Quantum and Atomic Physics

I. Wave/Particle Duality
- quantum energy, Planck's constant
- photons, photoelectric effect
- Bohr model, De Broglie wavelength
- electron diffraction, interference

II. Special Relativity
- simultaneity, time dilation
- relativistic mass, momentum, and energy

III. Nuclear Physics
- **nucleus structure, energy, strong force**
- **radiation/nuclear decay, weak force**
- **nuclear reactions**

Relativistic Energy for Particles and Nuclei

- One important application of $E = mc^2$ is analysis of interactions of particles and nuclei.
- The implication of the formulas is that a particle has energy that is directly proportional to its mass.
- Any interaction that *releases* energy will also result in a *reduction* in mass; an interaction that *requires* energy will result in an *increase* in mass.
- Many problems can be solved by determining a change in energy based on a change in mass such that $\Delta E = \Delta mc^2$.

$$^{226}_{88}\mathrm{Ra} \rightarrow \, ^{222}_{86}\mathrm{Rn} + \, ^{4}_{2}\mathrm{He}$$

$$^{14}_{6}\mathrm{C} \rightarrow \, ^{14}_{7}\mathrm{N} + e^{-} + \bar{n}$$

$$^{22}_{11}\mathrm{Na} \rightarrow \, ^{22}_{10}\mathrm{Ne} + e^{+} + n$$

$$^{1}\mathrm{H} + \, ^{1}\mathrm{H} \rightarrow \, ^{2}\mathrm{H} + e^{+} + \nu + \gamma$$

$$^{2}\mathrm{H} + \, ^{1}\mathrm{H} \rightarrow \, ^{3}\mathrm{He} + \gamma$$

$$^{3}\mathrm{He} + \, ^{3}\mathrm{He} \rightarrow \, ^{4}\mathrm{He} + \, ^{1}\mathrm{H} + \, ^{1}\mathrm{H} + \gamma$$

$$e^{+} + e^{-} \rightarrow 2\gamma$$

Notation – Nuclear/Particle Reactions

$${}^{A}_{Z}X_{N}$$

X = chemical symbol
Z = atomic number (number of protons)
A = mass number (number of nucleons)
N = number of neutrons

note: Z and N can be determined from X and Z and so these values are often omitted.

Example: ${}^{14}_{6}C_{8}$

This is carbon-14, which has
6 protons, 8 neutrons, and 14 nucleons.

Three types of Radiation

Alpha radiation occurs when an unstable nucleus emits a particle consisting of two protons and two neutrons bound together. This "alpha particle" is equivalent to the nucleus of a helium atom.

Beta radiation occurs when an unstable nucleus emits either an electron and an antineutrino or a positron (antielectron) and a neutrino. For β^- decay a neutron in the nucleus converts into a proton, for β^+ decay a proton converts into a neutron.

Gamma radiation occurs when an unstable nucleus undergoes a change in energy state and emits a high energy photon.

Example Nuclear Reactions

alpha decay

$$ {}^{226}_{88}\mathrm{Ra} \longrightarrow {}^{222}_{86}\mathrm{Rn} + {}^{4}_{2}\mathrm{He} + \gamma $$

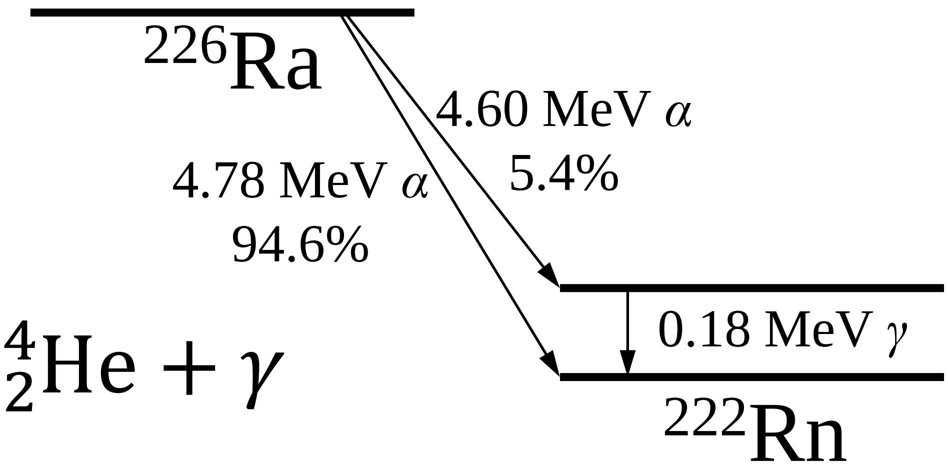
gamma decay

beta decay

$$ {}^{14}_{6}\mathrm{C} \longrightarrow {}^{14}_{7}\mathrm{N} + e^{-} + \bar{\nu} \qquad \beta^{-} $$

$$ {}^{13}_{7}\mathrm{N} \longrightarrow {}^{13}_{6}\mathrm{C} + e^{+} + \bar{\nu} \qquad \beta^{+} $$

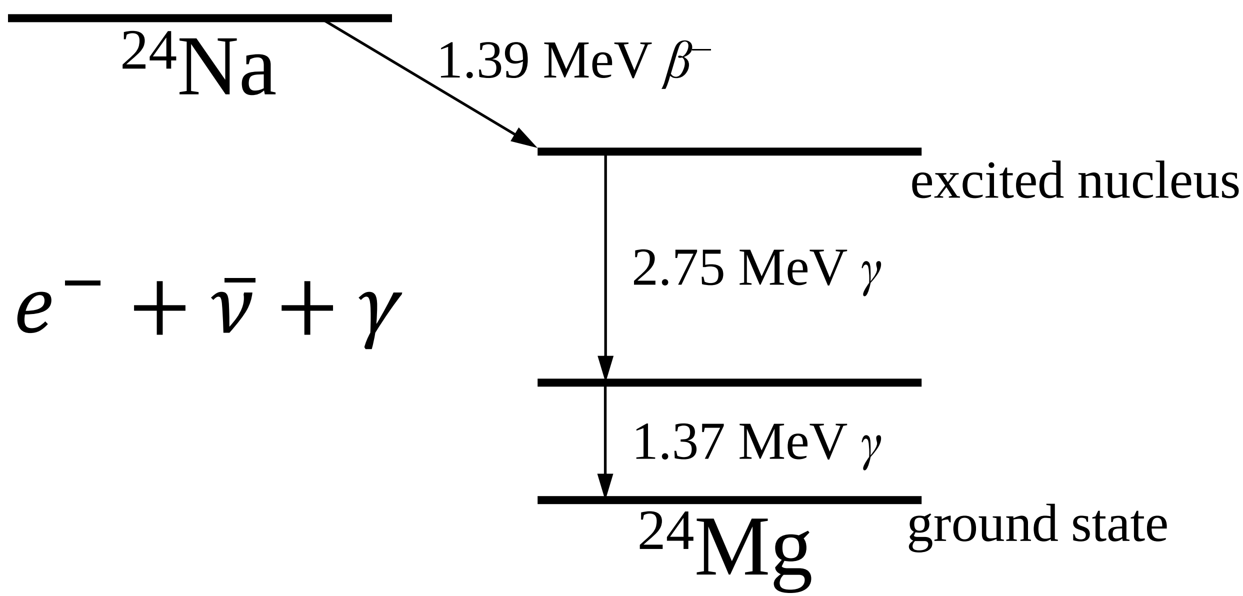
Example Nuclear Reactions



parent daughter

$$ {}^{226}_{88}\mathrm{Ra} \longrightarrow {}^{222}_{86}\mathrm{Rn} + {}^{4}_{2}\mathrm{He} + \gamma $$

Note: the recoil energy of the daughter is not shown in the energy level diagram.



$$ {}^{24}_{11}\mathrm{Na} \longrightarrow {}^{24}_{12}\mathrm{Mg} + e^{-} + \bar{\nu} + \gamma $$

$$^{226}_{88}\text{Ra} \rightarrow ^{222}_{86}\text{Rn} + ^{4}_{2}\text{He}$$

$$^{14}_{6}\text{C} \rightarrow ^{14}_{7}\text{N} + e^{-} + \bar{n}$$

$$^{22}_{11}\text{Na} \rightarrow ^{22}_{10}\text{Ne} + e^{+} + n$$

All nuclear reactions exhibit conservation of charge and conservation of nucleon (mass) number. Mass per se is *not* conserved when accounting for energy change associated with $E = mc^2$.

$$^{1}\text{H} + ^{1}\text{H} \longrightarrow ^{2}\text{H} + e^{+} + \nu + \gamma$$

$$^{2}\text{H} + ^{1}\text{H} \longrightarrow ^{3}\text{He} + \gamma$$

$$^{3}\text{He} + ^{3}\text{He} \longrightarrow ^{4}\text{He} + ^{1}\text{H} + ^{1}\text{H} + \gamma$$

$$e^{+} + e^{-} \longrightarrow 2\gamma$$

Find the energy released. A reference listing isotopes usually has the mass of a neutral *atom* (not just the *nucleus*). The value *includes* the mass of *all* the electrons bound to the atom.

$$^{226}_{88}\mathrm{Ra} \longrightarrow \, ^{222}_{86}\mathrm{Rn} + \, ^{4}_{2}\mathrm{He} + \gamma$$

$$^{14}_{6}\mathrm{C} \longrightarrow \, ^{14}_{7}\mathrm{N} + e^{-} + \bar{\nu}$$

Z	Name & Symbol		Mass (u)	Abundance	Half-life
0	(Neutron)	n	1.008665	β^-	10.4 min
1	Hydrogen	^1H	1.007825	99.985%	
	Deuterium	^2H	2.014102	0.015%	
	Tritium	^3H	3.016049	β^-	12.33 y
2	Helium	^3He	3.016029	0.000137%	
		^4He	4.002602	99.999863%	
6	Carbon	^{14}C	14.003242	β^-	5730 y
7	Nitrogen	^{14}N	14.003074	99.63%	
86	Radon	^{222}Rn	222.017570	α, γ	3.8235 d
88	Radium	^{226}Ra	226.025402	α, γ	1600 y

Find the energy released.

$$^{226}_{88}\text{Ra} \longrightarrow {}^{222}_{86}\text{Rn} + {}^{4}_{2}\text{He} + \gamma$$

226.025402 u   222.017570 u   4.002602 u

$\Delta m = (222.01757 + 4.002602) - 226.025402$

$\Delta m = -0.00523$ u

$E = mc^2 =$ 4.87 MeV

$$^{14}_{6}\text{C} \longrightarrow {}^{14}_{7}\text{N} + e^- + \bar{\nu}$$

14.003242 u   14.003074 u

$\Delta m = 14.003242 - 14.003074$

$\Delta m = -0.000168$ u

$E = mc^2 =$ 156 keV

A neutral nitrogen has seven electrons, but *this one* would have *six* (initially). The beta particle is a "seventh" electron, so use the mass of a neutral nitrogen!

Z	Name & Symbol		Mass (u)	Abundance	Half-life
0	(Neutron)	n	1.008665	β^-	10.4 min
1	Hydrogen	^{1}H	1.007825	99.985%	
	Deuterium	^{2}H	2.014102	0.015%	
	Tritium	^{3}H	3.016049	β^-	12.33
2	Helium	^{3}He	3.016029	0.000137%	
		^{4}He	4.002602	99.999863%	

Determine the mass defect for helium-4. What is the energy equivalence? How does it make sense to call this the "binding energy" of the helium nucleus? How is this similar to the binding energy of say the electron in a hydrogen atom (13.6 eV)?

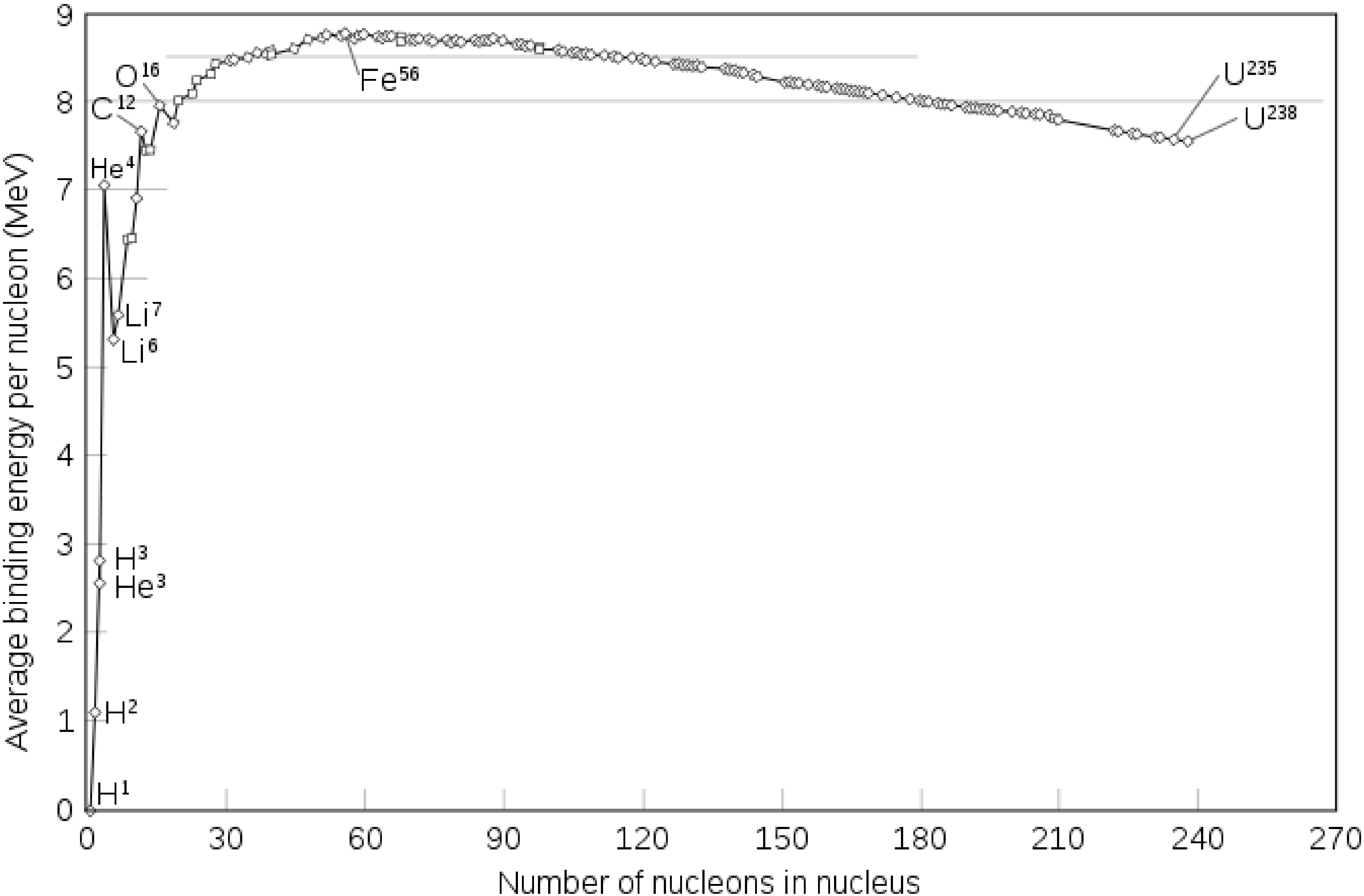
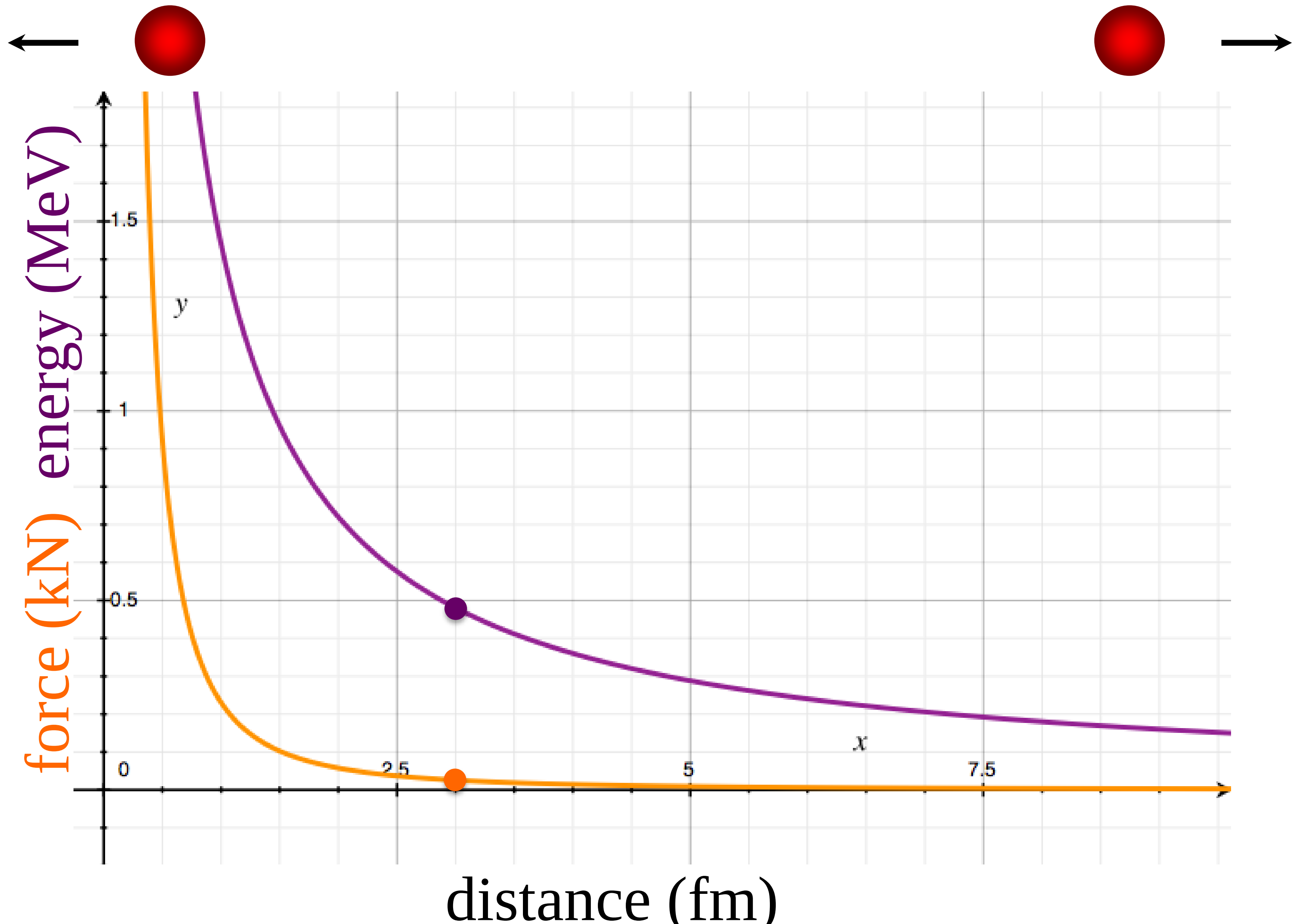
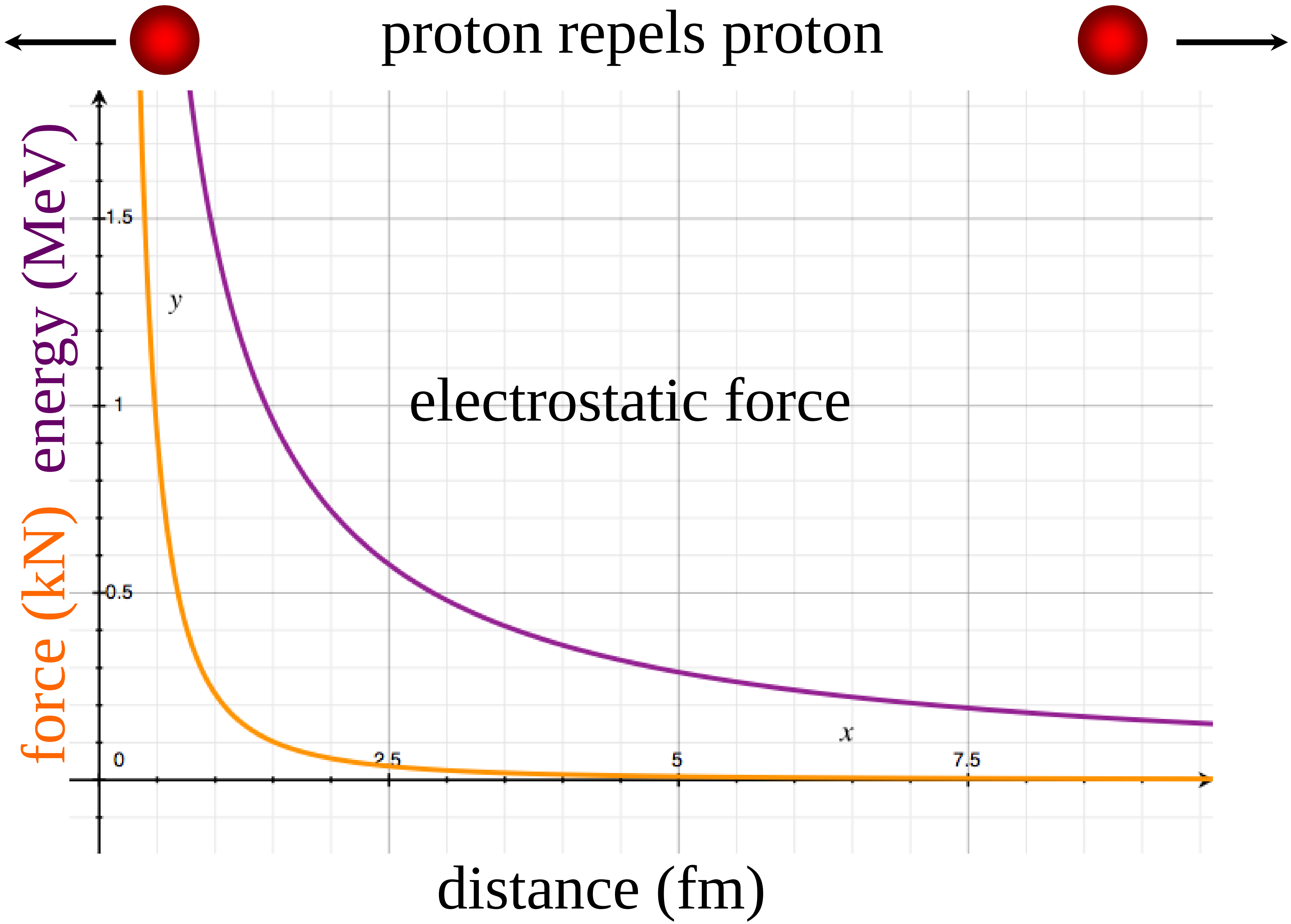
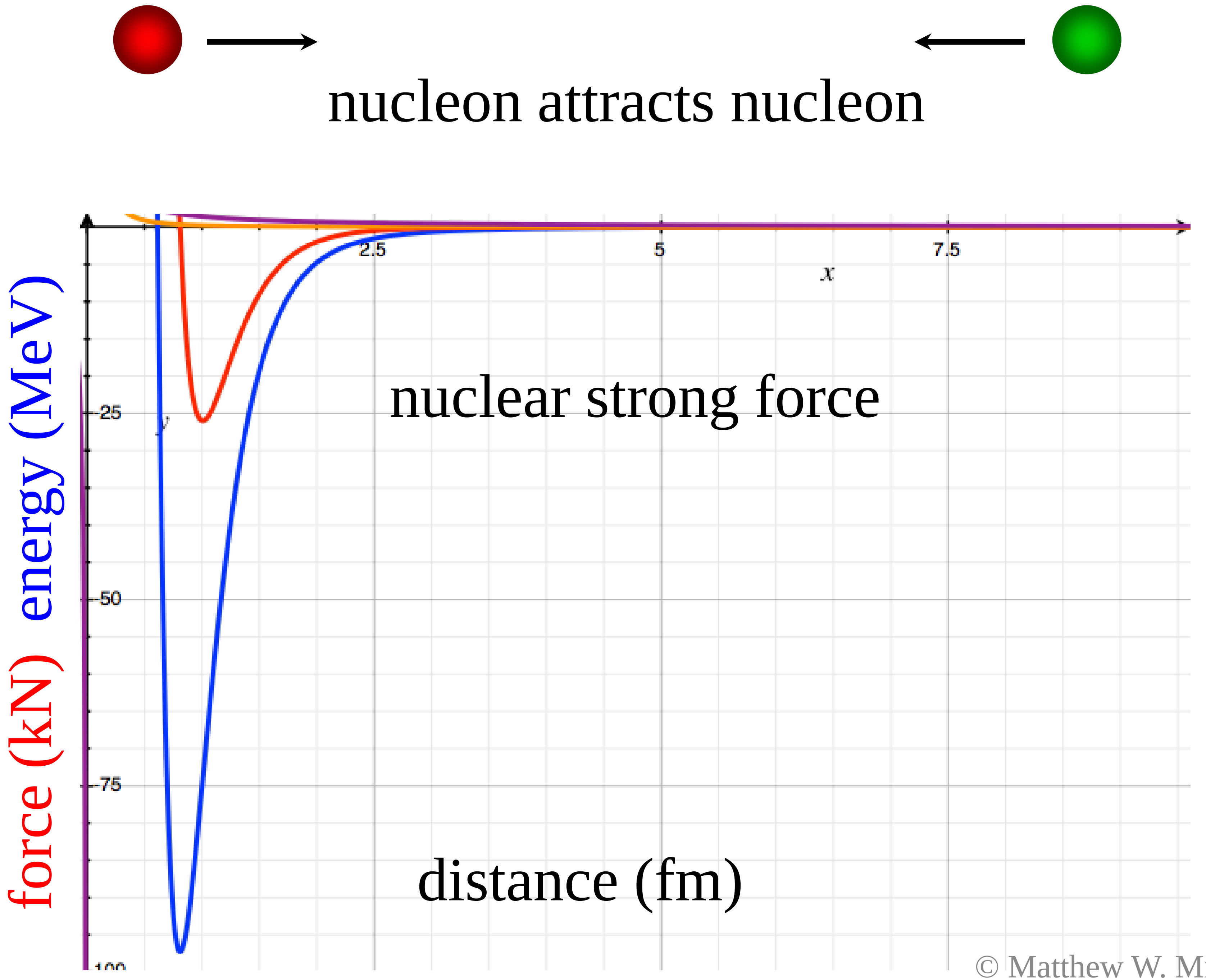


image credit: Wikipedia and Fastfission



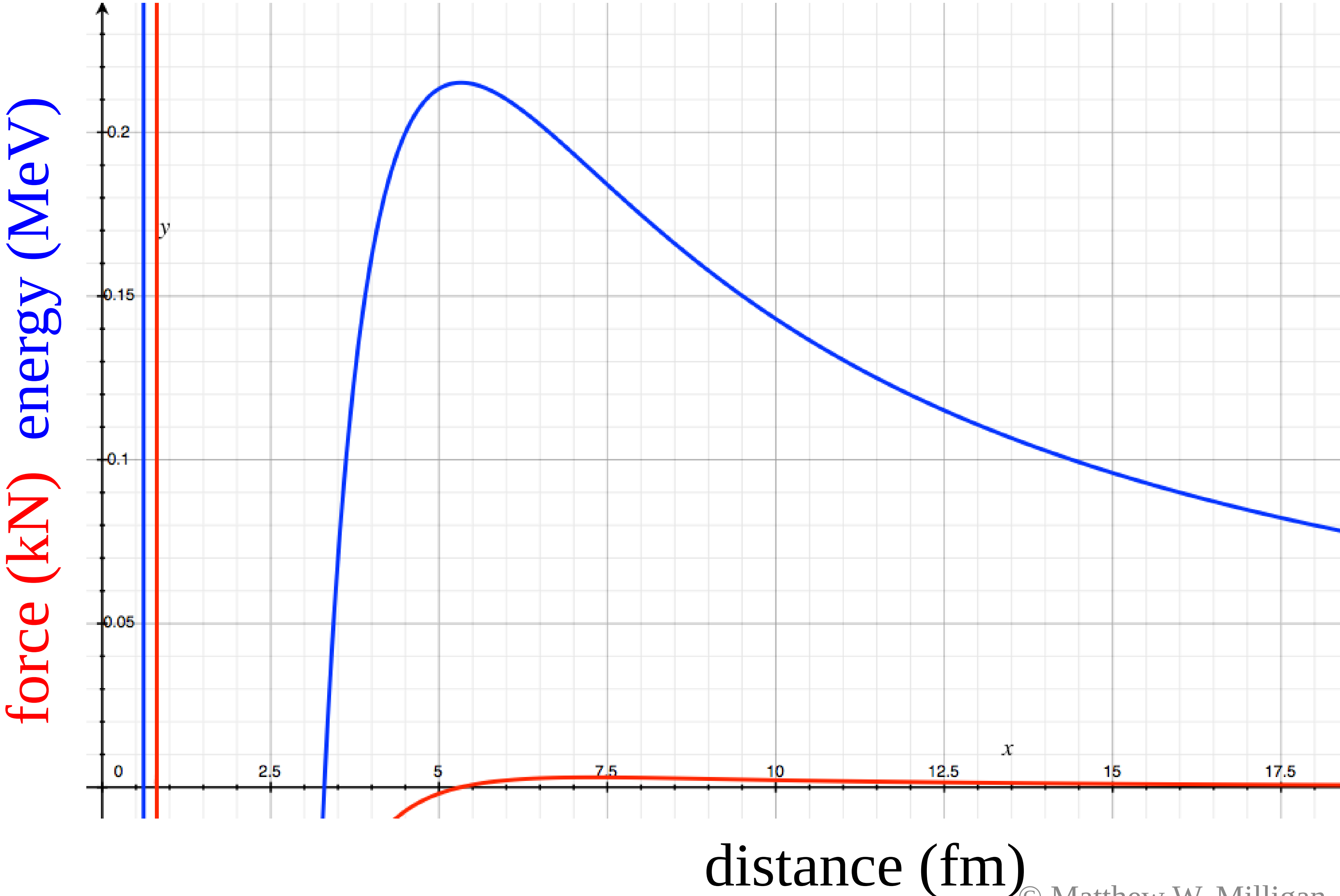
energy (MeV)
force (kN)
1.5
1
0.5
0
2.5
5
7.5
y
x
distance (fm)


proton repels proton
energy (MeV)
force (kN)
1.5
1
0.5
0
y
electrostatic force
2.5
5
7.5
x
distance (fm)

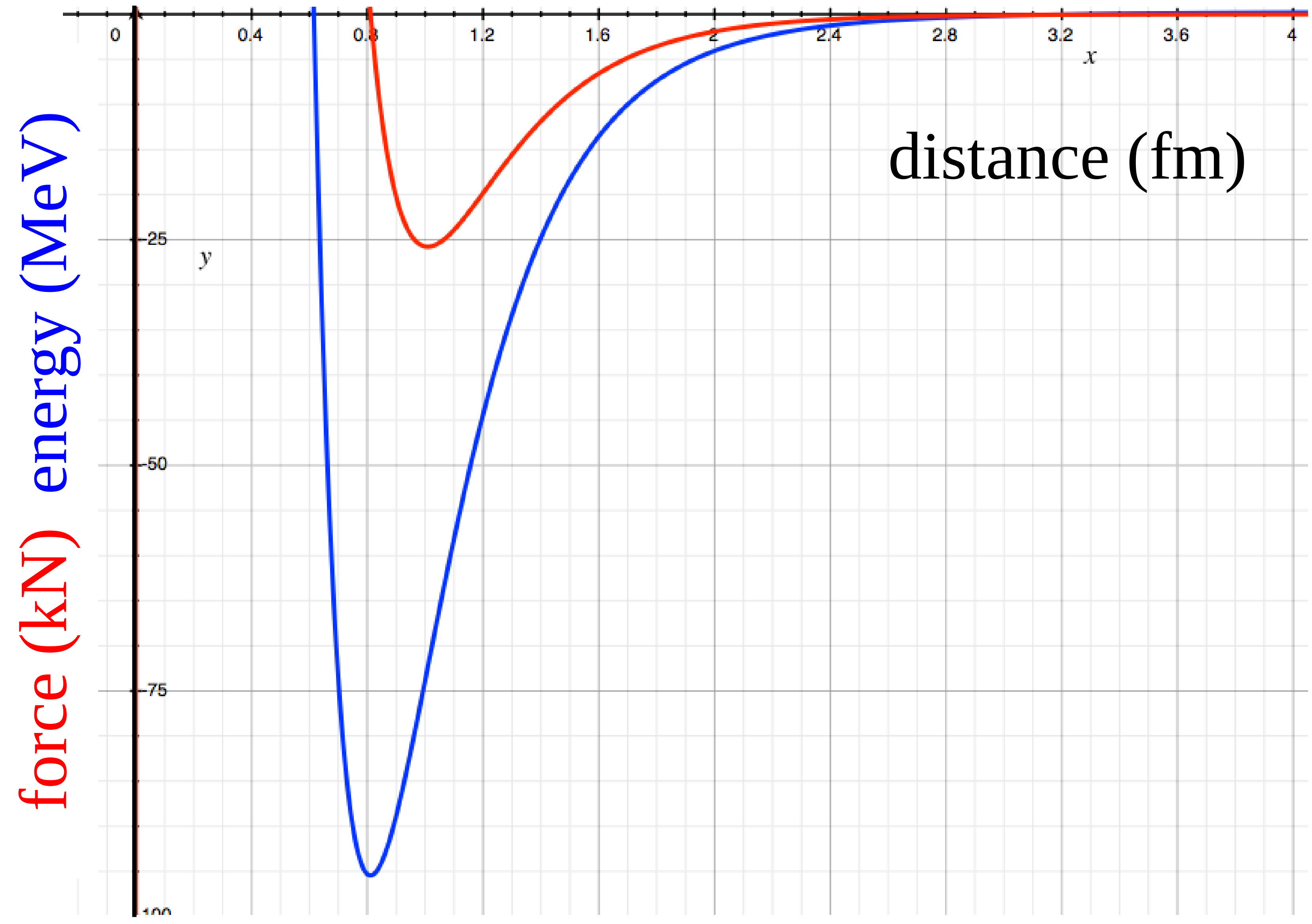

nucleon attracts nucleon
nuclear strong force
force (kN) energy (MeV)
distance (fm)
2.5
5
7.5
x
-25
-50
-75
100

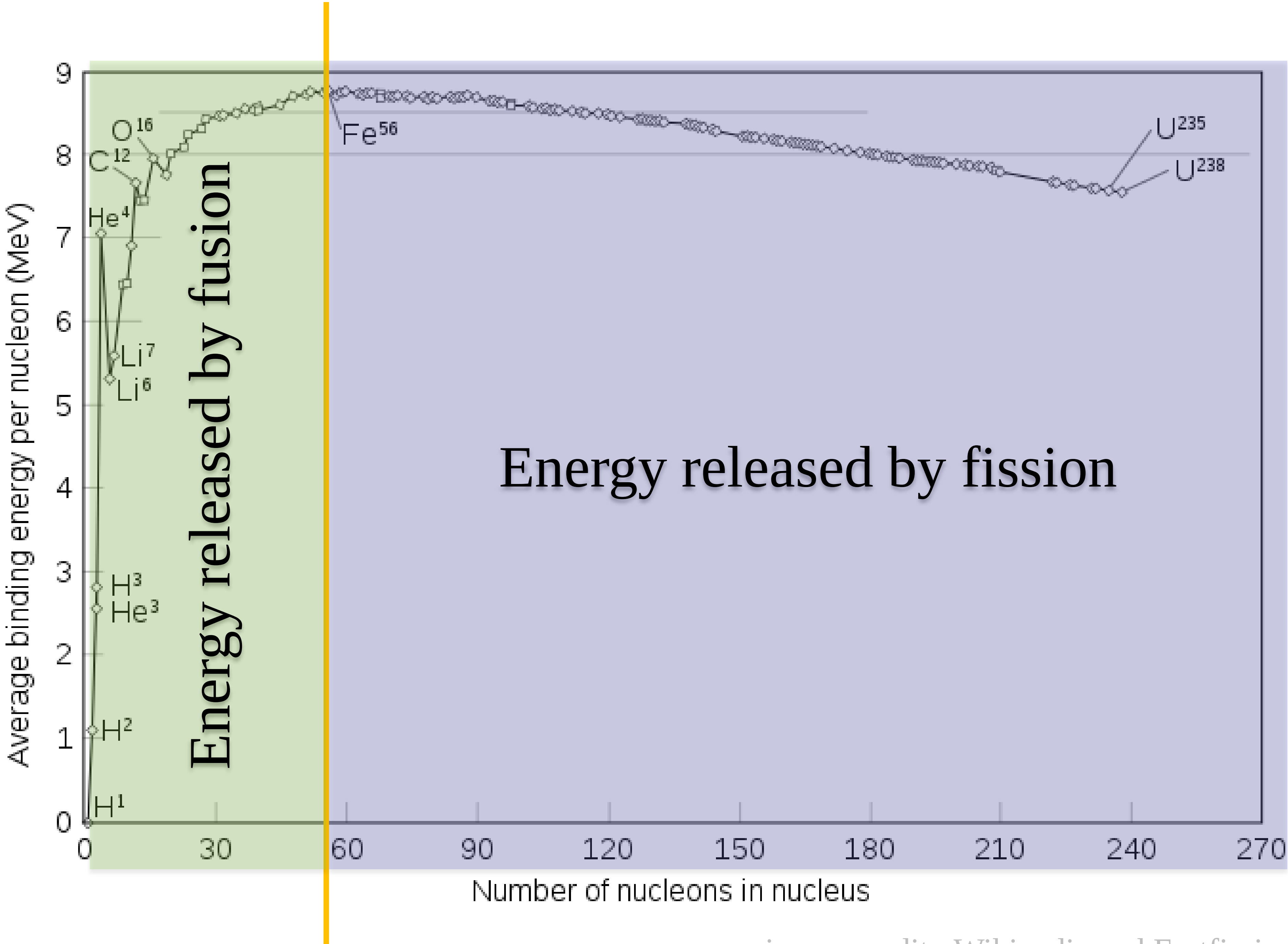
net force and energy for nucleons

net force and energy for nucleons



net force and energy for nucleons




Energy released by fusion
Energy released by fission
Average binding energy per nucleon (MeV)
Number of nucleons in nucleus
9
8
7
6
5
4
3
2
1
0
0
30
60
90
120
150
180
210
240
270
H1
H2
H3
He3
He4
Li6
Li7
C12
O16
Fe56
U235
U238