side

$$I = \frac{1}{2} MR^2$$

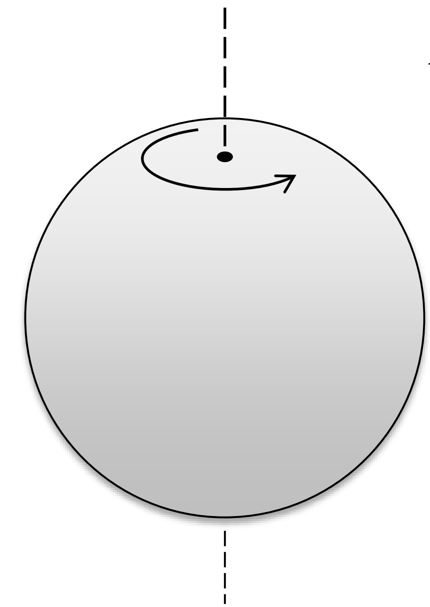


$$I = \frac{1}{4} MR^2 + \frac{1}{12} Mh^2$$

uniform solid cylinder,
axis through center
perpendicular to side

uniform solid sphere,
axis through center

$$I = \frac{2}{5} MR^2$$



$$I = \frac{2}{3} MR^2$$

hollow spherical shell,
axis through center