Rotation

I. Kinematics
- Angular analogs

II. Dynamics
- Torque and Rotational Inertia

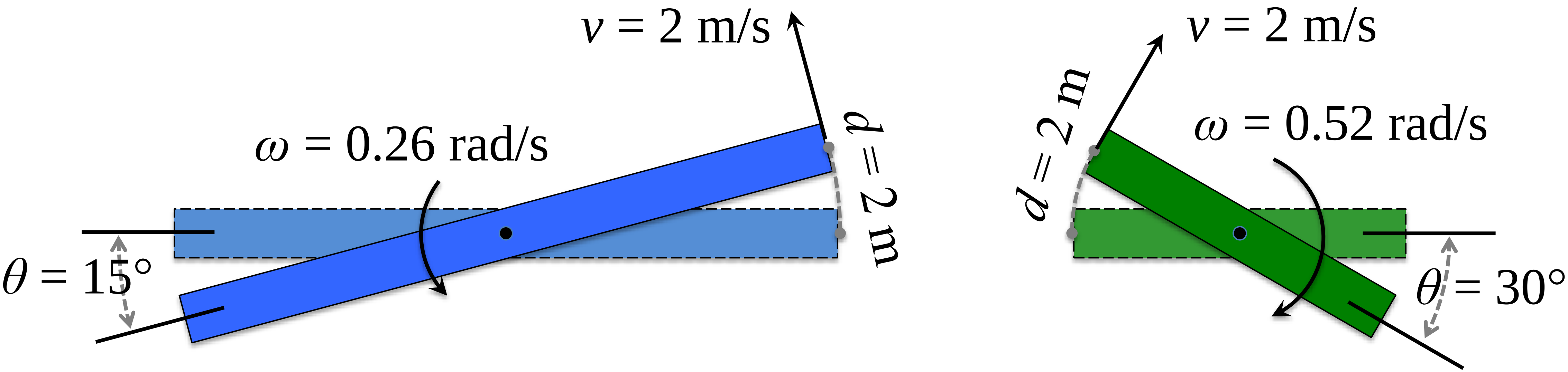
III. Work and Energy

IV. Angular Momentum
- Bodies and particles

V. Elliptical Orbits

	The student will be able to:	HW:
1	Define angular position, angular displacement, angular velocity, angular acceleration and solve related problems in fixed axis kinematics.	1 – 5
2	Define torque, lever arm (moment arm), and solve related problems.	6 – 8
3	Define rotational inertia (moment of inertia), use provided formulas of such to solve related problems.	9 – 11
4	Solve rotational dynamics problems using relation between torque, rotational inertia, and angular acceleration for fixed axis.	12 – 18
5	Define rotational kinetic energy and work and solve related problems.	19 – 22
6	Define angular momentum and angular impulse and solve related problems.	23 – 26
7	State and apply conservation of angular momentum to solve related problems.	
8	Analyze orbital motion, including elliptical orbits, using conservation of angular momentum and energy.	27 – 30

Consider two rotating objects – which is "moving faster"? The ends of the two rods have the same *speed* (in m/s), but the smaller rod is spinning more rapidly and has a greater *angular speed* (in deg/s or radians/s). This is rotational kinematics – the mathematical description of rotating objects.

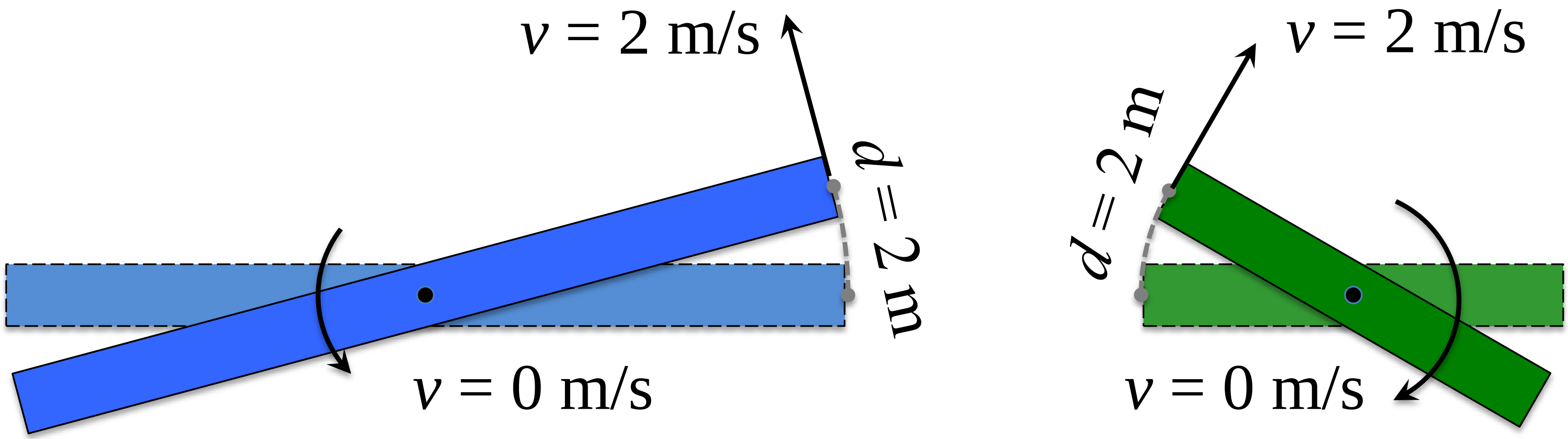


360° in 24 s
15 degrees/sec

360° in 12 s
30 degrees/sec

“Regular” Kinematics:

The ends of the two rods each happen to move a distance of 2 meters each second in this example and so the “linear speed” of each end is 2 m/s.

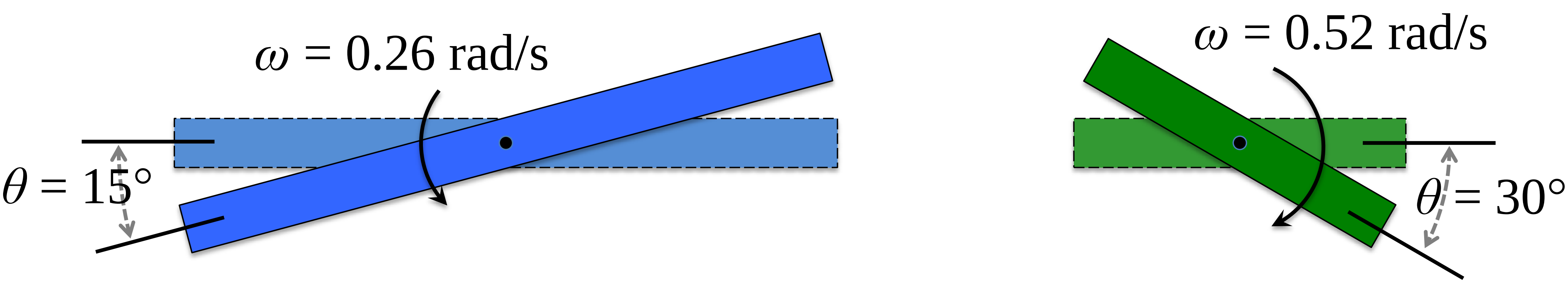


However, the center of each object moves zero distance per second and so the speed of the “whole object” for each one is zero in a sense.

360° in 24 s	360° in 12 s
15 degrees/sec	30 degrees/sec

Rotational or “Angular” Kinematics:

The angle through which each object turns is different each second. The smaller rod turns through an angle twice as large in an equal time. The angular speed of the shorter rod is twice that of the longer.



The angular speed describes the *whole* object – not the center, or the end, but rather the whole thing. This value is usually given in radians per second instead of degrees per second. (2π radians = 360 degrees)

15 degrees/s = 0.26 radians/s 30 degrees/s = 0.52 radians/s

$$v_x = v_{x0} + a_x t$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

$$x = x_0 + v_{x0} t + \frac{1}{2} a_x t^2$$

$$\omega = \omega_0 + \alpha t$$

$$v_x^2 = v_{x0}^2 + 2a_x (x - x_0)$$

$$\vec{a} = \frac{\sum \vec{F}}{m} = \frac{\vec{F}_{net}}{m}$$

$$\vec{\alpha} = \frac{\sum \vec{\tau}}{I} = \frac{\vec{\tau}_{net}}{I}$$

For every definition and equation concerning "regular" or "linear" motion there is a similar and analogous definition and equation concerning rotational motion!

$$\vec{p} = m\vec{v}$$

$$L = I\omega$$

$$\Delta\vec{p} = \vec{F}\,\Delta t$$

$$\Delta L = \tau\,\Delta t$$

$$K = \frac{1}{2} m v^2$$

$$K = \frac{1}{2} I \omega^2$$

Angular position is an indicator of the orientation of an object relative to a reference. Symbol: θ

Angular displacement is the net change in angular position. Symbol: $\Delta\theta$ or θ

Angular velocity is the rate of change in angular position. Symbol: ω

Angular speed is the magnitude of angular velocity. Symbol: ω

Angular acceleration is the rate of change in angular velocity. Symbol: α

> Except for angular speed, all of these are vectors. Direction can be described as clockwise (–) or counterclockwise (+) – often abreviated CW or CCW.

Translation		Rotation	
position	$\vec{r}$	angular position	$\vec{\theta}$
velocity	$\vec{v} = \dfrac{\Delta \vec{r}}{\Delta t}$	angular velocity	$\vec{\omega} = \dfrac{\Delta \vec{\theta}}{\Delta t}$
acceleration	$\vec{a} = \dfrac{\Delta \vec{v}}{\Delta t}$	angular acceleration	$\vec{\alpha} = \dfrac{\Delta \vec{\omega}}{\Delta t}$

constant acceleration:

$$\begin{cases} x = x_0 + v_0 t + \frac{1}{2} a t^2 \\ v = v_0 + a t \\ v^2 = v_0^2 + 2a\left(x - x_0\right) \end{cases}$$

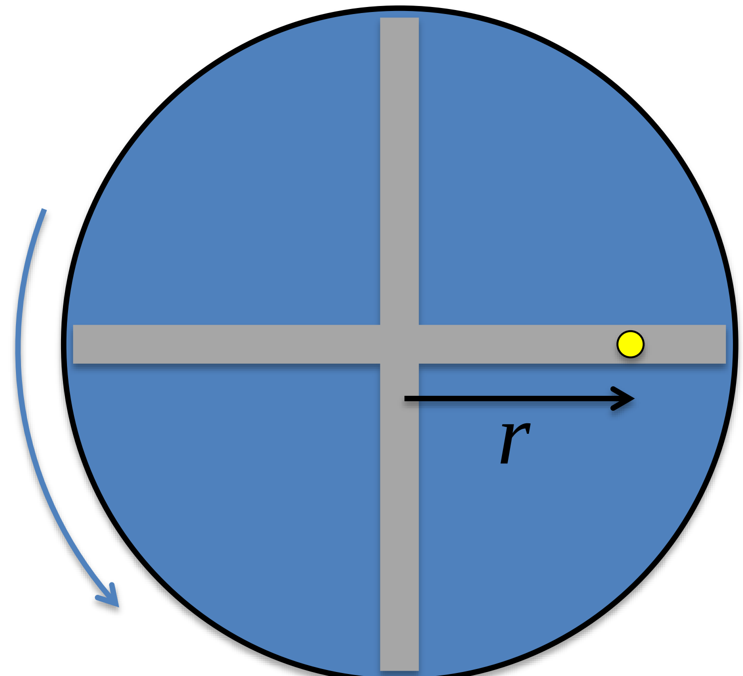
constant angular acceleration:

$$\begin{cases} \theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2 \\ \omega = \omega_0 + \alpha t \\ \omega^2 = \omega_0^2 + 2\alpha\left(\theta - \theta_0\right) \end{cases}$$

Angular speed
of entire disk:

$$\omega = \frac{\Delta\theta}{\Delta t}$$

$$\omega = \frac{2\pi}{T}$$



Linear speed
of yellow dot:

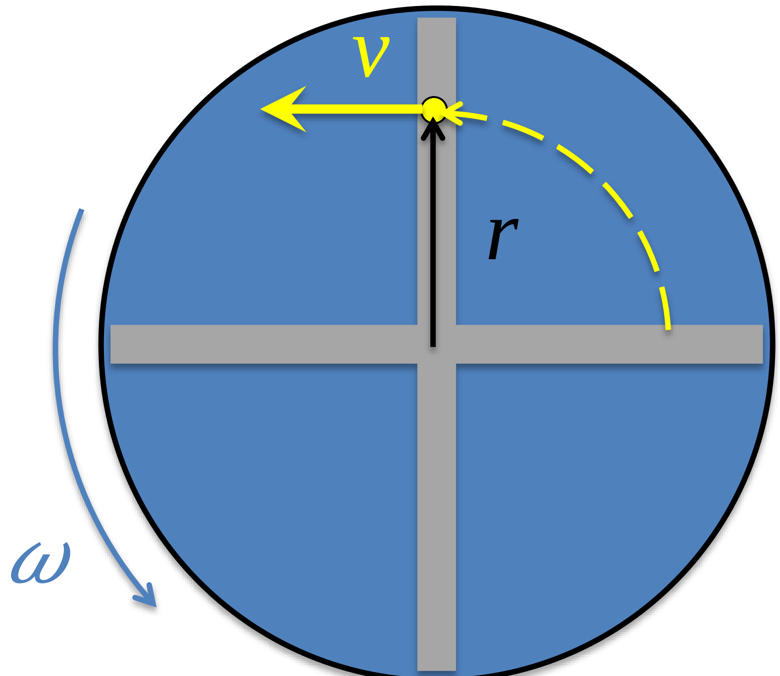
$$v = \frac{\Delta d}{\Delta t}$$

$$v = \frac{2\pi r}{T}$$

Angular speed of entire disk:

$$\omega = \frac{\Delta\theta}{\Delta t}$$

$$\omega = \frac{2\pi}{T}$$



Linear speed of yellow dot:

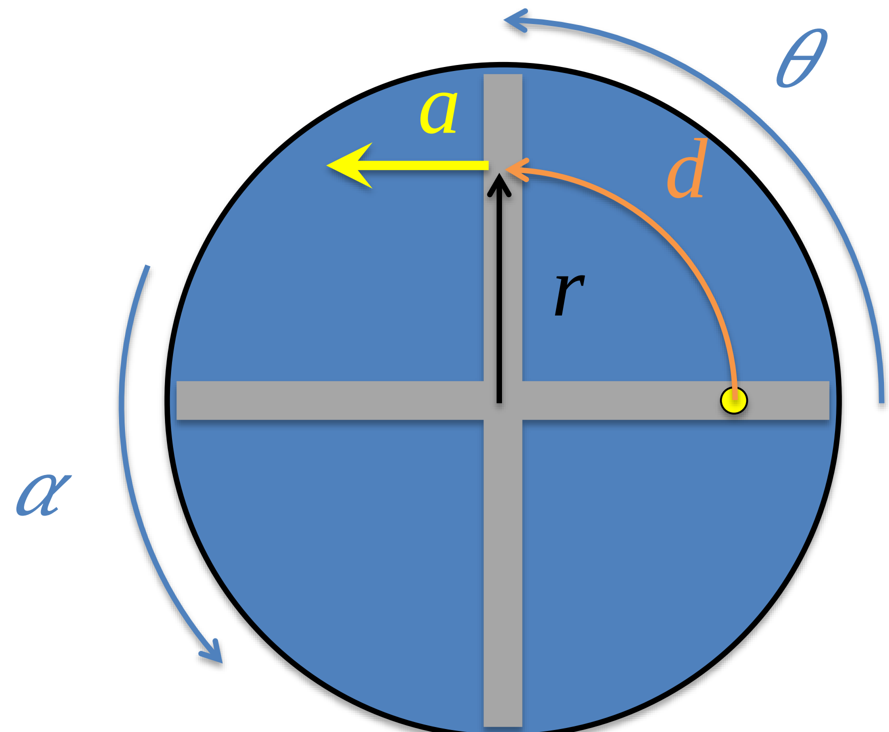
$$v = \frac{\Delta d}{\Delta t}$$

$$v = \frac{2\pi r}{T}$$

$$\boxed{v = \omega r}$$

Angular displacement and angular acceleration of entire disk:

$$\alpha = \frac{\Delta\omega}{\Delta t}$$



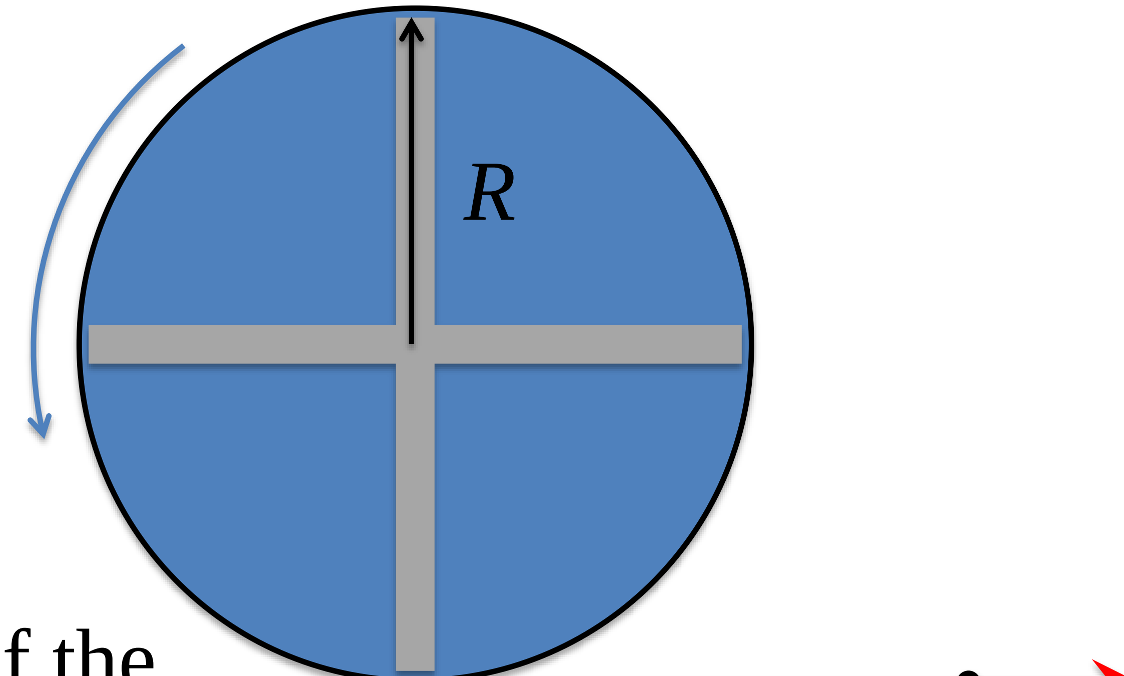
Linear distance and acceleration of yellow dot:

$$d = \theta r$$

$$a = \alpha r$$

Pulley rotating with parameters:
θ, ω, α

Distance, speed, and acceleration of the rope:



$d = \theta R$

$v = \omega R$

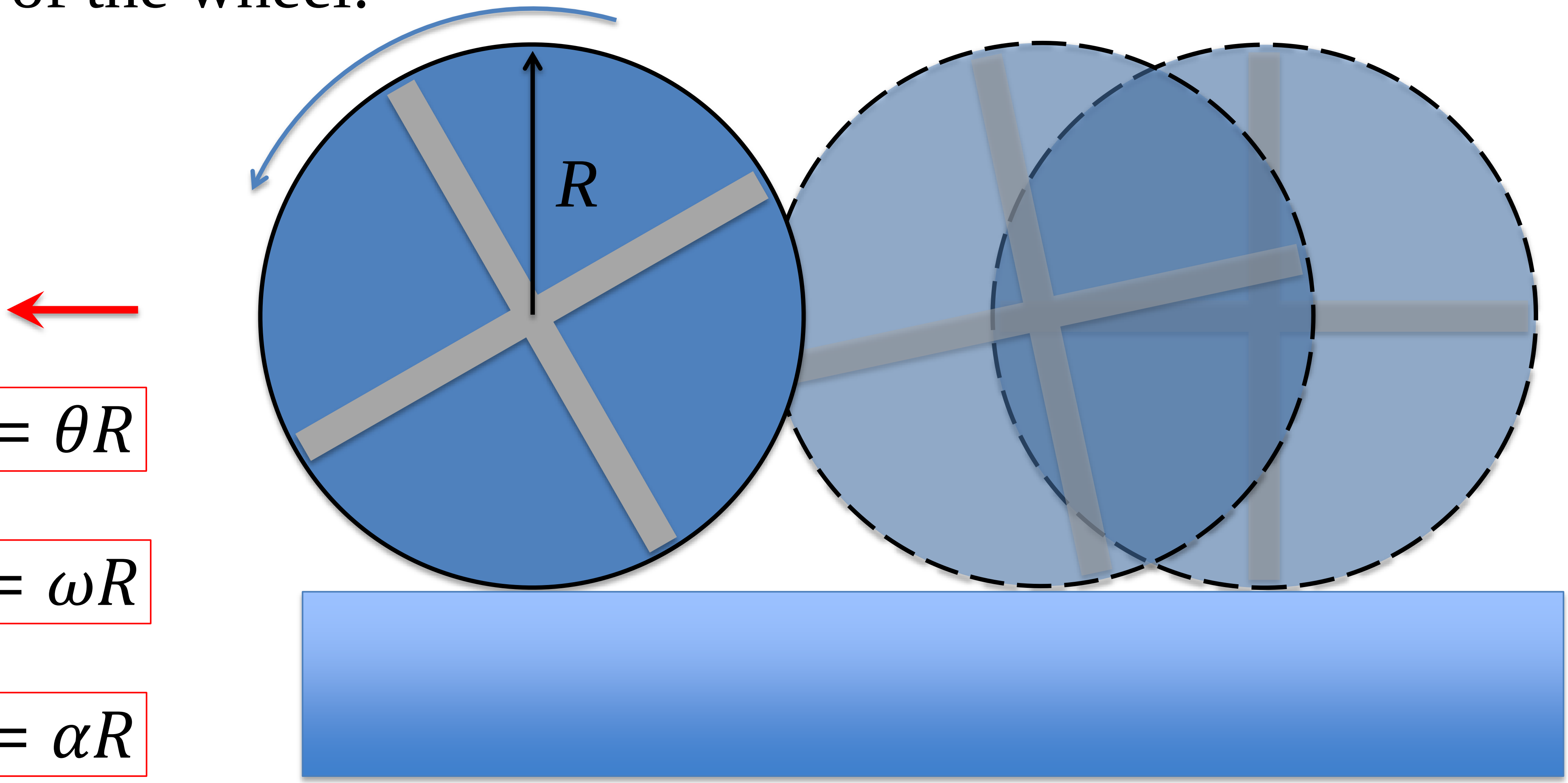
$a = \alpha R$

The linear motion of the rope is equivalent to the motion of a point on the pulley's circumference.

Rope unwinding

Distance, speed, and acceleration of the center of the wheel:

Rolling wheel rotates with parameters: θ, ω, α



$$d = \theta R$$

$$v = \omega R$$

$$a = \alpha R$$