Vector Addition and Subtraction

Learning to add all over again . . .

Vectors – 2-D Kinematics

I. **Vector Addition/Subtraction**
   - Graphical
II. Vector Components
   - Applications
III. Vector Addition/Subtraction
   - Numerical
IV. Relative Motion
V. Projectile Motion

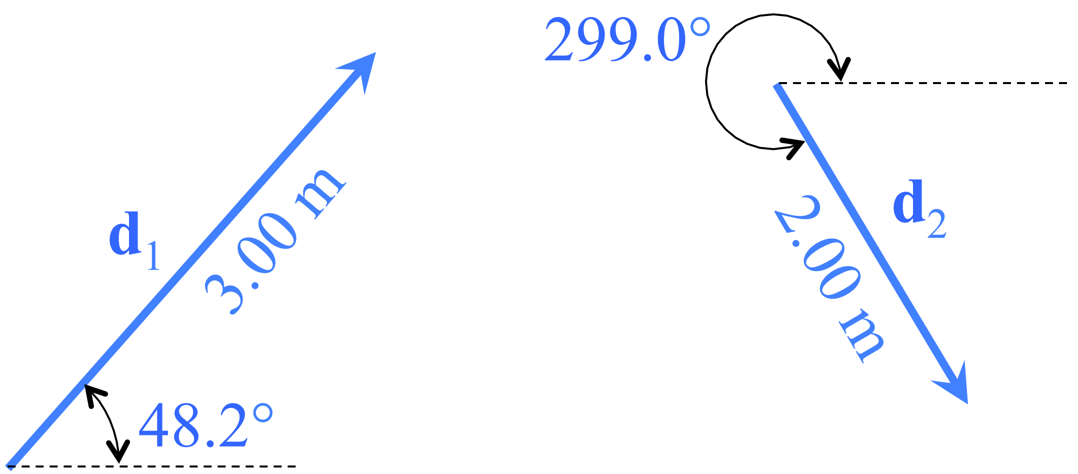
	The student will be able to:	HW:
1	Add or subtract vectors graphically and determine a vector's opposite.	1, 2
2	Calculate the components of a vector given its magnitude and direction.	3, 4
3	Calculate the magnitude and direction of a vector given its components.	5 - 9
4	Use vector components as a means of analyzing/ solving 2-D motion problems.	10 - 13
5	Add or subtract vectors analytically (using trigonometric calculations).	14, 15
6	Use vector addition or subtraction as a means of solving relative velocity problems.	16 - 20
7	State the horizontal and vertical relations for projectile motion and use the same to solve projectile problems and apply vector properties to projectile motion.	21 - 38

Vector Addition

- It is necessary to add vectors whenever two or more vectors occur either in sequence or simultaneously.
- The result of adding two vectors is a third vector that is equivalent to the combination of the two.
- To solve a vector addition problem both the *magnitude* and *direction* of the resulting vector must be found.

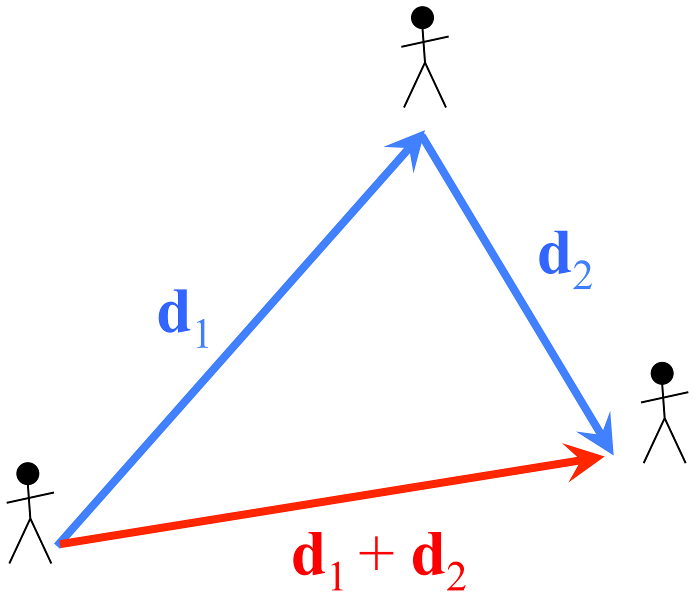
Example: adding displacements

Suppose a person undergoes two consecutive displacements: $\mathbf{d}_1$ = 3.00 m, 48.2° and $\mathbf{d}_2$ = 2.00 m, 299.0°. What is the resulting displacement of this person?



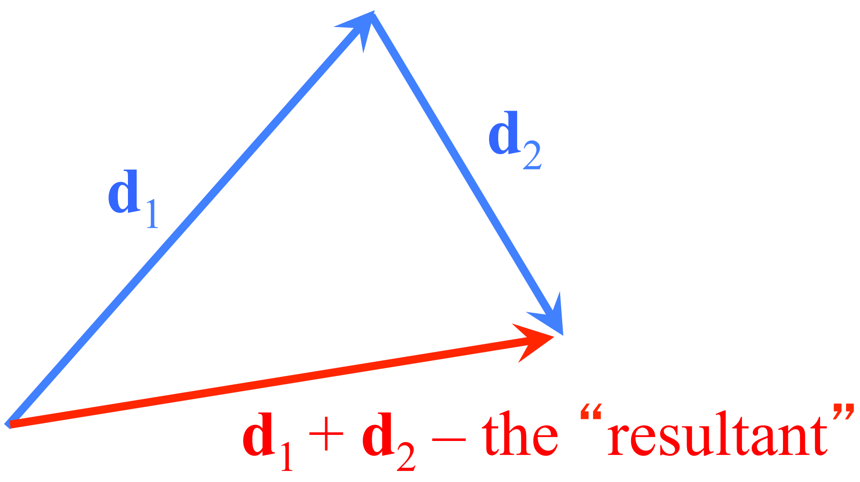
Example: adding displacements

The result is the displacement shown in red:



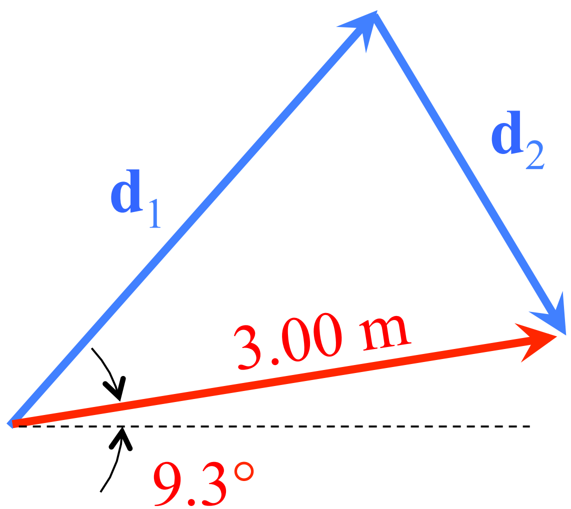
Example: adding displacements

This sum vector is sometimes called the *resultant*. Its magnitude and direction can be determined from the geometric figure.

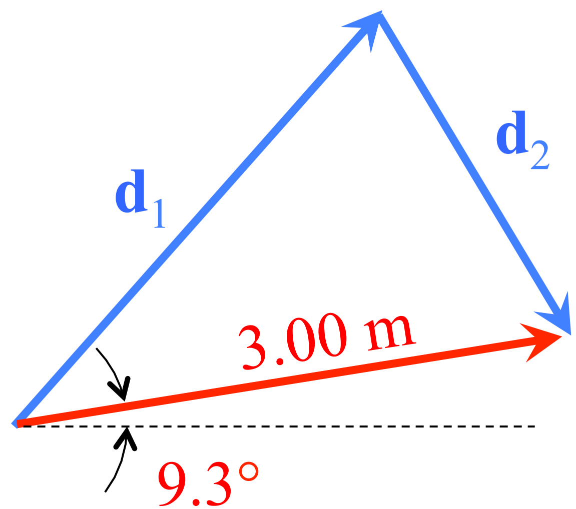


Example: adding displacements

This sum vector is sometimes called the *resultant*. Its magnitude and direction can be determined from the geometric figure.



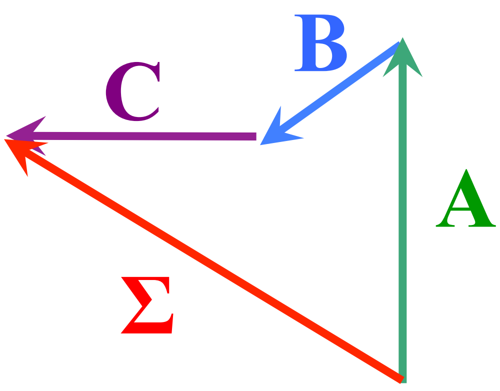
Therefore the sum of $\mathbf{d}_1$ = 3.00 m, 48.2° and $\mathbf{d}_2$ = 2.00 m, 299.0° is 3.00 m, 9.3°. This is a displacement that equals the combination of the two given displacements "put together".



(3.00 m, 48.2°) + (2.00 m, 299.0°) = (3.00 m, 9.3°)

Rule for Vector Addition

To add vectors, place the vectors head-to-tail. The resultant sum is the vector that extends from the tail of the first to the head of the last.



$$\vec{A} + \vec{B} + \vec{C} = \vec{\Sigma}$$

The graphical method of adding vectors (using ruler and protractor):

1. Draw and carefully measure a scale diagram of the vectors placed head to tail.
2. Draw and measure the resultant's length and angle.
3. Give answer magnitude and direction. (Adjust for the chosen scale of the diagram if necessary.)

Try these problems using the graphical approach – draw and measure:

1. $\mathbf{A}$ = 7.00 cm, 140.0º
   $\mathbf{B}$ = 6.00 cm, 270.0º
   $\mathbf{A} + \mathbf{B}$ = ?

2. $\mathbf{C}$ = 50.0 km, 325.0º
   $\mathbf{D}$ = 100.0 km, 70.0º
   $\mathbf{C} + \mathbf{D}$ = ? $\mathbf{D} + \mathbf{C}$ = ?

3. $\mathbf{E}$ = 3.70 m/s, 310.0º
   $\mathbf{F}$ = 7.90 m/s, 90.0º
   $\mathbf{E} + \mathbf{F}$ = ?

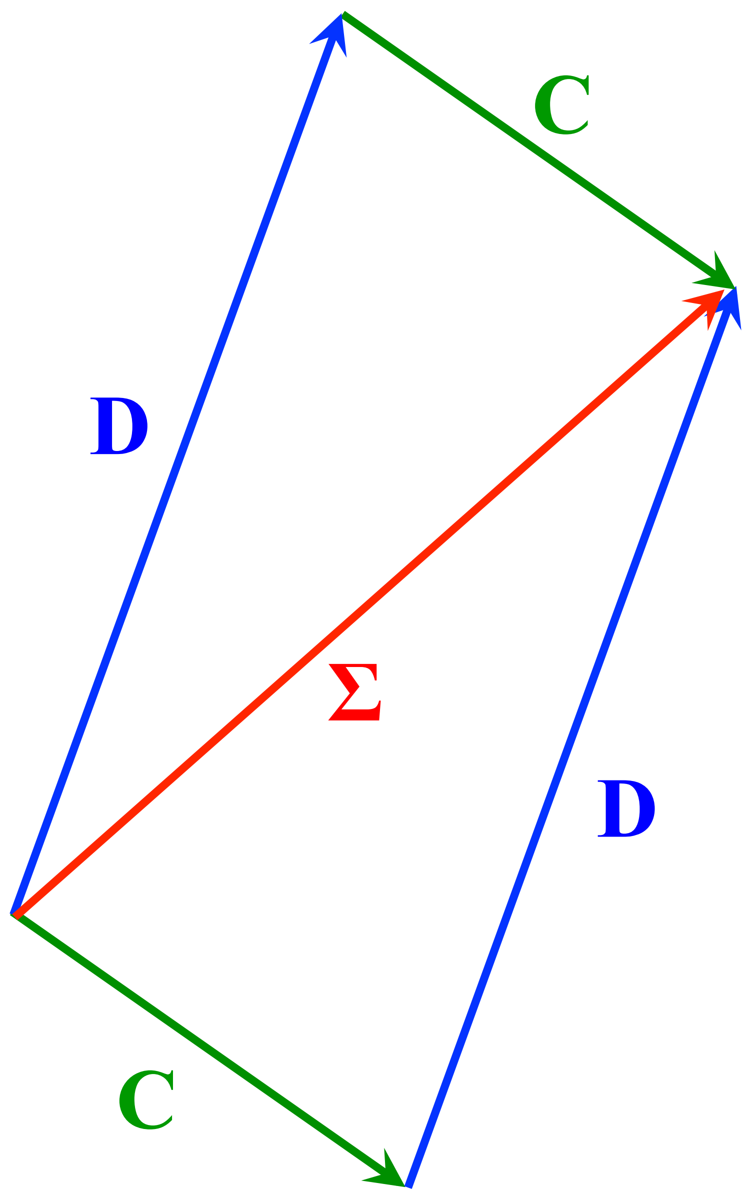
4. $\mathbf{G}$ = 16 m/s^2, 45.0º
   $\mathbf{H}$ = 3.0 m/s^2, 45.0º
   $\mathbf{G} + \mathbf{H}$ = ?

5. $\mathbf{I}$ = 105 m, 140.0º
   $\mathbf{J}$ = 35.0 m, 320.0º
   $\mathbf{I} + \mathbf{J}$ = ?

6. $\mathbf{K}$ = 40.0 m/s, 180.0º
   $\mathbf{L}$ = 40.0 m/s, 0.0º
   $\mathbf{K} + \mathbf{L}$ = ?

7. $\mathbf{M}$ = 5.70 cm, 320.0º
   $\mathbf{N}$ = 4.90 cm, 90.0º
   $\mathbf{O}$ = 6.00 cm, 210.0º
   $\mathbf{M} + \mathbf{N} + \mathbf{O}$ = ?

8. $\mathbf{P}$ = 10.0 m, 90.0º
   $\mathbf{Q}$ = 7.50 m, 0.0º
   $\mathbf{P} - \mathbf{Q}$ = ?

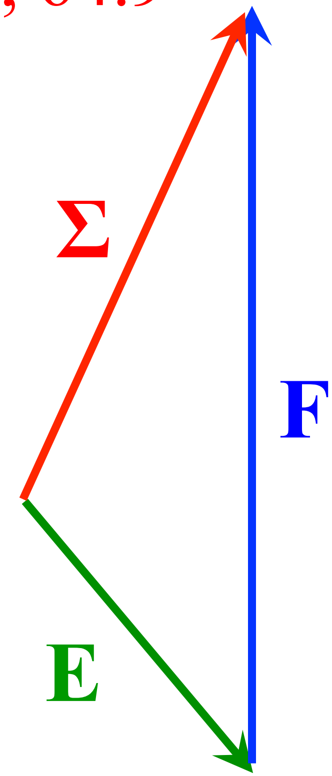


3. **C** = 50.0 km, 325.0º
D = 100.0 km, 70.0º

C + **D** = 99.6 km, 41.0º

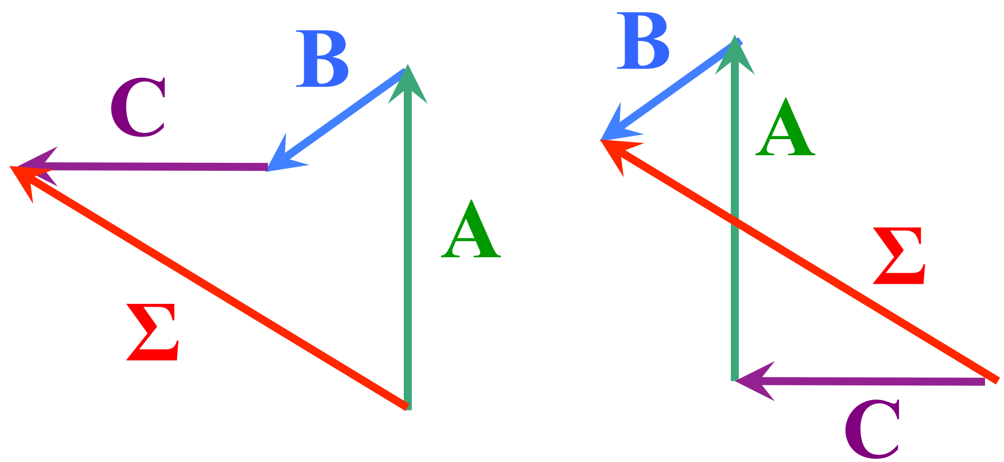
4. **E** = 3.70 m/s, 310.0º
F = 7.90 m/s, 90.0º

E + **F** = 5.60 m/s, 64.9º



Order of Vector Addition

The order of addition does not affect the resultant!
(commutative property)

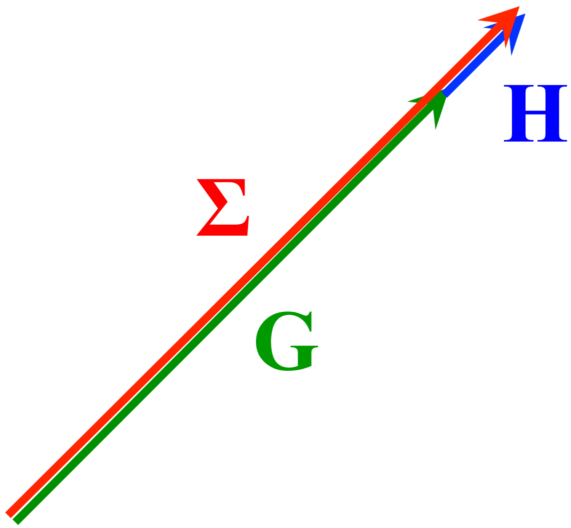


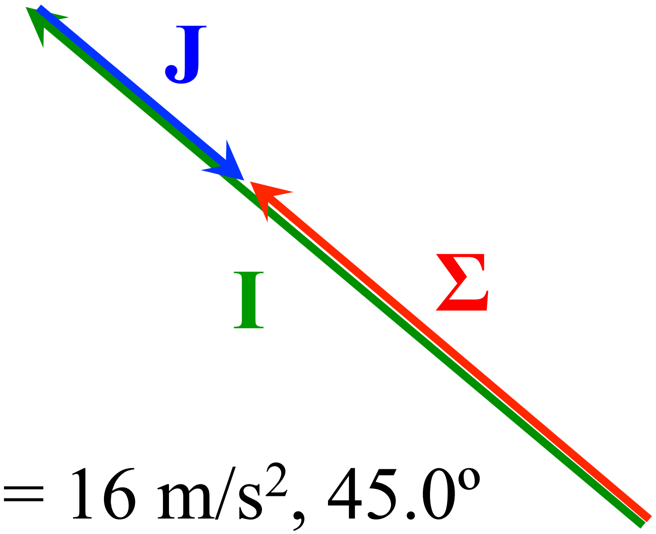
$$\vec{A} + \vec{B} + \vec{C} = \vec{C} + \vec{A} + \vec{B}$$

To add vectors, always place the vectors head-to-tail.
But the order of placeement will not affect the answer!

Collinear Vectors

- Vectors that lie on the same line may be added easily using "regular addition" – however, a vector that points in an **opposite direction** <u>must</u> be considered **negative**.

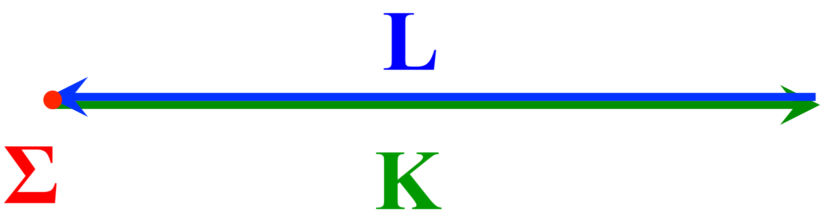




4. $\mathbf{G} = 16\ \text{m/s}^2,\ 45.0^{\circ}$

$\underline{\mathbf{H} = 3.0\ \text{m/s}^2,\ 45.0^{\circ}}$

$\mathbf{G} + \mathbf{H} = 19.0\ \text{m/s}^2,\ 45.0^{\circ}$



5. $\mathbf{I} = 105\ \text{m},\ 140.0^{\circ}$

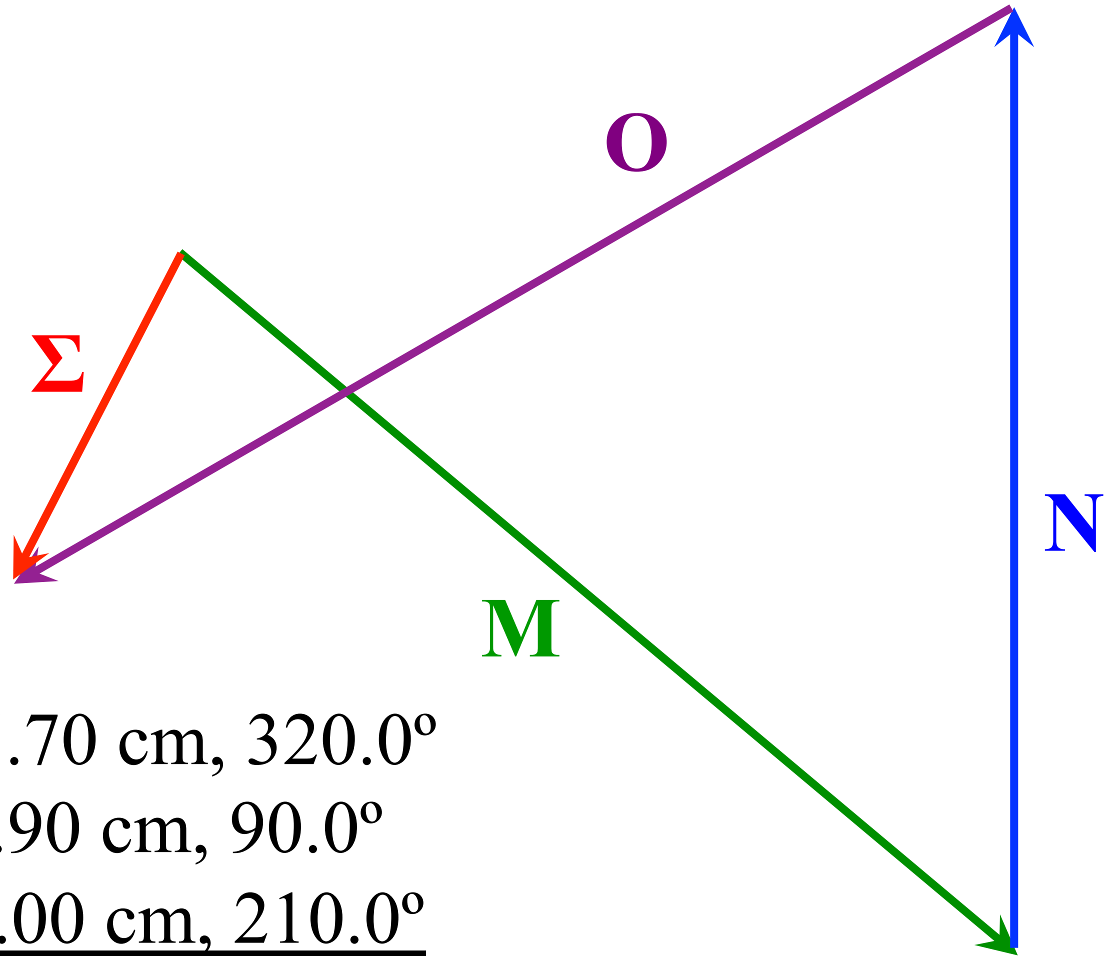
$\underline{\mathbf{J} = 35.0\ \text{m},\ 320.0^{\circ}}$

$\mathbf{I} + \mathbf{J} = 70.0\ \text{m},\ 140.0^{\circ}$

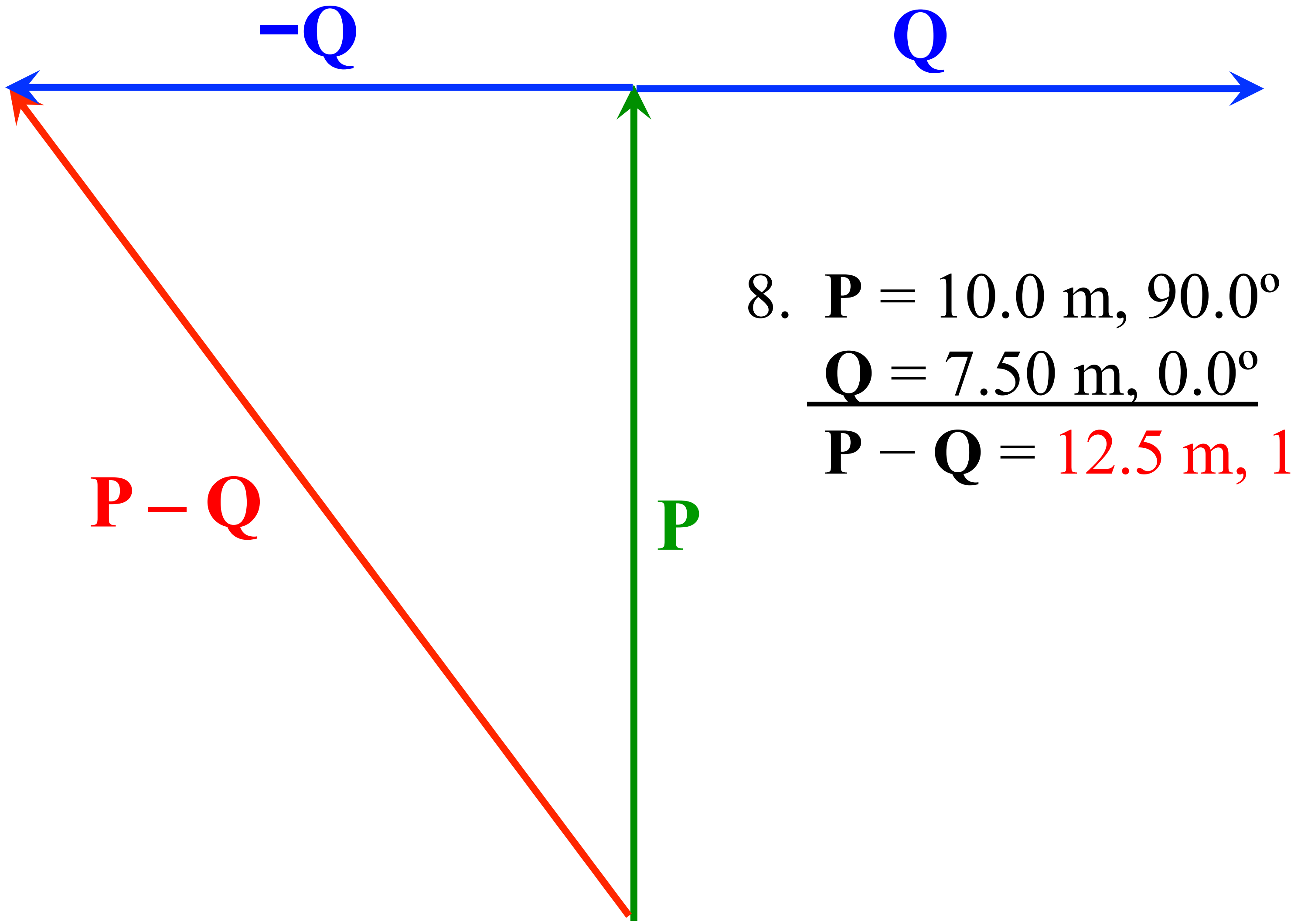
6. $\mathbf{K} = 40.0\ \text{m/s},\ 180.0^{\circ}$

$\underline{\mathbf{L} = 40.0\ \text{m/s},\ 0.0^{\circ}}$

$\mathbf{K} + \mathbf{L} = 0$



7. **M** = 5.70 cm, 320.0º
N = 4.90 cm, 90.0º
<u>**O** = 6.00 cm, 210.0º</u>
M + **N** + **O** = 1.96 cm, 243.6º



8. $\mathbf{P} = 10.0\ \text{m},\ 90.0^{o}$

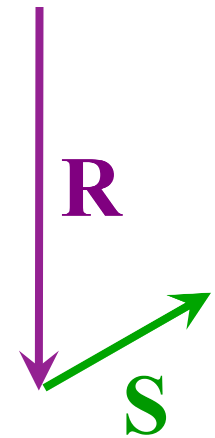
$\mathbf{Q} = 7.50\ \text{m},\ 0.0^{o}$

$\mathbf{P} - \mathbf{Q} = 12.5\ \text{m},\ 126.9^{o}$

Vector Subtraction

R = 20.0 m, 270.0º
S = 10.0 m, 30.0º

R − **S** = ?



To subtract a vector, add its opposite.

A vector's opposite has the same magnitude but opposite direction (differs by 180º).

Vector Subtraction

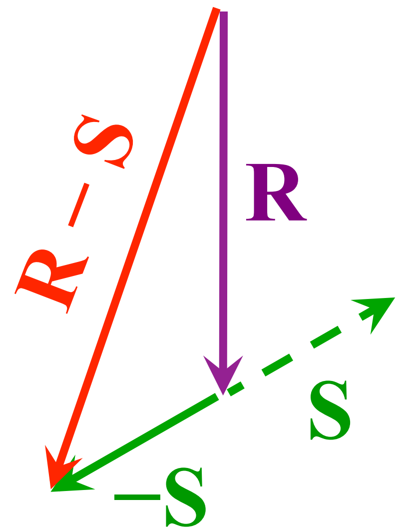
$\mathbf{R} = 20.0\text{ m}, 270.0^\circ$

$\mathbf{S} = 10.0\text{ m}, 30.0^\circ$

$-\mathbf{S} = 10.0\text{ m}, 210.0^\circ$

$\mathbf{R} - \mathbf{S} = \mathbf{R} + (-\mathbf{S})$

$\mathbf{R} - \mathbf{S} = 26.5\text{ m}, 109.1^\circ$



To subtract a vector, add its opposite.

A vector's opposite has the same magnitude but opposite direction (differs by 180º).

Vector Subtraction

R = 20.0 m, 270.0º

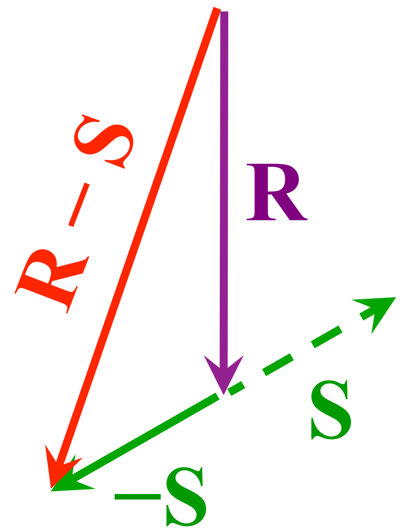
S = 10.0 m, 30.0º

<u>−**S** = 10.0 m, 210.0º</u>

R − **S** = **R** + (−**S**)

R − **S** = 26.5 m, 250.9º

S − **R** = ?



Vector Subtraction

$\mathbf{R} = 20.0\text{ m}, 270.0^{o}$

$-\mathbf{R} = 20.0\text{ m}, 90.0^{o}$

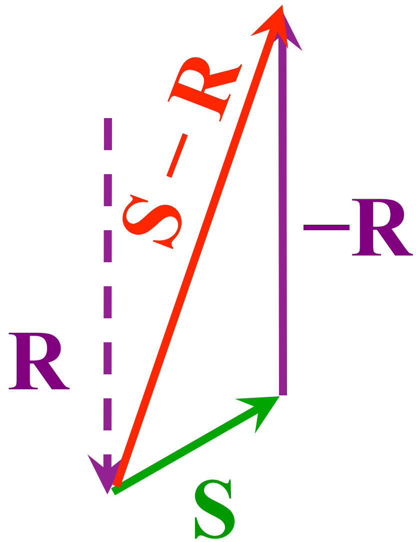
$\underline{\mathbf{S} = 10.0\text{ m}, 30.0^{o}}$

$\mathbf{R} - \mathbf{S} = \mathbf{R} + (-\mathbf{S})$

$\mathbf{R} - \mathbf{S} = 26.5\text{ m}, 250.9^{o}$

$\mathbf{S} - \mathbf{R} = \mathbf{S} + (-\mathbf{R})$

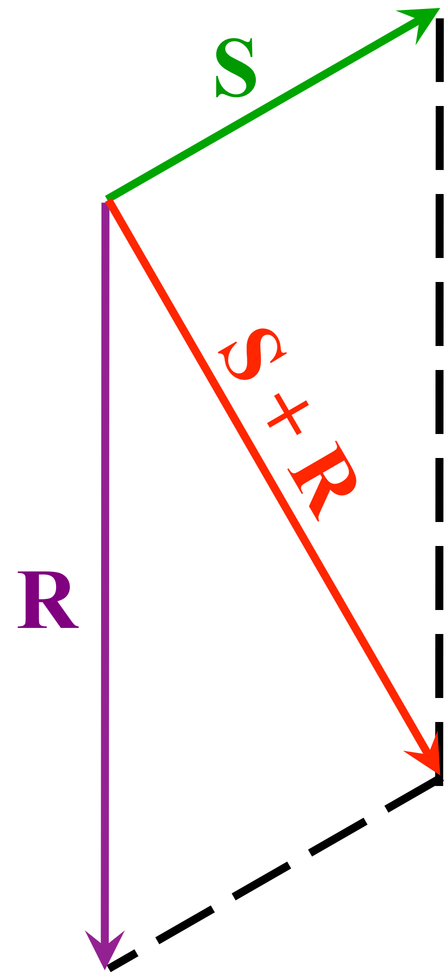
$\mathbf{S} - \mathbf{R} = 26.5\text{ m}, 70.9^{o}$



Parallelogram Rule

Vector addition and subtraction may also be visualized by the parallelogram formed by placing tail-to-tail…

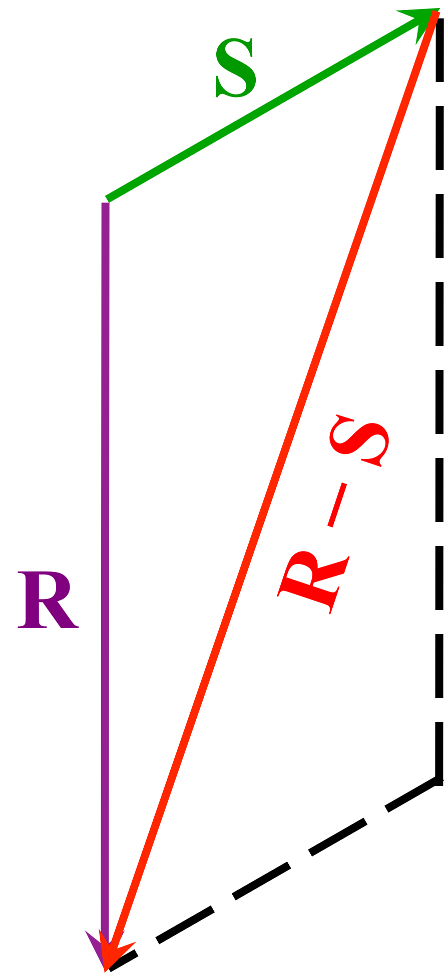
…the sum extends along a diagonal outward from the tails.



Parallelogram Rule

Vector addition and subtraction may also be visualized by the parallelogram formed by placing tail-to-tail…

…the difference is along a diagonal from head to head.



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