Circuits and Capacitors

I. Current, Power, Resistance
   - resistivity
   - internal resistance

II. Circuit Analysis
   - series and parallel
   - nodes, loops, switches

III. Capacitance
   - **parallel plate capacitor**
   - **capacitors in circuits**

	The student will be able to:	HW:
1	Define electric current and the ampere and solve problems relating current to charge and time and to power and voltage. ✓	1 – 3
2	Define resistance, resistivity, and the ohm and Ohm’s Law and solve related problems. ✓	4 – 10
3	Define and apply the concepts of internal resistance and emf to solve related problems with the standard model of the terminal voltage of voltaic cells. ✓	11 – 14
4	Determine resistance for series or parallel combinations of resistors, state and apply Kirchoff’s node and loop rules and solve related problems, including analysis circuits with multiple batteries, resistors, and switches. ✓	15 – 20
5	Define capacitance and relate to charge, voltage and energy to solve related problems involving capacitors in circuits at steady states of charge or discharge and qualitatively describe transitions of such states.	21 – 29
6	State the relation between capacitance, area, separation, and dielectric constant for parallel plate capacitors and solve related problems.	30 – 35

Capacitance refers to an electrical device's ability to store or retain charge. Capacitance is defined as the ratio of charge to voltage. This ratio specifies the relative charge capacity of the device.

$$\boxed{C = \frac{Q}{V}}$$

where: C = capacitance
Q = charge
V = potential difference

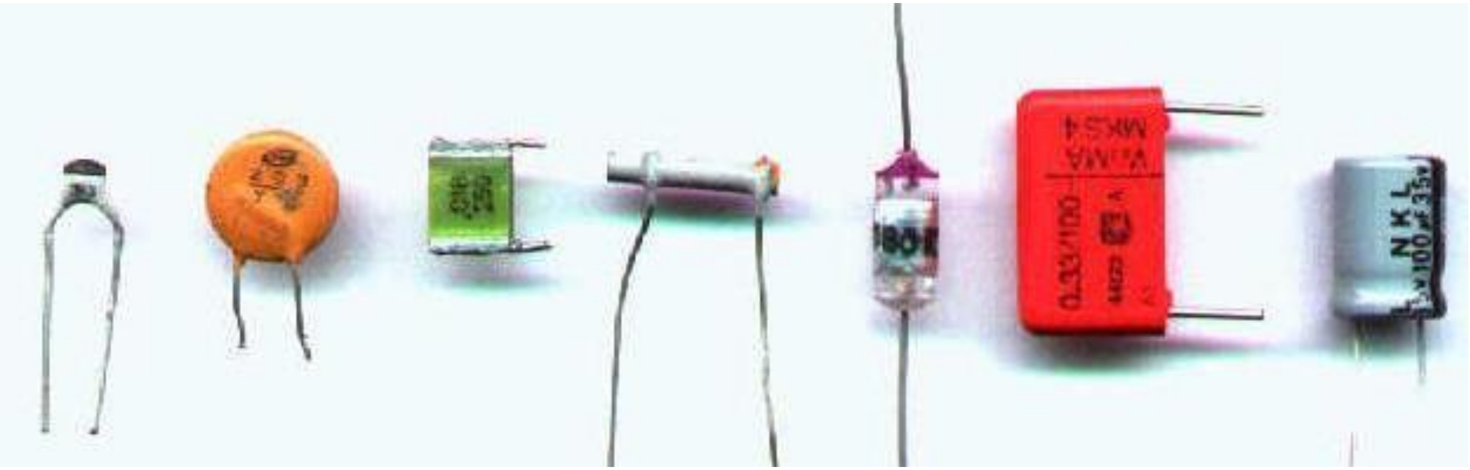
Capacitance – SI Units

1 farad = 1 coulomb per volt

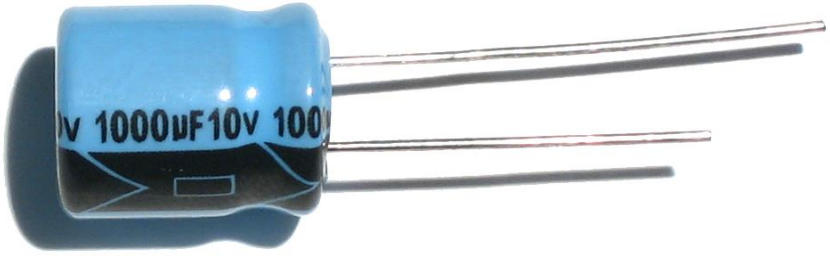
$$1\,\mathrm{F} = 1\,\frac{\mathrm{C}}{\mathrm{V}}$$

Typical capacitors have values in microfarads, nanofarads, or picofarads.

A **capacitor** is an electrical device designed specifically to provide a certain amount of capacitance. It consists of two conducting surfaces separated by an insulating material.



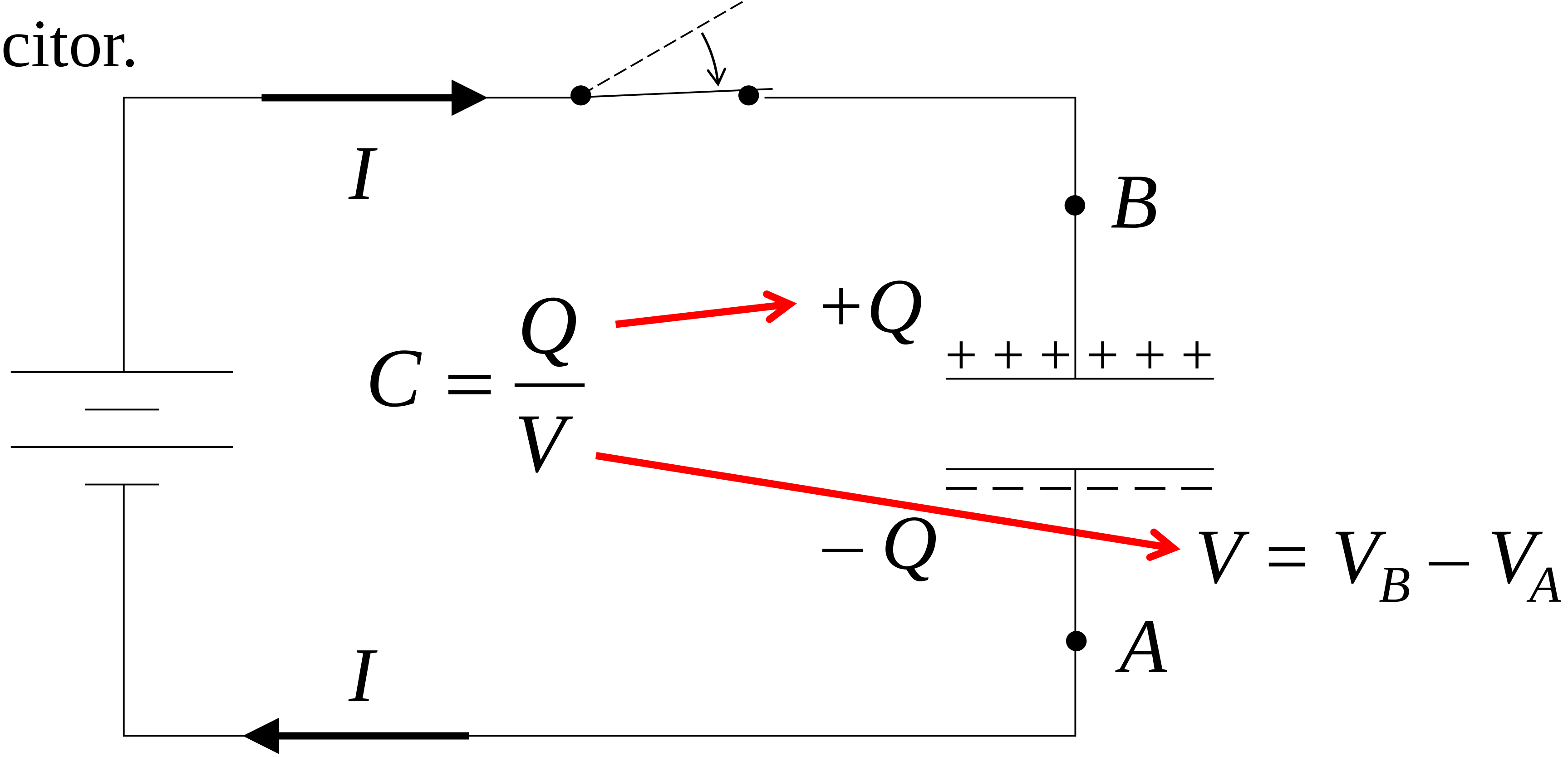
As voltage is applied to the two leads, equal and opposite charges develop on the two conducting surfaces within the capacitor.



electrolytic
capacitor

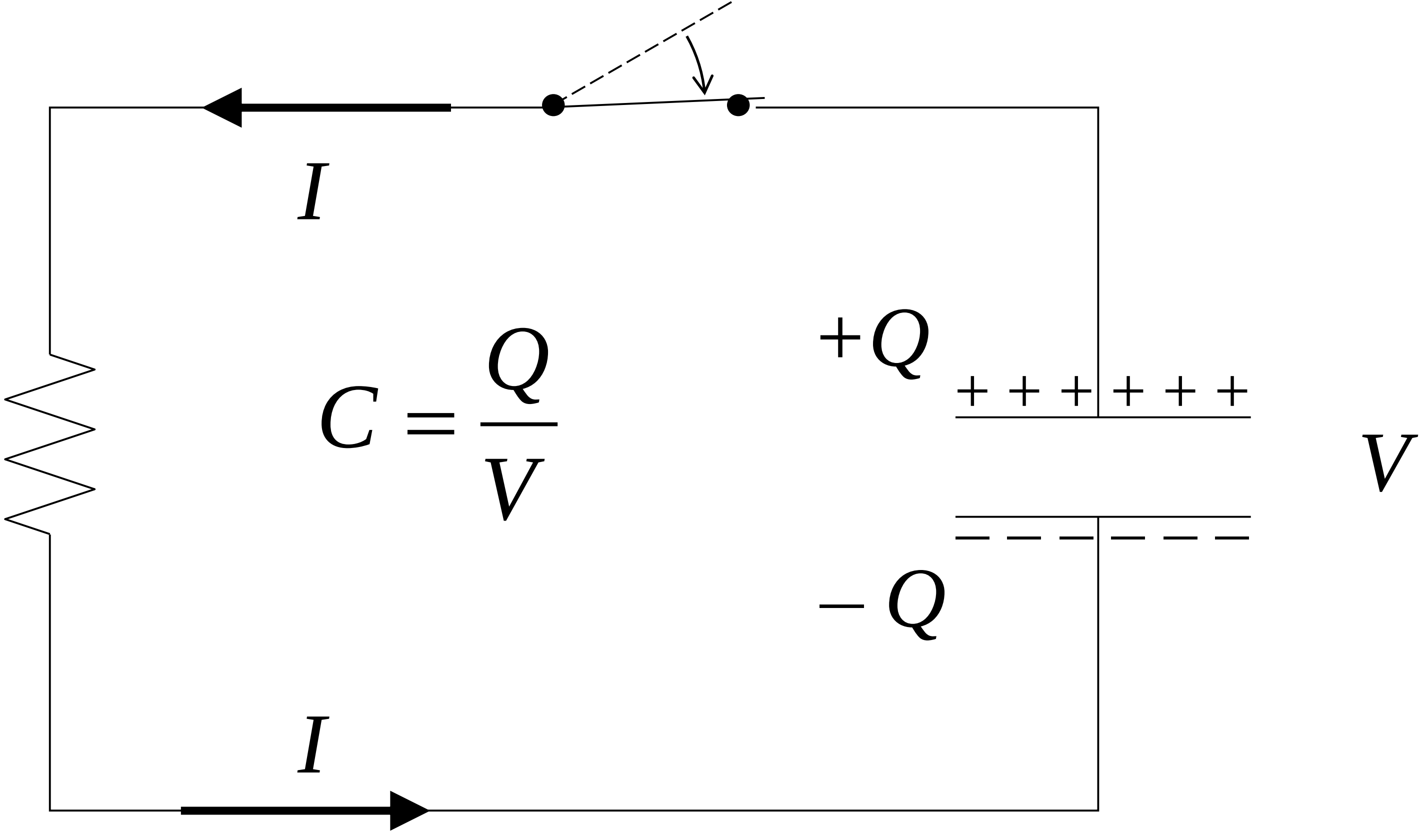
parallel plate
capacitor

When switch is closed equal current flows on both sides of the capacitor causing equal and opposite charge of amount Q to develop on the plates and energy to transfer from cell to capacitor.



At all times the ratio of charge Q to voltage V is constant and equal to the particular capacitance C, which depends on the characteristics of the capacitor design and materials.

When connected to a resistance the capacitor acts as a source of energy, potential, and current. The current depletes charge from both plates of the capacitor and its voltage drops.



At all times the ratio of charge Q to voltage V is constant and equal to the particular capacitance C, which depends on the characteristics of the capacitor design and materials.

The energy stored in a capacitor depends on the amount of charge stored and the potential difference across the two terminals.

$$\boxed{U = \frac{1}{2}CV^2}$$

$$\boxed{U = \frac{1}{2}\frac{Q^2}{C}}$$

where: U = potential energy
C = capacitance
Q = charge
V = potential difference

Behavior of Capacitors in Circuits: Charging

- If the charge amount is zero then voltage across a capacitor must be zero also – charge can “readily” flow to and from the plates.
- As plates become oppositely charged, it is as if a “current passes through” a capacitor (in spite of the gap inside).
- As charge builds up, so does the voltage and potential energy, and forces oppose the addition of more charge to the plates and current decreases.
- At some point, depending on the circuit, a voltage is reached that prevents further charge buildup and current drops to zero – a state of equilibrium.

Behavior of Capacitors in Circuits: Discharging

- A charged capacitor is a source of electric potential and energy and can drive current through a circuit somewhat like a battery.
- Unlike a battery, the voltage does not have any tendency to remain constant as current flows.
- As charge decreases so does the voltage and potential energy, and the forces causing the discharge lessen, decreasing the current.
- Discharge will continue until a new equilibrium voltage is reached, at which point forces tending to send charge into the capacitor are balanced with forces tending to send charge out.

The effective of equivalent capacitance of multiple connected capacitors:

parallel

$$C_{eq} = C_1 + C_2 + C_3 + \ldots$$

series

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots$$

Circuits and Capacitors

I. Current, Power, Resistance
  - resistivity
  - internal resistance

II. Circuit Analysis
  - series and parallel
  - nodes, loops, switches

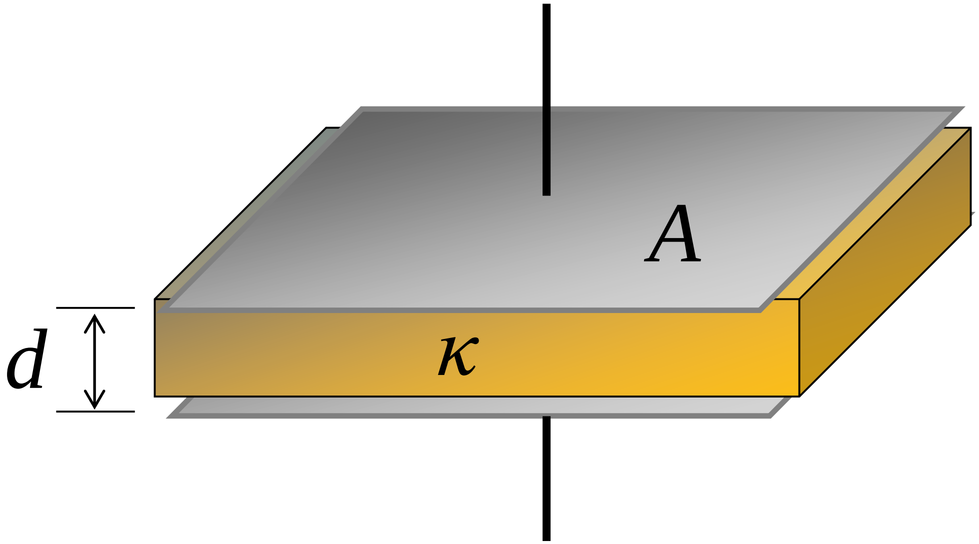
III. Capacitance
  - parallel plate capacitor
  - capacitors in circuits

	The student will be able to:	HW:
1	Define electric current and the ampere and solve problems relating current to charge and time and to power and voltage.	✓ 1 – 3
2	Define resistance, resistivity, and the ohm and Ohm’s Law and solve related problems.	✓ 4 – 10
3	Define and apply the concepts of internal resistance and emf to solve related problems with the standard model of the terminal voltage of voltaic cells.	✓ 11 – 14
4	Determine resistance for series or parallel combinations of resistors, state and apply Kirchoff’s node and loop rules and solve related problems, including analysis circuits with multiple batteries, resistors, and switches.	✓ 15 – 20
5	Define capacitance and relate to charge, voltage and energy to solve related problems involving capacitors in circuits at steady states of charge or discharge and qualitatively describe transitions of such states.	✓ 21 – 29
6	State the relation between capacitance, area, separation, and dielectric constant for parallel plate capacitors and solve related problems.	30 – 35

The capacitance of a parallel-plate capacitor is given by:

$$\boxed{C = \frac{\kappa \varepsilon_0 A}{d}}$$

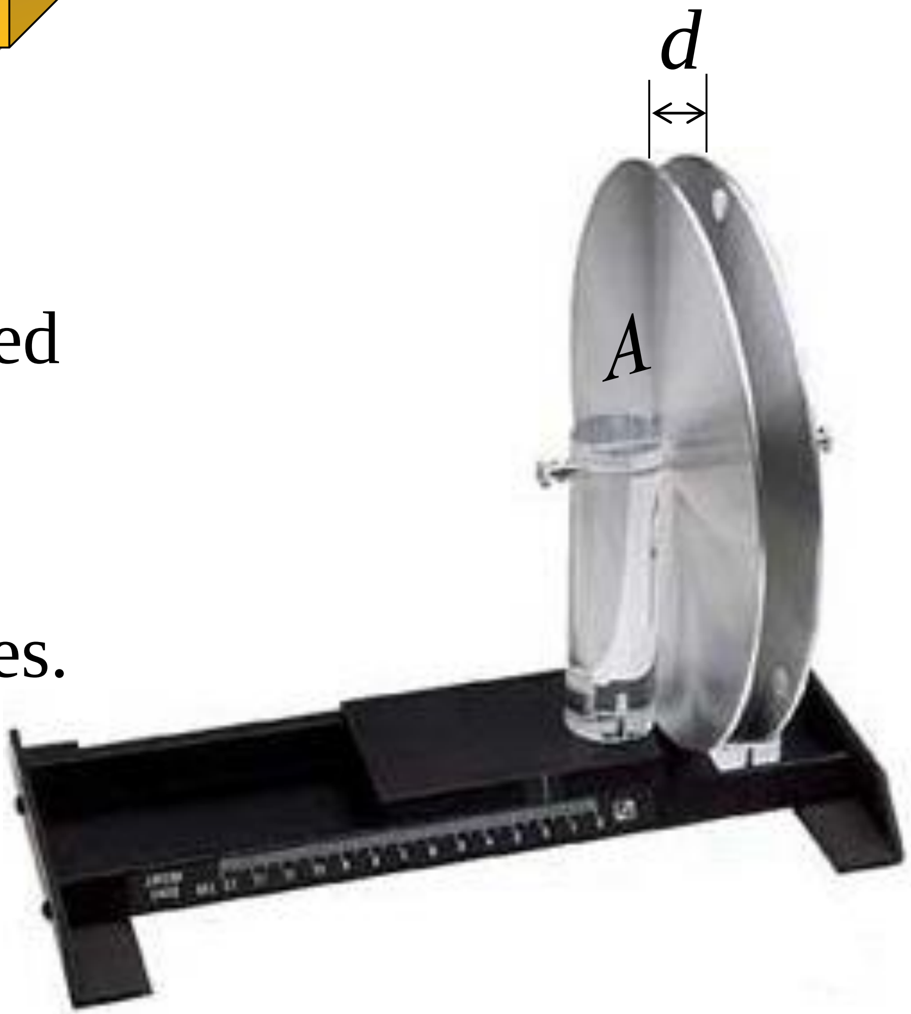
where: C = capacitance
A = area of either plate
d = distance separating plates
κ = dielectric constant of material between plates



The area A can be any shape, but commonly is a rectangle or circle.

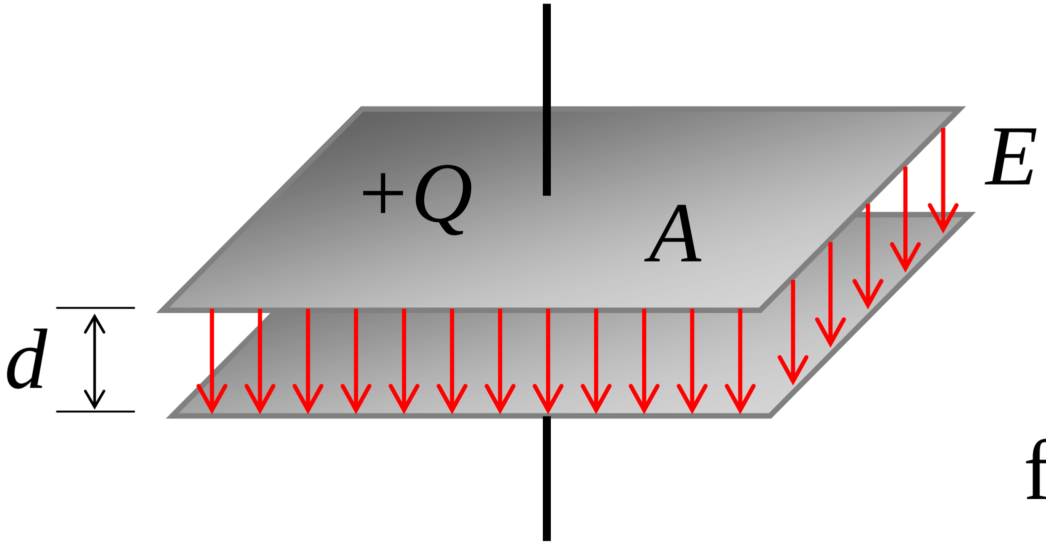
The insulating material between the plates is called the “dielectric” slab.

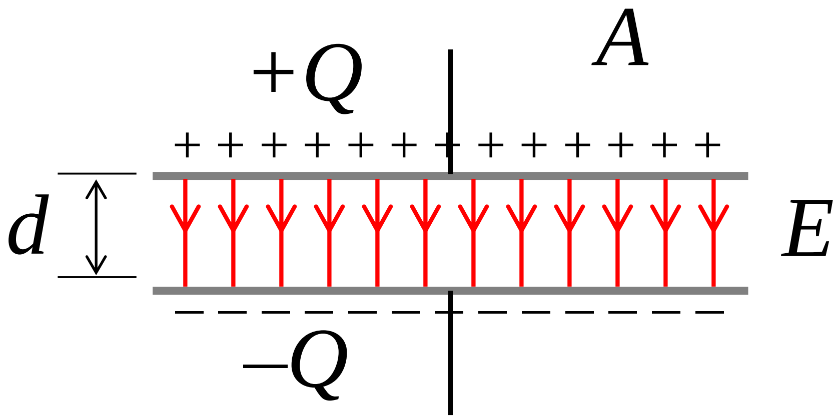
There can also be air or a vacuum between the plates.



Dielectric Constants and Strengths

Material	Dielectric Constant, κ	Dielectric Strength (MV/m)
Vacuum	1	N/A
Air	1.0006	3
Pyrex	5	14
Paper	3.7	16
Teflon	2.1	60
Paraffin	2.3	11
Polystyrene	2.6	24
Strontium Titanate	230	8

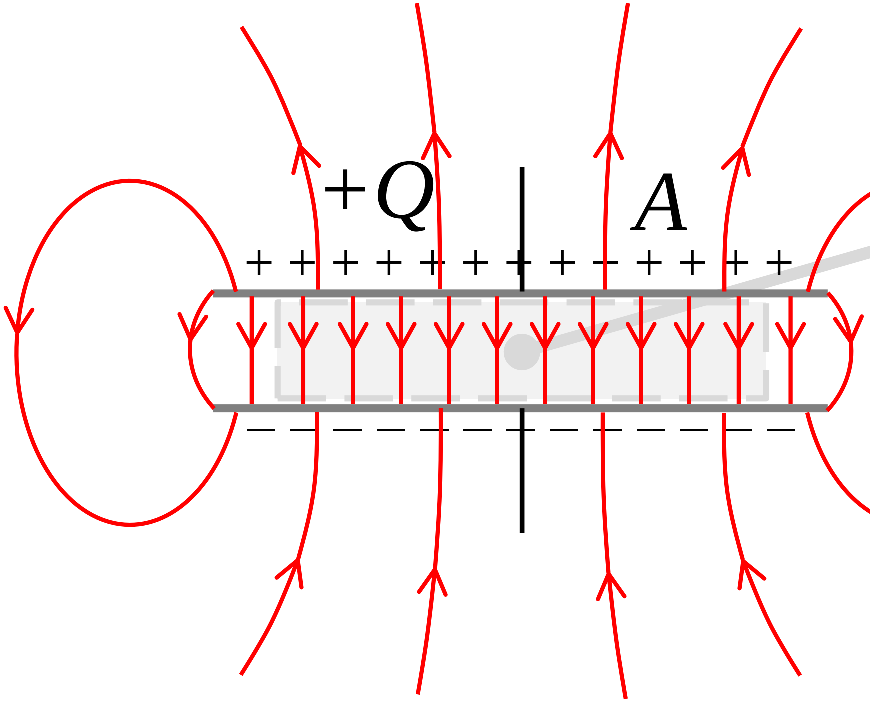




$$C = \frac{Q}{V} \qquad C = \frac{\kappa \varepsilon_0 A}{d}$$

Derive an expression for the electric field in terms of Q, A, d, κ, and appropriate physical constants…

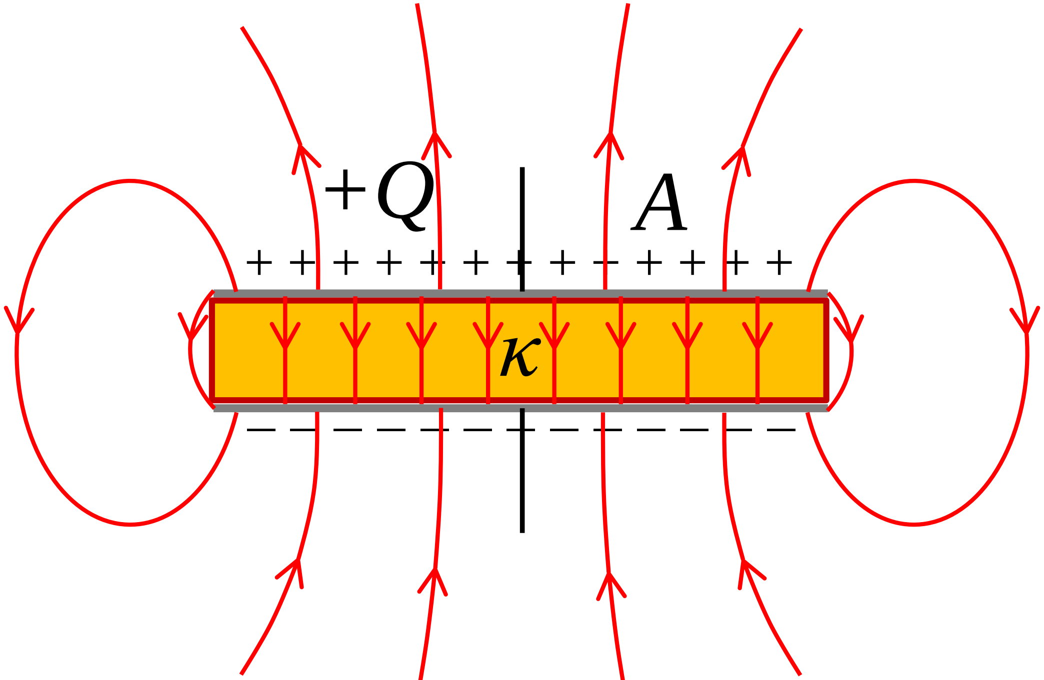
$$\boxed{E = \frac{Q}{k e_0 A}}$$



$$E = \frac{Q}{k e_0 A}$$

most accurate for
the shaded region

This expression applies *only* to the region *between* two oppositely charged surfaces separated by a relatively small gap. In reality the field weakens and curves near the edges of the plates and *outside* of the capacitor it is very weak and often assumed to be essentially nonexistent.



$$E = \frac{Q}{\kappa \epsilon_0 A}$$

gives the net electric field existing in the material between plates

If the gap is filled with air or a vacuum the value of κ can be taken to be 1.00. Presence of a dielectric material causes the electric field to be weakened by the factor κ, which is a particular value depending on the characteristics of the insulating material. This is due to induced charges in the dielectric which create an opposing field. This allows greater charge to be stored on the plates for a given voltage, and hence increases capacitance by the factor κ.